Baryon Distribution Amplitudes:

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Outline

Form factors from Baryon DAs via LCSR

- Basic idea
- Example: Em form factors of the nucleon
- L.H.S. of the sum rule
- R.H.S. of the sum rule
- Combining L. & R.H.S. of the sum rule

Current activities - including higher twist DAs:

- Literature - Overview
- Determining the DA
- Nucleon form factors
- Decays of Baryons

Outlook

Form factors from Baryon DAs via LCSR

To describe the transition of a baryon B to a baryon N via the current j_μ

$$B(P') \xrightarrow{j_\mu} N(P)\,, \qquad\qquad P' = P - q$$

Start with a correlation function

$$\boxed{T_\mu(P,q) = \int d^4x e^{-ipx} \langle 0 | T\{\eta(0) j_\mu(x)\} | N(P)\rangle}$$

- Interpolating field η: "creating B from the vacuum"
  e.g. $\eta_{\mathrm{CZ}}(x) \;\; = \varepsilon^{ijk} \left[u^i(x)(C\not{z})\, u^j(x)\right] \;\; (\gamma_5 \not{z}) \;\; d^k_\delta(x)$ $\Rightarrow$ B = Proton
  or $\eta_{\mathrm{Ioffe}}(x) = \varepsilon^{ijk} \left[u^i(x)(C\gamma_\nu)\, u^j(x)\right] (\gamma_5 \gamma^\nu)\, d^k_\delta(x)$ $\Rightarrow$ B = Proton
- Current j_μ
  e.g. $j^{\mathrm{em}}_\mu(x) = e_u \bar{u}(x)\gamma_\mu u(x) + e_d \bar{d}(x)\gamma_\mu d(x)$ $\Rightarrow$ em form factors
  e.g. $j^{\mathrm{weak}}_\mu(x) = \bar{u}(x)\gamma_\mu(1-\gamma_5)d(x)$ $\Rightarrow$ weak decay

Now: Express T_μ in two different ways

Example: EM form factors of the Nucleon

- **Rosenbluth-formula** (1955) for elastic e^--N scattering (1 photon exchange)

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta_e}{2}\right]$$

$$Q^2 = -q^2,\ \tau = \frac{Q^2}{4M^2c^2},\ \theta_e = \text{ scattering} - \text{angle of } e^-$$

- Electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ **Sachs form factors**
- Interpretation in **Breit frame**:
  - $G_E(Q^2)$ Fourier transform of electric charge distribution
  - $G_M(Q^2)$ Fourier transform of magnetization density
- Relation of Dirac and Pauli form factors to Sachs form factors

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \qquad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2).$$

- $G_M^p(0) = \mu_p = 2.79,\ G_M^n(0) = \mu_n = -1.91.$
- Def.: $\langle P', s|j_\mu^{\rm em}|P\rangle = \bar{N}(P')\left[\gamma_\mu F_1 - i\frac{\sigma_{\mu\nu}q^\nu}{2m_p}F_2\right]N(P)$

L.H.S. of the sum rule

Inserting a full set of one-particle states in the correlation function we obtain

$$T_\mu(P,q) = \frac{1}{m_p^2 - (P')^2} \sum_s \langle 0|\eta_p|P',s\rangle\langle P',s|j_\mu^{\rm em}|P\rangle + \dots$$

where the dots represent contributions of higher resonances.

The arising matrix elements read

- $\langle 0|\eta_{\rm CZ}|P\rangle = f_N P z \not{z} N(P)$ or $\langle 0|\eta_{\rm Ioffe}|P\rangle = \lambda_1 m_p N(P)$
  with the non-perturbative constants f_N, λ_1
- $\langle P',s|j_\mu^{\rm em}|P\rangle = \bar{N}(P')\left[\gamma_\mu F_1 - i\frac{\sigma_{\mu\nu}q^\nu}{2m_p}F_2\right]N(P)$

$$\Rightarrow T_\mu = T_\mu(F_1,F_2)$$

Which η to use?

