Quintessential Perturbations during scaling regime

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Abstract

We investigate the evolutions of perturbations while the background follows the scaling. We show that the perturbed scalar field also scales with the dominant fluid perturbations. The perturbations accompany the adiabatic mode and the isocurvature mode between the fluid and field.

Reference

- J. Hwang and H. Noh, Physical Review D 64, 103509.

Introduction

- Present day acceleration of the universe: explained by Cosmological constant, Quintessence...
- An exponential potential supports a scaling attractor regime: the density of the field follows the density of the dominating fluid in the background.
- We study the evolutions of perturbations in a fluid-field system with an exponential potential.
  - The perturbed scalar field also scales the perturbed density of the fluid.
  - While the background evolutions follow the scaling solutions the perturbations accompany both the adiabatic and the isocurvature modes.

Equations of Fluid and Field System

- A system of a multiple ideal fluid and a field without direct interactions.
- The energy-momentum tensors:

$$ds^2 = -a^2 (1 + 2\alpha) d\eta^2 - 2a^2 \beta_{,\alpha} d\eta dx^\alpha + a^2 \left[g^{(3)}_{\alpha\beta} (1 + 2\varphi) + 2\gamma_{,\alpha|\beta} \right] dx^\alpha dx^\beta, \tag{1}$$

$$T^0_0 = -(\bar{\mu} + \delta\mu), \quad T^0_\alpha = -\frac{1}{k} (\mu + p) v_{,\alpha},$$
$$T^\alpha_\beta = (\bar{p} + \delta p) \delta^\alpha_\beta + (\frac{1}{k^2} \nabla^\alpha \nabla_\beta + \frac{1}{3} \delta^\alpha_\beta) \pi^{(s)}. \tag{2}$$

$\bar{\mu} = \Sigma_l \bar{\mu}_l$, $\delta\mu = \Sigma_l \delta\mu_l$, and similarly for $\bar{p}$, δp, $(\mu + p)v$ and $\pi^{(s)}$. The sum includes the field contribution which can be expressed in terms of fluid variables:

$$\mu_\phi \equiv \frac{1}{2} \dot{\phi}^2 + V, \quad \delta\mu_\phi \equiv \dot{\phi} \delta\dot{\phi} - \dot{\phi}^2 \alpha + V_{,\phi} \delta\phi,$$
$$p_\phi \equiv \frac{1}{2} \dot{\phi}^2 - V, \quad \delta p_\phi \equiv \dot{\phi} \delta\dot{\phi} - \dot{\phi}^2 \alpha - V_{,\phi} \delta\phi,$$
$$v_\phi \equiv \frac{k}{a} \frac{\delta\phi}{\dot{\phi}}, \quad \pi^{(s)}_\phi \equiv 0. \tag{3}$$

- Background evolutions:

$$H^2 = \frac{8\pi G}{3} \mu + \frac{\Lambda}{3} - \frac{K}{a^2}, \tag{4}$$
$$\dot{\mu}_i + 3H (\mu_i + p_i) = 0, \tag{5}$$
$$\ddot{\phi} + 3H \dot{\phi} + V_{,\phi} = 0. \tag{6}$$

- The energy-momentum conservation of individual fluid and the equation of motion of a scalar field give:

$$\dot{\delta}_i + 3H(c_i^2 - w_i)\delta_i = (1 + w_i)\,(\kappa - 3H\alpha - \frac{k}{a}v_i), \quad (7)$$

$$\frac{[a^4(\mu_i + p_i)v_i]^{\cdot}}{a^4(\mu_i + p_i)} = \frac{k}{a}(\alpha + \frac{\delta p_i}{\mu_i + p_i}), \quad (8)$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \frac{k^2}{a^2}\delta\phi + V_{,\phi\phi}\delta\phi$$
$$= \dot{\phi}\,(\kappa + \dot{\alpha}) + \left(2\ddot{\phi} + 3H\dot{\phi}\right)\alpha, \quad (9)$$

where $\delta_i \equiv \delta\mu_i/\mu_i$, $w_i \equiv p_i/\mu_i$, and $c_i^2 \equiv \dot{p}_i/\dot{\mu}_i$.

