(LCDAs for) η and η' mesons

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Outline

1. Decay constants and η-η' mixing
   - Matrix elements of octet and singlet currents
   - Phenomenological values
2. Sensitivity to LCDAs in 2-photon processes
   - Definitions
   - CLEO measurement of $P\gamma$ form factors
   - Phenomenological constraints on 2-particle DAs
3. More applications
4. Outlook

Decay constants and η-η' mixing

Matrix elements of (local) octet and singlet currents

[Kaiser/Leutwyler, hep-ph/9806336; TF/Kroll/Stech, hep-ph/9802409]

Definition of 4 independent decay constants

$$\langle 0|J^a_{\mu 5}|P(p)\rangle = i\, f^a_P\, p^\mu$$

- $a = 8, 0$ for octet and singlet axial-vector currents
- $P = \eta, \eta'$ for physical meson states
- isospin breaking neglected

Two-angle parametrization

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} \equiv \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_0 \\ \sin\theta_8 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} f_8 \\ f_0 \end{pmatrix}$$

- $SU(3)_F$ limit: $\theta_8 = \theta_0 = 0^\circ$, $f_8 = f_\pi = f_K \neq f_0$

Phenomenological values (I)

- Change to quark-flavor basis $(u\bar{u} + d\bar{d}$ vs. $s\bar{s})$
- Assume *single* mixing angle $\phi_{u,d} = \phi_s = \phi$ (to be justified a-posteriori)

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} \equiv \begin{pmatrix} f_q \cos\phi & -f_s \sin\phi \\ f_q \sin\phi & f_s \cos\phi \end{pmatrix}$$

- Exploit relation between masses and decay constants from $\partial^\mu J^a_{\mu 5}$
- Study various processes with η and η'

Result: [TF/Kroll/Stech, hep-ph/9802409]

$$\phi = 39.3^\circ \pm 1^\circ\,, \quad f_q/f_\pi = 1.07 \pm 0.02\,, \quad f_s/f_\pi = 1.34 \pm 0.06$$

+ systematic effects from $\phi_q \neq \phi_s$ etc.

Central values translated to octet-singlet basis:

$$\theta_8 = -21.2^\circ\,, \quad \theta_0 = -9.2^\circ\,, \quad f_8/f_\pi = 1.26\,, \quad f_0/f_\pi = 1.17\,.$$

- similar/updated analysis in [Escribano/Frere, hep-ph/0501072], but larger values for f_s and f_8 !

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Phenomenological values (II)

- Use two-photon widths, $\Gamma[\eta(\eta') \to \gamma\gamma]$
- Use meson masses, pion and kaon decay constants.
- Use large-N_c relations
- Use lattice value for topological susceptibility

Result:

[Shore, hep-ph/0601051]

$$\theta_8 = -20.1^\circ \pm 0.7^\circ\,, \qquad f_8/f_\pi = 1.13 \pm 0.02\,,$$
$$\theta_0 = -12.3^\circ \pm 3.0^\circ\,, \qquad f_0/f_\pi = 1.15 \pm 0.05\,.$$

Central values translated to quark-flavour basis:

$$\phi_q = 38.4^\circ\,, \quad \phi_s = 38.3^\circ\,, \quad f_q/f_\pi = 1.07\,, \quad f_s/f_\pi = 1.21\,.$$

Main discrepancies for f_8 respectively f_s $\Rightarrow$ increase error for f_s in FKS!

Sensitivity to LCDAs in 2-photon processes

Definitions of two-particle DAs (leading-twist only)

see, for instance, [Kroll/Passek-Kumericki, hep-ph/0210045] and references therein

Light-cone matrix elements: $(n^2 = 0,\ n \cdot p = 1)$

$$\langle P(p)|\bar{q}(\lambda n)\, \not{n}\gamma_5\, \lambda^a\, [\lambda n, 0]\, q(0)|0\rangle = -i\, f_P^a \int_0^1 du\, e^{i\,u\,\lambda}\, \phi_P^a(u,\mu)$$

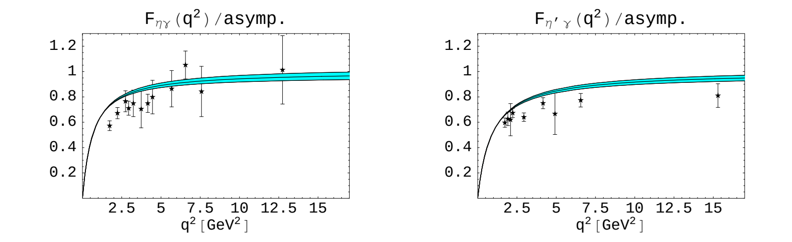
$$n_\mu n^\nu\, \langle P(p)|G^{\mu\alpha}(\lambda n)\, [\lambda n, 0]\, G_{\alpha\nu}(0)|0\rangle = -\mathcal{N}_g\, i\, f_P^0 \int_0^1 du\, e^{i\,u\,\lambda}\, \phi_P^g(u,\mu)$$

Gegenbauer expansion:

$$\phi_P^a(x,\mu) = 6x(1-x)\left[1 + \sum_{n=2,4,\ldots} B_{Pn}^a(\mu)\, C_n^{3/2}(2x-1)\right]$$

$$\phi_P^g(x,\mu) = x^2(1-x)^2 \sum_{n=2,4,\ldots} B_{Pn}^g(\mu)\, C_{n-1}^{5/2}(2x-1)$$

- different conventions for normalization $\mathcal{N}_g$ of 2-gluon DA
- ϕ_P^g and ϕ_P^0 mix under evolution



