Pion distribution amplitude from the lattice

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Pion distribution amplitude

$$\phi(x, Q^2) = \int^Q dk_\perp \Psi(x, k_\perp)$$

- non-perturbative physics - lattice QCD / other
- pion em form factor/factorization
- asymptotic behaviour is known

$$\phi_{\rm as}(x) \stackrel{Q^2 \to \infty}{=} a_0\, x(1-x)$$

$$\phi(x, Q^2) = \phi_{\rm as}(x) \left[1 + \sum_n a_{2n}(Q^2)\, C_{2n}^{3/2}(\xi) \right]$$

equivalently:

$$\langle \xi^n \rangle = \int d\xi\, \xi^n \phi(\xi, Q^2)$$

Lattice observable

OPE relates the moments of the distribution to QCD matrix elements

$$\langle 0|O_{\mu_0\ldots\mu_m}(0)|\pi(p)\rangle = f_\pi\, p_{\mu_0}\ldots p_{\mu_m}\,\langle\xi^m\rangle$$

where

$$O_{\mu_0\ldots\mu_m}(x) = (-i)^n \bar{\psi}(x)\,\gamma_{\mu_0}\gamma_5\ \overleftrightarrow{D}_{\mu_1}\ \ldots\ \overleftrightarrow{D}_{\mu_m}\ \psi(x)$$

can be computed by lattice QCD - first principle calculation
with **controlled** statistical and systematic errors

$$\begin{aligned} C_2^Q(t) &= \sum_{\mathbf{x}} e^{i\mathbf{p}\mathbf{x}}\langle Q(\mathbf{x},t)\,\bar{\psi}\gamma_5\psi(0)\rangle \\ &\stackrel{t\to\infty}{\longrightarrow} \frac{C}{2E}\,\langle 0|Q(0)|\pi(p)\rangle\, e^{-E(\mathbf{p})t}, \end{aligned}$$

the second moment of the PDA on the lattice is then given by:

$$R = \left.\frac{C_2^O(t)}{C_2^A(t)}\right|_{\mathbf{p}=(1,1,0)} = p_1 p_2 \langle\xi^2\rangle^L$$

Lattice observable - 2

related to the continuum $\overline{MS}$ definition:

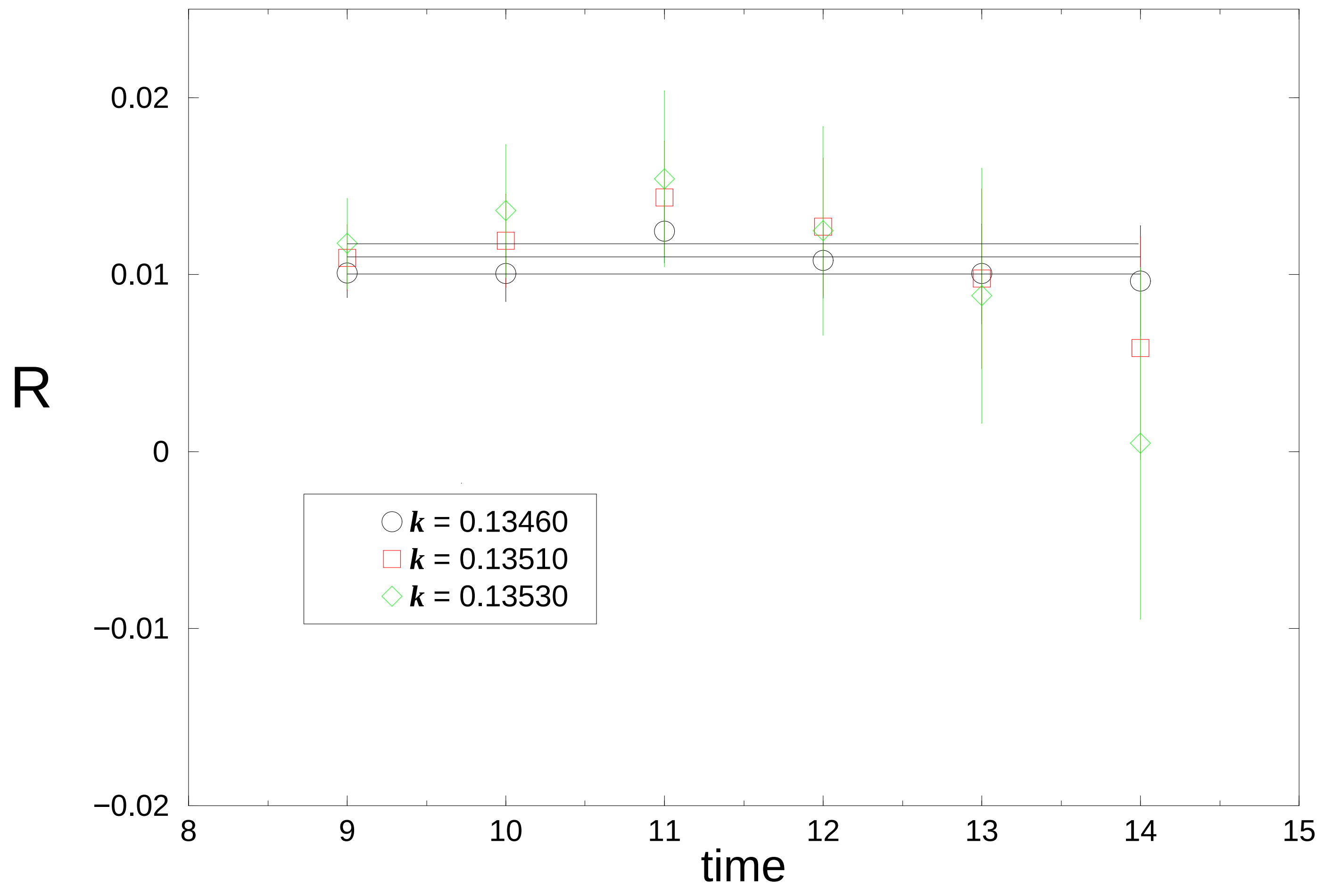
$$\langle\, \xi^2 \rangle^L = \frac{Z_A}{Z_O} \langle\, \xi^2 \rangle^{\overline{MS}}$$

