Warm inflation has no η-problem

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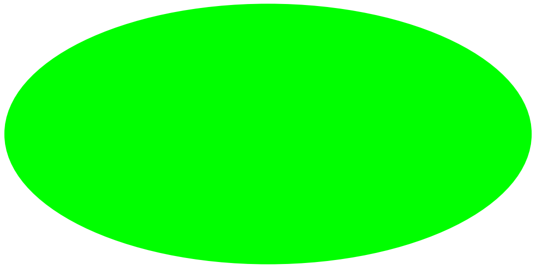
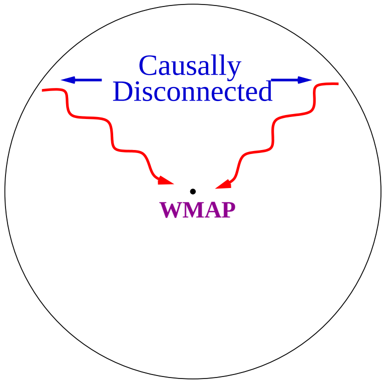
Overview

- Review of inflation
- Developing first principles theory of inflation (dissipative effects)
- Primordial Fluctuations during inflation
- Results: Solving η-problem with warm inflation

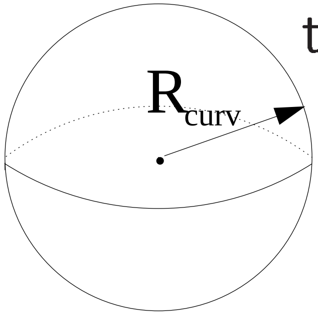
Shortcomings of Standard Cosmology

- Horizon problem - universe homogeneous within 10^{60+} causally disconnected regions





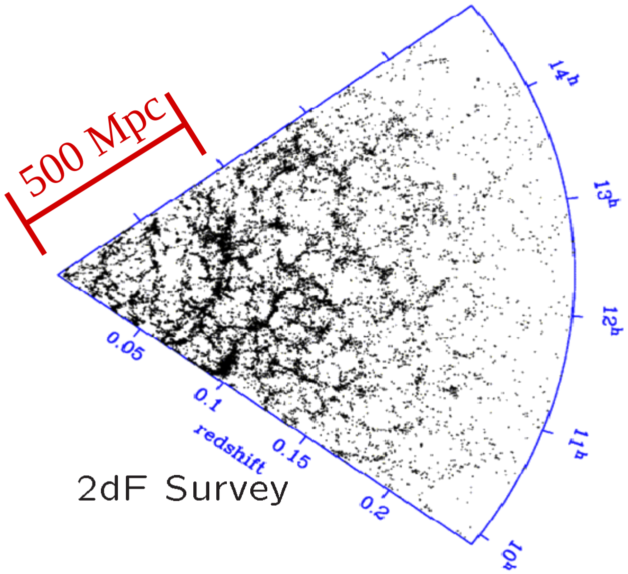
- Flatness problem - present universe close to flat, requires highly tuned initial conditions on initial energy density



$$R^2_{\mathrm{curv}} = H_0^{-2}/|\Omega - 1|$$

$$\Omega \equiv \frac{\rho}{\rho_c}; \ \frac{\rho - \rho_c}{\rho} \lesssim 10^{-50}$$

- Formation of structure problem - nonrandom correlations at scales larger than the BB model can explain



