Constraints on the twist-2 pion DA from the measurements of pion form factors

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Pion distribution amplitude of twist 2

- Formal definition at $(x_2 - x_1)^2 \to 0$:

$$\langle 0|\bar{u}(x_2)[x_2, x_1]\gamma_\mu\gamma_5 d(x_1)|\pi^-(p)\rangle = i f_\pi p_\mu \int\limits_0^1 du e^{-iupx_1 - i\bar{u}px_2}\varphi_\pi(u,\mu) + twist4 + ...$$

- Gegenbauer Expansion:

$$\varphi_\pi(u,\mu) = 6u\bar{u}\left[1 + \sum_{n=1} a_{2n}^\pi(\mu) C_{2n}^{3/2}(u - \bar{u})\right],$$

The ongoing hunt for $a_{2n}^\pi(\mu_0)$, $\mu_0 \sim 1$ GeV

Constraining Gegenbauer coefficients

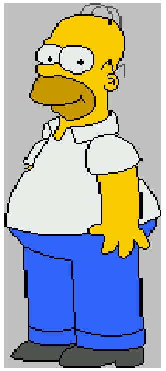
- Form factors: $F_\pi(Q^2)$, $F_{\pi\gamma\gamma*}(Q^2)$, $f^+_{B\pi}(q^2)$, ..., provided there is factorization

$$F(Q^2) = \int du\, T_{hard}(Q^2, u, \mu)\varphi_\pi(u, \mu) + \{\text{tw}\, 4, 6, ... \sim [1/Q^2]^k\}$$

*calculate T_{hard}, incl. $O(\alpha_s)$, $O(\alpha_s^2)$
*estimate tw 4,6,... , are they small enough ?
*measure the form factor at large enough Q^2
* obtain intervals of $a^\pi_{2,4,..}$ depending on the adopted model/ansatz for $\varphi_\pi(u, \mu)$

$\Rightarrow$ Pion form factors yield constraints in the “space” of a^π_{2n}

How many Gegenbauer coeffs contribute?



one: $\varphi_\pi(u,\mu) = 6u(1-u)[1 + a_2(\mu) C_2^{3/2}(2u-1)]$

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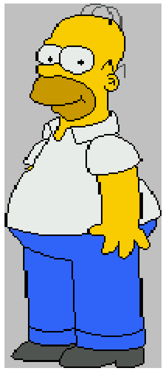


one: $\varphi_\pi(u,\mu) = 6u(1-u)[1 + a_2(\mu) C_2^{3/2}(2u-1)]$

or maybe a few: $\varphi_\pi(u,\mu) =$

$$6u(1-u)[1 + a_2(\mu) ... + a_4(\mu) ... + a_6(\mu) ...]$$

or infinitely many ? $a_2^\pi, ..., a_{1000}^\pi, ...$

- a recent model of n-behavior of a_{2n},

inspired by conformal expansion and asymptotics [Ball-Talbot, 2005]

Calculating Gegenbauer coefficients

- QCD sum rules for $a_{2n}^{\pi}(\mu_0) \sim \langle 0|O_{2n}^{tw\,2}|\pi\rangle|_{\mu_0}$
from $\langle 0|T\{O_{2n}^{tw\,2}(x)j_\pi(0)\}|0\rangle$, [Chernyak-Zhitnisky (1984)]

  - only a_2^{π} accessible within the standard (SVZ) local condensate OPE;

  - models of nonlocal condensates, [Mikhailov,Radyushkin(1986)]
. yield a_{2n}^{π} at $n > 2$, predict $a_{2n} \to 0$ at large n
  - recent "improved NLC"[Bakulev, Pimikov(2006)]

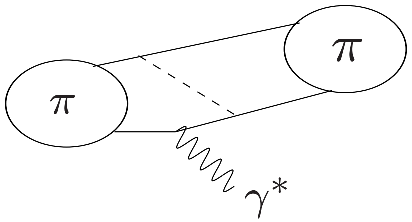
- lattice QCD: recent encouraging results on a_2^{π} , are $a_{4,6,..}^{\pi}$ accessible?

