Applications of B-Meson LCDAs

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Outline

1. 2-particle DAs of the B-meson (definition)
2. B-meson LCDAs in QCD factorization
   - Factorization theorems for exclusive amplitudes
   - The role of λ_B and σ_B
3. B-meson LCDAs in SCET sum rules
   - Factorization Theorem for Correlation functions
   - Sum rules for factorizable and non-factorizable currents
4. Summary/Outlook

2-particle DAs of the B-meson [Grozin/Neubert, hep-ph/9607366]

- Definition: (position space, $z^2 = 0$)

$$\langle 0|\bar{q}_\beta(z)\, P(z,0)\, b_\alpha(0)|B(v)\rangle_{\rm HQET}$$

$$= -\frac{if_B(\mu)m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\tilde{\phi}_+^B(v\cdot z,\mu)+\frac{\tilde{\phi}_-^B(v\cdot z,\mu)-\tilde{\phi}_+^B(v\cdot z,\mu)}{v\cdot z}\,\not{z}\right\}\gamma_5\right]_{\alpha\beta}$$

- Definition: (momentum space, $n_\pm v = 1$, $n_\pm^2 = 0$)

$$\langle 0|\bar{q}(\lambda n_-)\, P(\lambda,0)\, \not{n}_\mp\gamma_5\, b(0)|B(v)\rangle_{\rm HQET} = if_B(\mu)m_B\int_0^\infty d\omega\, e^{-i\lambda\omega}\,\phi_\pm^B(\omega,\mu)$$

The phenomenological parameters λ_B and σ_B [Braun/Ivanov/Korchemsky]

Define:

$$\lambda_B^{-1}(\mu_0) \equiv \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega, \mu_0)$$

$$\sigma_B(\mu_0) \equiv \lambda_B(\mu_0) \int_0^\infty \frac{d\omega}{\omega}\, \ln\left[\frac{\mu_0}{\omega}\right] \phi_+^B(\omega, \mu_0)$$

- λ_B determines leading term in QCDF (see below)
- $\phi_-^B(0) \approx \lambda_B^{-1}$ (WW relation, see also talk by Nils Offen)
- σ_B can be used to compare different models for the shape of $\phi_+^B(\omega)$

- In exclusive *B*-decays one needs $\phi_\pm^B(\omega)$ at hard-collinear scale

$$\lambda_B(\mu_{\rm hc}) \quad \text{and} \quad \sigma_B(\mu_{\rm hc}) \quad \text{with} \quad \mu_{\rm hc} \sim \sqrt{m_b \Lambda_{\rm had}} \simeq 1.5\ \text{GeV}$$

Renormalization-group evolution for ϕ_+^B

[Lange/Neubert, hep-ph/0303082; Lee/Neubert, hep-ph/0509350]

$$\phi_+^B(\omega,\mu) = e^{V-2\gamma_E g}\,\frac{\Gamma(2-g)}{\Gamma(g)}\int_0^\infty \frac{d\omega'}{\omega'}\,\phi_+^B(\omega',\mu_0)\left(\frac{\omega_>}{\mu_0}\right)^g \frac{\omega_<}{\omega_>}\,{}_2F_1\left(1-g,2-g;2;\frac{\omega_<}{\omega_>}\right)$$

where $\omega_> = \max(\omega,\omega')$ and $\omega_< = \min(\omega,\omega')$

$$V = V(\mu,\mu_0) = -\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\left[\Gamma_{\rm cusp}(\alpha)\int_{\alpha_s(\mu_0)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} + \gamma(\alpha)\right]$$

$$g = g(\mu,\mu_0) = \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\,\Gamma_{\rm cusp}(\alpha)$$

B-meson LCDAs in QCD factorization

Factorization Theorem for Decay Amplitudes (leading power)

[Beneke/Buchalla/Neubert/Sachrajda 99; Korchemsky/Pirjol/Yan 99; Beneke/TF 00; Bauer/Pirjol/Stewart 02; Lunghi/Pirjol/Wyler 02; Bosch/Hill/Lange/Neubert 03; Beneke/TF 03; Lange/Neubert 03; Becher/Hill/Neubert 05; ...]

