B-meson Distribution Amplitude in QCD: Facts and Fancy

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based on

Kawamura, Kodaira, Qiao, Tanaka *Phys. Lett.* **B523** *(2001) 111 [Err.–ibid.* **B536** *(2002) 344]*

Lange, Neubert *Phys. Rev. Lett.* **91** *(2003) 102001*

Braun, Ivanov, Korchemsky *Phys. Rev.* **D69** *(2004) 034014*

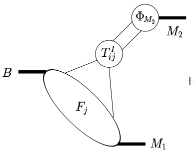
Outline

QCD factorization for hadronic decays

- Heavy quark expansion methods ($m_b \gg \Lambda_{QCD}$)
- Soft-collinear factorization (final state particle energies $\gg \Lambda_{QCD}$)

BBNS approach:

$$\langle M_1 M_2 | O_i | B \rangle = F^{B\to M_1}(0) \int_0^1 du\, T^{(1)}(u) \Phi_{M_2}(u) + \int_0^\infty d\omega \int_0^1 du\, dv\, T^{(2)}(\omega, u, v) \Phi_B(\omega) \Phi_{M_1}(u) \Phi_{M_2}(v)$$



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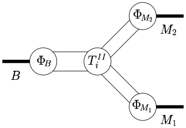
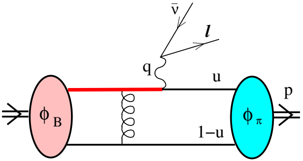


u, v — momentum fractions
ω — light quark energy in B-meson
$\Phi_{M,B}$ — distribution amplitudes

Heavy-to-light form factors

Szczepaniak, Henley, Brodsky '90



$$\sim \int_0^1 du \frac{1}{(1-u)^2}\Phi_\pi(u)$$

- Gluon does not 'want' to be hard
- Naive collinear factorization breaks down

Light Cone Sum Rules

Bagan, Ball, V.B '98

- Additive soft contribution
- Separation scheme-dependent

$$f_+(q^2) = \frac{f_\pi}{f_B^{\rm stat}} \frac{\sqrt{m_B}\omega_0^2}{4E^2}\Big\{\Phi'_\pi(0)\exp\Big[-\frac{\alpha_s C_f}{2\pi}\ln^2\frac{m_B x}{2\omega_0}\Big]$$
$$-\frac{\alpha_s C_f}{2\pi}\Big[\Big(1-\ln\frac{2\omega_0}{\mu}\Big)\int_0^1 du\frac{\Phi_\pi(u)-\bar u\Phi'_\pi(0)}{\bar u^2} + \Big(1-x-\ln\frac{2\omega_0}{\mu}\Big)\int_0^1 du\frac{\Phi_\pi(u)}{\bar u}\Big]\Big\}$$

Here: E is pion energy in B rest frame; $q^2 = m_B^2(1-x)$; $x = 2Em_B$; $\mu = \mu_{\overline{MS}}$
ω_0 is an effective IR cutoff (interval of duality); $\bar u = 1-u$

- Note $f_+ \sim \frac{\sqrt{m_B}}{E^2}$

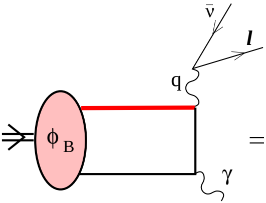
Heavy-to-light form factors — *continued*

Soft-Collinear Effective Theory (SCET) *Beneke, Feldmann '01*

$$f_+(q^2) = C_+(q^2)\zeta_\pi(q^2) + \int_0^\infty d\omega \int_0^1 du\, T_+(q^2;\omega,u)\Phi_B(\omega)\Phi_\pi(u)$$

- C_+ and T_+ are dominated by scales m_B and $\sqrt{m_B\Lambda}$
- The form factor $\zeta_\pi(q^2)$ is universal, i.e. the same for $f_+(q^2)$ and $f_-(q^2)$
- The form factor $\zeta_\pi(q^2)$ is a mixture of hard-collinear, collinear and soft subprocess

Singular case: $B \to \ell\nu\gamma$ *Descotes-Genon, Sachrajda '02–'03*



$$= \int_0^\infty \frac{d\omega}{\omega} T(\omega, E_\gamma;\mu_F)\Phi_B(\omega,\mu_F)$$

- No soft contributions
- Similar to $\pi\gamma^*\gamma$

- This should serve to define and determine B-meson distribution amplitude

B–Meson Distribution Amplitude

Definition

(?)Grozin, Neubert '97

$$\langle 0| \Big[\bar{q}(tn)\, \not{n}[tn,0]\Gamma h_v(0) \Big]_R |\bar{B}(v)\rangle = -\frac{i}{2} f_B^{\rm stat}(\mu)\, {\rm Tr}\left[\gamma_5\, \not{n}\Gamma P_+\right]\ \Phi_B(t,\mu)$$