Determine $f_N, \lambda_1, \dots$

R.H.S. of the sum rule

Do all possible Wick-contractions in T_μ:

$$T_\mu(P,q) \propto \int d^4x e^{-ipx} \ldots \; \Gamma_1^{\alpha\beta}\Gamma_2^{\delta\gamma} 4\langle 0|\epsilon_{ijk} u_\alpha^i u_\beta^j d_\gamma^k |N(P)\rangle$$

and insert the Baryon DA

$$4\langle 0|\epsilon_{ijk} u_\alpha^i u_\beta^j d_\gamma^k |N(P)\rangle = \sum \Gamma_3^{\alpha\beta}\Gamma_4^{\gamma} F$$

with Dirac structures Γ_i and 24 distribution amplitudes F
In the end all DAs can be reduced to eight non-perturbative parameters

$$\Rightarrow T_\mu = T_\mu(f_N, \lambda_1, \lambda_2, V_1^d, A_1^u, f_1^d, f_1^u, f_2^d)$$

Determine $f_N, \lambda_1, \ldots$

Combining L.H.S. and R.H.S.

In order to combine the two expressions for T_μ we perform first a projection

$$\boxed{\Rightarrow \Lambda_+ T_z = pz\,(m_p \mathcal{A}_1^{\mathrm{em}} + \not{q}_\perp \mathcal{B}_1^{\mathrm{em}})\,N^+(P)}$$

with $z^2 = 0 = qz; p_\mu = P_\mu - z_\mu m_p^2/(2Pz); \Lambda^+ = \not{p}\not{z}/(2pz); \Lambda^+ N = N^+$

$$\mathcal{B}_1^{\mathrm{em}} = \frac{\lambda_1 F_2^{\mathrm{em}}}{m_p^2 - P'^2} = -2e_d \int\limits_0^1 \frac{dx_3}{(q - x_3 P)^2} \int\limits_0^{1-x_3} dx_1 120 x_1 x_2 x_3 \delta(1 - x_1 - x_2 - x_3) f_N + \ldots$$

To supress the contributions of higher resonances we perform a Borel trafo

$$\boxed{F_2^{\mathrm{em}} = 2e_d \frac{1}{\lambda_1} \int\limits_{x_0}^1 \frac{dx}{x} \int\limits_0^{1-x} dx_1 120 x_1 x_2 x \delta(1 - x_1 - x_2 - x) f_N + \ldots}$$

Current activities - including higher twist DAs

- The nucleon DA up to twist 6
  Braun, Fries, Mahnke, Stein, 2000
- LCSR: em form factors of the nucleon
  use η_{CZ}; x^2-corrections to V_1
  Braun, A.L., Mahnke, Stein, 2001
- LCSR: em form factors of the nucleon
  use isospin conserving $\eta_{Im.}$
  A.L., Wittmann, Stein, 2003
- LCSR: form factors of the nucleon, n $\to$ p
  use $\eta_{CZ}, \eta_{Im.}, \eta_{Ioffe}$; x^2-correct. to A_1, T_1
  estimate of non-perturbative parameters
  Braun, A.L., Wittmann, 2006
- LCSR: N $\to$ Δ
  Braun, A.L., Peters, Radyushkin, 2005
- LCSR: $\Lambda_b \to p\, l \bar{\nu}$
  x^2-corrections to V_1, A_1, T_1
  Huang, Wang, 2004
- LCSR: Scalar form-factor of the nucleon
  use η_{CZ}
  Z. Wang, Wan, Yang, 2006
- LCSR: axial and induced pseudoscalar form-factor of the nucleon
  use η_{CZ}
  Z. Wang, Wan, Yang, 2006
- LCSR: $\Lambda_c \to \Lambda\, l \bar{\nu}$
  Axial Λ-DAs of leading conformal spin
  Huang, Wang, 2006
- LCSR: $\Sigma \to$ N
  use η_{Ioffe}
  Z. Wang, 2006

