Studies of final-state photon radiation in Drell-Yan W boson production

Catherine Bernaciak[1] and Doreen Wackeroth[1]
State University of New York, University at Buffalo, Buffalo, NY USA

Abstract

We study the effects of multiple soft, collinear photon radiation off a final state lepton in the process of Drell-Yan W boson production as implemented in the Monte Carlo program WGRAD3. EW next-to-leading (NLO) effects are also included. We have compared our implementation with existing results in the literature and found agreement. Work is currently underway to include parton shower effects using POWHEG and mixed QED + QCD corrections up to NNLO.

Motivation

Motivations for Measurements of EW Observables

- Comparing direct measurements of M_W from LEP2 (W pair production) and the Tevatron (single W production) with indirect measurements from LEP1/SLD can yield signs of BSM physics - so far agreement
- The total W production cross-section is useful for detector calibration
- Precise predictions and measurements of M_W together with m_t constrain the value of M_H
- Anticipating experimental precision of $\delta M_W = 15$ MeV $\rightarrow$ necessary to help reduce systematic error with more precise theoretical tools
  - most recent DZero measurement: $M_W = 80.401 \pm 0.044 GeV$

Motivations for including mFSR in theoretical predictions

- Masses and widths of W and Z extracted from their kinematic distributions depend heavily on EW corrections, specifically:
  - collinear final state photon radiation $\Rightarrow \alpha \log \frac{Q^2}{m_l^2} \Rightarrow$ largest contribution at $\mathcal{O}(\alpha)$
  - WGRAD2[12]includes the complete $\mathcal{O}(\alpha)$ electroweak radiative corrections including one final-state photon. How much can mFSR shift the prediction for M_W?

[1] U.Baur, S.Keller, D.Wackeroth,Phys. Rev. D59, 013002 (1999)
[2] U.Baur, D.Wackeroth, Phys. Rev. D70, 073015 (2004)

Implementing mFSR using QED Structure Function Approach

$$\sigma_{mFSR} = \int_0^1 dz \underbrace{\int dx_1 dx_2 \sum_{x_1,x_2} q(x_1,Q^2)\,\bar{q}(x_2,Q^2)\,\sigma_\circ(x_1x_2S)}_{hadronic\ cross-section} D^{NS}(z,s)\Theta(\text{cuts})$$

- **Non-Singlet** Electron Structure Function - only photon bremstrahlung, no pair production

$$D^{NS}(z,s) = \delta(1-z) + \int_{m_l^2}^{s} \frac{ds'}{s'} \frac{\alpha(s')}{2\pi} \int_z^1 \frac{dy}{y} P(y) D^{NS}(\frac{z}{y}, s')$$

$$P(y) = \frac{1+y^2}{1-y} - \delta(1-y)\int_0^1 dx \frac{1+x^2}{1-x}$$

Altarelli-Parisi (AP) splitting function

- can be solved iteratively - Parton Shower (PS) method - implemented in HORACE[1]
- can be solved using Mellin Transform (MT) techniques
- a combination of the two – more on this later

[1] C.M. Carloni Calame et. al, JHEP 0612 (2006) 016

Mellin Transform Technique - obtain $D^{NS}(z,s)$ to infinite order

$$D^{NS}(z,s) = \delta(1-z) + \int_{m_l^2}^{s} \frac{ds'}{s'} \frac{\alpha(s')}{2\pi} \underbrace{\int_z^1 \frac{dy}{y} P(y) D^{NS}(\frac{z}{y}, s')}$$

Mellin Transform deconvolutes the above convolution

$$\begin{aligned} MT[D^{NS}] \Rightarrow M^{(\zeta)}(s) &= \int_0^1 dz\, z^{\zeta-1} D^{NS}(z,s) \\ &= 1 + \int_{m_l^2}^{s} dt \frac{\alpha(t)}{2\pi t} M^{(\zeta)}(t) P^{(\zeta)} \\ &= exp\left(\tfrac{1}{4}\eta(s) P^{(\zeta)}\right) \end{aligned}$$

$$\text{where } \eta = \int_{m_l^2}^{s} dz \frac{2\alpha(z)}{\pi z} \rightarrow \frac{2\alpha}{\pi} \log \frac{Q^2}{m_l^2}$$

Inverse Mellin Transform to obtain expression for $D^{NS}(z,s)$

$$MT^{-1}[MT[D^{NS}]] \Rightarrow D^{NS}(z,s) = \tfrac{1}{2\pi} exp\left(\tfrac{1}{2}\eta(s)\left(\tfrac{3}{4} - \gamma_E\right)\right)$$
$$\times \int_{-\infty}^{\infty} dt\, x^{-it-c} exp\left(\tfrac{1}{4}\eta(s)(-\Psi(it+c) - \Psi(it+c+2))\right)$$

can only be solved numerically - not useful for calculating cross-sections

Approximate, Finite Order Solutions to $D^{NS}(z,s)$

- Gribov soft photon approximation[1],$z \rightarrow 1$

$$D^{GL}(z,s) = \frac{exp(\frac{1}{2}\eta(s)(\frac{3}{4} - \gamma_E))}{\Gamma(1+\frac{1}{2}\eta(s))} \frac{1}{2}\eta(s)(1-z)^{\frac{\eta(s)}{2}-1}$$