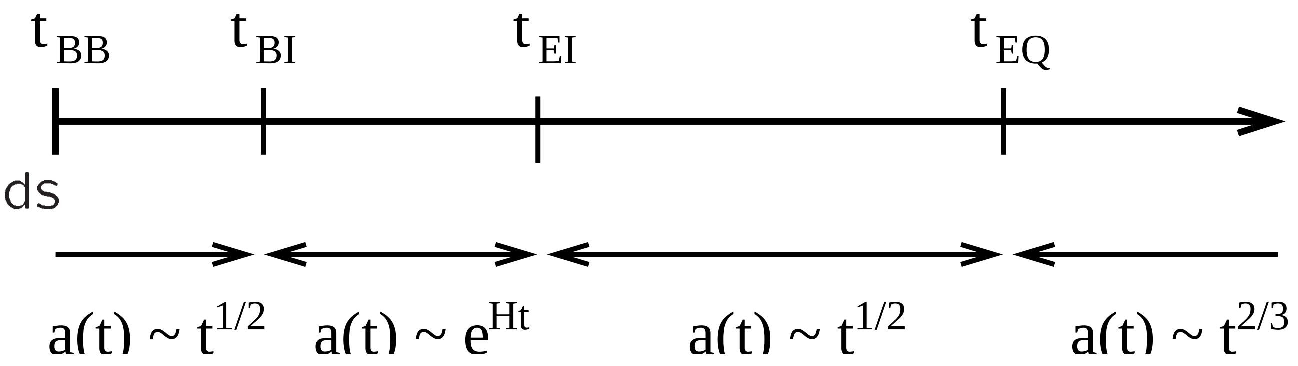
2dF Survey

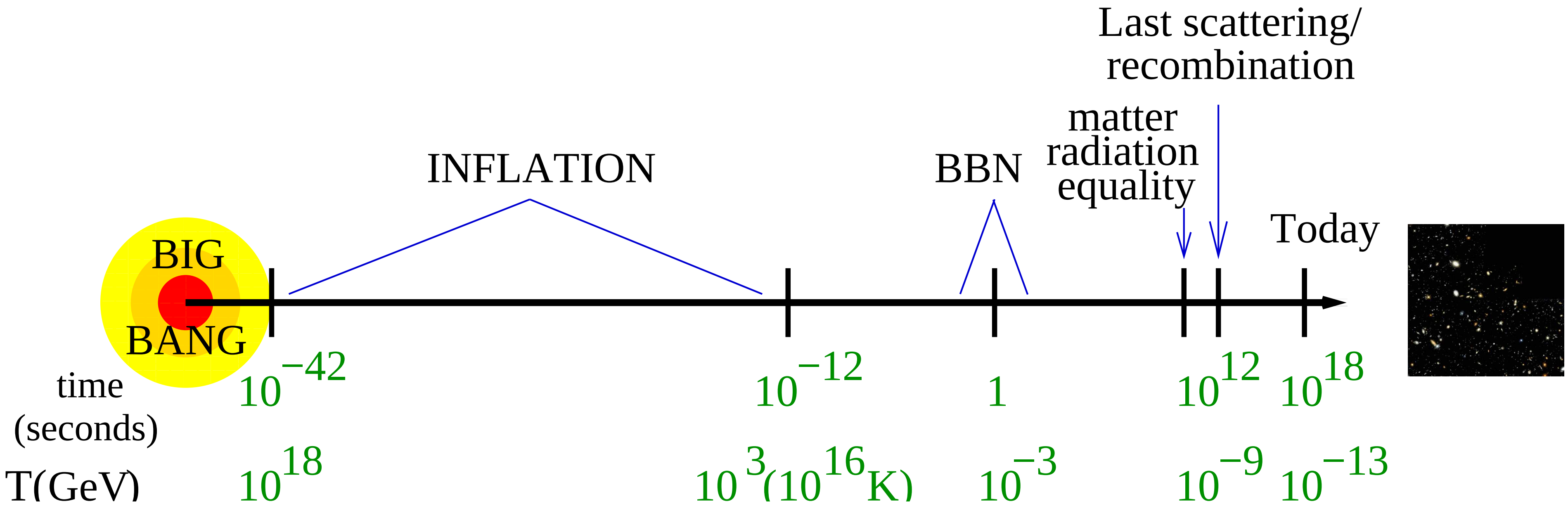
(Also: unwanted relics, breakdown classical ideal fluid approximation)

What is cosmic inflation

Short time interval
when universe expands
at accelerated rate

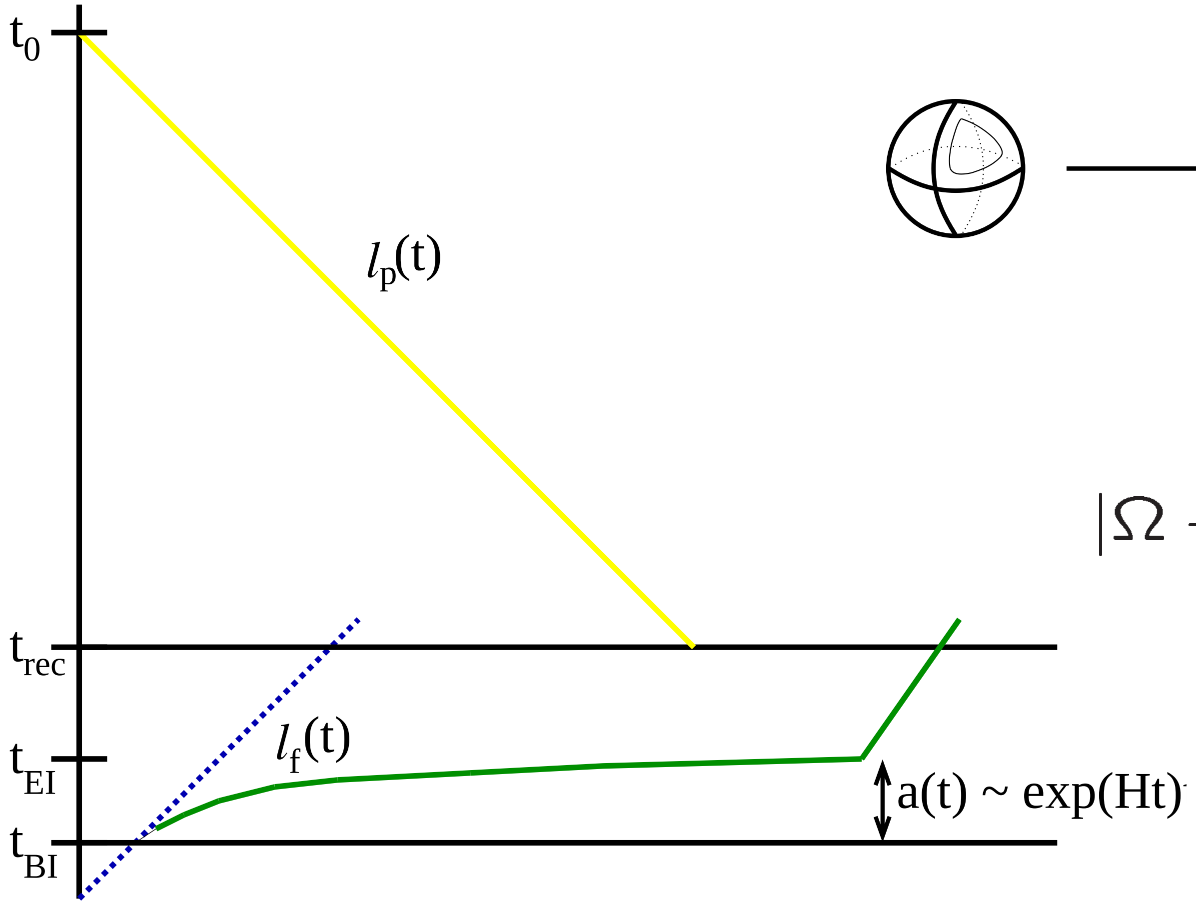
$a(t) \sim \exp(Ht)$



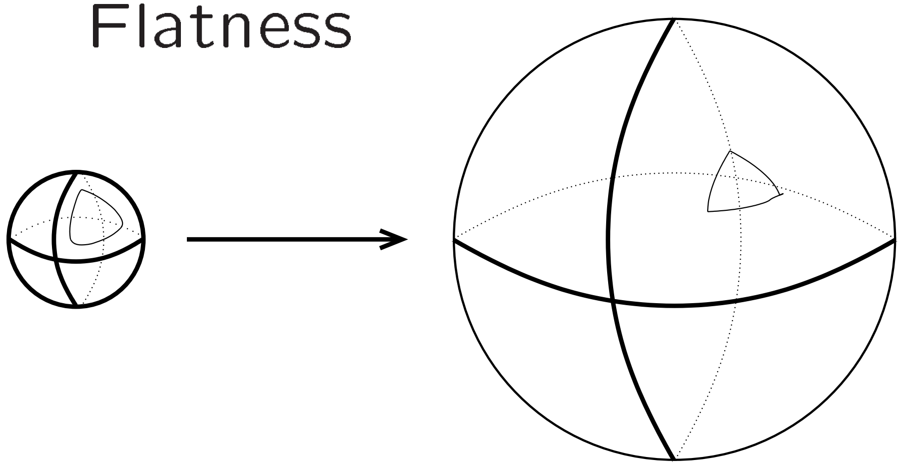


How inflation works

Horizon etc...

