Renormalon approach to Higher–Twist Distribution Amplitudes

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plan

- conformal expansion: why, why not
- cancellation of ambiguities in the OPE
- example: quadratic UV divergence of twist–4 operators
- renormalon model: results
- convergence of the conformal expansion
- endpoint behavior

Based on V.M. Braun, E. Gardi & S. Gottwald, *Nucl. Phys.* **B685** 171 (2004) [hep-ph/0401158]

Conformal expansion

any conformal–expansion–based ansatz,

advantages

- consistent with EOM at each truncation order $J_{\max}$
- preserved by leading–order QCD evolution
- high conformal spins are suppressed by $\propto J^{-\mathrm{const}\times\ln\mu_F^2}$

disadvantages

- practical only if converges fast
- convergence through evolution $\propto J^{-\mathrm{const}\times\ln\mu_F^2}$ holds only at high μ_F

Cancellation of ambiguities in the OPE

Consider a physical amplitude near the lightcone, depending on $\Delta \equiv x_1 - x_2$ where $|\Delta| \ll 1/\Lambda_{\rm QCD}$.

Factorization: short–distance $C^{(t)}$ $\otimes$ long–distance DA $\phi^{(t)}$

$$G(u,\Delta^2) = \underbrace{C^{(2)} \otimes \phi^{(2)}}_{\text{twist 2}} + \underbrace{\Delta^2 \sum C^{(4)} \otimes \phi^{(4)}}_{\text{twist 4}} + \mathcal{O}(\Delta^4)$$

Cancellation of μ_F dependence:

- logarithmic: at each twist $\Longrightarrow$ evolution equations
- power: between different twists $\Longrightarrow$ renormalons:
  $\delta_{\rm UV}(\text{twist 4}) = \Delta^2 \mu_F^2\, D^{(4\to 2)} \otimes \phi^{(2)} = -\delta_{\rm IR}(\text{twist 2})$

Three–particle twist–4 DA and EOM relations

$$\langle 0 \left| \bar{d}(-z)[-z, vz]\gamma_\nu\gamma_5 g G_{\mu\rho}(vz)[vz, z]u(z) \right| \pi^+(p)\rangle =$$

$$= f_\pi \int \mathcal{D}\alpha_i \, \mathrm{e}^{-ipz(\alpha_1-\alpha_2+\alpha_3 v)} \left\{ \frac{p_\nu}{pz}(p_\mu z_\rho - p_\rho z_\mu)\Phi_\parallel(\alpha_i) \right.$$

$$\left. + \left[p_\rho \left(g_{\mu\nu} - \frac{z_\mu p_\nu}{pz} \right) - p_\mu \left(g_{\rho\nu} - \frac{z_\rho p_\nu}{pz} \right) \right] \Phi_\perp(\alpha_i) \right\},$$

EOM relations between two– and three–particle DA:

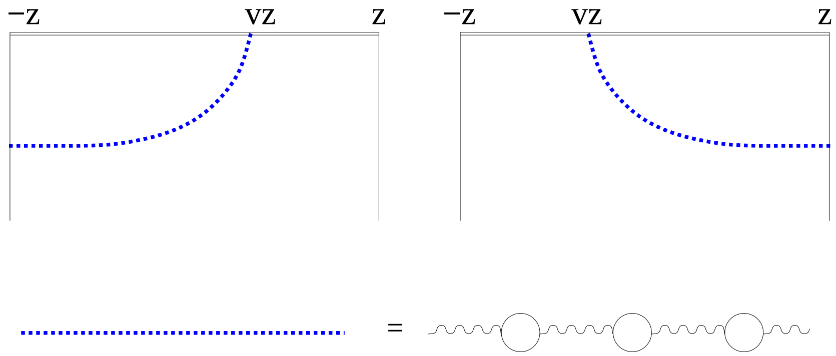
$$\phi_1^{(4)} + \phi_2^{(4)}(u) = \int_0^u d\alpha_1 \int_0^{1-u} d\alpha_2 \, \frac{\bar{u}\alpha_1 - u\alpha_2}{2\alpha_3^2} \left[2\Phi_\perp - \Phi_\parallel\right](\alpha_i),$$

where $\alpha_3 = 1 - \alpha_1 - \alpha_2$.