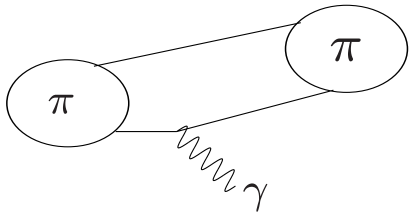
- to complete the list: "instanton liquid" models

A compilation of recent results

Method	$a_2^\pi(1\text{GeV})$	$a_4^\pi(1\text{GeV})$	Details	Ref.
F_π, LCSR $\oplus$ old Jlab data	0.24 ± 0.14 ± 0.08	-	assump.$a_4 = 0$	Bijnens, A.K. (2002)
F_π, LCSR $\oplus$ old Jlab,SLAC data	0.2 ± 0.03	-0.03± 0.06	R -model tw.4	Agaev(2005)
$F_{\pi\gamma\gamma^*}$,LCSR $\oplus$ CLEO data	0.31 0.44	-0.35 -0.40	central points R -model tw.4	Bakulev,Mikhailov, Stefanis[BMS](2006)
$F_{\pi\gamma\gamma^*}$,LCSR $\oplus$ CLEO data	0.27($\pm$ 0.10)	-0.3 ($\pm$ 0.2)	uncert. correl. R-model tw.4	Agaev(2006)
$f_{B\pi}$, LCSR	0.19± 0.19	≥ -0.07	$\frac{d\Gamma(B\to\pi l\nu_l)}{dq^2}$	Ball,Zwicky(2005)
QCD SR	$0.26^{+.21}_{-.09}$ 0.28±0.08	- -		A.K.,Mannel, Melcher (2004) Ball,Braun,Lenz(2006)
QCDSR,NLC	0.19± 0.06	-0.13∓ 0.09	$a_{n>4}$ small	BMS (2005)
Lattice (quench.)	0.34 ± 0.21 (from[BMS])	-		L. Del Debbio [UKQCD] talk at LC 2004
Lattice ($n_f = 2$)	0.27 ± 0.15 (LO evol.)	-	$a_2^\pi(2\text{GeV}) =$ $= .201 \pm .114$	QCDF/UKQCD Braun et al.(2006)

Pion e.m. form factor





"hard", factoriz. "soft", nonfact.

The QCD asymptotics:

$$F_\pi(Q^2)^{asympt} = \frac{8\pi\alpha_s f_\pi^2}{9Q^2}\left(\int_0^1 du \frac{\varphi_\pi(u,\mu)}{\bar{u}}\right)^2\Bigg|_{\mu\sim Q},$$

[Chernyak, Zhitnisky; Efremov,Radyushkin; Brodsky-Lepage (1977-1980)]

The form factor contains important (and even dominant !) nonasymptotic part at $Q^2 \leq 10$ GeV^2 (at least):

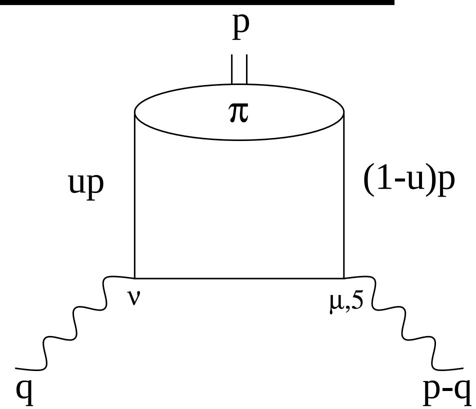
$$F_\pi(Q^2) = F_\pi(Q^2)^{asympt} + F_\pi(Q^2)^{nonas} = F_\pi(Q^2)^{hard} + F_\pi(Q^2)^{soft}$$

- combining $F_\pi(Q^2)^{hard}$ from PQCD and a model for $F_\pi(Q^2)^{soft}$?
- is QCD calculation of $F_\pi(Q^2)^{soft}$ possible?

a realistic solution : QCD light-cone sum rules

[V. Braun, I. Halperin (1999), V. Braun, M. Maul, A.K. (2000)]

Light-cone sum rule for $F_\pi^{em}(Q^2)$



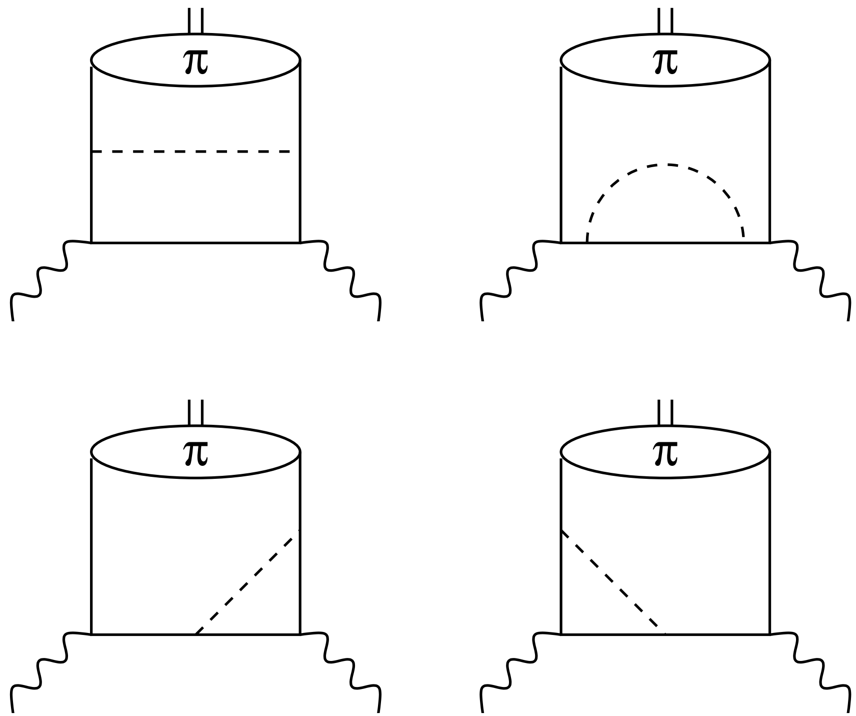
The correlation function:

