Moments of Meson Distribution Amplitudes from the lattice with $N_f = 2 + 1$ Domain Wall Fermions

DA06: Workshop on Light-Cone Distribution Amplitudes

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Project description

Plan:

UKQCD project for the computation of Meson DA's using lattice QCD

- 1st moments a_1^K, $a_1^{K^*}$
- 2nd moments a_2^π, a_2^K, a_2^ρ, $a_2^{K^*}$

Status:

- a_1^K (Phys. Lett. B 641, 67-74, 2006)
- preliminary results for a_2^π, a_2^K

People:

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Jonathan Flynn
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Jun-Ichi Noaki
Chris Sachrajda

<u>Edinburgh</u>
Peter Boyle
Robert Tweedie

Outline

- Definitions; Lattice essentials and systematics

  Further details already in Wolfram Schrör's talk
  (NPR, chiral extrapolation)

- Previous studies and comparison of UKQCD/QCDSF - UKQCD

- UKQCD-project - details

- Results

Definitions

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5\mathcal{U}(z,-z)s(-z)|K(p)\rangle = if_K p_\mu \int_0^1 du e^{i(2u-1)pz}\phi_K(u,\mu)$$

- Expand around the light cone $z^2 \to 0$:

$$\text{1st order:} \quad \underbrace{\langle 0|\,\bar{q}(0)\gamma_\mu\gamma_5\overset{\leftrightarrow}{D}_\nu s(0)\,|K(p)\rangle}_{O_{\mu\nu}} = f_K(ip_\mu)(ip_\nu)\overbrace{\int_0^1 du(2u-1)\phi_K(u,\mu)}^{\text{1st moment: } \langle\xi\rangle_K}$$

$$\text{2nd order:} \quad \underbrace{\langle 0|\,\bar{q}(0)\gamma_\mu\gamma_5\overset{\leftrightarrow}{D}_\nu\overset{\leftrightarrow}{D}_\rho s(0)\,|K(p)\rangle}_{O_{\mu\nu\rho}} = f_K(ip_\mu)(ip_\nu)(ip_\rho)\underbrace{\int_0^1 du(2u-1)^2\phi_K(u,\mu)}_{\text{2nd moment: } \langle\xi^2\rangle_K}$$

- re-construct DA from Gegenbauer momenta:

$$\phi(u,\mu) = 6u\bar{u}\left(1+\sum_{n\geq 1} a_n(\mu)C_n^{3/2}(2u-1)\right) \quad \text{where} \quad \begin{aligned} a_1 &= \tfrac{5}{3}\langle\xi\rangle, \\ a_2 &= \tfrac{7}{12}\left(5\langle\xi^2\rangle-1\right)\ldots \end{aligned}$$

- a_n have a single anomalous dimension, positive, increasing with n

Methods - Lattice

Lattice QCD

- Matrix elements in terms of Euclidean QCD Path-integral

  $\langle O[\bar\psi,\psi,A]\rangle = \frac{1}{Z}\int D\bar\psi D\psi DA\, O(\bar\psi,\psi,A)\, e^{-S_G(U)-S_q(\bar\psi,\psi,U)}$
- discretisation $\rightarrow$ Euclidean space time lattice $\rightarrow$ regulator $1/a$
- calculate integral by Monte Carlo method (statistical sampling)
- errors systematically improvable
- from first principles: tune coupling and quark mass and compute many quantities

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  no unique discretisation:

Glue:	Fermions:
Wilson,	Wilson,
IWASAKI, ...	DWF, ...

