1 Density perturbations from warm inflation

Based on astro-ph/0305015 (LMH Hall, IGM, A Berera)

Accurate treatment of density perturbations allowing for ϕ and T dependent friction terms.

1. improved treatment of the thermal fluctuations
2. numerical evolution of the classical fluctuations
3. peak in the spectrum for 'new inflation'

2 Introduction

In **warm inflation** (1985 and 1995) The inflaton field decays whilst inflating. It produces heat and friction, which prolongs the inflationary phase.

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V' = 0$$

Inflaton	ϕ	Radiation	ρ_r
Expansion	H	Friction	Γ
Potential	$V(\phi)$	Parameter	$r = \Gamma/3H$

Energy conservation requires

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2$$

In the Slow-roll approximation we take

$$3H\dot{\phi} + \Gamma\dot{\phi} + V' = 0$$

$$3H^2 = 8\pi GV$$

$$4H\rho_r = \Gamma\dot{\phi}^2$$

3 Slow-roll parameters

The validity of the slow-roll approximation depends on

$$\varepsilon = (16\pi G)^{-1}\left(\frac{V'}{V}\right)^2$$

$$\eta = (8\pi G)^{-1}\left(\frac{V''}{V}\right)$$

$$\nu = (16\pi G)^{-1}\left(\frac{V'}{V}\frac{\Gamma'}{\Gamma}\right)$$

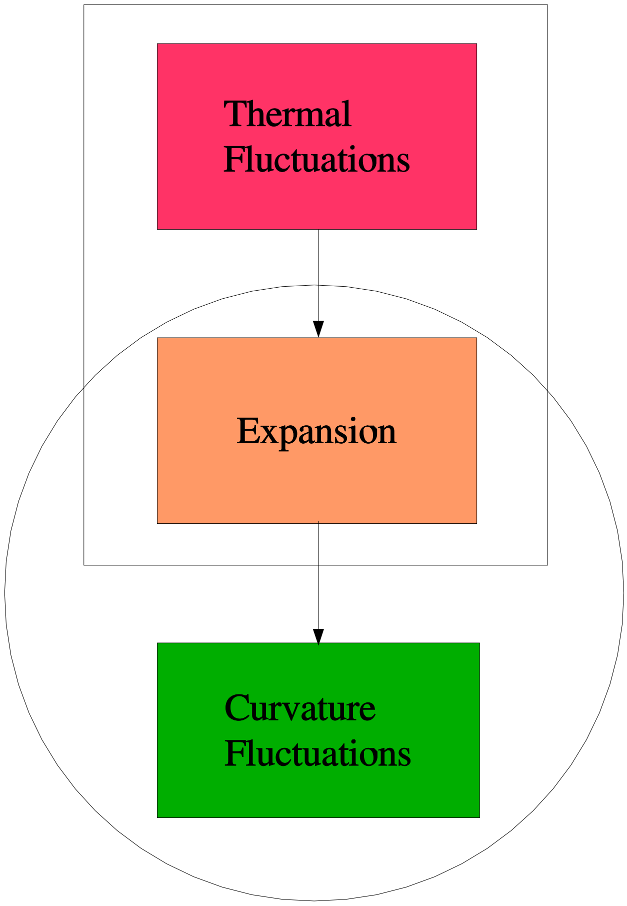
In the regime $r \gg 1$,

$$\begin{aligned}
\frac{1}{H}\frac{d\log H}{dt} &= -\frac{\varepsilon}{r} \\
\frac{1}{H}\frac{d\log\dot{\phi}}{dt} &= -\frac{1}{r}(\eta-\beta)
\end{aligned}$$

Inflation ends when $\epsilon = r$ or $\rho_r = V$.

4 Density fluctuations

Follow a mode with wave vector k through three stages:



5 Quantum fluctuations-homogenous expansion

Classical background value for the inflaton and a heat bath of ψ particles. Interactions $\mathcal{L}_I = \frac{1}{4} g^2 \phi^2 \psi^2$

$$\delta\ddot{\phi}_k + (3H + \Gamma)\delta\dot{\phi}_k + (k^2 e^{-2Ht} + m^2)\delta\phi_k = \xi_k$$

$$\langle \xi_k(t)\xi_{-k'}(t') \rangle = 2\Gamma T (2\pi)^3 \delta(k - k')\delta(t - t')e^{-3Ht}$$

$$\langle \delta\phi_k(t)\delta\phi_{-k'}(t') \rangle = P(2\pi/k)^3 \delta(k - k')$$

Exact solution gives

$$P \to \begin{cases} k e^{-Ht} T & k e^{Ht} \gg k_F \\ k_F T & k e^{Ht} \ll k_F \end{cases}$$

where $k_F = (\Gamma H)^{1/2}$. (Cold inflation $P = k^2 e^{-2Ht} + H^2$)