Determining the Nucleon DA upto twist 6 I

$$
\begin{aligned}
&4\langle 0|\,\varepsilon^{ijk} u^i_\alpha(a_1 x) u^j_\beta(a_2 x) d^k_\gamma(a_3 x)\,|P\rangle = \\
&\quad \mathcal{S}_1 M C_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{S}_2 M^2 C_{\alpha\beta}\left(\not{x}\gamma_5 N\right)_\gamma\, \mathcal{P}_1 M\left(\gamma_5 C\right)_{\alpha\beta} N_\gamma + \mathcal{P}_2 M^2\left(\gamma_5 C\right)_{\alpha\beta}\left(\not{x} N\right)_\gamma \\
&+\left(\mathcal{V}_1 + \frac{x^2 m_N^2}{4}\mathcal{V}_1^M\right)\left(\not{P} C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{V}_2 M\left(\not{P} C\right)_{\alpha\beta}\left(\not{x}\gamma_5 N\right)_\gamma + \mathcal{V}_3 M\left(\gamma_\mu C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma \\
&+\mathcal{V}_4 M^2\left(\not{x} C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{V}_5 M^2\left(\gamma_\mu C\right)_{\alpha\beta}\left(i\sigma^{\mu\nu}x_\nu\gamma_5 N\right)_\gamma + \mathcal{V}_6 M^3\left(\not{x} C\right)_{\alpha\beta}\left(\not{x}\gamma_5 N\right)_\gamma \\
&+\left(\mathcal{A}_1 + \frac{x^2 m_N^2}{4}\mathcal{A}_1^M\right)\left(\not{P}\gamma_5 C\right)_{\alpha\beta} N_\gamma + \mathcal{A}_2 M\left(\not{P}\gamma_5 C\right)_{\alpha\beta}\left(\not{x} N\right)_\gamma + \mathcal{A}_3 M\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(\gamma^\mu N\right)_\gamma \\
&+\mathcal{A}_4 M^2\left(\not{x}\gamma_5 C\right)_{\alpha\beta} N_\gamma + \mathcal{A}_5 M^2\left(\gamma_\mu\gamma_5 C\right)_{\alpha\beta}\left(i\sigma^{\mu\nu}x_\nu N\right)_\gamma + \mathcal{A}_6 M^3\left(\not{x}\gamma_5 C\right)_{\alpha\beta}\left(\not{x} N\right)_\gamma \\
&+\left(\mathcal{T}_1 + \frac{x^2 m_N^2}{4}\mathcal{T}_1^M\right)\left(i\sigma_{\mu P} C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma + \mathcal{T}_2 M\left(i\sigma_{xP} C\right)_{\alpha\beta}\left(\gamma_5 N\right)_\gamma + \mathcal{T}_3 M\left(\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}\gamma_5 N\right)_\gamma \\
&+\mathcal{T}_4 M\left(P^\nu\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\varrho}x_\varrho\gamma_5 N\right)_\gamma + \mathcal{T}_5 M^2\left(x^\nu i\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma^\mu\gamma_5 N\right)_\gamma + \mathcal{T}_6 M^2\left(i\sigma_{xP} C\right)_{\alpha\beta}\left(\not{x}\gamma_5 N\right)_\gamma \\
&+\mathcal{T}_7 M^2\left(\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\nu}\,\not{x}\gamma_5 N\right)_\gamma + \mathcal{T}_8 M^3\left(x^\nu\sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\sigma^{\mu\varrho}x_\varrho\gamma_5 N\right)_\gamma
\end{aligned}
$$

BFMS 2000

* The 24 functions $\mathcal{F}^{(i)} = \mathcal{S}_i, \mathcal{P}_i, \mathcal{A}_i, \mathcal{V}_i, \mathcal{T}_i$ can be related to **8 LCDAs** of twist-3 to twist-6.
* In leading conformal spin we have 3 parameters: $\lambda_1, \lambda_2, f_N$
* in NL conformal spin we have 5 parameters: $V_1^d, A_1^u, f_1^d, f_1^u, f_2^d$
* $\mathcal{V}_1^M$: BLMS 2001
* $\mathcal{A}_1^M, \mathcal{T}_1^M$: Huang, Wang 2004; BLW 2006.

