Evolution of Primordial Magnetic Fields in the Early Universe*

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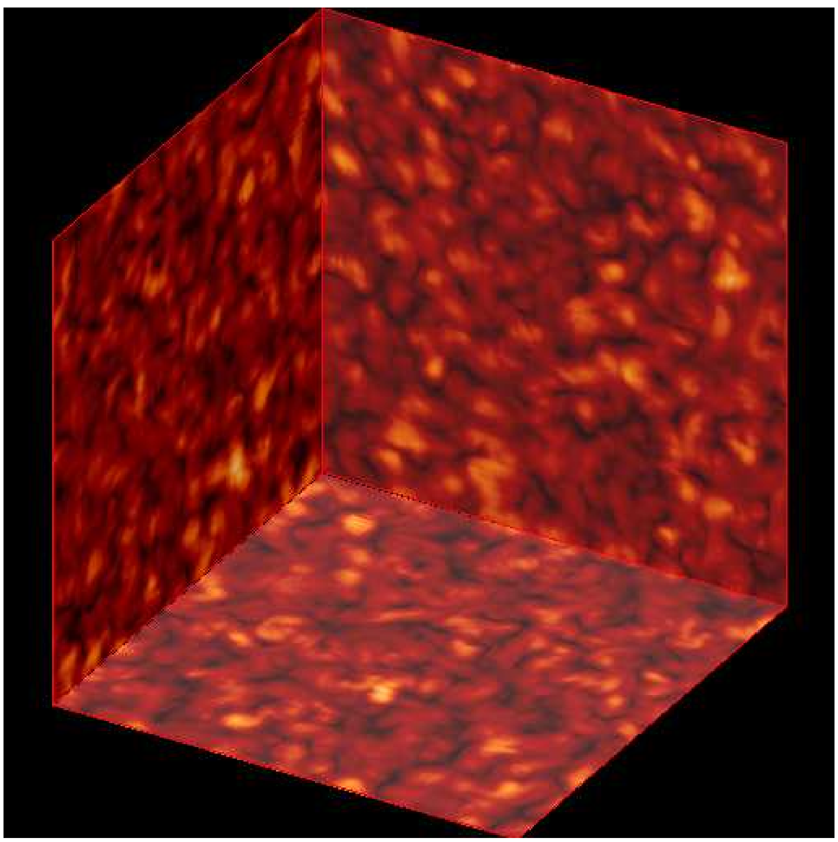
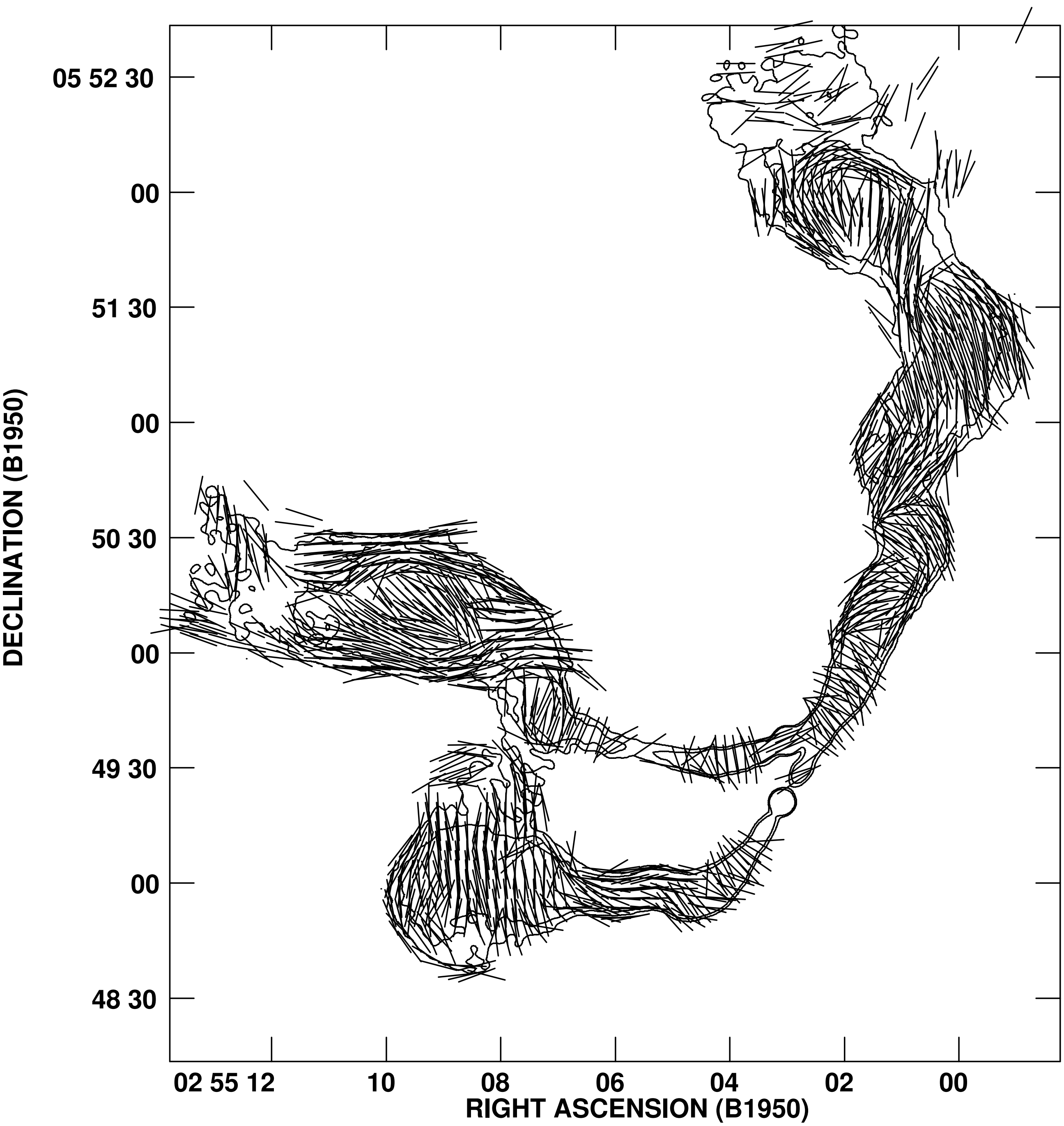