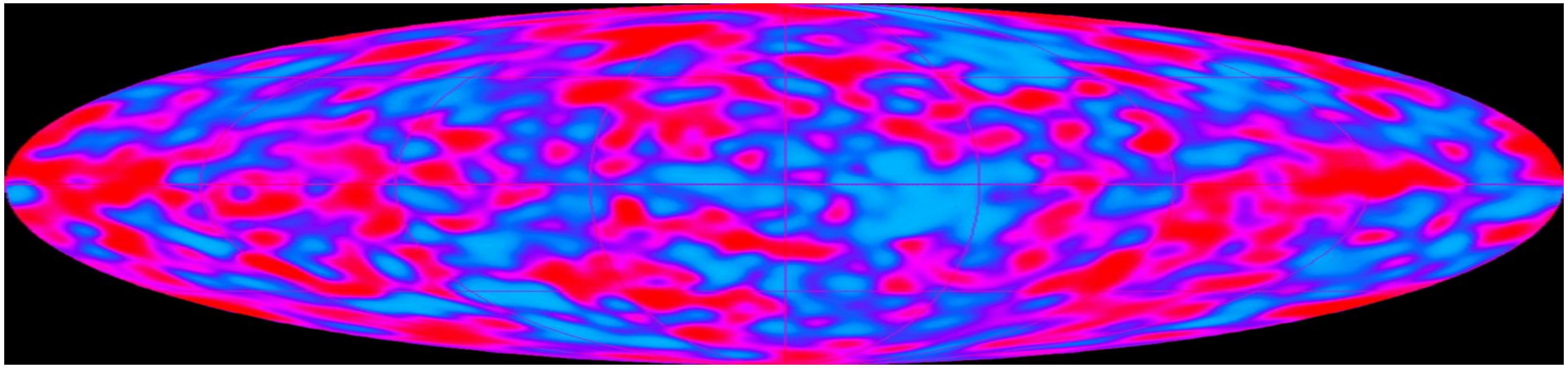


Flatness



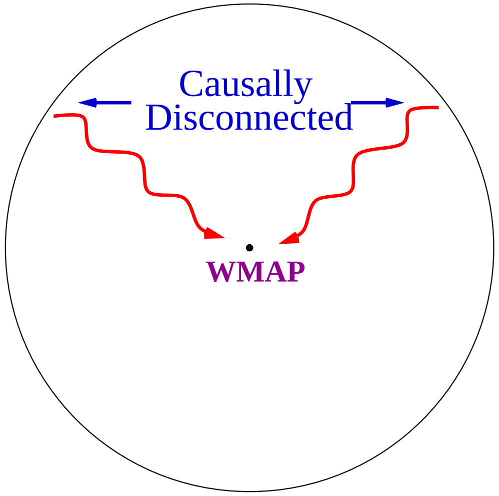
$$|\Omega - 1| \sim \exp(-2Ht)$$

Inflation

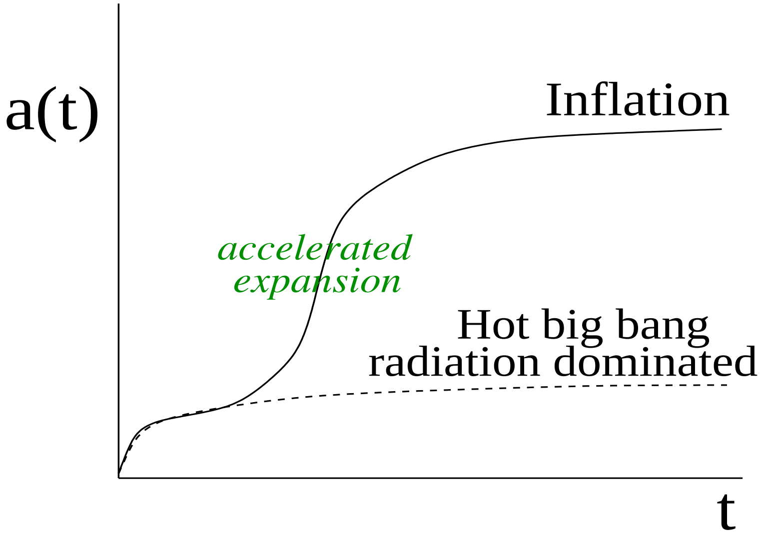


The puzzle of cosmological initial conditions:

$\frac{\Delta T}{T} < 10^{-5}$ in CMBR without causal contact



The inflation solution (1981+)

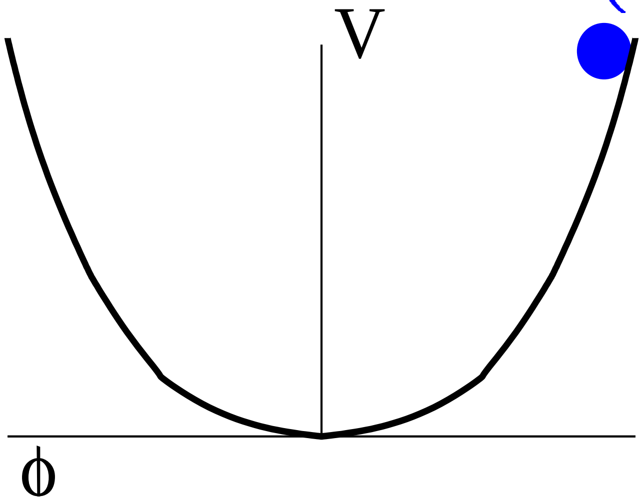


Accelerated expansion $\ddot{a}(t) > 0$

implies $\rho + 3p < 0$

How to get from high energy physics?

