Models for Leading Twist DAs

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What do we know about T2 DAs?

Standard procedure: construct model for ϕ as truncated conformal expansion:

$$\phi(u,\mu^2) \approx 6u(1-u)\left(1+\sum_1^{n_{\max}} a_n(\mu^2) C_n^{3/2}(2u-1)\right)$$

- for π, η_8, ρ, ω, ϕ: odd a_i vanish due to G-parity
- $n_{\max}$ typically 2 or 4
- model closed under renormalisation, obtain correct limit for $\mu \to \infty$, i.e. asymptotic DA $6u(1-u)$
- constrain $a_{2,4}^{\pi}$ from experimental data for π EM and $\gamma\gamma^* \to \pi$ (see talk by N. Stefanis)
- or calculate $a_{1,2}^{\pi,K,\ldots}$ from non-perturbative methods (QCD sum rules/lattice) (talks Thu afternoon)

Why can one truncate?

Actual observable is convolution integral

$$\int_0^1 du\, \phi(u) T(u)$$

with pertubatively calculable amplitude $T(u)$:

contributions from large-order a_n "washed out" for "smooth" T due to oscillatory behaviour of $C_n^{3/2}$:

(a) $\int du\, \phi(u)\, \sqrt{u} \sim \sum \frac{1}{n^3}\, a_n$: strong suppression of a_n

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(b) $\int du\, \phi(u) \ln u \sim \sum \frac{1}{n^2} a_n$: strong suppression of a_n

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(c) $\int du\, \phi(u)\, \frac{1}{u} \sim \sum (-1)^n a_n$: no suppression!

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Truncated conformal expansion OK for $T \sim (\sqrt{u}, \ln u)$, but not necessarily for $1/u$! (And certainly not for $1/u^2$.)

Models beyond Conformal Expansion

Define

$$\Delta \equiv \frac{1}{6}\int du \frac{\phi(u)+\phi(1-u)}{u} = \sum_n a_{2n}\,;$$

"worst case scenario" with no suppression of higher a_n

- features in non-leptonic B decays and $\gamma\gamma^* \to \pi$

Idea (Ball/Talbot 05): define model for ϕ in terms of

1. $\Delta \equiv \sum_{\rm even} a_n$ (constrained to be 1.2 ± 0.2 at $\mu = 1.2$ GeV, for π from $\gamma\gamma^* \to \pi$)
2. power-like fall-off behaviour of a_n in n ($a > 1, b > 0$ arbitrary):

$$a_n = \frac{1}{(n/b+1)^a} \quad \text{or} \quad a_n = \frac{(-1)^{n/2}}{(n/b+1)^a}$$

Models beyond Conformal Expansion

Series in Gegenbauers can be summed explicitly:

$$\phi^{\pm}_{a,b}(\Delta) = 6u\bar{u} + \frac{\Delta - 1}{\Delta^{\pm}_{a,b} - 1}\left(\tilde{\phi}^{\pm}_{a,b}(u) - 6u\bar{u}\right), \quad \text{valid for } a > 1 \text{ and } b > 0$$

with

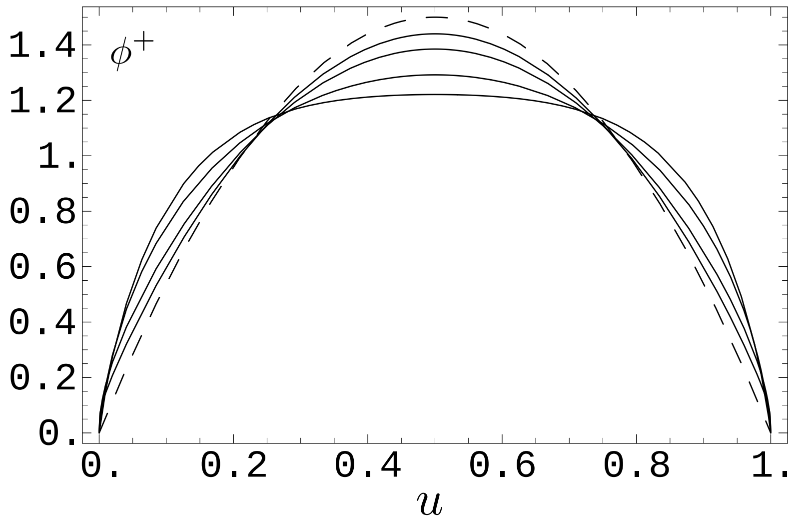
$$\tilde{\phi}^{+}_{a,b}(u) = \frac{3u\bar{u}}{\Gamma(a)} \int_0^1 dt(-\ln t)^{a-1} \left(f(2u-1, t^{1/b}) + f(2u-1, -t^{1/b})\right)$$