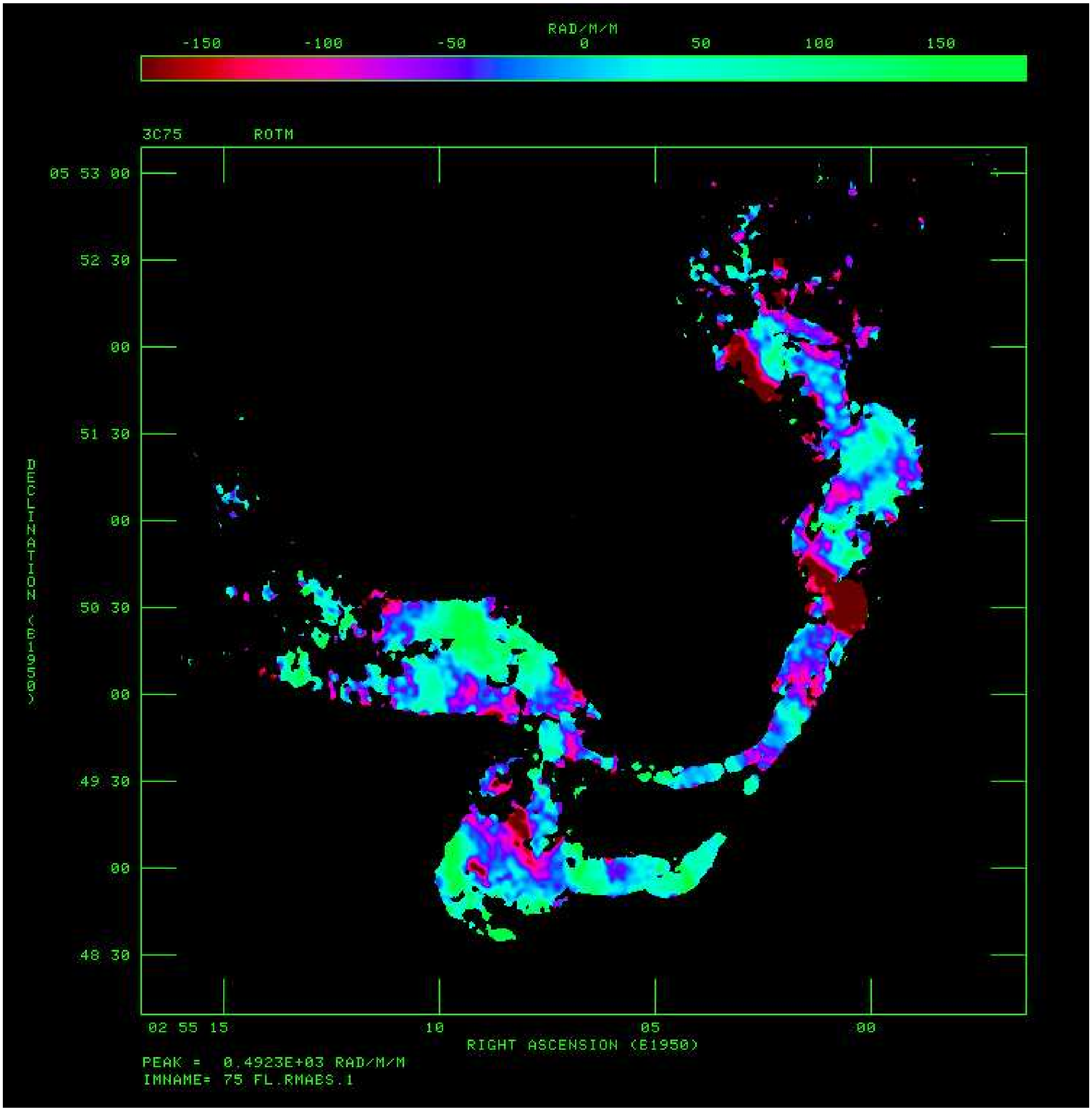
*astro-ph/0306211, R. B. and K. Jedamzik