$$T_{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\langle 0|T\{\left(\bar{d}(0)\gamma_\mu\gamma_5 u(0) j_\nu^{em}(x)\right\}|\pi(p)\rangle\,,$$

the form factor “embedded’ in the dispersion relation

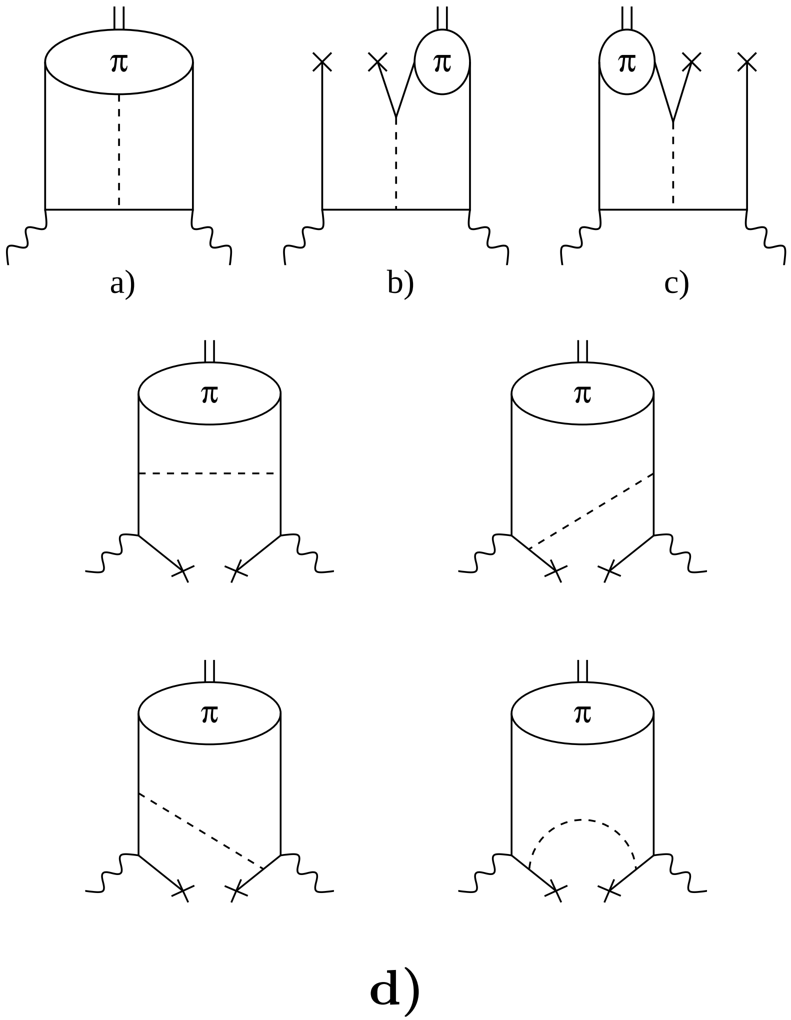
$$T_{\mu\nu}(p,q) = 2if_\pi(p-q)_\mu p_\nu F_\pi(Q^2)\frac{1}{m_\pi^2-(p-q)^2} + \int ds\, \frac{\rho_{\mu\nu}(s)}{s-(p-q)^2}\,.$$

$O(\alpha_s)$ diagrams [V. Braun, M. Maul, A.K. (2000)]



OPE diagrams: (a) twist-4, (b,c,d) twist-6 contributions

(twist 6 factorized into local condensate)