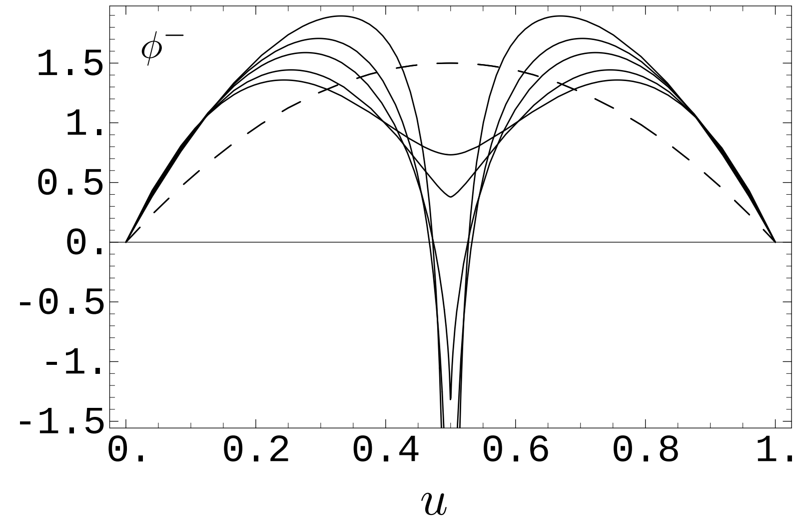
f: generating function of Gegenbauer polynomials:

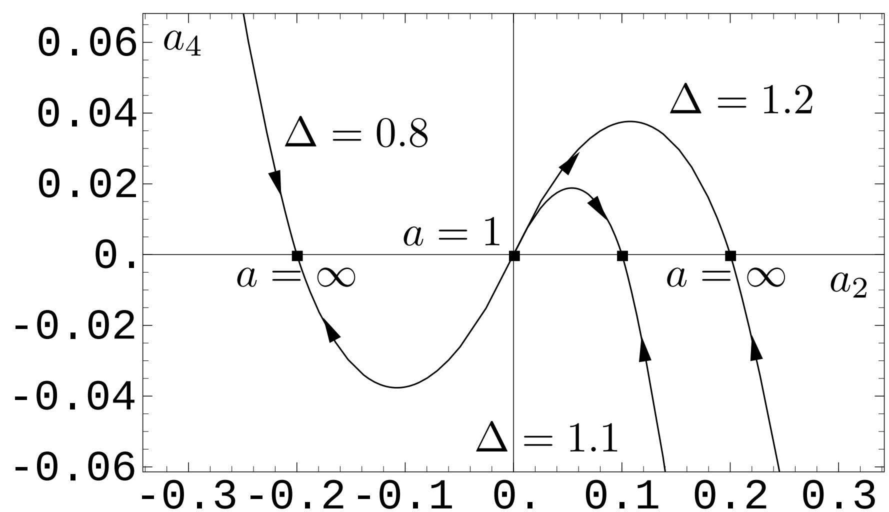
$$f(\xi, t) = \frac{1}{(1 - 2\xi t + t^2)^{3/2}} = \sum_{n=0}^{\infty} C_n^{3/2}(\xi) t^n ,$$

$$\Delta^{+}_{a,b} = \left(\frac{b}{2}\right)^a \zeta(a, b/2), \quad \Delta^{-}_{a,b} = \left(\frac{b}{4}\right)^a \{\zeta(a, b/4) - \zeta(a, 1/2 + b/4)\}$$