Motivation

- Observation of μG magnetic fields in galaxies and galaxy clusters
  - Zeeman splitting of spectral lines
  - synchrotron emission from free electrons
  - Faraday rotation measurements (RM)
- Origin of these fields is still unclear
- **possible explanations:** primordial, galactic dynamo, Biermann battery, ...
- How do primordial magnetic fields evolve with time in the early universe?



Magnetic field vectors for 3C75 (Abell 400), Eilek and Owen, astro-ph/0109177



Rotation measure distribution in fAbell 400, Eilek and Owen, astro-ph/0109177

MHD in the Early Universe

Non relativistic MHD equations in the expanding universe

$$\begin{aligned}
\frac{\partial \tilde{\rho}}{\partial \tilde{t}} + \nabla \cdot (\tilde{\rho}\,\tilde{\mathbf{v}}) &= 0 \\
\frac{\partial \tilde{\mathbf{v}}}{\partial \tilde{t}} + (\tilde{\mathbf{v}} \cdot \nabla)\,\tilde{\mathbf{v}} + \frac{1}{\tilde{\rho}}\nabla\,\tilde{p} + \frac{1}{4\pi\tilde{\rho}}\tilde{\mathbf{B}} \times (\nabla \times \tilde{\mathbf{B}}) &= \tilde{\gamma}\,\tilde{\mathbf{v}} \\
\frac{\partial \tilde{\epsilon}}{\partial \tilde{t}} + \nabla \cdot (\tilde{\epsilon}\,\tilde{\mathbf{v}}) + \tilde{p}\,(\nabla \cdot \tilde{\mathbf{v}}) &= \tilde{\Gamma}\,\tilde{\epsilon} \\
\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} - \nabla \times (\tilde{\mathbf{v}} \times \tilde{\mathbf{B}}) &= \tilde{\eta}\,\nabla^2\,\tilde{\mathbf{B}}
\end{aligned}$$

– γ and Γ account for interaction with background particles (e.g. neutrinos, photons).
– η resistivity

$\tilde{}=$ conformal rescaled variables (e.g. Brandenburg, *et al.* PLB **392** (1997) 395, Subramanian and Barrow, PRD **58** (1998) 083502)

- radiation dominated universe:

$$\tilde{\mathbf{v}} = \mathbf{v} \qquad \tilde{\rho} = \rho\, a^4 \qquad \tilde{p} = p\, a^4$$
$$\tilde{\epsilon} = \epsilon\, a^4 \qquad \tilde{\mathbf{B}} = \mathbf{B}\, a^2 \qquad d\tilde{t} = a^{-1}\, dt$$

$$\Rightarrow \tilde{t} \propto a \propto T^{-1}$$

- matter dominated universe:

$$\tilde{\mathbf{v}} = \mathbf{v}\, a^{1/2} \qquad \tilde{\rho} = \rho\, a^3 \qquad \tilde{p} = p\, a^4$$
$$\tilde{\epsilon} = \epsilon\, a^4 \qquad \tilde{\mathbf{B}} = \mathbf{B}\, a^2 \qquad d\tilde{t} = a^{-3/2}\, dt$$

$$\Rightarrow \tilde{t} \propto \ln a \propto \ln T$$

we use an isothermal equation of state with speed of sound: $c_s^2 \equiv \partial p / \partial \rho$