Braun & Filyanov ('90)

Quadratic UV divergence of three–particle twist–4 operators

Consider a $q\bar{q}g$ twist–4 operator on the lightcone ($z^2 = 0$):



Andersen ('99)

BGG ('04)

$$\left\langle q_2 \left| \bar{d}(-z)\Gamma g G_{\alpha\beta}(vz)u(z) \right| q_1 \right\rangle = \mathrm{e}^{-i(q_1+q_2)z} \left(g_{\lambda\alpha} g^{\mu}_{\beta} - g_{\lambda\beta} g^{\mu}_{\alpha}\right)$$

$$\times \frac{4\pi^2 C_F}{\beta_0} \int_0^\infty dw\, \mathrm{e}^{\frac{5}{3}w} (-\Lambda^2)^w\, B^{\lambda}_{\mu}(w,q,z,v)$$

$$B^{\lambda}_{\mu} \equiv \bar{d}_{q_2}\gamma_\mu\gamma_\rho\Gamma u_{q_1}\, I^{\lambda\rho}(q_2,(1+v)z) - \bar{d}_{q_2}\Gamma\gamma_\rho\gamma_\mu u_{q_1}\, I^{\lambda\rho}(q_1,(1-v)z)$$

where $$I^{\lambda\rho}(q,z) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{k^\lambda (q+k)^\rho}{(k^2)^{(1+w)}\,(q+k)^2}\, \mathrm{e}^{-ikz}$$

Twist $4 \to 2$ operator–mixing relations

$B^{\lambda}_{\mu}(w,q,z,v)$ has a pole at $w=1$ owing to UV divergence:

$$I^{\lambda\rho}(q,z)\Big|_{w=1} = \frac{-i}{32\pi^2(1-w)}\int_0^1 da\, a\, \mathrm{e}^{iqz(1-a)} \times\left[iq^{\lambda}z^{\rho}(1-a) - iq^{\rho}z^{\lambda}a + g^{\lambda\rho} + \frac{1}{2}a(1-a)q^2 z^{\lambda}z^{\rho}\right].$$

$$O^{\nu}_{\mu\rho} = \bar{d}(-z)\gamma^{\nu}\gamma_5 g G_{\mu\rho}(vz)u(z)\,; \qquad y^{\pm} = z\Big(1-(1\pm v)(1-a)\Big)$$

$$\delta_{\mathrm{UV}}\left\{z^{\mu}g^{\rho}_{\nu}\,O^{\nu}_{\mu\rho}\right\} = k\Lambda^2\int_0^1 da(1-a)\left[\bar{d}(-y^{+})\not{z}\gamma_5 u(z) - \bar{d}(-z)\not{z}\gamma_5 u(y^{-})\right]$$

$$\delta_{\mathrm{UV}}\left\{z^{\mu}z_{\nu}\,O^{\nu}_{\mu\rho}\right\} = -k\Lambda^2\, z_{\rho}\int_0^1 daa\left[\bar{d}(-y^{+})\not{z}\gamma_5 u(z) - \bar{d}(-z)\not{z}\gamma_5 u(y^{-})\right]$$

Renormalon model: twist–4 DA in terms of twist–2 DA

Using the twist $4 \to 2$ operator mixing with the definitions of 3-particle twist–4 DA (l.h.s) and twist–2 DA (r.h.s):

$$\delta_{\text{UV}}\left\{\Phi_{\perp}(\alpha_1,\alpha_2,\alpha_3)\right\} = -\frac{1}{2} ik\Lambda^2 \left[\frac{\phi_\pi(\alpha_1)}{1-\alpha_1} - \frac{\phi_\pi(\alpha_2)}{1-\alpha_2}\right],$$

$$\delta_{\text{UV}}\left\{\Phi_{\parallel}(\alpha_1,\alpha_2,\alpha_3)\right\} = -ik\Lambda^2 \left[\frac{\alpha_2\phi_\pi(\alpha_1)}{(1-\alpha_1)^2} - \frac{\alpha_1\phi_\pi(\alpha_2)}{(1-\alpha_2)^2}\right]$$