Scalar field (“inflaton”) dynamics



$$\rho = \frac{\dot{\phi}}{2} + \mathrm{V}(\phi) + \frac{(\nabla\phi)^2}{2R^2}$$

$$p = \frac{\dot{\phi}}{2} - \mathrm{V}(\phi) - \frac{(\nabla\phi)^2}{6R^2}$$

- Cold inflation: Just Choose $\mathrm{V}(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} + \mathrm{V}'(\phi) = 0$$

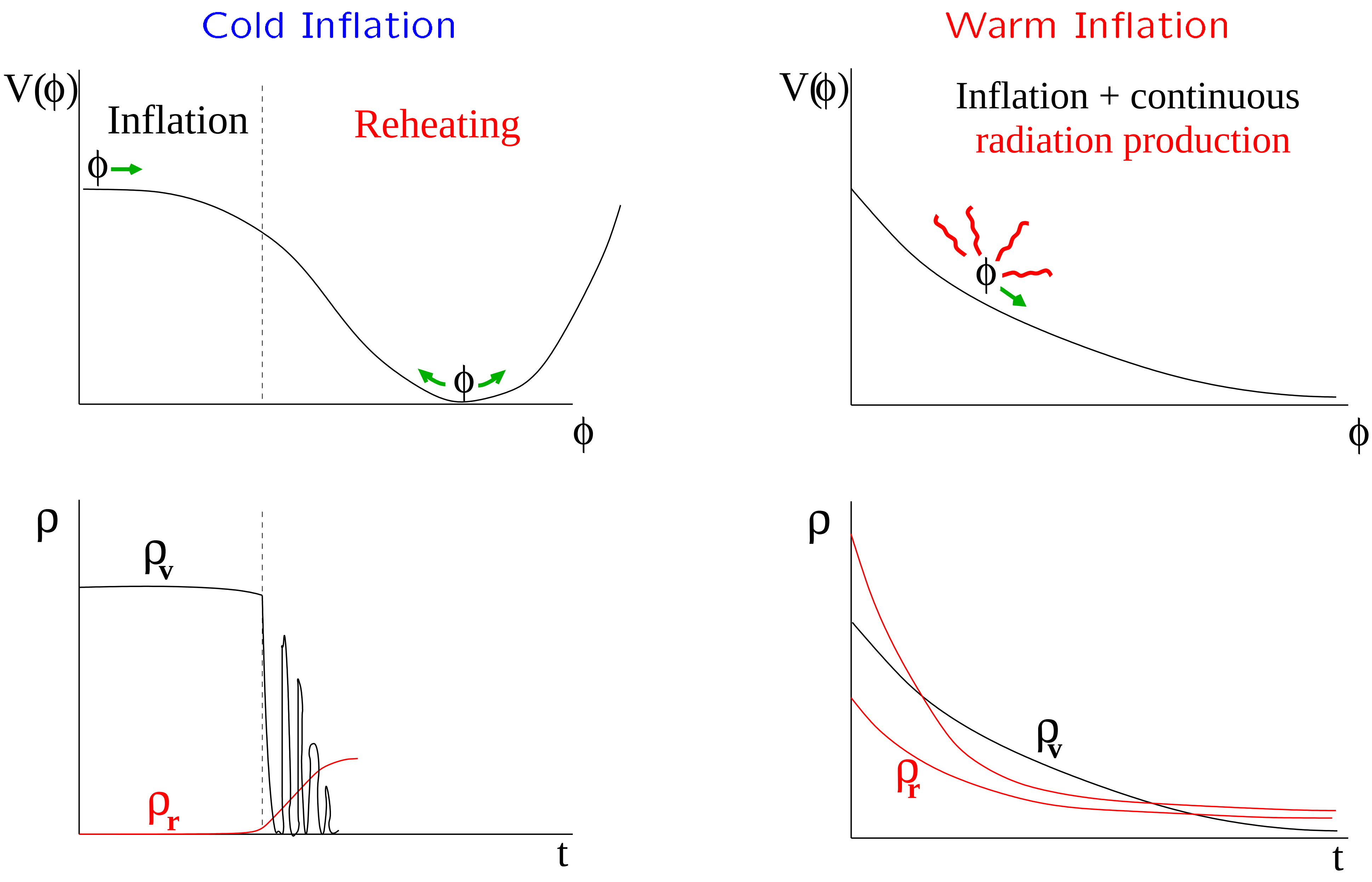
Potential energy dominated $3H\dot{\phi} \gg \ddot{\phi}$, “slow-roll”

- Warm Inflation: Υ important

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + \mathrm{V}'(\phi) = 0$$

Slow-roll now means $\Upsilon\dot{\phi} \gg 3H\dot{\phi}, \ddot{\phi}$, overdamped