Helicity

Definition of magnetic helicity:

$$\mathcal{H} \equiv \frac{1}{V} \int d^3x \, \mathbf{A} \cdot \mathbf{B}$$

– measure of knots and twists of magnetic field lines (complexity)
  field theory: Chern-Simons number (CP violation)
– Ideal MHD: H conserved, $\dot{H} = -2\,\eta \int d^3x \, \mathbf{B} \cdot (\nabla \times \mathbf{B})$
– Dissipates very slowly: $(\dot{H}/H)/(\dot{E}/E) \sim l_{\text{diss}}/L \ll 1$
– Helical magnetic fields gives rise to inverse cascade
  (e.g. Christensson, *et al.* PRE **64** (2001) 056405; Sigl, PRD **66** (2002) 123002)
– Helicity density estimate from magnetogenesis during EW PT
  $\mathcal{H} \sim 10^2 \, n_B \ll \mathcal{H}_{\text{max}} \approx L \, B^2$ (Vachaspati, PRL **87** (2001) 251302)

different damping regimes depend on the mean free path of interacting background particles (e.g. neutrinos, photons)

$$l_{\text{mfp}} \sim 1/\sigma\, n$$

- $l_{\text{mfp}} < L$: diffusion regime

$$\gamma\, \mathbf{v} = \nu\, \nabla^2\, \mathbf{v} \approx l_{\text{mfp}}\, \nabla^2\, \mathbf{v}$$

- $l_{\text{mfp}} > L$: free-streaming regime (drag force)

$$\gamma\, \mathbf{v} \approx -\alpha_{\text{drag}}\, \mathbf{v}$$

strength of viscosity: Reynolds number

$$R_e = \begin{cases} V\,L/\nu & \text{diffusion} \\ V/(\alpha_{\text{drag}}\,L) & \text{free-streaming} \end{cases}$$

⇒ turbulent regime: $R_e \gg 1$
⇒ viscous regime: $R_e < 1$

(almost) perfect conductivity: $\sigma \approx \eta^{-1}$

$$P_r = \frac{\nu}{\eta} \gg 1$$

⇒ damping of MF due to fluid viscosity / turbulence

Damping of magnetic fields

$$\dot{E}_{\mathsf{mag}} \approx E_{\mathsf{mag}} \frac{V}{L} = E_{\mathsf{mag}} \, \tau^{-1} \qquad ; \qquad \tau = ?$$

- turbulence: $R_e \gg 1 \quad ; (V_{\mathsf{A}} = B/\sqrt{4\pi(\rho + p)} \propto E_{\mathsf{mag}}^{1/2})$

$$\tau \approx \frac{L}{V_{\mathsf{A}}}$$

- viscous regime: $R_e < 1$

$$\tau \approx \frac{L}{V_{\mathsf{A}}} \Big/ R_e(V = V_{\mathsf{A}})$$

$\Rightarrow$ damping law: $\quad \tau(V_{\mathsf{A}}, L) \lesssim t_H \approx H^{-1}(T)$

- Assuming blue energy spectrum, i.e. $E_{\text{mag}} \approx E_g \, (l/L_g)^{-n}$ (E_{mag} comoving energy density)

$$E_{\text{mag}} \propto t^{-p} \quad \text{and} \quad L_{\text{com}} \propto t^{q}$$

e.g. with $p = 2\,n/(n+2) = 1.2$ and $q = 2/(n+2) = .4$ for $n = 3$ in the turbulent regime.

- Maximal helical fields, i.e. $\mathcal{H} \approx L \, E_{\text{mag}}$, in the turbulent regime:

$$p = 1/3 \quad \text{and} \quad q = 1/3$$

maximal helical state is reached when $L_{\text{com}}(T) \sim L^{\text{max}} \simeq L_g \, h_g^{-1/(n-1)}$

Ambipolar diffusion

$$\rho_i \left(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i\right) = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \rho_i \, \alpha_{\text{in}} \, (\mathbf{v}_i - \mathbf{v}_n)$$

$$\rho_n \left(\frac{\partial \mathbf{v}_n}{\partial t} + \mathbf{v}_n \cdot \nabla \mathbf{v}_n\right) = -\rho_n \, \alpha_{\text{ni}} \, (\mathbf{v}_n - \mathbf{v}_i)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}_i \times \mathbf{B})$$