Determining the Nucleon DA upto twist 6 II

Definition of the non-perturbative parameters

$$\begin{aligned}
\langle 0|\varepsilon^{ijk}\left[u^i(0)(C\not{z})\,u^j(0)\right] \quad (\gamma_5\not{z}) \ \ d^k_\delta(0)|P\rangle &= f_N pz\not{z}N(P) \\
\langle 0|\varepsilon^{ijk}\left[u^i(0)(C\gamma_\mu)\,u^j(0)\right] \quad (\gamma_5\gamma^\mu) \ \ d^k_\delta(0)|P\rangle &= \lambda_1 m_N N(P) \\
\langle 0|\varepsilon^{ijk}\left[u^i(0)(C\sigma_{\mu\nu})\,u^j(0)\right] \quad (\gamma_5\sigma^{\mu\nu}) \ \ d^k_\delta(0)|P\rangle &= \lambda_2 m_N N(P) \\
\\
\langle 0|\varepsilon^{ijk}\left[u^i(0)(C\not{z})\,u^j(0)\right] \quad (\gamma_5\not{z}) \ \ (iz\vec{D}d^k_\delta)(0)|P\rangle &= f_N V_1^d (pz)^2\not{z}N(P) \\
\ldots &\quad \ldots
\end{aligned}$$

Determining the Nucleon DA upto twist 6 III

- **Leading twist:** f_N, V_1^d, A_1^d
  - QCD SR: Chernyak, Zhitnitsky 1984; King, Sachrajda 1987; Gari,Stefanis 1987; Chernyak, Ogloblin, Zhitnitsky 1988,1989; Bolz, Kroll 1996.

$$f_N = (5.0 \pm 0.5) \cdot 10^{-3}\text{GeV}^2 \qquad A_1^u = 0.38 \pm 0.15 \qquad V_1^d = 0.23 \pm 0.03$$

  - Lattice: Martinelli, Sachrajda 1989; Aoki et al. 2006.
  - Asymptotic: $A_1^u = 0$ $V_1^d = 1/3$
  - LCSR: BLW 2006 $A_1^u = 0.13$ $V_1^d = 0.30$
- **Higher twist**
  - leading conformal spin: λ_1, λ_2 - QCD SR: BFMS 2000, BLW 2006

$$\lambda_1 = -(2.7 \pm 0.9) \cdot 10^{-2}\text{GeV}^2 \qquad \lambda_2 = (5.4 \pm 1.9) \cdot 10^{-2}\text{GeV}^2$$

  - next-to-leading conformal spin: f_1^d, f_1^u, f_2^d

Method	f_1^d	f_1^u	f_2^d	authors
QCD SR	0.40 ± 0.05	0.07 ± 0.05	0.22 ± 0.05	BFMS 2000, BLW 2006
LCSR	0.33	0.09	0.25	BFMS 2000, BLW 2006
asymptotic	0.30	0.10	$4/15$	

LCSR for the nucleon form factors

- EM - form factors using η_{CZ}: BLMS 2001
  - Higher twist is important
- EM - form factors using isospin conserving η_{IM}: LWS 2004
  - η_{CZ} leads to unphysical isospin violating effects
- EM and weak form factors, compare different ηs: BLW 2006
  - surprisingly good description of data