Two basic inflation pictures



Road to Prediction

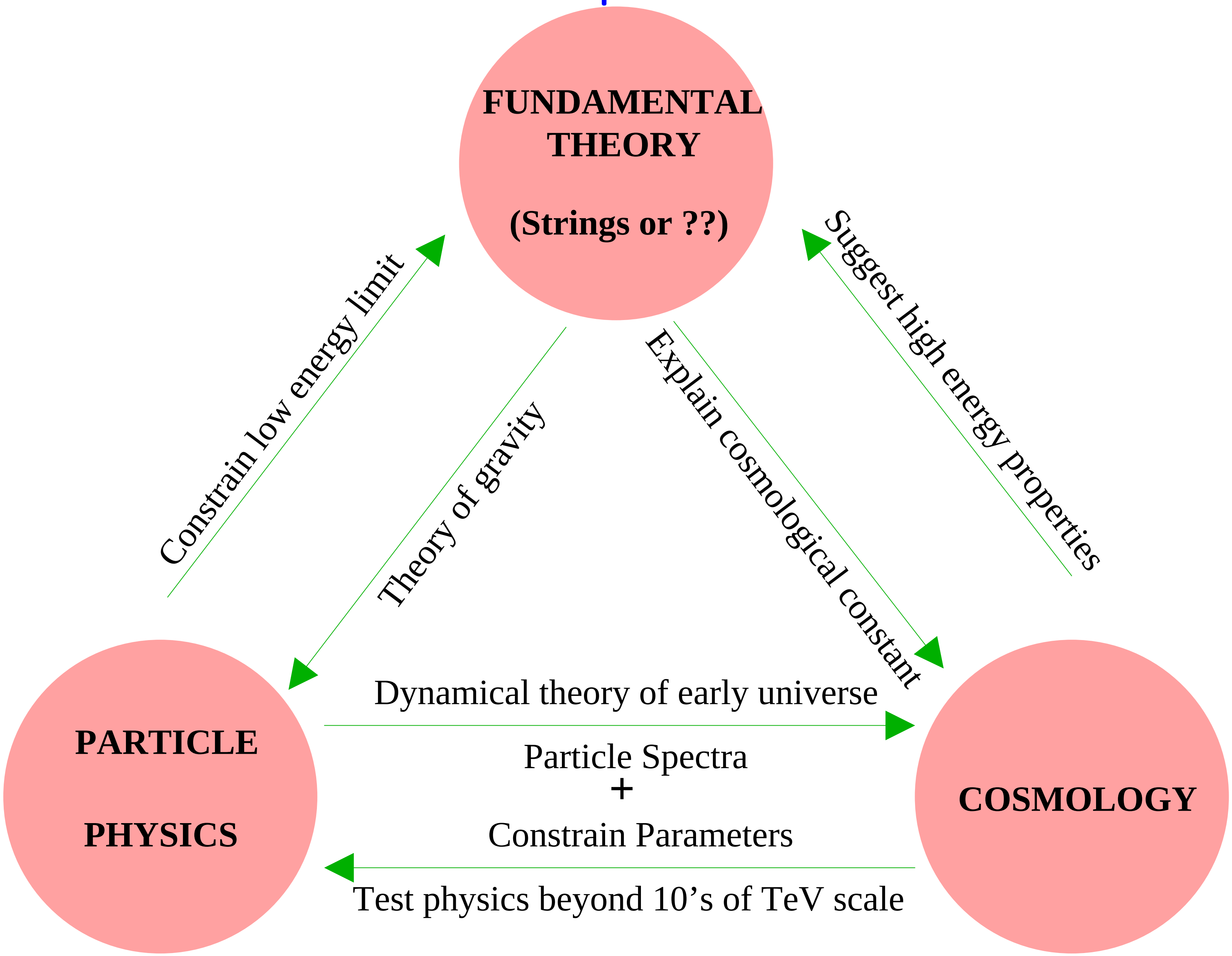
(Ingredients to build theory of the early universe)

- Particle physics model
- Nonequilibrium (dissipative) dynamics
- Further issues
- Observational tests

Role of particle physics

Feature	Particle Physics	Cosmology
Spontaneous symmetry breaking	mass generation	scalar fields (inflaton)
Standard model	explains collider experiments	particle spectrum (visible + dark)
Supersymmetry	solves hierarchy problem	explains flat potentials
Strings/Planck scale physics	explains gravity	solves cosmological constant problem

The BIG picture



Further issues

- Quantum-to-classical problem: any initial quantum coherent physics eventually must become classical
  CI: an issue for explaining density fluctuations
  WI: macroscopic physics classical from onset

- Entering inflation phase problem:
  initial mode fluctuations inhibit vacuum energy domination
  - damping can alleviate this problem

- Fine tuned potential problem:
  naturally explain ultraflat potentials necessary for inflation

- Dark matter, baryogenesis

- Cosmological constant problem:
  why vacuum energy today is almost zero

Energetics

- Consider GUT scale inflation: $M \sim 10^{15}$GeV

$$\Rightarrow \qquad \rho_v \sim M^4 \sim 10^{60}\text{GeV}^4 \quad \Rightarrow \quad H \sim 10^{10}\text{GeV}$$

  T $> H$ requires > 1 part in $\sim 10^{20}$ of $\rho_v \to \rho_r$
  (influences structure formation)
  T > 1GeV requires > 1 part in $\sim 10^{60}$ of $\rho_v \to \rho_r$
  (makes reheating unnecessary)

- Inflation pictures
  1. Cold inflation: basic assumption is no dissipation
  2. Warm inflation: radiation production inherent (AB, PRL **75**, 3218 (1995))

- Theoretical consideration
  Equipartition Hypothesis of Statistical Mechanics: *Scalar field should distribute its energy evenly amongst all degrees of freedom.*

  Dynamical Question: *Will relevant time scales during inflation prohibit the minute' radiation production given above?*

Scalar Field (Φ) Dynamics

$$\mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{m_\phi^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4$$

$$\mathcal{L}_I = -\Phi \sum_{j=1}^{N_\psi} h_j \bar{\psi}_j \psi_j, \; -\frac{1}{2} \sum_{j=1}^{N_\chi} g_j^2 \Phi^2 \chi_j^2 + \cdots$$

- Decompose into background φ and fluctuations ϕ: $\Phi = \varphi + \phi$
- Effective equation of motion (EOM) for $\varphi(t)$:

$$\ddot{\varphi}(t) + 3H\dot{\varphi}(t) + \xi R \varphi(t) + m_\phi^2 \varphi(t) + \frac{\lambda}{6}\varphi^3(t) + \varphi(t) \sum_{j=1}^{N_\chi} g_j^2 \langle \chi_j^2 \rangle + \cdots = 0$$

with $\langle \phi^2 \rangle$, $\langle \phi^3 \rangle$ and $\langle \chi_j^2 \rangle$ etc... evaluated perturbatively

Closed Time Path approach - Goal

Compute Observable

$$\langle \hat{O}(t) \rangle \equiv \frac{\mathrm{Tr}(\hat{\rho}(t)\hat{O})}{\mathrm{Tr}(\hat{\rho}(t))}$$

Thermal initial state at $T^{<}$:

$$\rho(T^{<}) = \exp(-\beta H) = U(T^{<} - i\beta, T^{<})$$

($U(t,t') \equiv \exp[-iH(t-t')]$, time evolution operator)