$F_\pi(Q^2)$ from LCSR

$$F_\pi(Q^2) = F_\pi^{(2)}(Q^2) + F_\pi^{(2,\alpha_s)}(Q^2) + F_\pi^{(4)}(Q^2) + F_\pi^{(6)}(Q^2)\,,$$

$$F_\pi^{(2)}(Q^2) = \int_{u_0^\pi}^1 du\, \varphi_\pi(u,\mu) \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right)\,, \qquad u_0^\pi = Q^2/(Q^2+s_0^\pi)$$

$$F_\pi^{(2,\alpha_s)}(Q^2) = \frac{\alpha_s C_F}{\pi}\int_0^1 du\, \varphi_\pi(u,\mu)\Big[\Theta(u-u_0^\pi)\mathcal{F}_{\rm soft}(u,M^2) + \Theta(u_0^\pi - u)\mathcal{F}_{\rm hard}(u,M^2)\Big]\,,$$

- “hard” and “soft” contributions to $F_\pi(Q^2)$ identified as different terms of the OPE/LCSR

- rely on quark-hadron duality in the pion channel
  (s_0^π taken from QCD SR (SVZ) calculation of f_π)

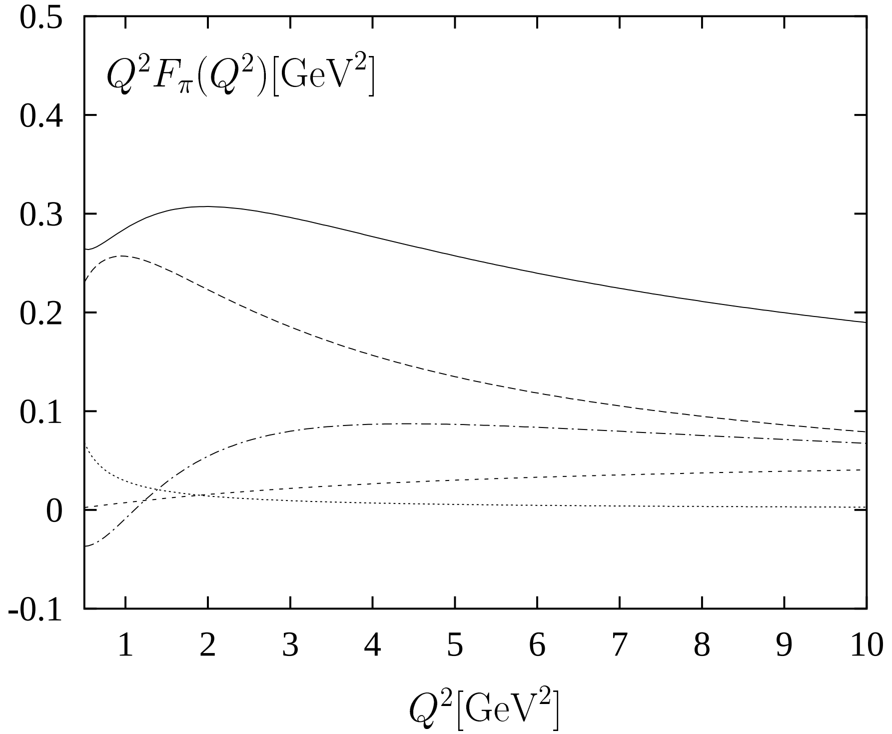
- the "hard" part of $F_\pi^{(2,\alpha_s)}(Q^2)$ contains the CZ-ER-BL asymptotic term with a correct normalization

- twist-4 part [Bijnens, A.K.(2002)] depends basically on one parameter $\delta_\pi^2 = 0.18 \pm 0.06 \text{ GeV}^2$, matrix element of a $\bar{q}qG$ operator, calculated with 2-point QCD SR [Ball,Braun,Lenz (2006) and earlier.]

- renormalon model of twist-4 part [Braun, Gardi, Gottwald(2004)], included in LCSR [Agaev(2005)]

- twist-6 factorized into quark condensate squared (a small correction !)