- The metric parts are described by Einstein's equations:

$$\kappa \equiv 3\,(-\dot{\varphi} + H\alpha) + \frac{k^2}{a^2}\chi, \quad (10)$$

$$-\frac{k^2 - 3K}{a^2}\varphi + H\kappa = -4\pi G\delta\mu, \quad (11)$$

$$\kappa - \frac{k^2 - 3K}{a^2}\chi = 12\pi G\frac{a}{k}(\mu + p)v, \quad (12)$$

$$\dot{\chi} + H\chi - \alpha - \varphi = 8\pi G\frac{a^2}{k^2}\pi^{(s)}, \quad (13)$$

$$\dot{\kappa} + 2H\kappa + (3\dot{H} - \frac{k^2}{a^2})\alpha = 4\pi G(\delta\mu + 3\delta p). \quad (14)$$

These equations are in a gauge-ready form. That is, we have not chosen the temporal gauge condition so that we could *use the freedom as an advantage* in handling problems in diverse situations; the variables are all spatially gauge-invariant [Bardeen 1988, Hwang 1991].

Perturbed equations

- Two component system: scalar field and dominant ideal fluid (f)
- $w \equiv \frac{p_f}{\mu_f} = \text{const}$
- As a gauge condition, $v_f = 0$: the comoving gauge based on the dominant fluid component

$$\begin{aligned}
&\ddot{\delta}_f + (2 - 3w) H \dot{\delta}_f + [w \frac{k^2}{a^2} - 6w \left(\dot{H} + H^2 \right) \\
&\quad -4\pi G \mu_f (1+w)(1+3w)] \delta_f \\
&\quad = 8\pi G \left[2w \dot{\phi}^2 \delta_f + (1+w) \left(2\dot{\phi} \delta\dot{\phi} - V_{,\phi} \delta\phi \right) \right], \quad (15) \\
&\delta\ddot{\phi} + 3H \delta\dot{\phi} + (\frac{k^2}{a^2} + V_{,\phi\phi}) \delta\phi \\
&\quad = \frac{1}{1+w} \left[(1-w) \dot{\phi} \dot{\delta}_f - 2w \left(\ddot{\phi} + 3H\dot{\phi} \right) \delta_f \right], \quad (16)
\end{aligned}$$

where $\delta_f = \delta_{f v_f}$ and $\delta\phi = \delta\phi_{v_f}$.

- All gauge invariant variables:

$$\delta_{f v_f} \equiv \delta_f + 3(1+w) \frac{aH}{k} v_f, \quad \delta\phi_{v_f} \equiv \delta\phi - \dot{\phi} \frac{a}{k} v_f. \quad (17)$$

Scaling background

- The scalar field with a potential $V = V_0 e^{-\lambda\phi}$: attractor solution of the dominant component.
- We set $8\pi G \equiv 1$.
- In a flat background without Λ

$$\mu_\phi = \frac{2}{1-w} V = \frac{1}{1+w}\dot{\phi}^2, \quad \Omega_\phi = \frac{3(1+w)}{\lambda^2},$$
$$\phi' = \frac{3(1+w)}{\lambda}\frac{a'}{a} = \frac{6(1+w)}{1+3w}\frac{1}{\lambda\eta}, \tag{18}$$

and $\Omega_f = 1 - \Omega_\phi$; a prime indicates the time derivative based on η.

Case with $w = \frac{1}{3}$

- Perturbed equations:

$$\delta_r'' + [\frac{k^2}{3} - (1 - \frac{8}{3\lambda^2})\frac{2}{\eta^2}]\delta_r = \frac{16}{3\lambda}(\frac{2}{\eta}\delta\phi' + \frac{1}{\eta^2}\delta\phi), \quad (19)$$

$$\delta\phi'' + \frac{2}{\eta}\delta\phi' + (k^2 + \frac{4}{\eta^2})\delta\phi = \frac{2}{\lambda}(\frac{1}{\eta}\delta_r' - \frac{1}{\eta^2}\delta_r). \quad (20)$$

- In the large scale limit: $\delta_r \propto \delta\phi \propto \eta^q$ where

$$q = 2, \quad -1, \quad \frac{1}{2}\left(-1 \pm \sqrt{1 - 16\Omega_r}\right). \quad (21)$$