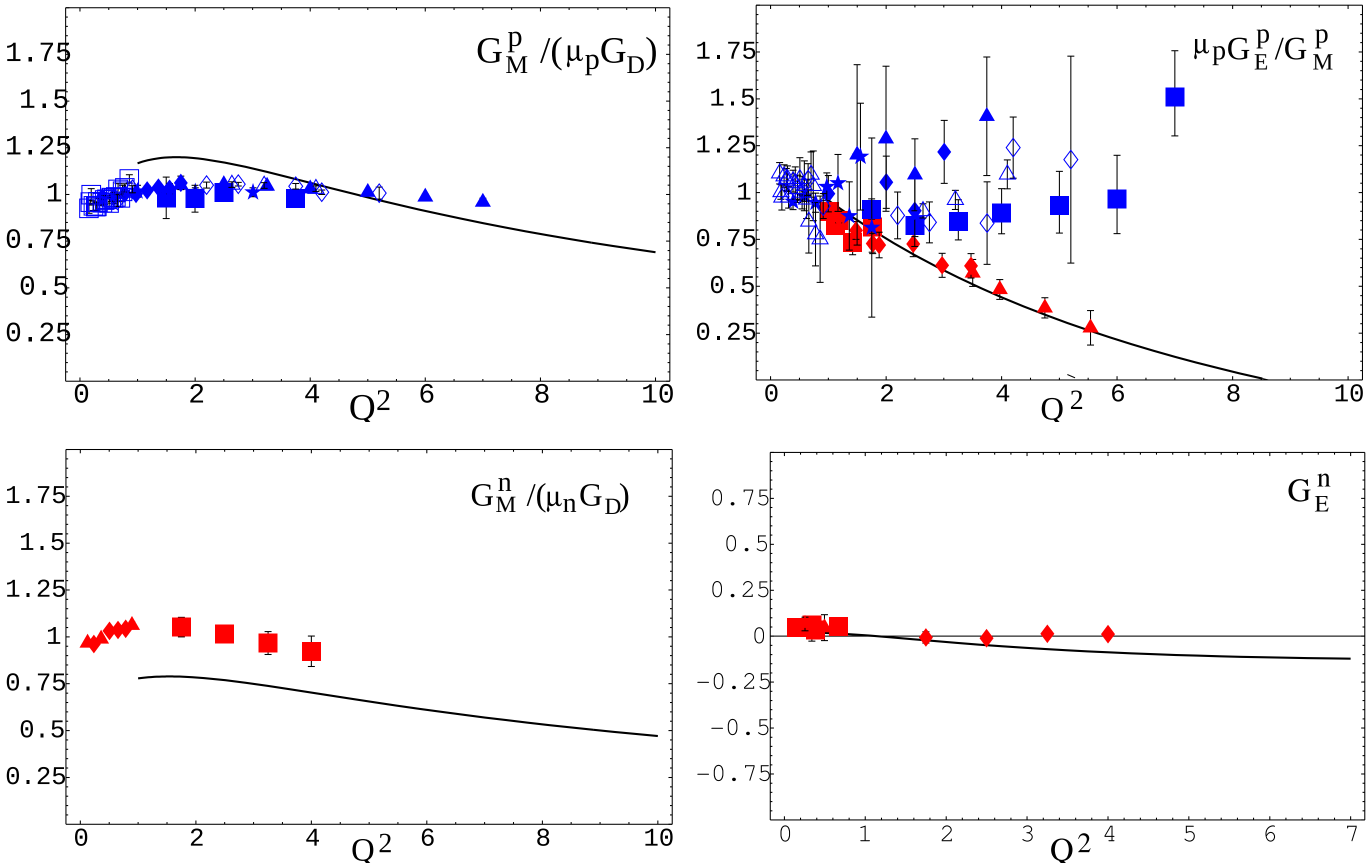
$\Rightarrow \eta_{\mathrm{Ioffe}}$ seems to be the best choice

- Scalar form factor of the nucleon using η_{CZ}: Wang, Wan 2006
- Axial and ps form factor of the nucleon using η_{CZ}: Wang, Wan 2006

* $\Rightarrow$ use η_{Ioffe} to compare experiment and LCSR
* FF depend on 5 Parameters $\lambda_1/f_N, A_1^u, V_1^d, f_1^d, f_1^u$

Nucleon electromagnetic form factors

$$\langle N(P')|j_\mu^{\rm em}(0)|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu F_1(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N}F_2(Q^2)\right]N(P)$$