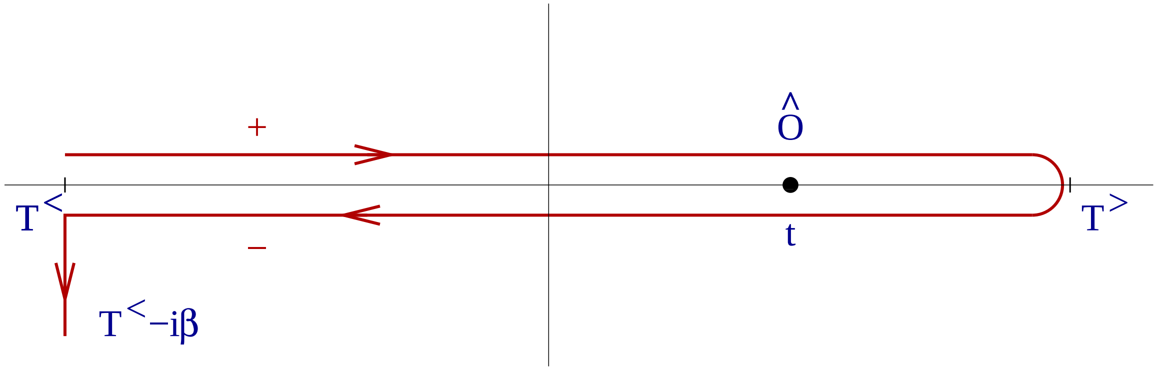
Thus

$$\langle \hat{O}(t) \rangle = \frac{\mathrm{Tr}[U(T^{<} - i\beta, T^{<})\, U(T^{<}, t)\, \hat{O}\, U(t, T^{<})]}{\mathrm{Tr}[U(T^{<} - i\beta, T^{<})]}$$

Also add large positive time $T^{>}$

$$\langle \hat{O}(t) \rangle = \frac{\mathrm{Tr}[U(T^{<} - i\beta, T^{<})\, U(T^{<}, T^{>})\, U(T^{>}, t)\, \hat{O}\, U(t, T^{<})]}{\mathrm{Tr}[U(T^{<} - i\beta, T^{<})]}$$

Closed Time Path approach - Method



Can express as a path integral

recall $\quad U(t,t') \equiv \int \mathcal{D}\Phi \exp\left(i \int_{t'}^{t} d^4x \mathcal{L}[\Phi]\right)$

$$
\begin{aligned}
Z[J^+, J^-, J^\beta] &= \mathrm{Tr}[U(T^< - i\beta, T^<; J^\beta) U(T^<, T^>; J^-) U(T^>, T^<; J^+)] \\
&= \int \mathcal{D}\Phi^+ \mathcal{D}\Phi^- \mathcal{D}\Phi^\beta \exp\left(i \int_{T^<}^{T^>} d^4x [\mathcal{L}^{J^+}[\Phi^+] - \mathcal{L}^{J^-}[\Phi^-]] + i \int_{T^<}^{T^< - i\beta} d^4x \mathcal{L}^{J^\beta}[\Phi^\beta]\right)
\end{aligned}
$$

e.g. scalar field theory:

$$
\mathcal{L}^J[\Phi] = \frac{1}{2}[\partial_\mu\Phi\partial^\mu\Phi - m^2\Phi^2] - \frac{\lambda}{4!}\Phi^4 + J\Phi
$$

Estimates of dissipation from QFT dynamics

Conditions on inflaton field: Overdamped motion & ultraflat potential

Direct Decay Models: $\phi \to \bar{\psi}\psi, \chi\chi$ requires $m_\phi > 2m_{\psi,\chi}$

$T \sim (\rho_r/g^*)^{1/4} \sim \lambda^{1/2} m_\phi/10$, $\rho_v^{1/4} \sim 10^{15-16}$ GeV,

$\lambda \sim 10^{-(10-16)}$, $m_\phi \sim 10^{(10-13)}$GeV

$$\Rightarrow \quad \boxed{T \sim 10 - 10^7 \text{GeV}}$$

Sets lower bound on radiation production from generic QFT models consistent with inflation

$\Rightarrow$ Reheating redundant & No gravitino overproduction in SUSY versions

Indirect Decay Models: $\phi \to \chi \to \bar{\psi}\psi$ requires $m_\phi < 2m_\psi < m_\chi$

For same parameter regime: $\boxed{T \lesssim 10^{14} \text{ GeV}}$

$\Rightarrow$ Copious radiation production

Very interesting regime

Robust dissipative mechanism

(AB and R. Ramos, PRD **63**, 103509 (2001) & hep-ph/0308211)

Lagrangian (encompasses reheating/hybrid inflation models):

$$\mathcal{L}_I = -\frac{1}{2} g^2 \phi^2 \chi^2 - g' \phi \bar{\psi}_\chi \psi_\chi - h \chi \bar{\psi}_d \psi_d,$$

Regime $m_\chi > 2m_\psi > m_\phi$: $\Longrightarrow$ $\phi \to \chi \to \psi_d$:
copious radiation production

e.g. GUT scale $\quad \mathrm{V}^{1/4}(\varphi) \sim 10^{15}\mathrm{GeV} \Longrightarrow \mathrm{T} \sim 10^{14}\mathrm{GeV}$

Physical Picture

- The evolution of $\varphi(t)$ changes χ-mass $\Longrightarrow$ χ-particle production from mixing $+/-$ frequency modes
(If left intact, quantum correlations of χ-field persist)

- Coupling χ to lighter ψ_d-field $\Longrightarrow$ χ-particles decay to ψ_d-particles, decohere χ-field