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  QCDOC-computer by UKQCD/RBC

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Systematic reduction of errors

- statistical
- quenched / partially quenched / full QCD
- discretisation errors (cut-off effects)
- finite volume errors
- extrapolation in the light physical quark mass
- (non-)perturbative renormalisation

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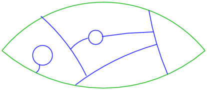
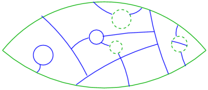


quenched QCD



full QCD

partially quenched:
- different masses for sea and valence quarks
- e.g. 2 dynamical + 1 quenched fermions

Nowadays $N_f = 2, 2 + 1$ ('full QCD') standard

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- finite volume errors
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  - $a \approx 0.1fm \rightarrow 1/a \approx 2\text{GeV}$

    naive estimate of cut-off effects $\begin{array}{lcl} O(a\Lambda_{\rm QCD}) & \approx & 13\% \\ O(a^2\Lambda^2_{\rm QCD}) & \approx & 1.5\% \end{array}$
  - $O(a)$-improvement
  - chirally symmetric action automatically $O(a)$-improved
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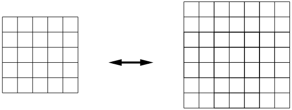
  lattice-box $\approx$ 2fm

  - problems for light hadrons ($m_\pi L > 3$)
  - test:



- extrapolation in the light physical quark mass
- (non-)perturbative renormalisation

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History of lattice results on distribution amplitudes

1985	A.S. Kronfeld and D.M. Photiadis	lattice operators
1986	S. Gottlieb and A.S. Kronfeld	a_2^π, pert. subtraction
1987	G. Martinelli and C.T. Sachrajda	a_2^π, non-pt. subtraction
1988	T.A. DeGrand and R.D. Loft	a_2^π, check the latter two
1991	D. Daniel, R. Gupta and D.G. Richards	a_2^π, dynamical
2000	L. Del Debbio, M. Di Pierro, A. Dougall and C.T. Sachrajda	a_2^π, quenched
2006	QCDSF/UKQCD	a_1^K, a_2^π, a_2^K
2006	UKQCD	a_1^K (a_2^π, a_2^K)

UKQCD/QCDSD - UKQCD

Comparison of major features of the UKQCD/QCDSF and UKQCD computations for a_1^K, a_2^π, a_2^K

	UKQCD/QCDSF	UKQCD
gluon action	Wilson	IWASAKI
fermion action	improved Wilson	Domain Wall
cut-off	1.6GeV-2.6GeV	1.6GeV
scaling study	yes$^\pi$/no^K	not yet
volume	1.5-2.2fm	1.9/2.9fm
pion mass	$>$ 600MeV	$>$ 400MeV
unitarity	unitary$^\pi$ partial quenching^K	unitary
renormalisation	non-perturbative	perturbative

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Domain Wall Fermions

Discretisation of fermion action - free case:

$$\partial_\mu\psi(x) = \frac{1}{a}\left(\psi(x + a\hat{\mu}) - \psi(x)\right) \qquad \partial^*_\mu\psi(x) = \frac{1}{a}\left(\psi(x) - \psi(x - a\hat{\mu})\right)$$

Naive	$D = \frac{1}{2}\left\{\gamma_\mu(\partial^*_\mu + \partial_\mu)\right\}$	16 'doublers' at $\frac{1}{a}sin\,(ap_\mu)$
Wilson	$D = \frac{1}{2}\left\{\gamma_\mu(\partial^*_\mu + \partial_\mu) - a\,\partial^*_\mu\partial_\mu\right\}$	doublers get mass $\propto a^{-1}$ one physical fermion chiral symmetry broken

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Domain Wall Fermions (by Kaplan, very nice in Lüscher's hep-lat/0102028):

- five dimensional formulation
- tower of fermions with mass $\approx a^{-1}$
- one physical massless mode localised on domain wall
  with definite chirality

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Pro/Contra:

+ (approximate) chiral symmetry on the lattice
+ no unphysical operator mixing (renormalisation)
+ automatic $O(a)$-improvement (reduced cut-off effects)
- expensive

UKQCD Simulation

We consider

- $O_{\{\mu\nu\}}$ with $\mu \neq \nu$ at $|\vec{p}| = 2\pi/L$ $\rightarrow \langle\, \xi \,\rangle$