$\hookrightarrow$ O(a)-improved, quenched calculation with 1-loop pert. th. matching

Systematic errors

- dynamical fermions
- light quark mass
- NP renormalization
- finite-volume effects
- continuum limit
- precision (?)

Lattice result



Matching

$$Z_{O,\overline{\mathrm{MS}}} = 1 + \frac{\alpha_s}{4\pi}[\gamma^{(0)} \log\left(\frac{\mu^2}{\lambda^2}\right) + d_{\mathrm{cont}}],$$

$$Z_{O,\mathrm{latt}} = 1 + \frac{\alpha_s}{4\pi}[\gamma^{(0)} \log\left(\frac{1}{a^2\lambda^2}\right) + d_{\mathrm{latt}}],$$

$$Z_M \equiv \left(\frac{Z_{O,\overline{\mathrm{MS}}}}{Z_{O,\mathrm{latt}}}\right)$$

$$= 1 + \frac{\alpha}{4\pi}[\gamma^{(0)} \log(a^2\mu^2) + (d_{\mathrm{cont}} - d_{\mathrm{latt}})],$$

Z_M	α_s			
	0.1255 (boosted)	0.12499 ($q = \pi/a$)	0.14019 ($q = 2/a$)	0.17299 ($q = 1/a$)
Z_1 ($O_{\sigma\mu\nu}$)	1.451	1.449	1.518	1.680

Results at $Q^2 = (2.4\ \mathrm{GeV})^2$

- $a^{-1} = 2.67\ \mathrm{GeV}$
- $m_{\mathrm{PS}} = 490\ \div\ 748\ \mathrm{GeV}$
- $L = 1.7\ \mathrm{fm}$, $ML \simeq 5$
- $\langle \xi^2 \rangle^L = 0.185 \pm 0.032$
- evolution [Bakulev et al 2003]

source	a_2	a_4	$\langle \xi^2 \rangle$	$\langle \xi^4 \rangle$
asympt	0.00	0.00	0.2	0.09
Chernyak:1983	0.44	0.25	0.35	0.21
Braun:1988	0.28	0.13	0.30	0.16
Bakulev:2001	0.14	-0.08	0.25	0.11
Dalley:2001	0.08	0.02	0.23	0.11
Gronberg:1997	0.19	-0.14	0.27	0.11
Bakulev:2002	0.23	-0.22	0.28	0.11
Aitala:2000	0.16	0.02	0.25	0.12
this work			0.27	

Summary and perspectives