- Entire process backreacts on φ $\Longrightarrow$ dissipative effects

φ-effective equation of motion

$$
\begin{aligned}
\ddot{\varphi}(t) &+ 3H(t)\dot{\varphi}(t) + \xi \mathsf{R}(t)\varphi(t) + \frac{dV_{\rm eff}(\varphi(t))}{d\varphi(t)} \\
&+ g^4\varphi(t)\int_{t_0}^{t} dt'\varphi(t')\dot{\varphi}(t')K(t,t') = 0,
\end{aligned}
$$

where

$$
K(t,t') = \int_{-\infty}^{t'} dt'' \int \frac{d^3q}{(2\pi)^3} \sin\left[2\int_{t''}^{t} d\tau\,\omega_\chi(\tau)\right] \times \frac{\exp\left[-2m_\chi\Gamma_\chi \int_{t''}^{t} d\tau/\omega_\chi(\tau)\right]}{4\omega_\chi(t)\omega_\chi(t'')},
$$

$$
\omega_\chi(\tau) = \left[\mathbf{q}^2\frac{a(t)}{a(\tau)} + m_\chi^2(\tau) + 2(6\xi - 1)H\right]^{1/2},
$$

$$
m_\chi = g\varphi \qquad \Gamma_\chi \approx \frac{h^2 m_\chi}{8\pi} \qquad H = \sqrt{\frac{8\pi V_{\rm eff}}{3m_p^2}} \qquad a(t) \approx \exp(Ht)
$$

Adiabatic-Markovian Approximation

For $\frac{\dot{\varphi}}{\varphi} < H, \Gamma_\chi$; $H < \Gamma_\chi, m_\chi$; φ-effective EOM becomes:

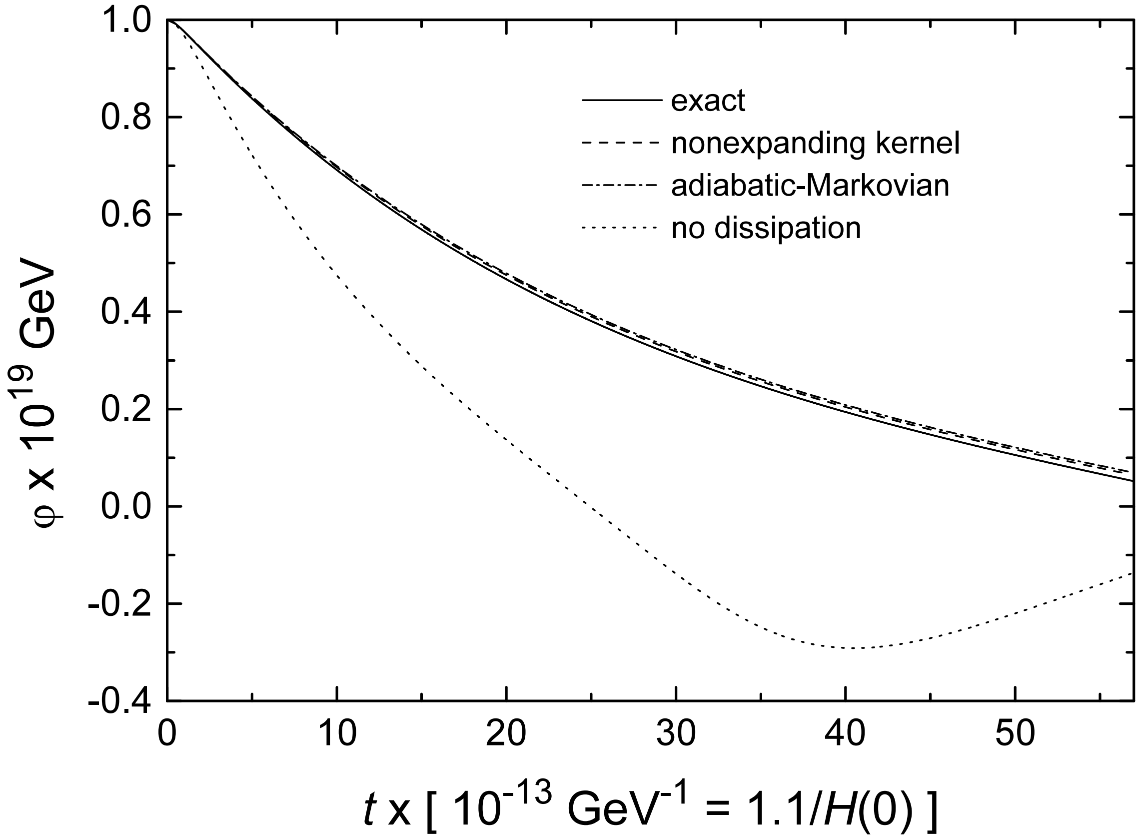
$$\ddot{\varphi} + [3H + \Upsilon(\varphi)]\dot{\varphi} + \xi \mathsf{R} \varphi + \frac{dV_{\mathrm{eff}}(\varphi)}{d\varphi} = 0,$$

where

$$\Upsilon(\varphi) = \frac{g^4 \varphi^2(t) \Gamma_\chi}{32\pi \sqrt{m_\chi^2 + \Gamma_\chi^2} \sqrt{2 m_\chi \sqrt{m_\chi^2 + \Gamma_\chi^2} + 2 m_\chi^2}}.$$

Results - comparing approximations

Evolution of $\varphi(t)$ for $V = \frac{\lambda}{4}\varphi^4$, $\lambda = 10^{-13}$, $g = h = 0.37$, $\xi = 0$, $\varphi(0) = m_p$, $\dot{\varphi}(0) = 0$.