- v_μ is the heavy quark velocity, n_μ is the light-like vector, $n^2 = 0$, such that $n \cdot v = 1$,
- $P_+ = \frac{1}{2}(1+\not{v})$ is the projector on upper components of the heavy quark spinor
- $|\bar{B}(v)\rangle$ is the $\bar{B}$-meson state in the heavy quark effective theory (HQET)

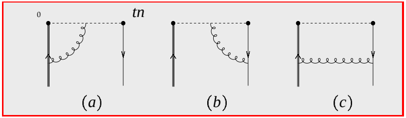
- **All divergences are assumed to be dimensionally regularized**

In momentum space

$$\Phi_B(\omega,\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\, \Phi_B(t-i0,\mu)$$

- ω is the light quark energy in the b–quark rest frame

Renormalization, One loop



$$O_+(t) = \bar{q}(tn)\, \not{n}[tn,0]\Gamma\, h_v(0) \qquad O_+^{\rm ren}(\omega,\mu) = \int d\omega'\, Z_+(\omega,\omega';\mu)\, O_+^{\rm bare}(\omega')$$

in momentum space *Lange, Neubert '03*

$$Z_+(\omega,\omega';\mu) = \delta(\omega-\omega') + \frac{\alpha_s C_F}{4\pi} Z_+^{(1)}(\omega,\omega';\mu) + \ldots$$

$$Z_+^{(1)}(\omega,\omega';\mu) = \left(\frac{4}{\hat\epsilon^2} + \frac{4}{\hat\epsilon}\ln\frac{\mu}{\omega} - \frac{5}{\hat\epsilon}\right)\delta(\omega-\omega') - \frac{4}{\hat\epsilon}\left[\frac{\omega}{\omega'}\frac{\theta(\omega'-\omega)}{\omega'-\omega} + \frac{\theta(\omega-\omega')}{\omega-\omega'}\right]_+$$

in coordinate space *V.B., Ivanov, Korchemsky '04*

$$O_+^{\rm ren}(t,\mu) = O_+^{\rm bare}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat\epsilon^2} + \frac{4}{\hat\epsilon}\ln(it\mu)\right) O_+^{\rm bare}(t) - \frac{4}{\hat\epsilon}\int_0^1 du\, \frac{u}{\bar u}\left[O_+^{\rm bare}(ut) - O_+^{\rm bare}(t)\right] - \frac{1}{\hat\epsilon} O_+^{\rm bare}(t)\right\}$$

- Note $\ln(it\mu)$ which is non-analytic at $t \to 0$
- quasilocality

Renormalization, One loop — *continued*

what is the meaning of $\ln(it\mu)$?

- comes from the term $\sim (\mu t)^\epsilon/\epsilon^2$ in the heavy vertex
- large for $t \gg 1/\mu$ — physical, typical Sudakov double logs
- large for $t \ll 1/\mu$ — artefact, since $t \gg 1/m_b$ make sense

what are the consequences ?

renormalization does not commute with the short-distance expansion

$$[\bar{q}(tn)\,\not{n}\,[tn,0]\Gamma\, h_v(0)]_R \quad \neq \quad \sum_{p=0}^{\infty}\frac{t^p}{p!}[\bar{q}(0)(\overleftarrow{D}\cdot n)^p h_v(0)]_R$$

$\Rightarrow$ $\Phi_B(\omega,\mu)$ develops a radiative tail at large light-quark momenta

$$\Phi_B(\omega,\mu) \sim -\frac{\ln\omega}{\omega}$$

$\Rightarrow$ All integer moments $\int dk\, k^p \phi_+(k,\mu)$ for $p = 0, 1, 2, \ldots$ are not related to matrix elements of local operators and in fact do not exist

is this acceptable?

- unclear, quantity of prime interest is rather

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\Phi_+(\omega,\mu)$$

- can the large-momentum tail be removed by a finite renormalization?

Lange – Neubert Evolution Equation

Conjecture: autonomous evolution *Lange, Neubert '03*

$$\mu\frac{d}{\mu}\Phi_+(\omega,\mu) = -\int_0^\infty d\omega'\gamma_+(\omega,\omega',\mu)\Phi_+(\omega',\mu)$$

$$\gamma_+(\omega,\omega',\mu) = \left[4\ln\frac{\mu}{\omega}-5\right]\delta(\omega'-\omega) - 4\left[\frac{\omega}{\omega'}\frac{\theta(\omega'-\omega)}{\omega'-\omega}+\frac{\theta(\omega-\omega')}{\omega-\omega'}\right]_+$$