  $O_{\{\mu\nu\rho\}}$ with $\mu \neq \nu \neq \rho$ at $|\vec{p}| = \sqrt{2}\, 2\pi/L$ $\rightarrow \langle\, \xi^2 \,\rangle$
- Lattice parameters

$N_f = 2 + 1$ DWF, IWASAKI gauge action generated by RBC/UKQCD-collaboration

$(L/a)^3 \times (T/a) \times L_s$	$16^3 \times 32 \times 16$ $24^3 \times 64 \times 16$
a^{-1}	1.6 GeV
L	1.9/2.9 fm
am_s	0.04
am_l	0.03 / 0.02 / 0.01

Remainder:

- Perturbative renormalisation (NPR under way)
- Bare results
- Comparison, summary and outlook

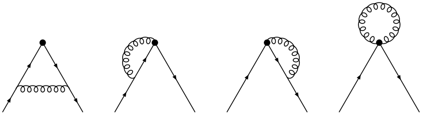
Perturbative Renormalisation

- Perturbative Renormalisation

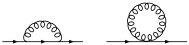
  compare 1-loop amputated Green's function with desired operator inserted in both schemes (lattice and $\overline{\mathrm{MS}}$)

  for the 1st moment:

  operator $O^5_{\mu\nu}$ ($\rightarrow V$):



  self energy ($\rightarrow \Sigma$):



Perturbative Renormalisation

Matching condition: $O_{\rho\mu}^{\overline{\rm MS}}(\mu) = Z_{O_{\rho\mu}} O_{\rho\mu}^{\rm latt}(a)$

$$Z_{O_{\rho\mu}} = \frac{1}{(1-w_0^2)Z_w}\left[1 + \frac{g^2 C_{\rm F}}{16\pi^2}\left(-\frac{8}{3}\ln(\mu^2 a^2) + \Sigma_1^{\overline{\rm MS}} - \Sigma_1 + V^{\overline{\rm MS}} - V\right)\right]$$

- Σ and V are contributions from wave-function and vertex graphs, resp.

Result: $Z_{O_{\rho\mu}} = \frac{1}{0.9082}\left[1 - \frac{g^2 C_{\rm F}}{16\pi^2} 5.2509\right]\left[1 + \frac{g^2 C_{\rm F}}{16\pi^2}\left(-\frac{8}{3}\ln(\mu^2 a^2) - 0.6713\right)\right]$

- for this project divide by Z_A since we want to renormalise
- dependent on renormalised coupling

$$\frac{Z_{O_{\mu\nu}}}{Z_A}(1.6\mathrm{GeV}) = 1.28 \pm 0.05$$

Remainder:

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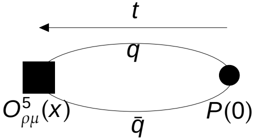
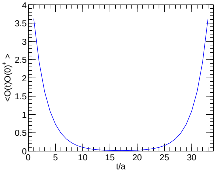
- Bare results
- Comparison, summary and outlook

Bare lattice correlation functions

Time dependence of correlation functions (e.g. 2pt function):

$$\langle O_{\rho\mu}(t)P^\dagger(0)\rangle = \sum_{n=1}^{\infty}\langle 0|O_{\rho\mu}|n\rangle\langle n|P^\dagger|0\rangle e^{-E_n t} \stackrel{t\to\infty}{=} \langle 0|O_{\rho\mu}|1\rangle\langle 1|P^\dagger|0\rangle e^{-m_O t}$$