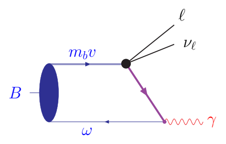
$$\begin{aligned}
\mathcal{A}_i(B \to \gamma + \text{lept.}) &= && + T_i^{\rm II}(\mu) \otimes \phi_B(\mu) \\
\mathcal{A}_i(B \to M + \text{lept.}) &= \xi_M(\mu) \cdot T_i^{\rm I}(\mu) && + T_i^{\rm II}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \\
\mathcal{A}_i(B \to MM') &= \xi_M(\mu) \cdot T_i^{\rm I}(\mu) \otimes \phi_{M'}(\mu) && + T_i^{\rm II}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \otimes \phi_{M'}(\mu)
\end{aligned}$$

- universal transition form factor ξ_M (“non-factorizable” !)
- two-particle LCDAs for B-meson and light hadrons M
- perturbative coefficient functions, $T_i^{\rm I}$ and $T_i^{\rm II} = H_i \otimes J$
- power-corrections induce more factorizable and non-factorizable terms

$B \to \gamma \ell \nu$ (large recoil) [Korchemsky/Pirjol/Yan, hep-ph/9911427; Descotes-Genon/Sachrajda, hep-ph/0209216; Lunghi/Pirjol/Wyler, hep-ph/0210091; Bosch/Hill/Lange/Neubert, hep-ph/0301123]

LO approximation for $B \to \gamma$ form factors:

$$f_V(E_\gamma) \simeq f_A(E_\gamma) \propto \frac{f_B}{E_\gamma} \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega)$$



- phenomenological constraint on λ_B
- similar role as $\pi \to \gamma$ form factor for $\langle u^{-1} \rangle_\pi$.

NLO corrections and resummation of logs:

- important to set the scale for $\phi_+^B(\omega, \mu)$ and $\lambda_B(\mu)$
- jet function contains logarithmic dependence on ω
- requires more information on shape of $\phi_+^B(\omega, \mu_0)$ $(\sigma_B(\mu_0), \ldots)$

Power-corrections are not negligible:

- factorizable power corrections (calculable, also requires photon DAs)
- non-factorizable power corrections (soft $B \to \gamma$ form factor, LCSR)

$B \to \pi(\rho, \ldots)$ form factors (large recoil)

Tree-level approximation:

- form factors fulfill symmetry relations [Charles et al. hep-ph/9812358]
- dominated by soft (endpoint) contributions $\Rightarrow$ non-factorizable
  (i.e. not expressed in terms of moments of $\phi_\pm^B(\omega)$ and $\phi_{\pi(\rho)}(u)$)

Radiative corrections:

- Vertex corrections (perturbative in QCD)
- Spectator interactions (perturbative in SCET):
  - NLO: expressed in terms of λ_B and $\langle u^{-1}\rangle_{\pi(\rho)}$ [Beneke/TF, hep-ph/0008255]
  - NNLO: also (weakly) sensitive to σ_B [Beneke/Yang hep-ph/0508250]

$\longrightarrow$ Plots

Power corrections (model dependent)

Corrections to symmetry relations:

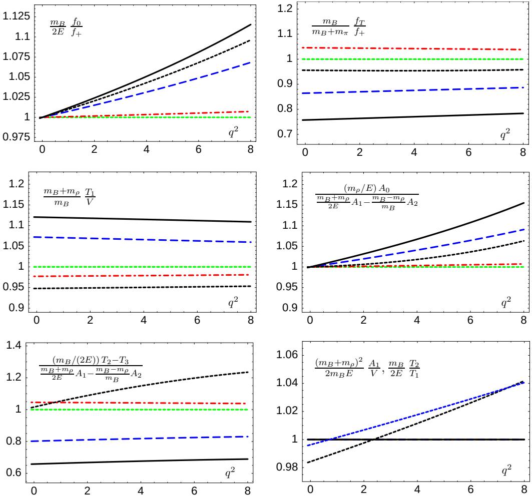
— symmetry limit

— full result

– – without NNLO spect. + no resum.

- – - without any spectator effects

- - - QCD sum rules



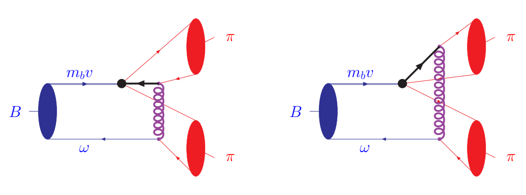
[from Beneke/Yang, hep-ph/0508250; using $\phi_+(\omega) = \omega/\lambda_B^2\, e^{-\omega/\lambda_B}$ with $\lambda_B = 0.35^{+0.15}_{-0.10}$ GeV]

Charmless non-leptonic decays $B \to \pi\pi(\pi K)$

0^{th} approximation: Naive factorization (form factor $\otimes$ decay constant)

$$\mathcal{A}(B \to M_1 M_2) \;\propto\; F_{B\to M_1}(q^2 = M_2^2) \cdot f_{M_2}$$

- leading spectator scattering corrections require λ_B and $\langle u^{-1}\rangle_{\pi(K)}$