Hierarchy of twists in LCSR

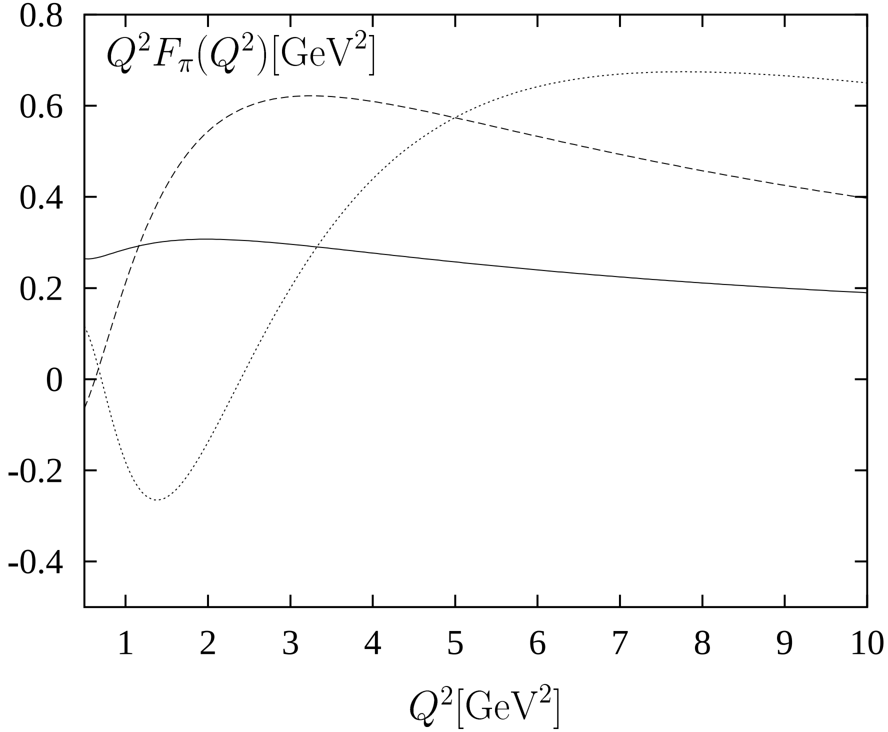


-total
-twist-2 LO
- twist-4
- twist-2 $O(\alpha_s)$
-twist-6

from [A.K., J.Bijnens, (2002)], asymptotic pion DA's

in renormalon model twist-4 DA contribution slightly enhanced
asympt.$<$ twist 4 contribution $<$ renorm.model

Interplay of Gegenbauer moments in LCSR for $F_\pi(Q^2)$



-coeff. at a_4^π
-coeff. at a_2^π
-asympt.

from [A.K., J.Bijnens, (2002)], asymptotic pion DA's

new Jefferson Lab data on $F_\pi(Q^2)$

[T.Horn et al. nucl-ex/0607005; V. Tadevosyan et al. nucl-ex/0607007]

- Electroproduction of pions: $\gamma^* N \to \pi N$,
new data allow a clean separation of $d\sigma_L/dt(Q^2, t)$ (contains F_π via pion exchange) and $d\sigma_T/dt(Q^2, t)$

- $Q^2_{max} = 2.45$ GeV^2 reached (old SLAC/Cornell data at even larger Q^2 not reliable), data at lower Q^2 update the older paper [Volmer et al.2000]

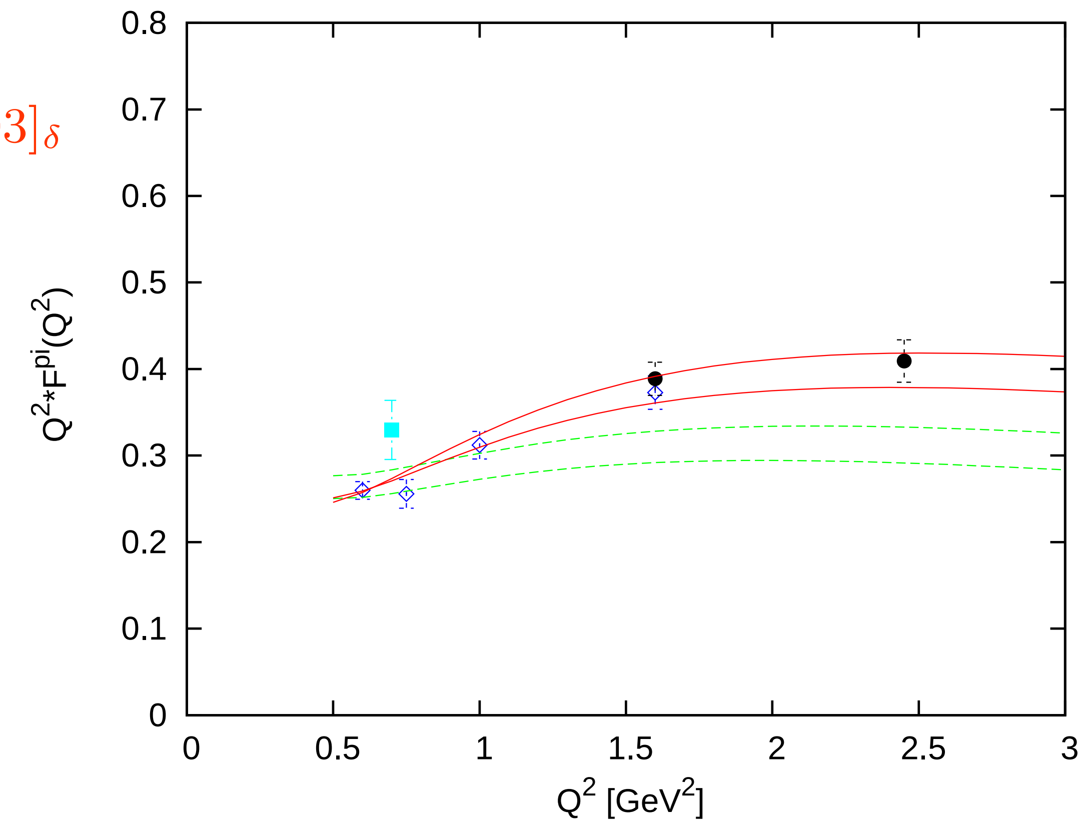
- form factor data model dependent ! (extraction of pion-exchange with an "antique" Regge model)