For large scales $\mu \sim m_b \to \infty$ obtain asymptotic behavior

$$\Phi_+(\omega,\mu) \sim \begin{cases} \omega; & \text{for } \omega \to 0 \\ \omega^{-1+g}; & \text{for } \omega \to \infty \end{cases} , \quad g = \frac{2C_f}{\beta_0}\ln\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \to \infty$$

- small-momentum behavior $\Phi_+(\omega,\mu) \sim \omega$ is supported
- unphysical rise at large light quark momenta, artefact

- Naive SCET factorization in $B \to \ell\nu\gamma$ is destroyed by the resummation

Equations of Motion

general parametrization compatible with Lorentz and heavy quark symmetry involves two two-particle DAs

$$\langle 0|\bar{q}(z)\Gamma h_v(0)|\bar{B}(v)\rangle = -\frac{i}{2} f_B m_B \,\mathrm{Tr}\left[\gamma_5 \Gamma P_+ \left\{\phi_+(t) - \frac{\not{z}}{2t}[\phi_+(t) - \phi_-(t)]\right\}\right], \quad t = vz$$

and four three-particle DAs *Kawamura et al. '02*

$$\langle 0|\bar{q}(z) g G_{\mu\nu}(uz) z^\nu \Gamma h_v(0)|\bar{B}(v)\rangle = \frac{1}{2} f_B m_B \,\mathrm{Tr}\left[\gamma_5 \Gamma P_+ \left\{(v_\mu \not{z} - t\gamma_\mu)\left(\Psi_A(t,u) - \Psi_V(t,u)\right) - i\sigma_{\mu\nu} z^\nu \Psi_V(t,u) - z_\mu X_A(t,u) + \frac{z_\mu}{t} \not{z} Y_A(t,u)\right\}\right]$$

that are not independent but related by EOM and in particular because of certain exact operator identities

Equations of Motion — *continued*

solving these equations, Kawamura *et al.* obtain in momentum space

$$\omega\frac{d\phi_-(\omega)}{d\omega}+\phi_+(\omega)=I(\omega)$$
$$\left(\omega-2\bar\Lambda\right)\phi_+(\omega)+\omega\phi_-(\omega)=J(\omega)$$

where $I(\omega)$ and $J(\omega)$ are certain integrals of three-particle functions

They define a “WW” approximation by setting $I(\omega)=J(\omega)=0$ and obtain, from the first eq.:

$$\phi_-^{(WW)}(\omega)=\int_\omega^\infty\frac{d\omega'}{\omega'}\phi_+^{(WW)}(\omega)$$

Beneke, Feldmann ‘01

whereas combining the both eqs:

$$\phi_+^{(WW)}(\omega) = \frac{\omega}{2\bar\Lambda^2}\theta(2\bar\Lambda-\omega)$$
$$\phi_-^{(WW)}(\omega) = \frac{2\bar\Lambda-\omega}{2\bar\Lambda^2}\theta(2\bar\Lambda-\omega)$$

adding gluon corrections they reproduce the relations between the moments of $\phi_+(\omega)$ and matrix elements of quark-gluon operators, derived earlier by Grosin and Neubert

Equations of Motion — *continued (2)*

? Results are striking, but are they consistent with the LN autonomous evolution ?

1. **Kawamura *et al.* use a different definition of the DA**
   - In their approach, nonlocal operators are defined as generating functions of local operators, with e.g. power divergences subtracted
   - Accepting this, it remains unclear whether separation of WW contributions is legitimate in the present context
2. **If a Lange–Neubert definition is used, how much of the results remain?**
   - The first equation of Kawamura *et al.* holds true $\rightarrow$ Beneke–Feldmann relation
   - Derivation of the second equation requires going off-light-cone and probably will have to be reconsidered
   - Autonomous evolution could justify the WW approximation
3. Further study needed

QCD Sum Rules: local duality limit

Grosin, Neubert, '97 *V.B., Ivanov, Korchemsky, '04*

To derive the sum rules consider the following correlation function in HQET:

$$i\int d^4x\, e^{-i\omega(vx)}\langle 0|\mathrm{T}\{\bar q(tn)\, \not n\, \Gamma_1[tn,0]h_v(0)\bar h_v(x)\Gamma_2 q(x)\}|0\rangle = -\frac{1}{2}\mathrm{Tr}\left[\not n\, \Gamma_1 P_+\Gamma_2\right] T(t,\omega)$$

$$T(t,\omega) = \frac{1}{2}F^2(\mu)\,\frac{1}{\bar\Lambda-\omega}\int_0^\infty dk\, e^{-ikt}\phi_+(k,\mu) + \ldots$$

- Sorry for the change in notation $\omega \to k$

local duality limit

$$\phi_+(k)^{\mathrm{LD}} = \frac{3}{4\omega_0^3}\theta(2\omega_0 - k)\, k(2\omega_0 - k)$$