- Exponentiation procedure improves soft, all-order solution with finite order terms
- **Additive Hybrid**[2]

$$\begin{aligned} D^{KF}_{(3)}(z,s) = D^{GL}(z,s) &- \frac{\eta}{4}(1-z) + \frac{\eta^2}{16}\Big(-2(1+z)\ln(1-z) - 2\frac{\ln z}{1-z} + \frac{3}{2}(1+z)\ln z \\ &- \frac{5}{2} - \frac{1}{2}z\Big) + \frac{\eta^3}{8}\Big[-\frac{1}{2}(1+z)\Big(\frac{9}{32} - \frac{\pi^2}{12} + \frac{3}{4}\ln(1-z) + \frac{1}{2}\ln^2(1-z) - \\ &\frac{1}{4}\ln z \ln(1-z) + \frac{1}{16}\ln^2 z - \frac{1}{4}Li_2(1-z)\Big) + \frac{1}{2}\frac{1+z^2}{1-z}\Big(-\frac{3}{8}\ln z + \frac{1}{12}\ln^2 z \\ &- \frac{1}{2}\ln z \ln(1-z)\Big) - \frac{1}{4}(1-z)\Big(\ln(1-z) + \frac{1}{4}\Big) + \frac{1}{32}(5-3z)\ln z\Big] \end{aligned}$$

- **Factorized Hybrid**[3]

$$\begin{aligned} D^{JW}_{(3)}(z,s) = D^{GL}(z,s)\Big[&\frac{1}{2}(1+z^2) + \frac{\eta}{8}\Big(-\frac{1}{2}(1+3z^2)\ln z - (1-z)^2\Big) \\ &\frac{\eta^2}{32}\Big((1-z^2) + \frac{1}{2}(3z^2 - 4z + 1)\ln z + \frac{1}{12}(1+7z^2)\ln^2 z + (1-z^2)Li^2(1-z)\Big)\Big] \end{aligned}$$

- **Both are implemented in WGRAD3**

[1] V.Gribov, L.Lipatov: Sov. J. Nucl. Phys. 15 (1972) 675, 938
[2] E.A. Kuraev, V.S. Fadin: Sov. J. Nucl. Phys. 41 (1985) 466
[3] S. Jadach, B.F.L. Ward: Comp. Phys. Commun. 56 (1990) 351

WGRAD2 $\Longrightarrow$ WGRAD3

- WGRAD2 is a parton level MC program that includes the complete $\mathcal{O}(\alpha)$ electroweak radiative corrections to $p\overset{(-)}{p} \rightarrow W^\pm \rightarrow \ell^\pm \nu$.
- at $\mathcal{O}(\alpha)$ large logs $\left(\sim \alpha \log \frac{Q^2}{m_l^2}\right)$ are present in WGRAD2 - must match to avoid double counting

Matching Procedure

- Extract $\mathcal{O}(\alpha)$, leading-log (LL), term from $D^{NS}(z,Q^2)$
- $D^{NS}_{LL}(z,Q^2) = \lim_{\delta_s \rightarrow 0}\left[\frac{\eta}{4}\frac{1+z^2}{1-z}\Theta(1-z-\delta_s) - \frac{\eta}{2}(\ln \delta_s + \frac{3}{4})\right]$
- subtract this term from the full mFSR cross-section and add the complete $\mathcal{O}(\alpha)$ cross-section

$$\sigma_{\text{WGRAD3}} = \sigma_{\text{WGRAD2}} + \sigma_{\text{mFSR}} - \sigma_{LL} - \sigma_\circ$$

Tuned Comparison

- HORACE and Brensing[1] et.al both contain full $\mathcal{O}(\alpha)$ + mFSR
  - Brensing et.al $\Rightarrow$ additive hybrid QED Structure Function
  - HORACE $\Rightarrow$ PS Approach

Comparison with Brensing et.al - agrees within statistical error

$\sqrt{s}$ =14 TeV	$p_{T,\mu} < 25$ (GeV)		$p_{T,\mu} < 50$ (GeV)	
	WGRAD3	Brensing et.al.	WGRAD3	Brensing et.al
$\sigma_\circ$ (pb)	4495.9(2)	4495.7(2)	27.590(2)	27.589(2)
δ_{mFSR} (%)	0.122(3)	$0.12^{+0.03}_{-0.02}$	0.320(3)	$0.31^{+0.08}_{-0.07}$

- Next: perform χ^2 analysis using M_T and p_T distributions to calculate ΔM_W due to mFSR - a la HORACE[2]
  - HORACE analysis with Run II cuts $\rightarrow \Delta M_W \sim 10$ MeV in muon channel, a few MeV in electron channel

[1] S. Brensing, S. Dittmaier, M. Kramer, A. Muck, Phys.Rev. D77 , 073006 (2008)
[2] C. Calame, G. Montagna, O. Nicrosini, M. Treccani, Phy. Rev. D69, 037301 (2004)

Current Work and Future Plans

Current Work

- implementation of all NLO QCD corrections
  - vertex corrections, self-energies
  - collinear and hard gluon radiation
- incorporation of mFSR (for both final state leptons) to Drell-Yan Z boson production
  - In order to improve the measurement of the W boson mass it is necessary to have precise predictions for Z boson observables.
  - This would be an update to ZGRAD2[1]

Future Work

- In addition to mFSR and QCD NLO corrections, we plan to model initial-state parton shower effects using POWHEG.[2]
- Ultimately include the complete set of mixed QED and QCD corrections up to NNLO
- With WGRAD3 one could then study effects on the W boson mass and other observables due to NNLO mixed QED + QCD corrections, initial-state parton shower and final state multiple, soft photon radiation.

[1] U. Baur, O. Brein,W. Hollik,C. Schappacher, D. Wackeroth,Phys.Rev. D65 (2002) 033007
[2] P.Nason, "A new method for combining NLO QCD with shower Monte Carlo algorithms," JHEP **0411** (2004) 040, [arXiv:hep-ph/0409146].

This work was supported by the NSF through an
LHC Theory Initiative Fellowship