Upon fixing the overall normalization (one parameter!) using

$$\langle 0|\bar{d}\gamma_\nu ig\widetilde{G}_{\mu\rho}u|\pi^+(p)\rangle = \frac{\delta^2}{3} f_\pi [p_\rho g_{\mu\nu} - p_\mu g_{\rho\nu}]; \quad \delta^2\big|_{\text{QCD}-\text{SR}} \simeq 0.2\ \text{GeV}^2,$$

UV ren. ambiguities translate into a model: $-ik\Lambda^2 \longrightarrow \delta^2/3$.

Renormalon model *vs.* conformal–expansion based model

3–particle twist–4 pion DA $\alpha_{3(g)} = 1 - \alpha_{1(q)} - \alpha_{2(\bar{q})}$

Renormalon model assuming asymptotic leading–twist DA:

$$\begin{aligned} \Phi_\perp(\alpha_i) &= \delta^2[\alpha_1 - \alpha_2]\,, \\ \Phi_\parallel(\alpha_i) &= 2\delta^2\alpha_1\alpha_2\left[\frac{1}{1-\alpha_1} - \frac{1}{1-\alpha_2}\right] \end{aligned}$$

First two orders in the conformal expansion ($J = 3,\ 4$):

$$\begin{aligned} \Phi_\perp^{\rm BF}(\alpha_i) &= 10\,\delta^2\,(\alpha_1 - \alpha_2)\alpha_3{}^2\,[1 + 6\,\epsilon\,(1 - 2\alpha_3)]\ , \\ \Phi_\parallel^{\rm BF}(\alpha_i) &= 120\epsilon\delta^2\alpha_1\alpha_2\alpha_3(\alpha_1 - \alpha_2) \end{aligned}$$

Braun & Filyanov ('90)

Qualitatively different for vanishing gluon momentum $\alpha_3 \to 0$.

Renormalon model for two–particle twist–4 DA

Using the 3-particle twist–4 DA with the EOM relations:

$$\phi_1^{(4)}(u) = \frac{\delta^2}{6}\int_0^1 dv\, \phi_\pi(v)\left\{\frac{1}{v^2}\left[u+(v-u)\ln\left(1-\frac{u}{v}\right)\right]\theta(v>u) + \frac{1}{\bar{v}^2}\left[\bar{u}+(u-v)\ln\left(1-\frac{\bar{u}}{\bar{v}}\right)\right]\theta(v<u)\right\},$$

$$\phi_2^{(4)}(u) = -\frac{\delta^2}{6}\int_0^1 dv\, \phi_\pi(v)\left\{\left(\frac{u}{v}\right)^2\theta(v>u) + \left(\frac{\bar{u}}{\bar{v}}\right)^2\theta(v<u)\right\}$$

In physical amplitudes $\delta_{\mathrm{UV}}(\text{twist } 4) + \delta_{\mathrm{IR}}(\text{twist } 2) = 0.$

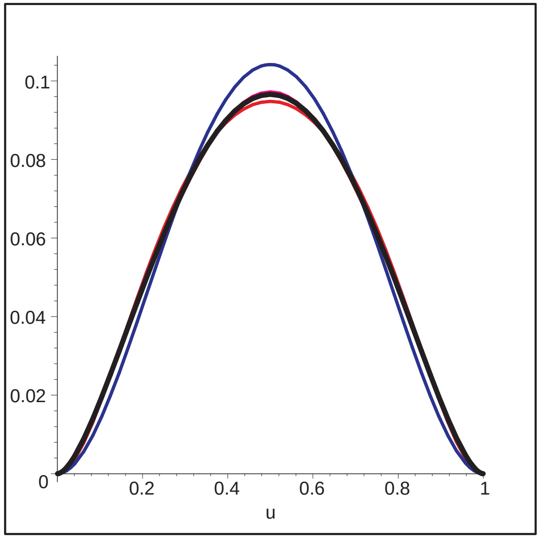
Indeed, the same expressions are obtained considering IR renormalons in twist–2 coefficient functions.