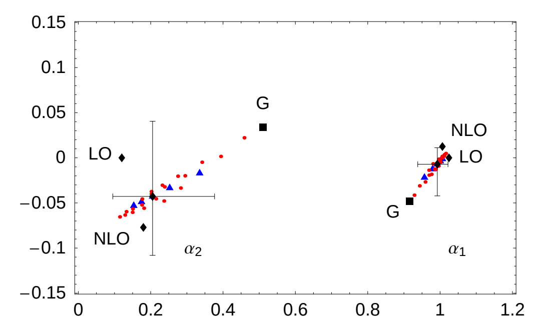
[BBNS 1999]

- Again, higher-order spectator effects sensitive to shape of $\phi_+^B(\omega)$

(non-factorizable) power corrections phenomenologically important !

Example: Tree-amplitudes $\alpha_{1,2}$ in QCDF for $B \to \pi\pi$

[from Beneke/Jäger, hep-ph/0512351]



- ■ **G:** good description of data
- ♦ QCDF
  - + NNLO result + hadr. uncertainties
  - ▲ variation of $\lambda_B = 0.2 - 0.5$ GeV
  - • add. variation from $\langle u^{-1} \rangle_\pi \in \{2.7, 3.9\}$

- colour-allowed tree amplitude α_1 close to 1 ($\pm$10%)
- colour-supressed tree amplitude α_2 rather uncertain

More applications

- $B \to K^*\gamma$, $B \to \rho\gamma$, ...

  [Bosch/Buchalla, hep-ph/0106081; Beneke/TF/Seidel, hep-ph/0106067]

  [Ali/Parkhomenko, hep-ph/0105302]

- $B \to K^*\ell^+\ell^-, \ldots$

  [Beneke/TF/Seidel, hep-ph/0412400; Ali/Kramer/Zhu, hep-ph/0601034]

- $B \to \pi\ell\nu\gamma$, ...

  [Cirigliano/Pirjol, hep-ph/0508095]

- ...

B-meson LCDAs in SCET sum rules

[De Fazio/TF/Hurth, hep-ph/0504088]

1. Integrate out hard fluctuations from QCD $\to$ $\mathrm{SCET_I}$
   $\to$ perturbative matching coefficients $C_i(\mu)$
2. Identify factorizable (J_1) and non-factorizable currents (J_0) in SCET

$$J_0 = \bar\xi_{\rm hc}\, h_v\,, \qquad J_1 = \bar\xi_{\rm hc}\, g\not{A}^\perp_{\rm hc}\, h_v$$

3. Study correlation functions with interpolating axial-vector current J_π

$$\Pi_{0,1}(p') = i \int d^4x\, e^{ip'x}\, \langle 0| T\left[J_\pi(x) J_{0,1}(0)\right] |B(v)\rangle$$

Factorization theorem $\quad [(p')^2 \equiv m_B(n_- p') = \mathcal{O}(m_B\Lambda)]$

$$\Pi_0(p',\mu) = \int_0^\infty d\omega\, T_0(n_-p',\omega,\mu)\, \phi_-^B(\omega,\mu) + \alpha_s\, \phi^B_{bqg} + \text{power corrections}$$

$$\Pi_1(p',\mu) = \int_0^\infty d\omega\, T_1(n_-p',\omega,\mu)\, \phi_+^B(\omega,\mu) + \text{power corrections}$$

Factorization Theorem for Correlation functions ($B \to \pi$ form factor)

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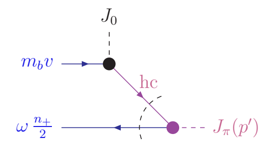
Leading contributions to Π_0 and Π_1

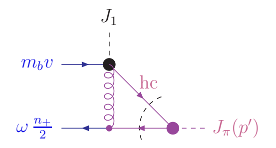
- non-factorizable current:

$$\Pi_0(n_-p') = f_B m_B \int_0^\infty d\omega \, \frac{\phi_-^B(\omega)}{\omega - n_-p' - i\eta}$$



- factorizable current:

$$\Pi_1(n_-p') = -\frac{\alpha_s C_F}{4\pi} f_B m_B^2 \int_0^\infty d\omega \, \frac{\phi_+^B(\omega)}{\omega} \times \ln\left[1 - \frac{\omega}{n_-p' + i\eta}\right]$$



- perturbative corrections can be systematically included
- no large logs if $\mu^2 = \mathcal{O}(p'^2) = \mathcal{O}(\omega\, n_+p') = \mathcal{O}(m_b\Lambda)$ (hard-collinear scale in $\mathrm{SCET_I}$)
- heavy-quark power counting – fixed by SCET Feynman rules