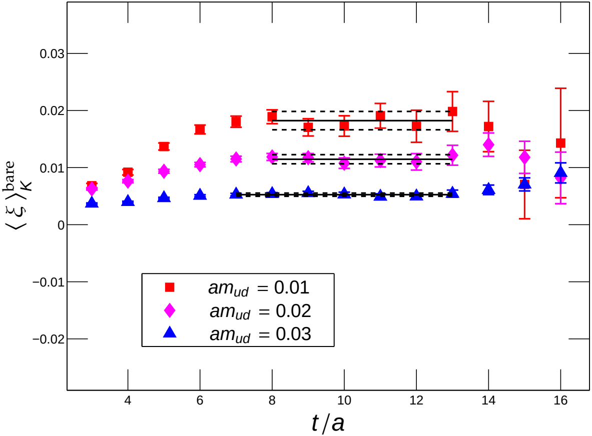
Cancel time-dependence and correlation effects; e.g. 1st moment $\langle\, \xi \,\rangle$:

$$\frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle O^5_{\rho\mu}(x)P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle A_\nu(x)P^\dagger(0)\rangle} \stackrel{t\to\infty}{=} \frac{(ip_\rho)(ip_\mu)}{ip_\nu}\langle\, \xi \,\rangle$$

Bare results - $\langle \xi \rangle_K$

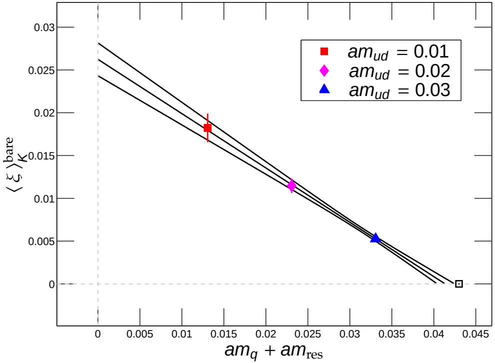
$16^3 \times 32 \times 16$, $am_{u,d} = 0.03; 0.02; 0.01$, $am_s = 0.04$,
300 measurements/source, 2, 2 and 4 pos. of the source
folded in T-direction

$$\frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle O^5_{\rho\mu}(x) P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle A_\nu(x) P^\dagger(0)\rangle} \sim$$



$$\langle \xi \rangle_K^{\rm bare} = 0.0053(3);\ 0.012(1);\ 0.018(2)$$

Bare results - χ extrapolation



$\langle\xi\rangle_K^{\rm bare} = 0.0262(23)$

Chen and Stewart (2004) studied the mass behaviour in χPT.
At lowest non trivial order: $\langle\xi\rangle_K \propto (m_s + m_q)$ (no logs).
$\rightarrow$ Our result shows clear signs of partonic SU(3)-breaking effects
$\rightarrow$ extrapolation is compatible with iso-spin symmetry ($\langle\xi\rangle_\pi = 0$)

Bare results - $\langle \xi^2 \rangle_K$ - PRELIMINARY

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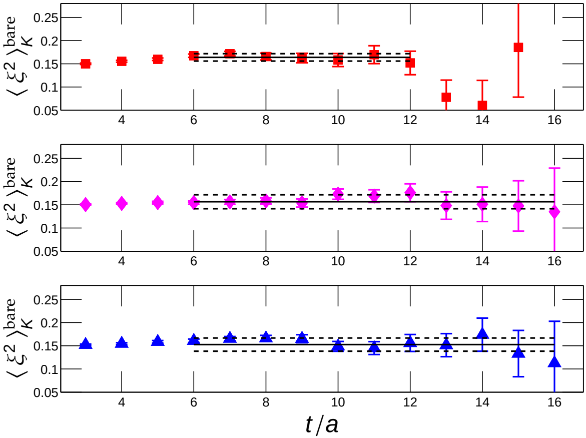
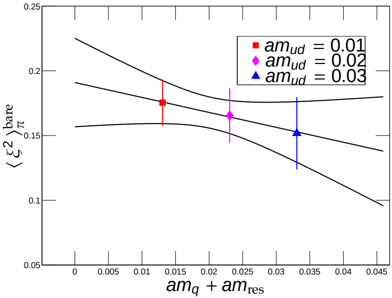
300 measurements/source, 2, 4 and 4 pos. of the source