Fitting the new JLab data on $F_\pi(Q^2)$

assuming $a_{n\leq 4} = 0$

$a_2^\pi(1\text{GeV}) = 0.16 \pm [0.03]_M \pm [0.03]_\delta$

preliminary !



– – – – – – asymptotic DA, – – – – – - with fitted a_2^π

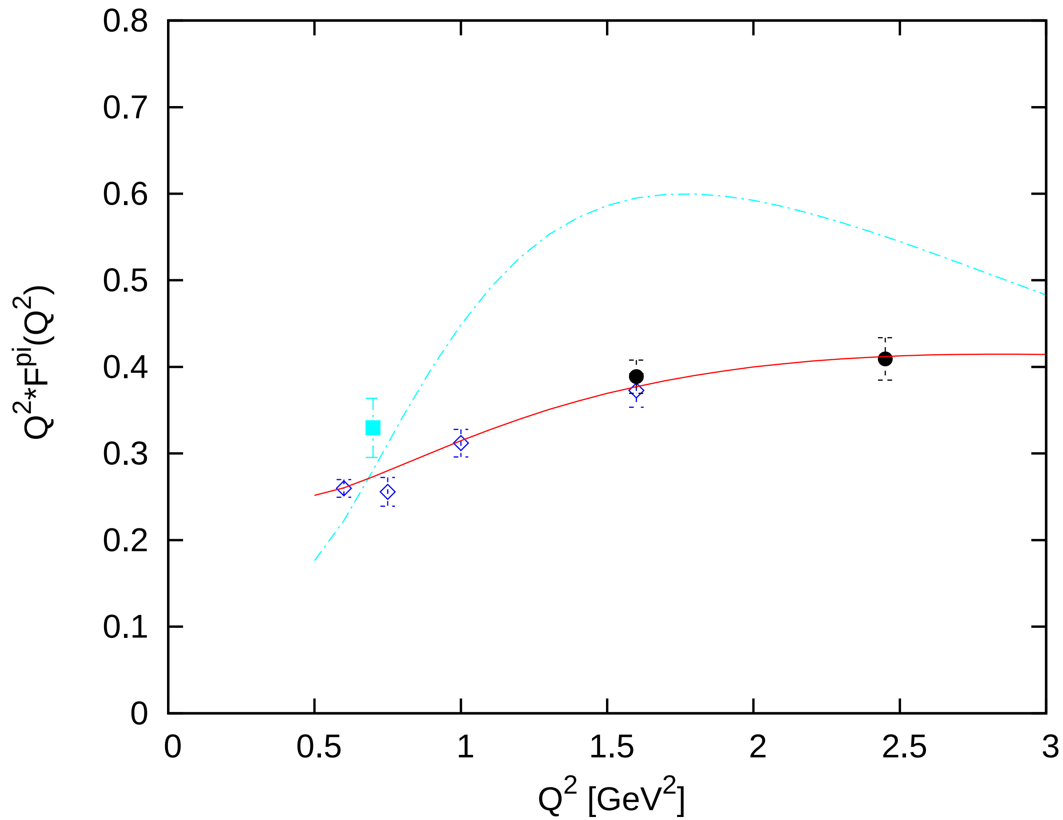
assuming $a^{\pi}_{n\leq 4} = 0$:

$a^{\pi}_{2}(1\text{GeV}) = 0.18$,

$a^{\pi}_{4}(1\text{GeV}) = +0.04$

(central point, errors correlated)

preliminary !