Models beyond Conformal Expansion





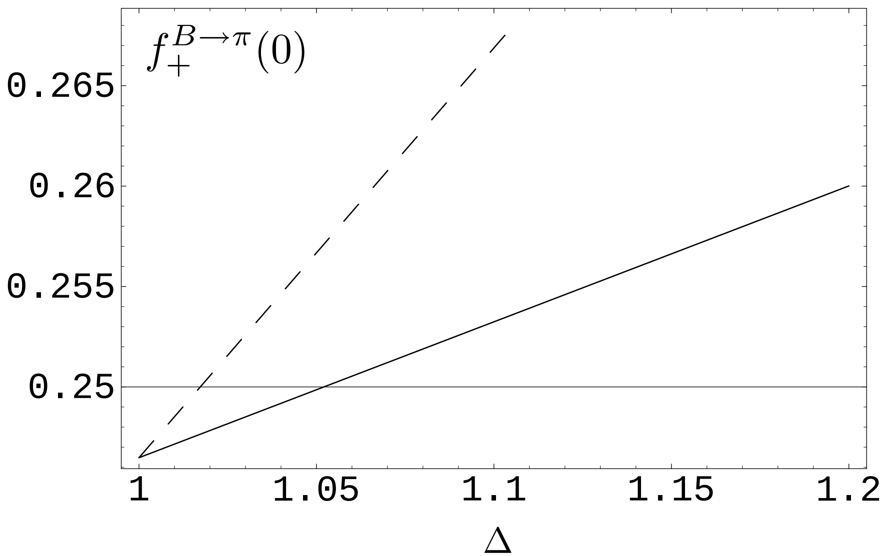


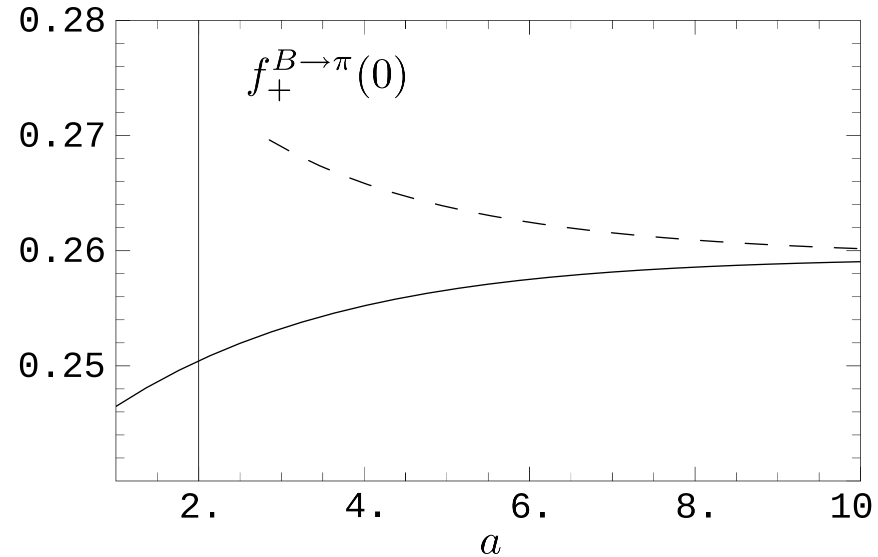
(Ball/Talbot 05)

How does it work?

- similar to parametrisation of parton distribution functions in DIS:
  - choose suitable model defined at low scale in terms of a few parameters
  - use RG scaling to get PDF at higher scales
  - fit model parameters to experimental results
- evolution equation: $\mu^2 \frac{\partial}{\partial \mu^2} \phi(u,\mu^2) = \int_0^1 dv\, V(u,v,\mu^2)\, \phi(v,\mu^2)$;
  kernel V known to 2-loop accuracy for $\pi, \rho_{||}$ (Mikhailov/Radyushkin 85) and 1-loop accuracy for $\rho_\perp$ (Ball/Talbot 05)
- relation between (a_2, a_4) and (Δ, a) is one-to-one (at a fixed scale μ) (see p.5): difference in resulting observables/convolutions is, if nothing more, estimate of truncation error of conformal expansion
- experimental data: far fewer than in DIS...

