Higher Twist Meson DAs

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DA06, Durham Sep 30 06

Why Higher Twist?

- power corrections to various processes: phenomenologically relevant for small and moderate momentum transfer/mass scales
  - QCD sum rules on the light-cone for B and D decays
  - some power corrections in QCD factorisation of non-leptonic and radiative B decays
  - EM π form factor, $\pi\gamma$ transition form factor etc.
- leading order corrections in special cases, e.g. CP asymmetry in $B \to K^*\gamma$
- complete results available for twist 3 & 4 of light (pseudoscalar and vector) mesons

What DAs?

Two-particle DAs:

$$\langle 0|\bar{\psi}_2(x)[x,-x]\Gamma\psi_1(-x)|M\rangle \sim \int_0^1 du e^{ipx(2u-1)}\phi(u)$$

Pseudoscalar mesons:

Twist	$(\mu\nu)$	$\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi$	(μ)	$\bar{\psi}\gamma_\mu\gamma_5\psi$	$\bar{\psi}\gamma_5\psi$
3	$\cdot\star$	ϕ_σ			ϕ_p
4			$\cdot\ x^2$	$\mathbb{A}$	

For vector mesons, there are more DAs, reflecting the two possible polarisations.

What DAs?

3-particle DAs:

$$\langle 0|\bar\psi_2(x)[x,ux]\Gamma G(ux)[ux,-x]\psi_1(-x)|M\rangle \sim \int \prod_{i=1}^{3} d\alpha_i \delta(1-\sum\alpha_1) e^{-ipx(-\alpha_1+\alpha_2+u\alpha_3)} \mathcal{T}(\alpha_i)$$

Pseudoscalar mesons:

Twist	$(\mu\nu\alpha\beta)$	$\bar\psi\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta}\psi$	$(\mu\alpha\beta)$	$\bar\psi\gamma_\mu\gamma_5 G_{\alpha\beta}\psi$	$\bar\psi\gamma_\mu\widetilde{G}_{\alpha\beta}\psi$
3	$\cdot\perp\cdot\perp$	$\mathcal{T}$			
4			$\cdot\cdot *$	$\mathcal{A}_\parallel$	$\mathcal{V}_\parallel$
			$\perp\perp\cdot$	$\mathcal{A}_\perp$	$\mathcal{V}_\perp$

Vector mesons:

Twist	$(\mu\nu\alpha)$	$\bar\psi\widetilde{G}_{\mu\nu}\gamma_\alpha\gamma_5\psi$	$\bar\psi G_{\mu\nu}\gamma_\alpha\psi$	$(\mu\nu\alpha\beta)$	$\bar\psi G_{\mu\nu}\sigma_{\alpha\beta}\psi$	$(\mu\nu)$	$\bar\psi G_{\mu\nu}\psi$	$\bar\psi\widetilde{G}_{\mu\nu}\gamma_5\psi$
3	$\cdot\perp\cdot$	$\mathcal{A}$	$\mathcal{V}$	$\cdot\perp\cdot\perp$	$\mathcal{T}$			
4	$\cdot\perp\perp$	$\widetilde{\Phi}$	Φ	$\perp\perp\cdot\perp$	$T_1^{(4)}$	$\cdot\perp$	S	$\widetilde{S}$
	$\cdot * \cdot$	$\widetilde{\Psi}$	Ψ	$\cdot\perp\perp\perp$	$T_2^{(4)}$			
				$\cdot * \cdot\perp$	$T_3^{(4)}$			
				$\cdot\perp\cdot *$	$T_4^{(4)}$			

A Headcount (or Bodycount?)

Pseudoscalars:

T3: 2 two-particle, 1 three-particle DA

T4: 2 two-particle, 5 three-particle DA

Vectors:

T3: 4 two-particle, 3 three-particle DA

T4: 4 two-particle, 12 three-particle DA

A Headcount (or Bodycount?)

Pseudoscalars:

T3: 2 two-particle, 1 three-particle DA

T4: 2 two-particle, 5 three-particle DA

Vectors:

T3: 4 two-particle, 3 three-particle DA

T4: 4 two-particle, 12 three-particle DA

Fortunately the number of independent DAs is smaller!