– – – – – - with fitted a^{π}_{2}, a^{π}_{4}

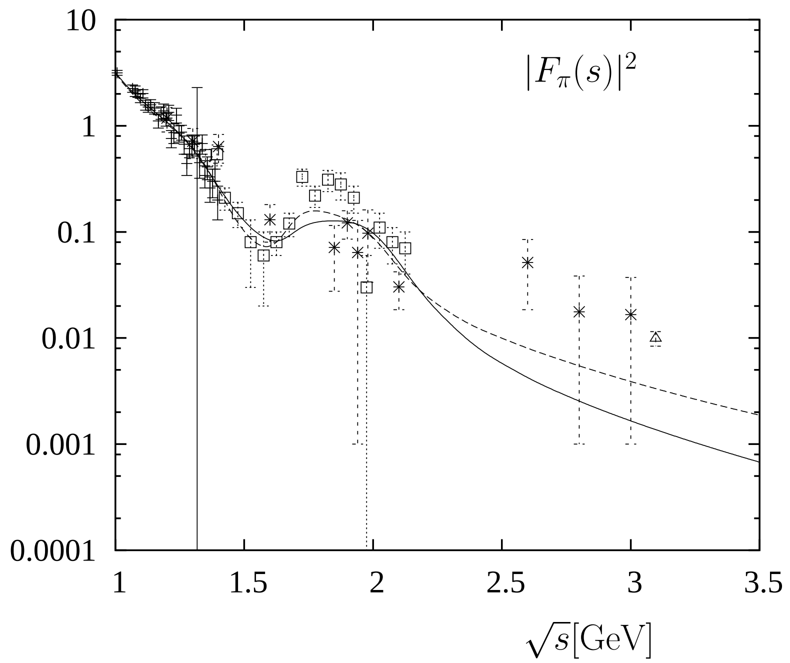
– – – – – BMS(2006) "Best fit point" ($a^{\pi}_{2} = .44$, $a^{\pi}_{4} = -.40$)

Is $F_\pi(Q^2)$ at larger Q^2 accessible ?

- Experiment: future CEBAF upgrade, promise $Q^2 \sim 5-6$ GeV^2

- analytic continuation from timelike region,
need a reliable model of timelike data
at large $s = -Q^2$

Example: an ansatz based on
dual-resonance model,
fitted to the timelike data
(C. Bruch, A.K., J.Kühn (2005))



continuation $s \rightarrow= -Q^2$:
at $Q^2 > 3$ GeV^2 large uncertainties
from the choice of resonance ansatz
(Breit-Wigner,Gounaris-Sakurai etc.)

- more sophisticated but less model-dependent continuation:
dispersion bounds [Geshkenbein(2000)] :
relating $F_\pi(Q^2)$ with integrals over measured $|F_\pi(s)|^2$: still large uncert.

- a recent unexpectedly large $F_\pi(s)$ at $s = 13.48$ GeV^2
(measured by CLEOc slightly below $\psi(2S)$ resonance)

$$[sF_\pi(s)]_{s=13.48} = 1.01 \pm 0.11 \pm 0.07\text{GeV}^2$$

[T.K. Pedlar et al. (CLEO Collab.) hep-ex/0510005]

a typical LCSR prediction at $Q^2 = 13.48$ GeV^2:

$$[Q^2 F_\pi(Q^2)]_{Q^2=13.48} = 0.2 - 0.3 \text{ GeV}^2$$

an indication that the onset of “true asymptotics” is even higher than expected ?

Extracting φ_π from $F_{\pi\gamma\gamma^*}(Q^2)$

- The LCSR relation for $F_{\pi\gamma\gamma^*}(Q^2)$, derived [A.K. (1998)]
from a very similar correlation function: $\gamma^*\gamma^* \to \pi^0$

$$\int d^4x e^{-iqx}\langle \pi^0(p)|T\{j_\mu^{em}(x)j_\nu^{em}(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta F^{\gamma^*\pi}(Q^2,(p-q)^2).$$

at Q^2 and $|(p-q)^2|$ sufficiently large,
employ light-cone OPE in terms of the same diagrams $(\gamma_\mu\gamma_5 \to \gamma_\mu)$

$$F^{\pi\gamma^*\gamma^*}(Q^2,(p-q)^2) = \frac{\sqrt{2}f_\pi}{3}\int\limits_0^1 \frac{du\ \varphi_\pi(u)}{\bar{u}Q^2 - u(p-q)^2} + O(\alpha_s) + O(twist4) + ...$$

twist-2 [Brodsky-Lepage(1979)]; $O(\alpha_s)$ [Braaten (1983)]; twist-4.

LCSR for $F_{\rho\pi\gamma^*}$

$$F^{\rho\pi\gamma^*}(Q^2) = \frac{f_\pi}{3f_\rho}\int_{u_0}^1 \frac{du}{u}\varphi_\pi(u)\exp\left(-\frac{Q^2(1-u)}{uM^2}+\frac{m_\rho^2}{M^2}\right) + O(\alpha_s) + O(\text{twist } 4)$$

- this form factor is an independent object to use for φ_π extraction
($O(\alpha_s)$ has the same $1/Q^4$ asymptotics as the soft part)

- $F^{\rho\pi\gamma^*}(Q^2)$ can be measured in $\gamma^* N \to \rho N$ (e.g., at JLab)
(pion exchange dominates at small t)