6 Classical Perturbations

Consider the pertrubed metric in a shear-free gauge

$$ds^2 = -(1+2\varphi)dt^2 + a^2(1-2\varphi)dx^2$$

Perturbed matter fields,

Inflaton	$\delta\phi$
Radiation	$\delta\rho_r$
Velocity	$k^{-1}(p+\rho)v_{,i}$

Curvature perturbation: $\mathcal{R} = \varphi - k^{-1}aHv$
Entropy perturbation: $e = \delta p - c_s^2\delta\rho$

For $k < aH$: The entropy perturbation decays and $\mathcal{R} \to$ const.
For $k > aH$: The perturbation equations couple. (Inflaton equation almost decouples if $\Gamma_{,T} = 0$)

7 Numerical Results

Lisa's programme integrates the cosmological perturbation equations for initial data set by the thermal fluctuations at the freezeout time.

The plots shown here use

$$V = \mu^2\phi^2\left(\log\left(\frac{\phi^2}{\phi_0^2}\right) - 1\right) + V_0$$

$$\begin{aligned} \mu &= 6.71 \times 10^{10} GeV \\ \phi_0 &= 0.2 m_{pl} \\ V_0 &= (4.49 \times 10^{14} GeV)^4 \end{aligned}$$

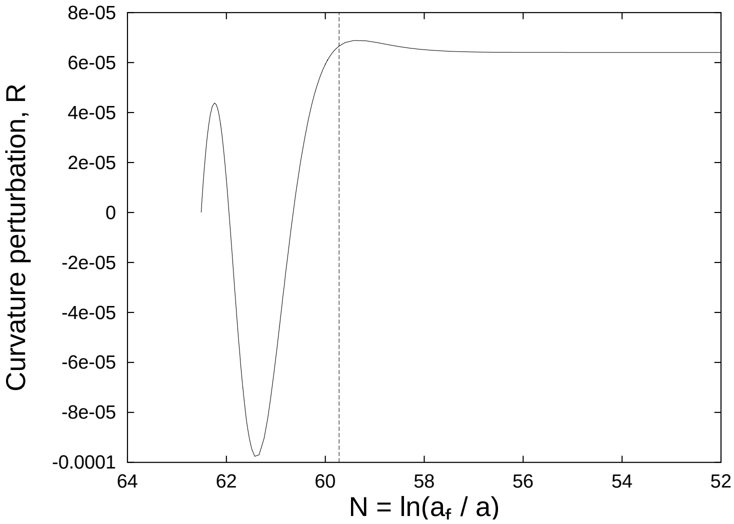
There are three models with reheating term $\Gamma \propto \phi^b T^c$,

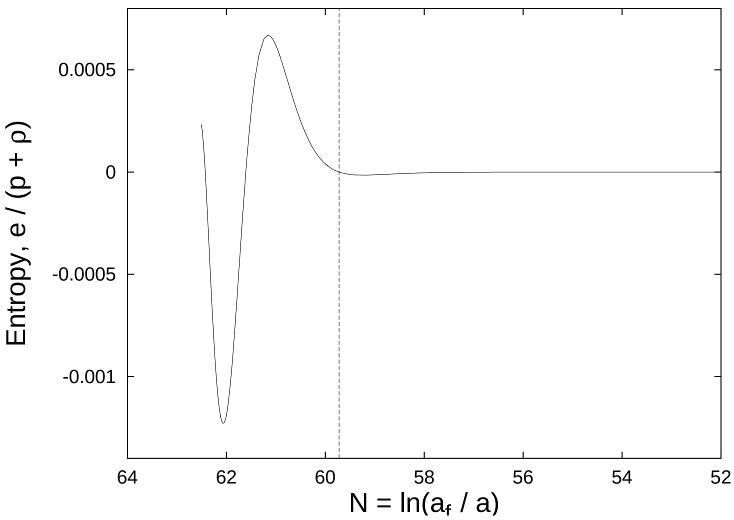
model	I	II	III
b	0	1	2
c	0	0	-1
Tf(10^{10}GeV)	2200	1100	4600

The temperature at the end of inflation is T_f.