Reduce DOF by applying exact operator relations

$\rightsquigarrow$ integral relations for DAs.

The Tools: Conformal Expansion and Exact Operator Relations

- step 1: operator relation, follows from QCD equations of motion:

$$\partial_\mu\{\bar q(x)\gamma_\mu\gamma_5 s(-x)\} = -i\int_{-1}^{1} dv\, \bar q(x) x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu\gamma_5 s(-x) + (m_s+m_q)\bar q(x) i\gamma_5 s(-x)$$

- step 2: take matrix elements $\rightsquigarrow$ relation between DAs:

$$\psi_{K;4}(u) = m_K^2\{2\phi^p_{3;K}(u) - \phi_{2;K}(u)\} + \frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar u} d\alpha_2\, \frac{2\left(\Phi_{4;K}(\underline{\alpha}) - 2\Psi_{4;K}(\underline{\alpha})\right)}{1-\alpha_1-\alpha_2}$$

- step 3: replace DAs on r.h.s. by their conformal expansion:

$$\begin{aligned}
\psi_{4;K}(u) &= \psi^{T4}_{4;K}(u) + \psi^{WW}_{4;K}(u)\\
\psi^{T4}_{4;K}(u) &= \frac{20}{3}\,\delta^2_K C_2^{1/2}(2u-1) + 5\left\{5\theta_1^K - \theta_2^K\right\} C_3^{1/2}(2u-1)\,,\\
\psi^{WW}_{4;K}(u) &= m_K^2\left\{1 + 6\rho_+^K(1+6a_2^K) - 18\rho_-^K a_1^K\right\} C_0^{1/2}(2u-1)\\
&\quad + m_K^2\left\{-12\kappa_{4K} - \frac{9}{5}\,a_1^K + 27\rho_+^K a_1^K - 3\rho_-^K(1+18a_2^K)\right\} C_1^{1/2}(2u-1)\\
&\quad + \left\{m_K^2\left(1 + \frac{18}{7}\,a_2^K + 30\rho_+^K a_2^K - 6\rho_-^K a_1^K\right) + 60\,\frac{f_{3K}}{f_K}\,(m_s+m_q)\right\} C_2^{1/2}(2\ldots\\
&\quad + \left\{m_K^2\left(\frac{9}{\ldots}\,a_1^K + \frac{16}{\ldots}\,\kappa_{4K} - 9\rho_-^K a_2^K\right) + 20\,\frac{f_{3K}}{\ldots}\,(m_s+m_q)\lambda_{3K}\right\} C_3^{1/2}(2u\ldots
\end{aligned}$$

The Tools: Conformal Expansion and Exact Operator Relations

- conformal expansion for T2 DAs:

$$\phi(u,\mu^2) = 6u(1-u)\left(1+\sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)\right) :$$

partial wave expansion in conformal spin $\equiv$ conformal expansion

- symmetry group of massless QCD:

$SL(2,R)$, collinear subgroup of the conformal group

symmetry anomalous, broken at NLO

fields with fixed Lorentz-spin projection on the light-cone have definite conformal spin $j = 1/2(l+s)$

$\rightarrow$ good quantum number in LO QCD

l = canonical mass-dimension, s = Lorentz-spin projection

The Tools: Conformal Expansion and Exact Operator Relations

- asymptotic DA of m-particle state with lowest possible conformal spin:

$$\phi_{as}(\alpha_1, \alpha_2, \ldots, \alpha_m) \propto \alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\ldots\alpha_m^{2j_m-1}$$

  Higher waves given by polynomials in m variables α_i ($\sum_i \alpha_i = 1$), which are orthogonal over weight function ϕ_{as}

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Where do we stand now?