- Interpolation formula: ($P \to \gamma\gamma$, VDM, pQCD) [TF/Kroll, hep-ph/9805294]

$$F_{P\gamma}(Q^2) = \frac{6 C_q f_P^q}{Q^2 + 4\pi^2 f_q^2} + \frac{6 C_s f_P^s}{Q^2 + 4\pi^2 f_s^2}\,, \qquad C_q = \frac{5}{9\sqrt{2}}\,, \quad C_s = \frac{1}{9}$$

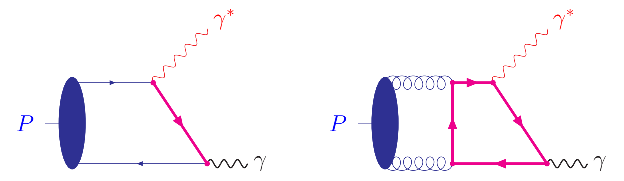
- Varying $f_s = (1.34 \pm 0.12)\, f_\pi$ (FKS, doubled error)

$\to$ pions (not shown), η and η' behave similarly,
i.e. no anomalous behaviour of η' in 2-photon processes

Phenomenological constraints on 2-particle DAs

- $P\gamma$ form factors at large Q^2 to leading-twist accuracy:

$$
\begin{aligned}
F_{P\gamma}(Q^2) &= F^8_{P\gamma}(Q^2) + F^0_{P\gamma}(Q^2) \\
&= T_8^{q\bar{q}}(u,Q^2) \otimes \phi_P^8(u) + T_0^{q\bar{q}}(u,Q^2) \otimes \phi_P^0(u) + T_0^{gg}(u,Q^2) \otimes \phi_P^g(u)
\end{aligned}
$$



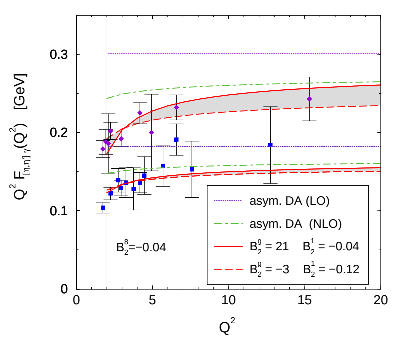
$T_{8,0}^{q\bar{q}} = \mathcal{O}(1)$ and $T_0^{gg} = \mathcal{O}(\alpha_s)$

Fit to CLEO and L3 data (I)

[Kroll/Passek-Kumericki, hep-ph/0210045]

$$B_2^8 = -0.04 \pm 0.04\,, \quad B_2^0 = -0.08 \pm 0.04\,, \quad B_2^g = 9 \pm 12 \qquad \text{(at } \mu_0 = 1 \text{ GeV)}$$

- NLO, incl. mixing between ϕ_P^0 and ϕ_P^g under evolution
- truncate Gegenbauer expansion after $n = 2$
- simplifying asssumption $\phi_\eta^a = \phi_{\eta'}^a \equiv \phi_a$



Fit to CLEO and L3 data (II)

[Agaev/Stefanis, hep-ph/0307087]

$$B_2^q \geq 0.15\,, \quad B_2^g = \in \{16, 20\} \qquad \text{(at } \mu_0 = 1 \text{ GeV)}$$

- incl. (model-dependent) estimates of power corrections (decreases result)

$\Rightarrow$ larger results for Gegenbauer coefficients

- also gives result for $\theta_8 = \theta_0$

More applications

- Electroproduction of η, η' mesons:
  - factorization in terms of proton GPD and meson DAs
  - ϕ_P^g already appears at leading order α_s

[e.g. Huang/Jakob/Kroll/Passek-Kumericki, hep-ph/0309071]

- Non-leptonic $B \to \eta(\eta')$ decays
  - also requires soft (non-factorizable) $B \to \eta(\eta')$ form factors

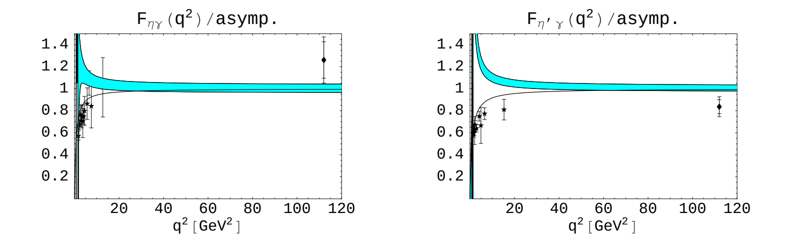
[e.g. Beneke/Neubert, hep-ph/0210085]

- Charmonium decays into η and η'
- …

Outlook

- Update of phenomenological analysis for decay constants ?
  (include new data on *time-like* transition form factors from BaBar ? $\longrightarrow$)
- η-η' mixing and sum rules ?
  (analyses in [Ball/Zwicky, hep-ph/0406232; Ball, hep-ph/9812375] favour $B_2^\eta \simeq 0.1 - 0.2$)
- η-η' mixing and lattice QCD ?
  (for first result, see e.g. [hep-lat/0401005; hep-lat/0111056])
- ...

BaBar measurement of *time-like* $P\gamma$ form factors [hep-ex/0605018]



- experimental result for form-factor ratio at $q^2 = 112$ GeV^2:

$$F_{\eta'\gamma} : F_{\eta\gamma} = 1.10 \pm 0.17$$

- using FKS values (and doubled error for f_s)

$$F_{\eta'\gamma} : F_{\eta\gamma} = 1.65 \pm 0.12$$

- smaller value of mixing angle ϕ ?
- important contribution from ϕ_0^g ?
- problem with factorization for time-like processes ?