- For the growing solution $\delta_r \propto \eta^2$,

$$\delta_r = \frac{5}{2}\delta_\phi = 5\lambda\delta\phi. \quad (22)$$

- The adiabatic and the isocurvature perturbation δ_v and S_{ij}

$$\delta_v = (1 - \frac{2}{5}\Omega_\phi)\delta_r, \quad S_{r\phi} = \frac{9}{20}\delta_r, \quad (23)$$

  where [Kodama and Sasaki 1984]

$$S_{ij} \equiv \frac{\delta\mu_i}{\mu_i + p_i} - \frac{\delta\mu_j}{\mu_j + p_j}. \quad (24)$$

- Also, we have

$$\frac{k^2 - 3K}{a^2}\varphi_\chi = \frac{1}{2}\mu\delta_v, \quad (25)$$

  where $\varphi_\chi \equiv \varphi - H\chi$; in the notation of [Bardeen 1980,Muhanov et al. 1992] we have $\varphi_\chi = \Phi_H = -\Psi$. Thus, vanishing δ_v implies vanishing φ_χ, thus isocurvature in the multi-component situation. Therefore, eq. (23) shows that the growing mode of perturbation during the scaling regime accompanies the adiabatic (δ_v) as well as the isocurvature ($S_{r\phi}$) modes; Some comments***

Case with $w = 0$

- In the large scale limit, we have solutions $\delta_c \propto \delta\phi \propto \eta^q$ where

$$q = 2, \quad -3, \quad \frac{3}{2}\left(-1 \pm \sqrt{1 - 8\Omega_c}\right). \tag{26}$$

- For the dominant solution $\delta_c \propto \eta^2$

$$\delta_c = -14\delta_\phi = \frac{7}{3}\lambda\delta\phi. \tag{27}$$

  We can show $\delta = (1 - \frac{15}{14}\Omega_\phi)\delta_c$, thus, the adiabatic and the isocurvature modes

$$\delta_v = (1 - \frac{9}{14}\Omega_\phi)\delta_c, \quad S_{c\phi} = \frac{15}{14}\delta_c. \tag{28}$$

- In the small scale limit, we have $\delta_c \propto \eta^q$ where $q = \frac{1}{2}(-1 \pm \sqrt{1 + 24\Omega_c})$, which lead to $q = 2, -3$ for vanishing Ω_ϕ, [Ferreira and Joyce 1998].

Case with general w

- In the large scale limit: $\delta_f \propto \delta\phi \propto \eta^q$ with

$$q = 2, \quad -\frac{3(1-w)}{1+3w},$$
$$\frac{3(1-w)}{2(1+3w)}\Big(-1 \pm \sqrt{1 - 8\frac{1+w}{1-w}\Omega_f}\Big). \tag{29}$$

- For the dominant solution $\delta_f \propto \eta^2$

$$\delta_f = -\frac{2(7+9w)}{1-24w-9w^2}\delta_\phi = \frac{7+9w}{3(1-w)}\lambda\delta\phi. \tag{30}$$

- The adiabatic and isocurvature modes:

$$\delta_v = \left[1 - \frac{9(1-w^2)}{2(7+9w)}\Omega_\phi\right]\delta_f,$$
$$S_{f\phi} = \frac{3(1-w)(5+3w)}{2(1+w)(7+9w)}\delta_f. \tag{31}$$

Also, we have

$$\varphi_v = \frac{2(5+3w)}{(1+w)(1+3w)^2}\left[1 - \frac{9(1-w^2)}{2(7+9w)}\Omega_\phi\right]\frac{1}{(k\eta)^2}\delta_f. \tag{32}$$

In the large-scale limit we have $\varphi_v = C(\mathbf{x})$ which will fix the normalizations of δ_f and others in terms of C.

Summary

- A system of a field and a fluid with constant w.
- We find that:
  - While the field scales with the fluid in the background, the perturbed field also scales with the perturbed fluid.
  - The ordinary growing and deaying solutions of the fluid without the field remain valid for the coupled system.
  - During the scaling the perturbations accompany both adiabatic and the isocurvature modes.

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