proton

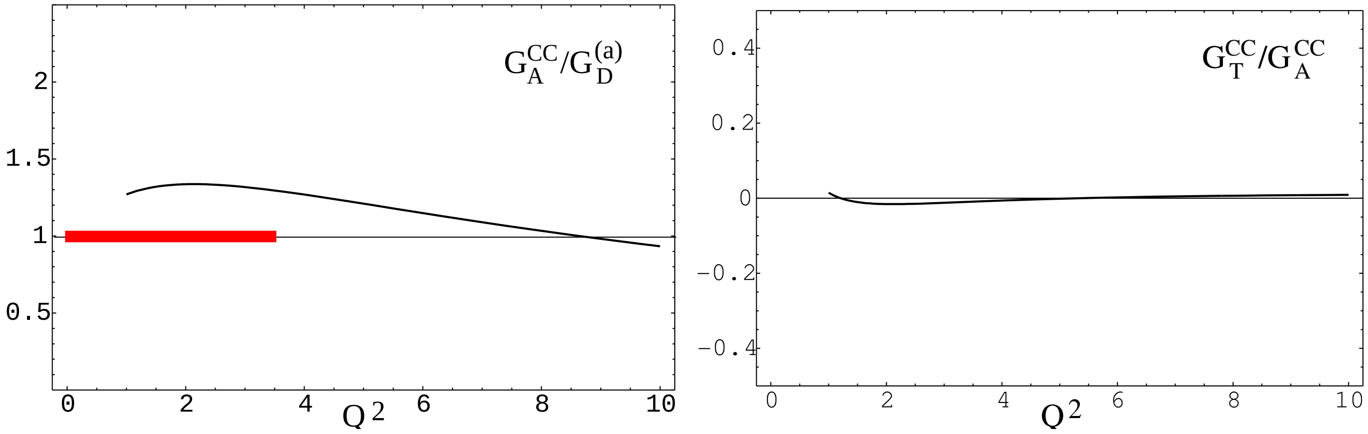
neutron

- **Leading order LCSR, BLW distribution amplitudes**

Braun, Lenz, Wittmann; PRD73 (2006) 094019

Nucleon axial vector form factors

$$\langle N(P')|A_\mu(0)|N(P)\rangle = \bar{N}(P')\left[\gamma_\mu G_A(Q^2) - \frac{q_\mu}{2m_N} G_P(Q^2) - i\frac{\sigma_{\mu\nu}q^\nu}{2m_N} G_T(Q^2)\right]\gamma_5 N(P)$$



charged current

- **Leading order LCSR, BLW distribution amplitudes**

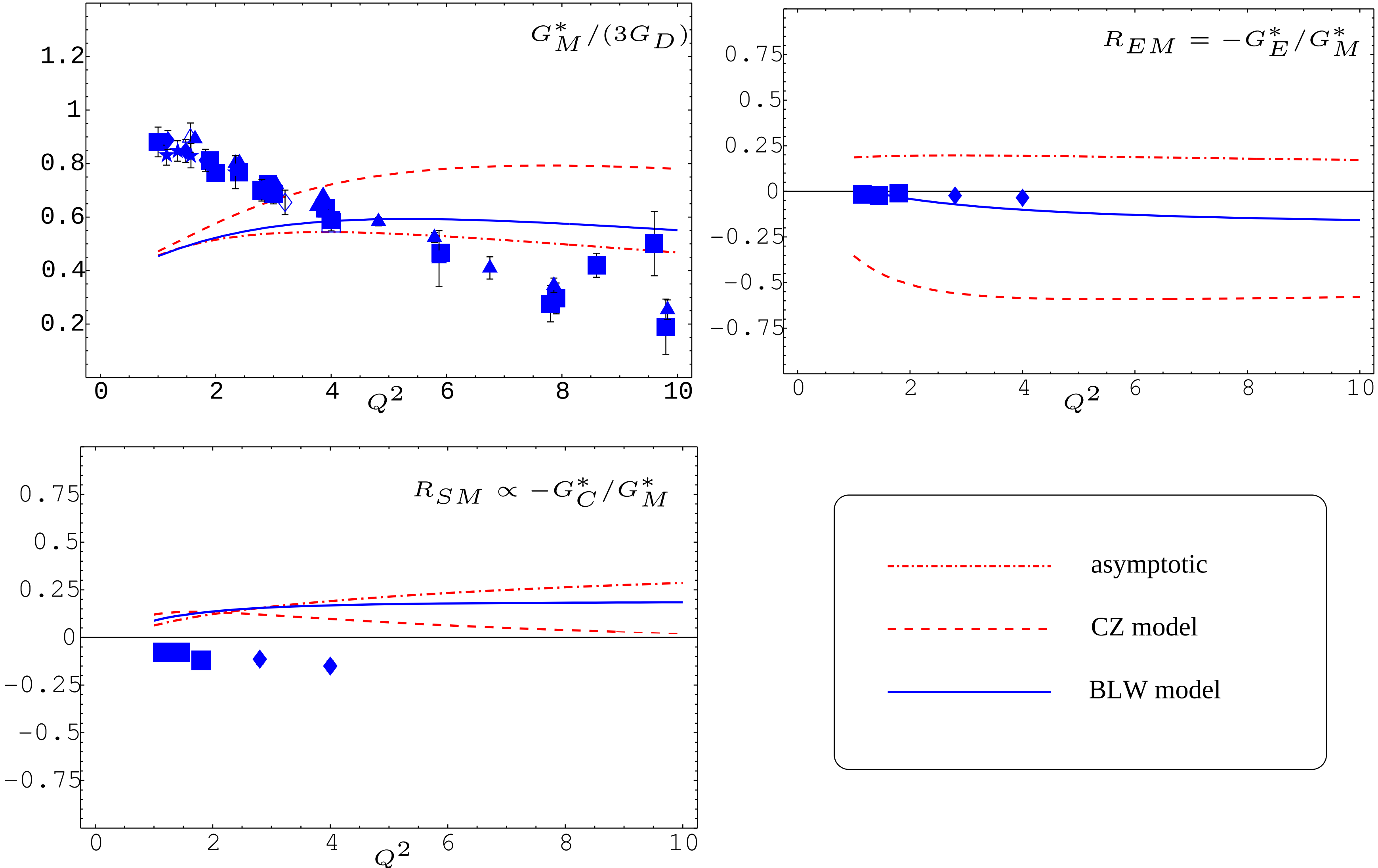
Braun, Lenz, Wittmann; PRD73 (2006) 094019