- resembles asymptotic pion DA, if rewritten in terms of the scaling variable $\xi = k/(2\omega_0)$
- does not agree with the WW result by Kawamura *et al.*

QCD Sum Rules: radiative corrections

V.B., Ivanov, Korchemsky, '04

- $\mathcal{O}(\alpha_s)$ radiative corrections calculated

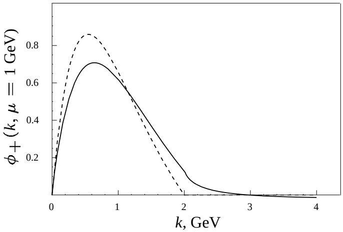


Figure: B-meson distribution amplitude $\phi_+(k, \mu = 1\text{ GeV})$ calculated from the sum rule in QCD perturbation theory to leading order (dashed curves) and next-to-leading order (solid curves) for the continuum threshold ω_0=1 GeV and two values of the Borel parameter $M = 0.3$ GeV (left panel) and $M = 0.6$ GeV (right panel).

as expected

$$\phi_+(k) \sim k \;\text{ for }\; k \to 0\,, \qquad \phi_+(k) \sim -\frac{1}{k}\ln(k/\mu) \;\text{ for }\; k \gg \mu\,,$$

QCD Sum Rules: nonperturbative corrections

V.B., Ivanov, Korchemsky, '04

- Problem: OPE produces an expansion in derivatives of $\delta(k)$

$$T^{\langle\bar{q}q\rangle}(t,\omega) = \frac{\langle\bar{q}q\rangle}{2\omega} \qquad \phi_+(k) \sim \ldots - \frac{1}{2}\langle\bar{q}q\rangle\delta(k)$$

- Write SR for the DA as a function of imaginary light-cone distance $t = -i\tau$
- Resum large logarithms — Sudakov factor

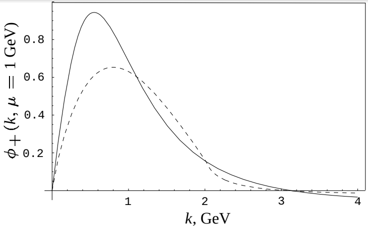
$$S(\tau,M,\mu) = \exp\left\{-\frac{\alpha_s C_F}{2\pi}\left[\ln^2(\tau\mu e^{\gamma_E}) + \frac{5\pi^2}{24} - 1 - \ln\frac{\mu e^{\gamma_E}}{2M} + \mathrm{Li}_2(-2\tau M)\right]\right\}$$

"improved" sum rule:

$$\begin{aligned}\frac{1}{2}F^2(\mu)e^{-\bar{\Lambda}/M}\varphi_+(\tau,\mu) &= \int_0^{\omega_0} ds\, e^{-s/M}\rho_{\rm pert}(s,\tau,\mu) - \frac{1}{2}\langle\bar{q}q\rangle\, S(\tau,M,\mu)\left\{1 + \frac{\alpha_s C_F}{2\pi}\left[2 - \ln(\tau\mu e^{\gamma_E})\right.\right.\\ &\left.\left. - \ln\frac{\mu e^{\gamma_E}}{2M} - \ln(1+2\tau M)\right] + \frac{1}{48}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{M\tau^2}{(1+2\tau M)^2} - \frac{1}{16}\frac{m_0^2}{M^2}(1+2\tau M)\right\}\end{aligned}$$

- not enough, introduce nonlocal condensate

QCD Sum Rules: Results



parameters:

$$\lambda_B(\mu) = \int_0^\infty \frac{dk}{k}\,\phi_+(k,\mu)$$

$$\sigma_B(\mu) = \lambda_B(\mu)\int_0^\infty \frac{dk}{k}\,\ln\frac{\mu}{k}\,\phi_+(k,\mu)$$

simple parametrization:

$$\phi_+(k, \mu = 1\text{ GeV}) = \frac{4\lambda_B^{-1}}{\pi}\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B - 1)}{\pi^2}\ln k\right]$$

$$\lambda_B^{-1}(\mu = 1\text{ GeV}) = 2.15 \pm 0.5\text{ GeV}^{-1} \qquad \lambda_B(\mu = 1\text{ GeV}) = 460 \pm 110\text{ MeV}$$

$$\sigma_B(\mu = 1\text{ GeV}) = 1.4 \pm 0.4$$

compare

λ_B	$=$	350 ± 150 MeV	*BBNS, DS′03*
λ_B	$=$	$\bar\Lambda \sim 400 - 500$ MeV	*KKQT′01*
λ_B	$=$	473 MeV	*KLS′01*
λ_B	$\simeq$	600 MeV	*BK′03*

to summarize:

Thank you for attention!