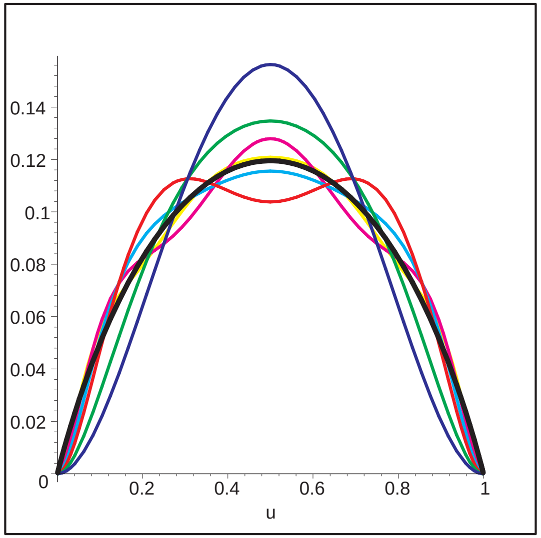
Convergence of the conformal expansion for 2-particle twist-4 DA

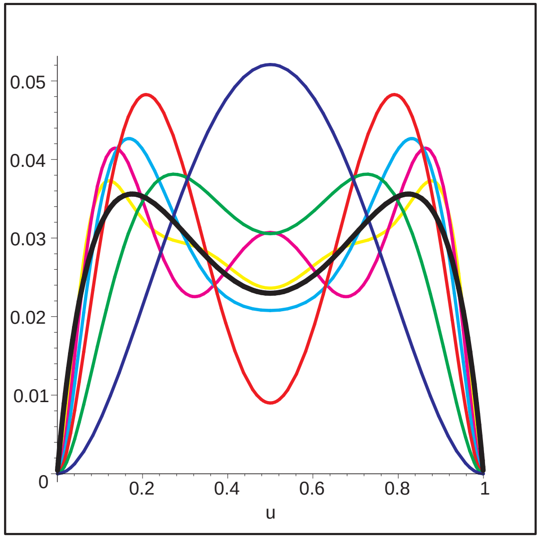
We can examine the convergence of the conformal expansion by expanding the renormalon model *in this basis*:

$$\phi_2^{(4)}(u) = -4\delta^2\, u^2(1-u)^2 \sum_{J=3,5,7,\ldots}^{\infty} \frac{2J-1}{J(J-1)^2(J-2)} P_{J-3}^{(2,2)}(2u-1)$$

the good $-\phi_2^{(4)}(u)$ | the bad $\phi_1^{(4)}(u)$ | the ugly $\phi_1^{(4)} + \phi_2^{(4)}$







Endpoint behavior of 2–particle twist–4 DA

Renormalon model (assuming asymptotic leading–twist DA):

$$\phi_1^{(4)}(u) = \delta^2 \left\{ \bar{u}\left[\ln(\bar{u}) - \mathrm{Li}_2(\bar{u})\right] + u\left[\ln(u) - \mathrm{Li}_2(u)\right] - u\bar{u} + \frac{\pi^2}{6} \right\}$$
$$\simeq \delta^2 \left[\left(\frac{\pi^2}{6} - 1 \right) u + \mathcal{O}(u^2) \right].$$

Its conformal expansion (at any truncation order):

$$\phi_1^{(4)}(u) = \delta^2 \left\{ \frac{5}{2} \left[u^2 \, \bar{u}^2 \right]_{J=3} + \cdots \right\} \simeq \delta^2 \, \mathcal{O}(u^2)$$

The expansion does not converge uniformly at the endpoints!

Conclusions

- Renormalons: model for higher–twist DA in terms of the leading–twist DA.
  - consistent with all EOM relations!
  - single parameter(!) for the entire set of twist–4 DA
  - Already available for π and ρ (BGG) and for K (BBL).
- Convergence of the conformal expansion
  - 3–particle DA: no convergence for fixed α_3.
  - 2–particle DA: converges away from endpoints, but qualitatively different — slower — endpoint behavior
  - High J contributions: renormalon model is an upper bound, since evolution $\propto J^{-\mathrm{const}\times\ln\mu_F^2}$