Application: $f_+^{B\to\pi}(0)$ from LCSRs





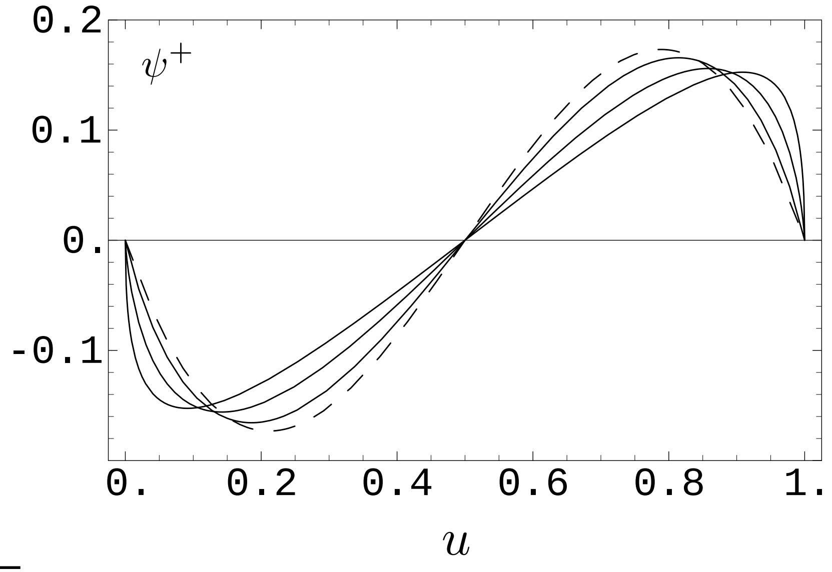
Solid: positive sign a_n

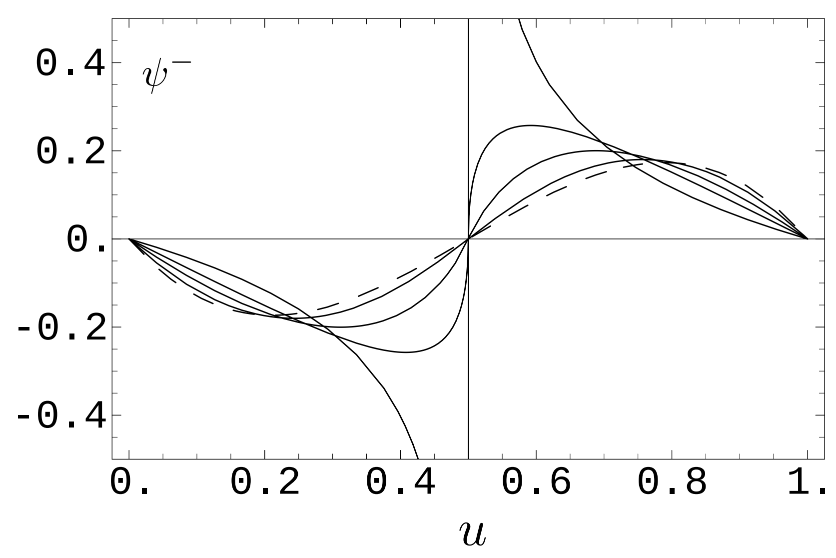
Dashed: alternating sign a_n

Models for odd a_n, normalised to a_1

$$\psi_c^{\pm}(u) = a_1 \left(\frac{3}{2}\right)^c \tilde{\psi}_c^{\pm}(u)$$ (c is power of fall-off)

with
$$\tilde{\psi}_c^{+}(u) = \frac{3u\bar{u}}{\Gamma(c)} \int_0^1 dt(-\ln t)^{c-1} \left(f(2u-1,\sqrt{t}) - f(2u-1,-\sqrt{t})\right)$$
$$\tilde{\psi}_c^{-}(u) = \frac{3u\bar{u}}{i\Gamma(c)} \int_0^1 dt(-\ln t)^{c-1} \left(f(2u-1,i\sqrt{t}) - f(2u-1,-i\sqrt{t})\right)$$





Summary & Conclusions

- alternative to truncated conformal expansion based on 2 parameters for symmetric part of $\phi(u)$:
  - first inverse moment
  - power of fall-off of a_{2n}
- and two parameters for antisymmetric part of $\phi(u)$:
  - a_1
  - power of fall-off of a_{2n+1}
- closed form at fixed low-energy scale, use evolution equation to go to higher scales
- deliverables: fast and safe numerical code for solution of evolution equation
- desirables 1: two-loop evolution kernel for $\rho_\perp$
- desirables 2: fix parameters from fits to experimental data