$\alpha_{\text{ni}} = \frac{\rho_i}{\rho_n} \alpha_{\text{in}} \approx X_e \, \alpha_{\text{in}}$ and $\mathbf{v}_D \equiv \mathbf{v}_i - \mathbf{v}_n \ll \mathbf{v}_i$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla \times \left(\frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi \, \rho_i \, \alpha_{\text{in}}} \times \mathbf{B}\right)$$

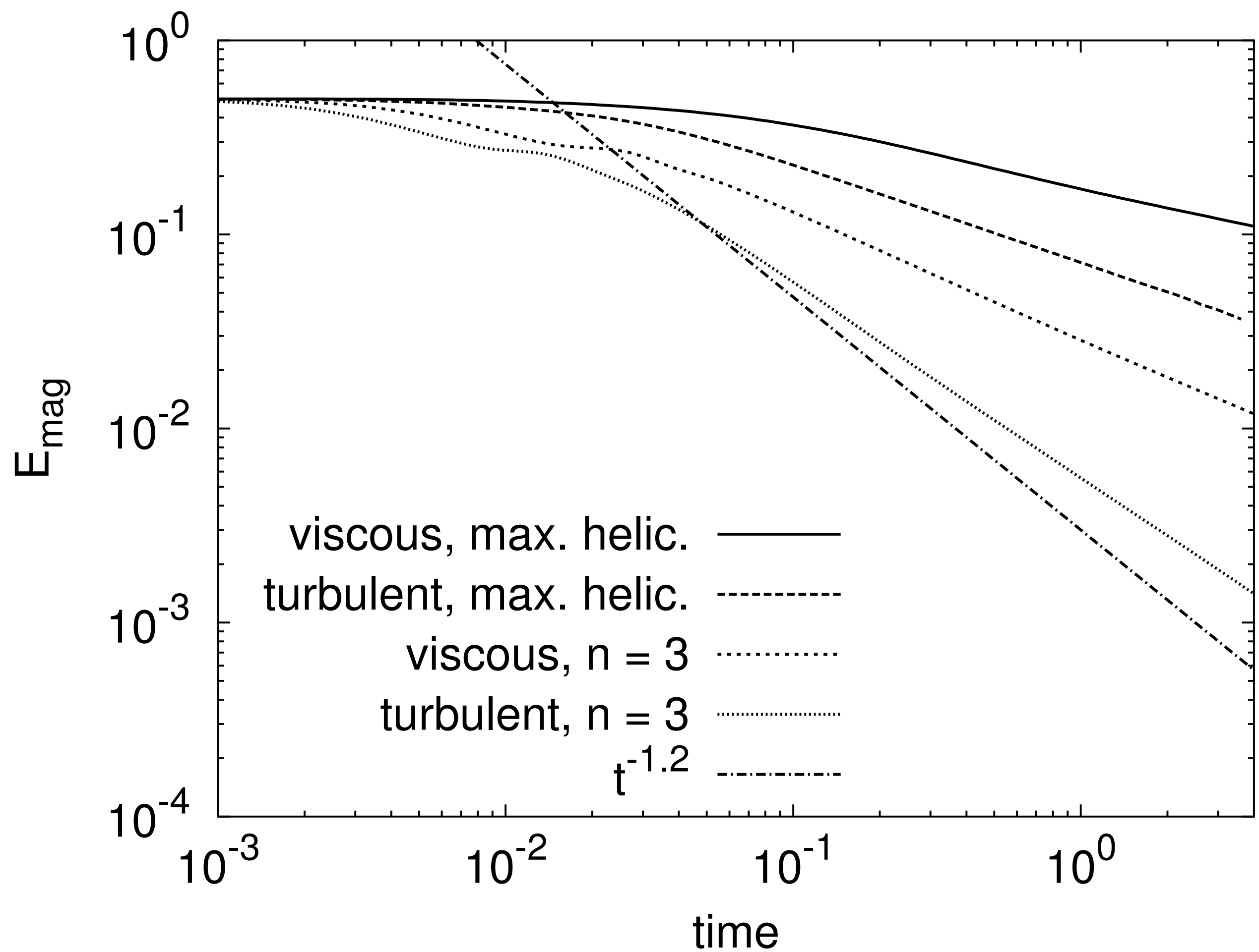
tight coupling, $v_i \sim v_n \Rightarrow R_{\text{amb}} \approx L \, \alpha_{\text{in}} \, X_e / v_A \gg 1$