Sum rule for factorizable and non-factorizable currents

Dispersion relations

$$\Pi(n_-p') = \frac{1}{\pi}\int_0^\infty d\omega' \, \frac{\mathrm{Im}\,[\Pi(\omega')]}{\omega' - n_-p' - i\eta}$$

- Borel transformation: $(n_-p') \longrightarrow \omega_M \equiv M^2/m_B$ (Borel parameter)
- Model continuum above threshold $\omega' > \omega_s \equiv s_0/m_B$
- Separation of spectrum into resonance and continuum re-introduces ratio of two distinct scales: $s_0/\mu_{\rm hc}^2$ and $M^2/\mu_{\rm hc}^2$!

Sum rule for factorizable current → form factor corrections

At one-loop:

$$\Delta F_\pi \propto \frac{f_B M^2}{f_\pi} \int_0^\infty \frac{d\omega}{\omega}\, \phi_+^B(\omega,\mu) \left\{ 1 - e^{-\omega_s/\omega_M}\, \theta(\omega-\omega_s) - e^{-\omega/\omega_M}\, \theta(\omega_s-\omega) \right\}$$

- Limit $\omega_s \sim \omega_M \ll \omega \sim \Lambda$ can be performed
- Use leading-order sum rule for f_π

⇒ Reproduces QCDF result for asymptotic value, $\langle u^{-1}\rangle_\pi = 3$ √

Sum rule for non-factorizable current → soft form factor

$$\xi_\pi \propto \frac{f_B}{f_\pi} \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \int_0^\infty d\omega \, t_0(\omega', \omega, \mu) \, \phi_-^B(\omega, \mu)$$

- At tree-level, $t_0(\omega', \omega, \mu) = \delta(\omega - \omega')$
  - limit $\omega_s \sim \omega_M \ll \omega$ can be performed
  - tree-level sum rule sensitive to $\phi_-^B(\omega = 0)$ [see also Khodjamirian/Mannel/Offen 2005]
- Beyond tree-level, $t_0(\omega', \omega, \mu)$ contains large logarithms $\ln^2 \left[\frac{\omega' m_B}{\mu^2}\right]$
  - perturbation theory breaks down for $\omega_s \to 0$
  - for finite (but small) ω_s, strong logarithmic dependence on ω_s
    $=$ remainder of endpoint divergences encountered in QCDF approach!
  - resummation of endpoint logs (?) goes beyond standard factorization theorems for correlation functions in SCET (and also in QCD)

[De Fazio/TF/Hurth, work in progress]

Summary/Outlook

Factorizable contributions to exclusive B-decays:

- dominant hadronic uncertainties from λ_B (and $\langle u^{-1}\rangle_M$)
- *shape* of $\phi_+^B(\omega)$ sub-leading effect

Correlation functions in SCET sum rules:

- also $\phi_-^B(\omega)$ (and 3-particle DAs) occur
- knowledge about $\phi_-^B(\omega)$ currently improving
  (beyond WW approximation, see Nils Offen's talk)
- need evolution equations for ϕ_-^B and ϕ_{bqg} too !

Sum rules for soft $B \to M$ form factors:

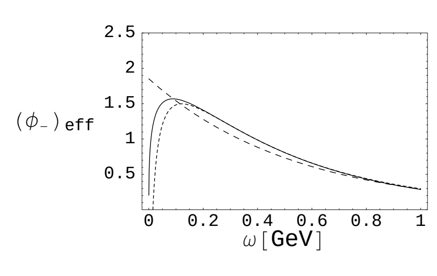
- Enhanced endpoint sensitivity for $s_0/\mu_{\rm hc}^2 \to 0$ beyond tree level !
- Is it possible to resum endpoint logarithms ?
- Is it justified to use "standard" values for s_0 and M^2
  in sum rules for non-factorizable quantities ?!

Backup Slides

Sum rule for ξ_π at NLO – PRELIMINARY ! –

- Absorb NLO kernel into "effective" distribution amplitude, such that

$$\xi_\pi = \frac{f_B\, m_B}{f_\pi\,(n_+p')} \int_0^{\omega_s} d\omega\, e^{-\omega/\omega_M}\, \phi_-^{\rm eff}(\omega, n_+p', \mu)$$



– – Tree level, $\phi_-^B(\omega)$

- - - NLO, fixed order α_s

— ad-hoc resummation of double logs $\ln^2\omega$

- typical value $\omega_s \sim 1\ {\rm GeV}^2/m_B \sim 0.2\ {\rm GeV}$

[from De Fazio/TF/Hurth, work in preparation]