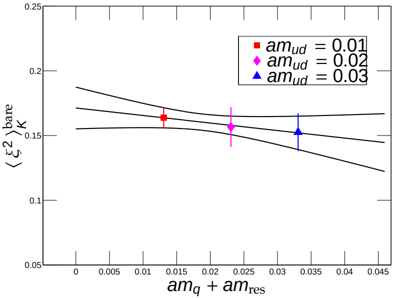
$$\frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle O^5_{\rho\mu\nu}(x) P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle A_\nu(x) P^\dagger(0)\rangle} \sim$$



$$\langle \xi^2 \rangle_K^{\rm bare} = 0.153(14); 0.157(15); 0.164(8)$$
$$\langle \xi^2 \rangle_\pi^{\rm bare} = 0.160(11); 0.165(11); 0.187(6)$$

Bare results - χ extrapolation - PRELIMINARY





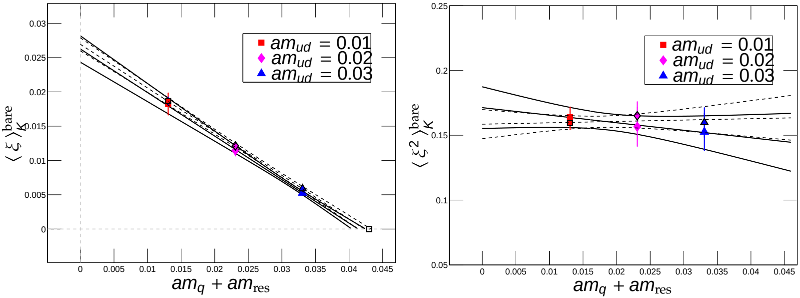
$\langle \xi^2 \rangle_\pi^{\rm bare} = 0.191(34)$ $\langle \xi^2 \rangle_K^{\rm bare} = 0.171(16)$

Remainder:

- perturbative renormalisation
- Bare results
- Finite volume effects
- Final results, comparison, summary and outlook

Finite volume errors

Comparison between 1.9fm and 2.9fm box ($16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$)



We don't find significant finite volume effects

Remainder:

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Final result ($16^3 \times 32 \times 16$ only)

$$a_1^K(2\mathrm{GeV})|_{\overline{\mathrm{MS}}} = \frac{Z_{O_{\mu\nu}}}{Z_A}(2\mathrm{GeV}) \times \frac{5}{3}\langle\, \xi \,\rangle_K^{\mathrm{bare}} = 0.055(5)$$

PRELIMINARY (contribution from mixing neglected):

$$a_2^K(2\mathrm{GeV})|_{\overline{\mathrm{MS}}} = \frac{Z_{O_{\mu\nu\rho}}}{Z_A}(2\mathrm{GeV}) \times \frac{5}{12}\left(5\langle\, \xi^2 \,\rangle_K^{\mathrm{bare}} - 1\right) = 0.217(31)$$

PRELIMINARY (contribution from mixing neglected):

$$a_2^\pi(2\mathrm{GeV})|_{\overline{\mathrm{MS}}} = \frac{Z_{O_{\mu\nu\rho}}}{Z_A}(2\mathrm{GeV}) \times \frac{5}{12}\left(5\langle\, \xi^2 \,\rangle_\pi^{\mathrm{bare}} - 1\right) = 0.241(51)$$

Comparison, Summary and Outlook

All results at $\mu = 2$ GeV

	UKQCD/QCDSF	UKQCD
a_1^K	0.0453(9)(29)	0.055(5)
a_2^K	0.175(18)(47)	0.217(31)
a_2^π	0.201(114)	0.241(54)

- Results compatible
- UKQCD numbers for a_2^K and a_2^π (right column) are very preliminary and mixing is neglected in the renormalisation!

Comparison, Summary and Outlook

UKQCD-project

- a_1^K, a_2^K, a_2^π done
- reducing systematics:

- finite volume effects	done
- non-perturbative renormalisation	under way
- cut-off effects	scaling study under way
- chiral extrapolation	$m_\pi \to 250$ MeV ?

- moments in the vector channel

General

- 'an independent calculation on the lattice would be both timely and useful' [Ball & Zwicky, 2006]
- results from two very different approaches in good agreement