- π and η_8 two- and three-particle DAs known to T4
  (Braun/Filyanov 90, Ball 98)
- K DAs known to T4 (Ball/Braun/Lenz 06)
- η' (i.e. η_0) (QCD anomaly) (??)
- ρ and ϕ DAs known to T4 (Ball/Braun/Koike/Tanaka 98, Ball/Braun 98)
- K^* DAs to T4 in preparation (Ball/Braun/Jones/Lenz 06)
- 2-particle T3 DAs of $K_1(1^+)$ mesons (K.C. Yang 05/06)
- $K^{(*)}$ more complicated than π, ρ etc. because of G-odd (antisymmetric) contributions.
- to NLO in conformal expansion, need 6 hadronic T3/4 parameters for K DAs
- ditto 16 T3/4 parameters for K^*
- all hadronic parameters determined from QCD sum rules
  (admittedly not their fi nest hour. . .)

Applications

CP violation in $B \to K^*\gamma$ ($B_s \to \phi\gamma$)

(Atwood/Gronau/Soni 97, Grinstein/Pirjol 05, Ball/Zwicky 06)

- $b \to s\gamma$ is actually either $b_R \to s_L\gamma_L$ (with, in the SM, a helicity factor m_b) or $b_L \to s_R\gamma_R$ (with, in the SM, a helicity factor m_s): γ dominantly left-polarised, γ_R suppressed by m_s/m_b
- entails a small time-dependent CP asymmetry (interference of γ_L/γ_R amplitudes):

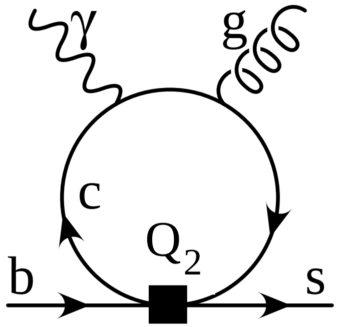
$$A_{\rm CP} = \frac{\Gamma(\bar{B}^0(t) \to \bar{K}^{*0}\gamma) - \Gamma(B^0(t) \to K^{*0}\gamma)}{\Gamma(\bar{B}^0(t) \to \bar{K}^{*0}\gamma) + \Gamma(B^0(t) \to K^{*0}\gamma)}$$
$$\approx -2\frac{m_s}{m_b}\sin(2\beta)\sin(\Delta m_B t) \approx -2\% \cdot \sin(\Delta m_B t)$$

- helicity suppression removed by new physics if spin flip can occur on virtual line (e.g. left-right symmetric model, MSSM): factor $m_{\rm virtual}/m_b$ instead of m_s/m_b
- this is a so-called (quasi) null test of the SM!

CP Asymmetry in $B_d \to K^*\gamma$ ($B_s \to \phi\gamma$)

Caveat emptor! No helicity suppression in 3-parton process $b \to s\gamma g$.

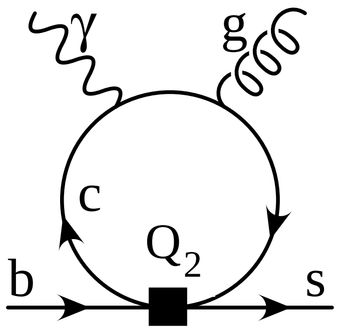
Contributes to $B \to K^*\gamma$
if B or K^* is in three-particle qqg state!



Estimated to increase A_{CP} to $\sim 10\%$
(Grinstein/Pirjol 05, dimensional estimate in SCET).

Can one do better?

CP violation in $B \to K^*\gamma$ ($B_s \to \phi\gamma$)



- calculate effective operator for soft-gluon emission in $1/m_c$ expansion
- calculate relevant hadronix matrix element from QCD sum rules on the light-cone
- LCSR contains T3 & 4 three-particle DAs
- result: $A_{\rm CP} = -(2 \pm 2)\% \sin(\Delta m_B t)$ (Ball/Zwicky 06)
- CP asymmetries in $B_d \to K^*\gamma$ and $B_s \to \phi\gamma$ remain near-perfect null test of SM!