8 Curvature and Entropy perturbations

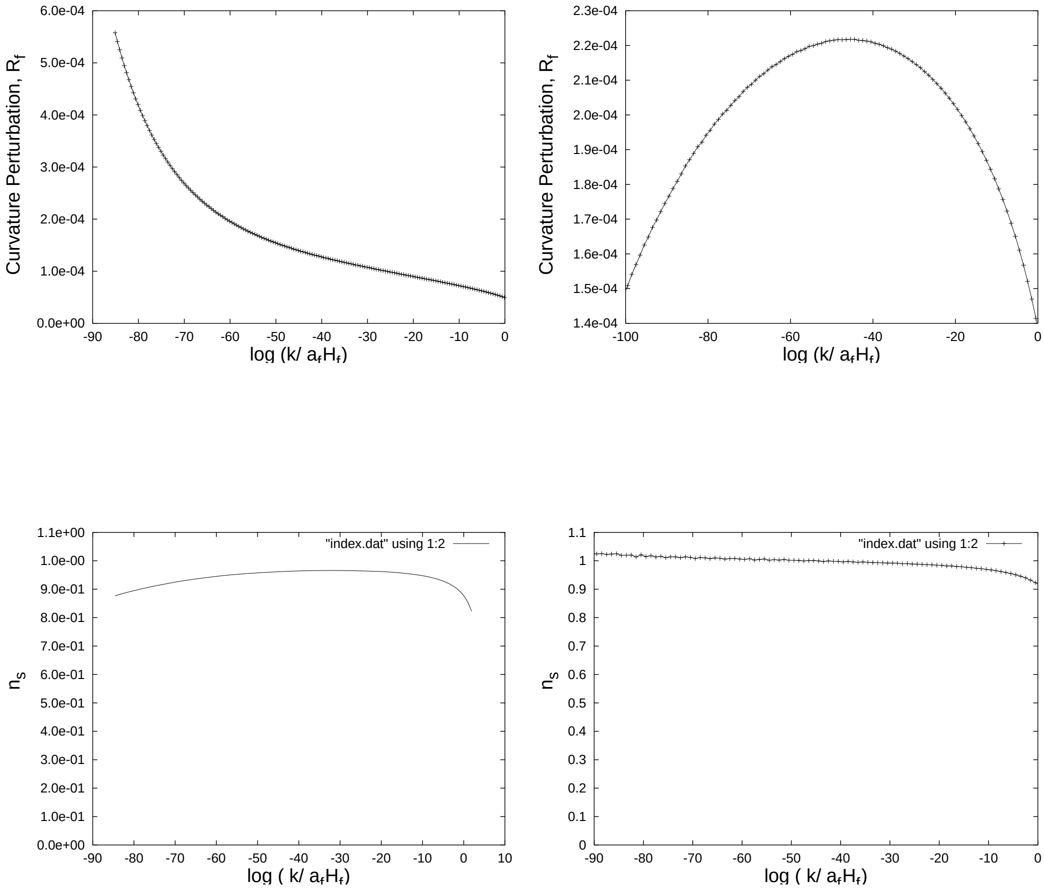
Single mode (model III).





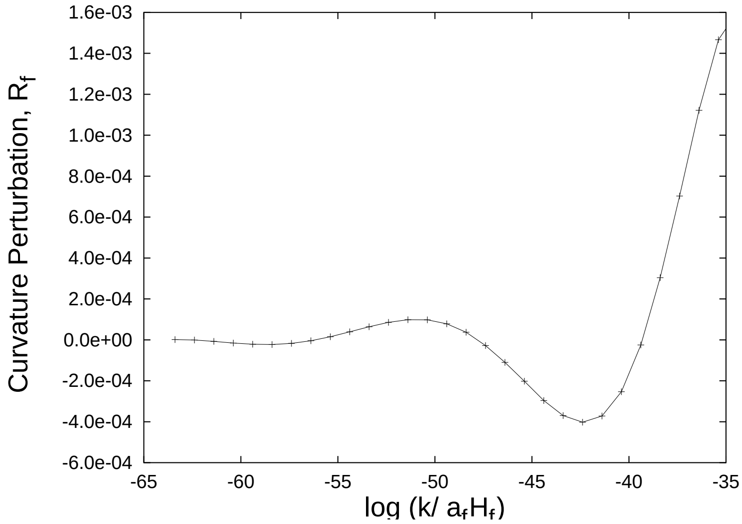
9 The effect of $\Gamma \equiv \Gamma(\varphi)$

Plots of perturbation amplitude and index against wavenumber for $\Gamma \propto \varphi^b$, $b = 0$ and $b = 1$.



10 The effect of $\Gamma \equiv \Gamma(T)$

The perturbation amplitude for Hosoya's damping term $\Gamma \propto T^c$ with $c = -1$.



11 General features

When $\Gamma_{,T} = 0$, we get an analytic result for the amplitude of curvature perturbations

$$P = \left(\frac{\pi}{4}\right)^{1/4} \left(\frac{k_F T}{H^2}\right) \frac{H^4}{\dot{\phi}^2}\Bigg|_{k=aH} \tag{1}$$

The spectral index can be expressed in terms of slow-roll parameters

$$n_s - 1 = \frac{1}{r}\left(-\frac{9}{4}\epsilon + \frac{3}{2}\eta - \frac{9}{4}\beta\right) \tag{2}$$

For 'new inflation' we can prove the following theorem

> If $V'(0) = 0$, $V''(0) < 0$, $V(\phi_0) = 0$, $V'(\phi_0) = 0$ and $b > 2/3$ then there exists a value of k for which $n_s = 1$.

This corresponds to a peak in the spectrum.

12 conclusions

1. Our computer programme will take any potential and friction term for warm inflation and output a primordial perturbation spectrum.
2. A spectral peak appears to be a feature of 'new inflation' potentials with friction terms which increase with ϕ.