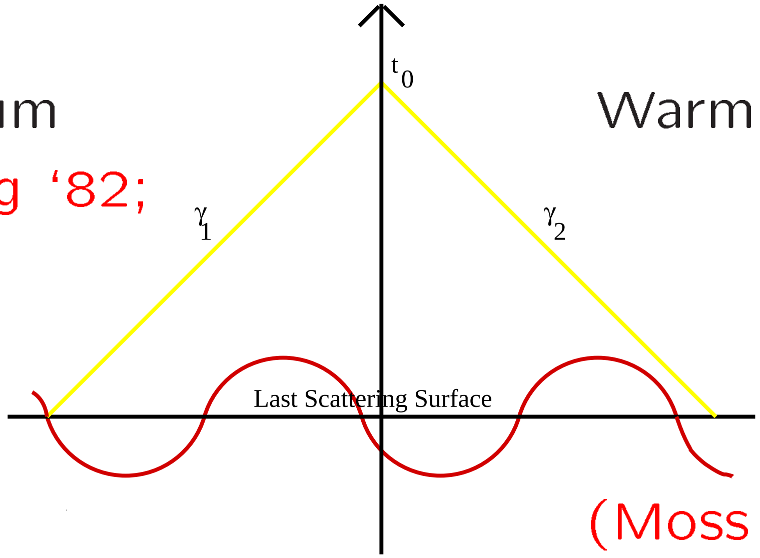


$T > H$ during inflation (so affects $\delta\varphi$) for e.g., $g = h > 10^{-2}$ or $h = 0.1, g > 10^{-3}$

Nature of the fluctuations

Thermal/quantum fluctuations leave “ripples” in early universe

- imprinted on CMBR at Last Scattering
- amplified by gravity to make structure



Cold inflation: Quantum

(Guth, Pi ‘82; Hawking ‘82;

Starobinsky ‘82;

Bardeen *et. al.* ‘83)

$$\delta\phi = \frac{H}{2\pi}$$

Warm Inflation: Thermal

$$\delta\phi = \frac{k_F^{1/2} T^{1/2}}{2\pi}$$

$\Upsilon < 3H : k_F = H$

(Moss ‘85, AB,Fang ‘95)

$\Upsilon > 3H : k_F = \sqrt{\Upsilon H}$

(AB ‘99)

$$\frac{\delta\rho}{\rho} = \frac{H\delta\phi}{\dot{\phi}}\Bigg|_{\text{HORIZON EXIT}}$$

Worked example - no η-problem

Model: $V = \frac{m^2}{2}\varphi^2$

$\Longrightarrow N_e \approx 2\sqrt{2}\frac{\varphi_0}{m}\frac{\Upsilon}{m_p}$, $T \approx \frac{m^{3/4} m_p^{1/4} \varphi_0^{1/4}}{\Upsilon^{1/4}}$, $\frac{\delta\rho}{\rho} \approx (\frac{\varphi_0}{m})^{3/8}(\frac{\Upsilon}{m_p})^{9/8}$

For $N_e = 60$, $\frac{\delta\rho}{\rho} = 10^{-5} \Longrightarrow \frac{\varphi_0}{m} \approx 6 \times 10^8$, $\frac{\Upsilon}{m_p} \approx 4 \times 10^{-8}$

e.g., $m = 10^9\text{GeV} \Longrightarrow \frac{H}{m} \approx 0.17$, $\frac{\varphi_0}{m_p} \approx 6 \times 10^{-2}$, $T \approx 10^4 m$

- No η-problem: $\underline{m > H}$
- No φ amplitude problem: $\underline{\varphi < m_p}$
- No graceful exit problem: inflation $\to$ RD automatic
- No quantum-to-classical trans. problem: $\delta\varphi$ classical

Observational tests for inflation

- Spectra of energy density fluctuations (scalar spectra)
- Spectra of gravitational waves (tensor spectra)
- Non-gaussian deviations
- Isocurvature fluctuations
- Present day cosmological constant
- Particle Spectra

EXPERIMENTS

- CMBR - COBE, MAP, Planck, Boomerang, Maxima
- Redshift surveys - Sloan, 2df ...
- Big Bang Nucleosynthesis
- Supernovae 1A
- Hubble Space Telescope

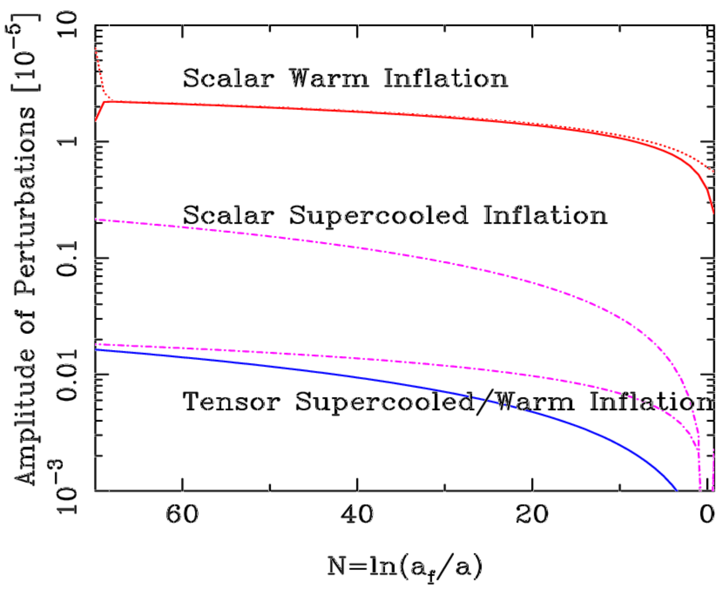
Signatures for inflation dynamics

(Taylor and AB, PRD **62**, 083517 (2000))

- Standard inflation has no interactions so universe is cold
- Warm inflation includes interactions

Amplitude of perturbations

$(V = \frac{1}{2}m^2\phi^2)$:



- Warm inflation predicts different tensor-to-scalar ratio than cold inflation.
- No consistency relation in warm inflation.
- Mechanism for isocurvature modes - which may be detectable by Planck.

Conclusion

Precision data requires robust precision theory

- Inflation most compelling early universe picture (Guth, Sato '81)
- Need dynamical theory of inflation - lack treatment of interactions (dramatically changes picture)
- Inflation in general can occur concurrent with radiation production, warm inflation (no η-problem)
  (AB '95, variation on the theme: de Oliveira & Ramos '98, Maia & Lima '99, Chimento *et al.* 2002)
  - Observational tests for radiation during inflation - tensor/scalar ratio, nongaussian, isocurvature
    (Thermal density perturbations - AB & Fang '95, Gupta *et al.* 2002, de Oliveira and Joras 2001, Hall *et al.* 2003)
  - Quantum field theory robust mechanism for warm inflation
    (AB, Ramos 2001,03; other dissp. studies of wi: AB, Gleiser, Ramos '98; Yokoyama, Linde '99; Moss 2001; Lawrie 2002)