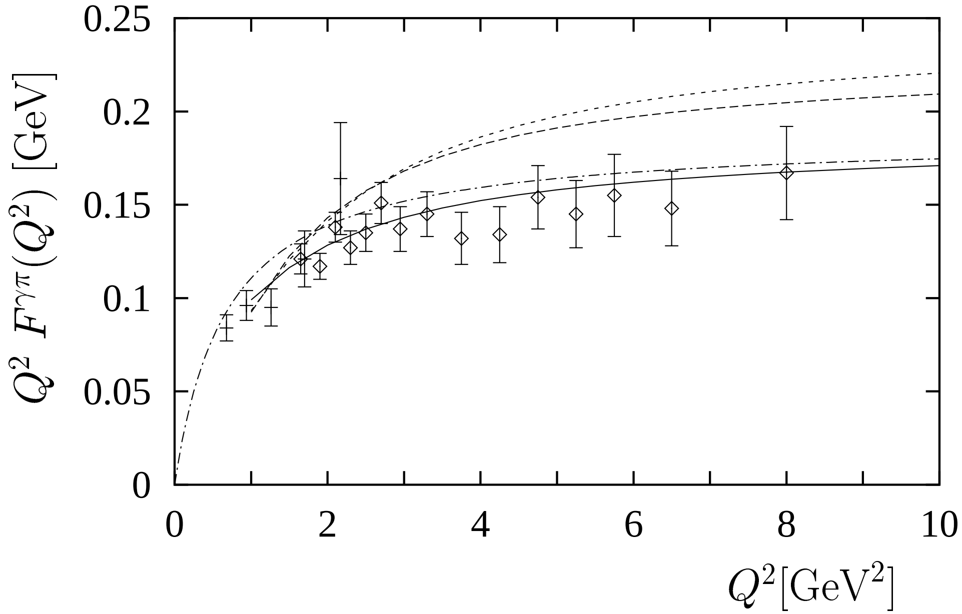
- Next step: dispersion relation for $F^{\pi\gamma^*\gamma^*}$ in $(p-q)^2$, with $F^{\rho\pi\gamma^*}_{LCSR}$ in the ρ- pole term, using duality ansatz:

$$F^{\pi\gamma^*\gamma^*}(Q^2,(p-q)^2) = \frac{\sqrt{2}f_\rho F^{\rho\pi\gamma^*}_{LCSR}(Q^2)}{m_\rho^2-(p-q)^2} + \frac{1}{\pi}\int_{s_0^\rho}^{\infty} ds\,\frac{\text{Im}F^{\gamma^*\gamma^*\pi}(Q^2,s)}{s-(p-q)^2}.$$

- finally: $(p-q)^2 \to 0$ safely, i.e. $F^{\pi\gamma^*\gamma^*}(Q^2,(p-q)^2) \to F^{\pi\gamma\gamma^*}(Q^2)$
this is not literally LCSR, but a dispersion relation, containing LCSR

LCSR vs CLEO data

data: [J.Gronberg et al., hep-ex/9707031] (essentially, V.Savinov)



from [A.K. Eur.Phys.J (1999)]

since then :

- $O(\alpha_s)$, Im- part from Braaten's result [Schmedding,Yakovlev,(1999)]

a uniform decrease by $\sim 15\%$ making room for nonasymptotic part

- fitting to CLEO data [Schmedding,Yakovlev(1999), Bakulev,Mikhailov,Stefanis($\leq$ 2006), Agaev (2006)] some improvements of the theoretical formula (tw4 renorm-model, NLO evolution)

- the LCSR relation mostly depends on the inverse moment:

$$\int_0^1 du \frac{\varphi_\pi(u,\mu)}{1-u} = 3\sum_n a_{2n}^\pi$$

difficult to separate $a_{2,4,..}^\pi$

- the LCSR relation can be and has to be improved further: e.g., calculate the "twist 6" term (factorized into $\langle \bar{q}q\rangle^2$)

- NNLO $O(\alpha_s^2)$ for both F_π and $F_{\pi\gamma\gamma^*}$ sum rules, a realistic task (with the help of experts in multiloop calculations),

Conclusions

* LCSR for $F_\pi(Q^2)$ fitted to new JLab measurement:
-yields a_2^π in the ballpark of QCD SR and lattice QCD calculations,
-indicates small a_4^π ,

* $F_{\rho\pi\gamma^*}(Q^2)$ deserves to be measured

* $F_{\pi\gamma\gamma^*}(Q^2)$, obtained from LCSR has a room for improvement:
twist 6, $O(\alpha_s)$ twist 4, $O(\alpha_s^2)$ tw2

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repeat CLEO measurements at BaBar/Belle;

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THANKS TO PATRICIA and ROMAN !,

LOOKING FORWARD FOR INTERESTING TALKS