LCSR for transitions

- Huang, Wang, 2004: $\Lambda_b \to p l \nu$
  - Interpolating field: $\eta_{\Lambda_b} = \epsilon_{ijk} u^i C \not{z} d^j \cdot \gamma_5 \not{z} b^k$ vs. $\eta_{\Lambda_b} = \epsilon_{ijk} u^i C \not{z} d^j \cdot \gamma_5 \not{z} h_v^k$
  - Determine f_Λ from QCD-SR
  - HQET $\approx$ 1/10 QCD!
- BLPR 2005: N $\to \Delta$
  - Disentangle N $\to N^*$ (spin 1/2)
  - Interpolating field: $\eta_\Delta = \epsilon_{ijk} \left(2 u^i C \gamma_\mu d^j \cdot \not{z} u^k + u^i C \gamma_\mu u^j \cdot \not{z} d^k \right)$
- BLW 2006: $n \to p$
- Huang, Wang, 2006: $\Lambda_c \to \Lambda l \nu$
  - Determine Λ-DA up to twist-6 and leading conformal spin
  - Interpolating field: $\eta_{\Lambda_c} = \epsilon_{ijk} u^i C \gamma_5 \not{z} d^j \cdot \not{z} c^k$
  - only tw-3 agrees with experiment
- Wang, 2006: $\Sigma \to$ N
  - Interpolating field: $\eta_\Sigma = \epsilon_{ijk} d^i C \gamma_\mu d^j \cdot \gamma_5 \gamma^\mu s^k$
  - Wang compared results for $Q^2 = 0$ with the data!

$N\Delta\gamma$ transition form factors

- **magnetic, electric and quadrupole form factors exist:**



Braun, Lenz, Radyushkin, Peters; PRD73 (2006) 034020

Outlook

- Nucleon DA
  - α_s corrections to LCSR for form factors of the nucleon,
    Compare LCSR to experiment and fit the non-perturbative parameters
    Braun, A.L., Passek-Kumericki, Peters, in progress
  - Lattice determination of the non-perturbative parameters
- Decay of heavy baryons
  - LCSR for $\Lambda_b \to p\, l \bar{\nu}$
    A.L., Wankerl, in progress
  - LCSR for $\Lambda_b \to \Lambda_c\, l + l$
    A.L., Rohrwild, planned
- LCSR for pion electro-production
  Braun, Ivanov, A.L., Peters, in progress