Exact Relations between T2 and T4 MEs

(Braun/Lenz 04, Ball/Zwicky 06)

Define T4 MEs κ_4 (G-odd):

$$\begin{aligned}
\langle 0|\bar{q}(gG_{\alpha\mu})i\gamma^\mu\gamma_5 s|K(q)\rangle &= iq_\alpha f_K m_K^2 \kappa_4(K)\,,\\
\langle 0|\bar{q}(gG_{\alpha\mu})i\gamma^\mu s|K^*(q,\lambda)\rangle &= e_\alpha^{(\lambda)} f_K^\parallel m_{K^*}^3 \kappa_4^\parallel(K^*)\,,\\
\langle 0|\bar{q}(gG_\alpha{}^\mu)\sigma_{\beta\mu} s|K^*(q,\lambda)\rangle &= f_K^\perp m_{K^*}^2 \left\{\kappa_4^\perp(K^*)(e_\alpha^{(\lambda)} q_\beta - e_\beta^{(\lambda)} q_\alpha) + \dots\right\}.
\end{aligned}$$

κ_4 related to T2 ME a_1:

$$\begin{aligned}
\frac{9}{5}a_1(K) &= -\frac{m_s-m_q}{m_s+m_q} + 4\,\frac{m_s^2-m_q^2}{m_K^2} - 8\kappa_4(K),\\
\frac{3}{5}a_1^\parallel(K^*) &= -\frac{f_K^\perp}{f_K^\parallel}\,\frac{m_s-m_q}{m_{K^*}} + 2\,\frac{m_s^2-m_q^2}{m_{K^*}^2} - 4\kappa_4^\parallel(K^*),\\
\frac{3}{5}a_1^\perp(K^*) &= -\frac{f_K^\parallel}{f_K^\perp}\,\frac{m_s-m_q}{2m_{K^*}} + \frac{3}{2}\,\frac{m_s^2-m_q^2}{m_{K^*}^2} + 6\kappa_4^\perp(K^*).
\end{aligned}$$

Exact Relations between T2 and T4 MEs

$$\frac{9}{5}a_1(K) \quad = \quad -\frac{m_s - m_q}{m_s + m_q} + 4\,\frac{m_s^2 - m_q^2}{m_K^2} - 8\kappa_4(K)$$

implies, for $m_q \to 0$: $\kappa_4(K) = -\frac{1}{8} + O(m_s)$ (Braun/Lenz 04)

These relations can be used to either constrain a_1 or κ_4 or to check the consistency of an independent calculation of either.

Where do they come from?

local expansion of non-local operator relations

$\leftrightarrow$ boundary conditions for differential equations for DAs

$\leftrightarrow$ conditions for 1st moment of T4 DAs

What the Future holds. . .

- finish calculation of K^* DAs (Ball/Braun/Jones/Lenz): Xmas present?
- find more interesting applications like $B \to K^*\gamma$?
- look into η'?
  - $J/\psi \to \eta^{(\prime)}\gamma$ sensitive to $\langle 0|G\widetilde{G}|\eta^{(\prime)}\rangle$: T4 (Ball/Frere/Tytgat 95)
  - systematic approach? Other channels?
- get better control over hadronic parameters from lattice?

Backup Slides

G Parity

G parity is the generalisation of C parity to multiplets of particles.
C parity applies only to neutral systems; in the pion triplet, only π^0 has a definite C parity. Generalise to G parity such that

$$\mathcal{G}\begin{pmatrix}\pi^+\\ \pi^0\\ \pi^-\end{pmatrix} = \eta_G \begin{pmatrix}\pi^+\\ \pi^0\\ \pi^-\end{pmatrix}$$

Since G parity is applied to a whole multiplet, charge conjugation has to see the multiplet as a neutral entity. Thus, only multiplets with an average charge of 0 will be eigenstates of G.

G is conserved by strong interactions. One has $G(\pi) = +1$, $G(\rho) = -1$.

G Parity of Operators

$\bar{q}\gamma_\mu G_{\alpha\beta} q$: $G(\bar{q}\gamma_\mu q) = -1$, $G(G_{\alpha\beta}) = -1$ $\Rightarrow$ G-even

$\bar{q}\gamma_\mu\gamma_5 G_{\alpha\beta} q$: $G(\bar{q}\gamma_\mu\gamma_5 q) = +1$, $G(G_{\alpha\beta}) = -1$ $\Rightarrow$ G-odd