Numerical simulations

- Numerical simulations with ZEUS-3D code: 3D MHD-solver, Eulerian mesh[1], staggered grid, optimized for vector computers
- Including dissipation terms: viscosity, resistivity and drag force
- Initial conditions: Gaussian random magnetic and velocity field with zero mean field and spectra $b_k \propto k^n$
- Initial magnetic field can be generated with maximal and fractional helicity due to the proper choice of a coordinate system in Fourier space, i.e. the complex unit polarization vectors $\{\mathbf{e}_+, \mathbf{e}_-, \mathbf{k}\}$
- Simulations of different damping regimes: turbulent, viscous diffusion, free-streaming

[1] we use up to 512^3 grid points

Evolution of comoving magnetic energy



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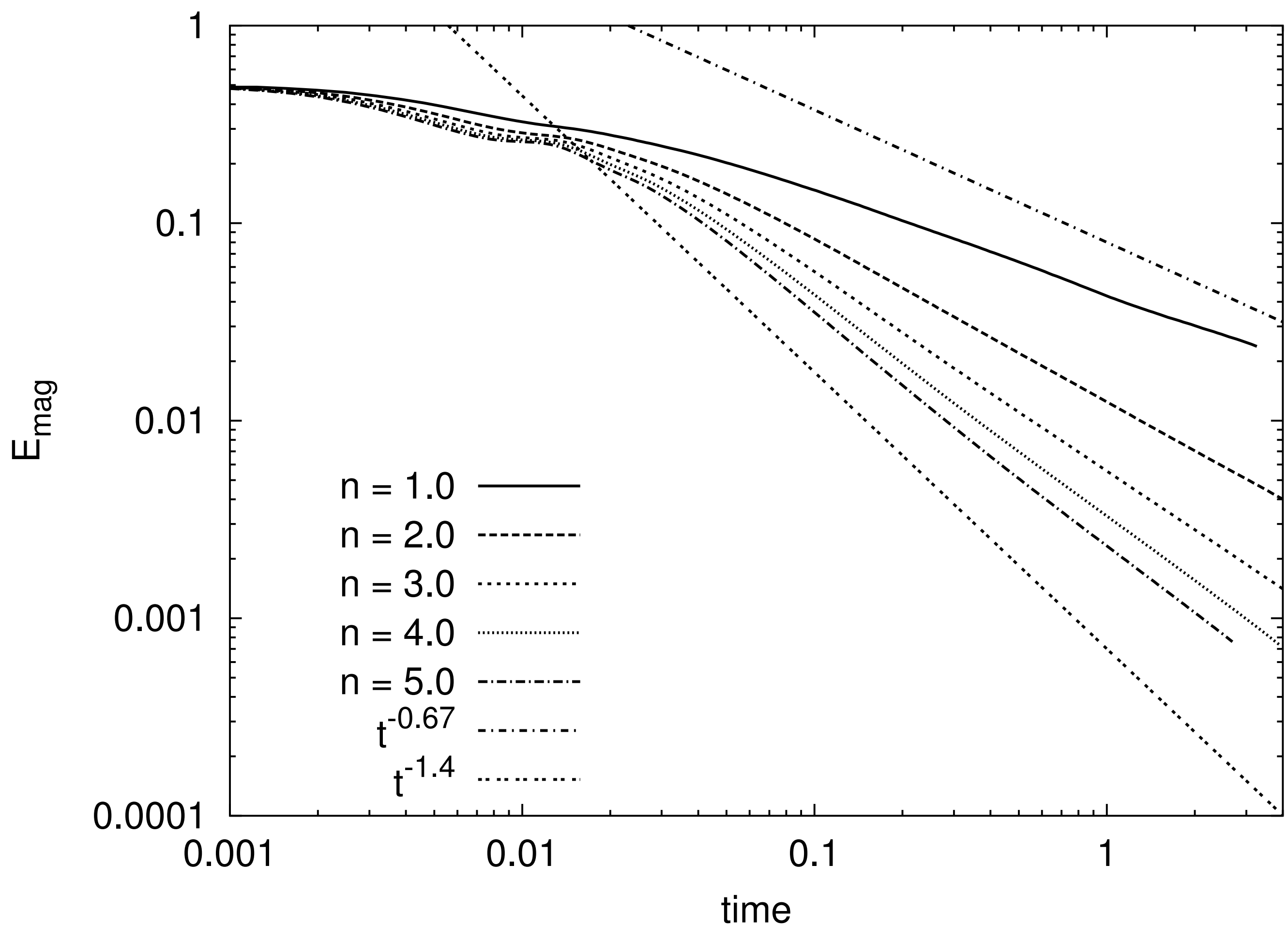
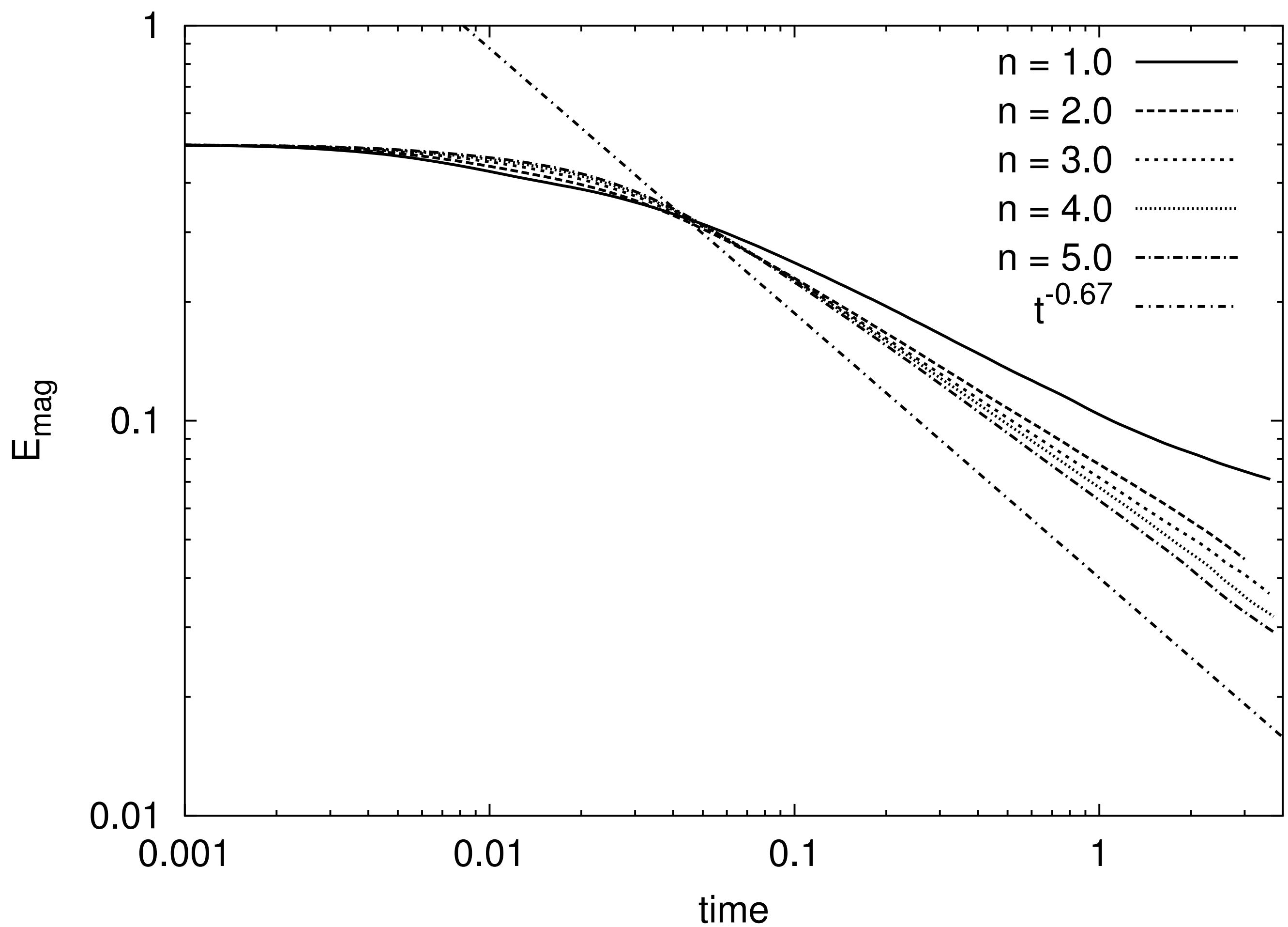
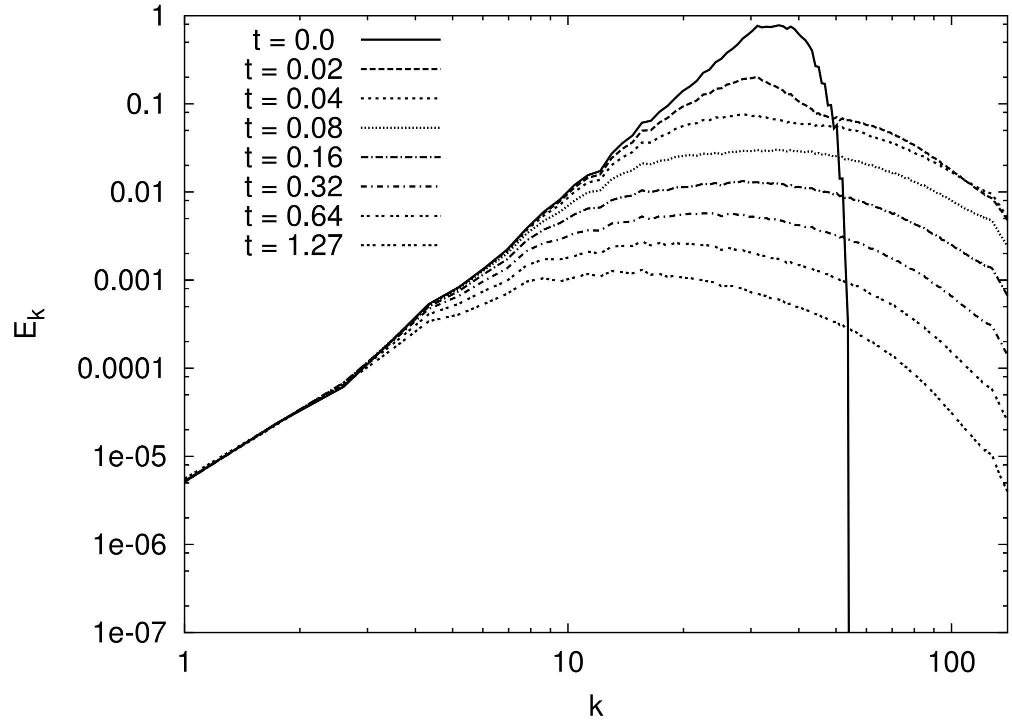
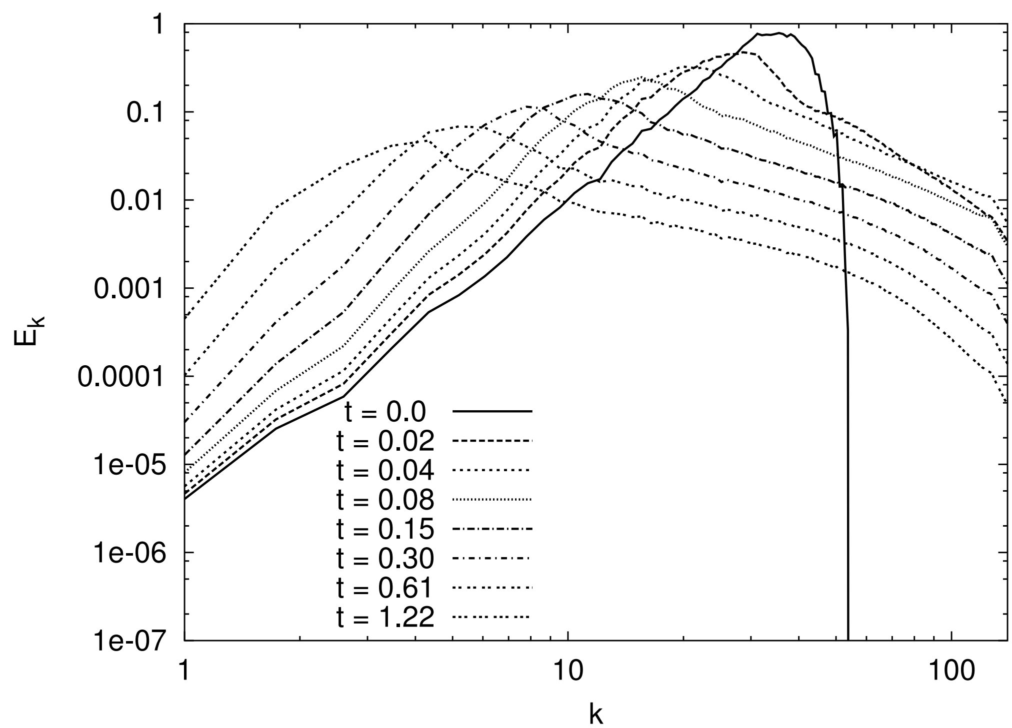


Evolution of maximal helical fields

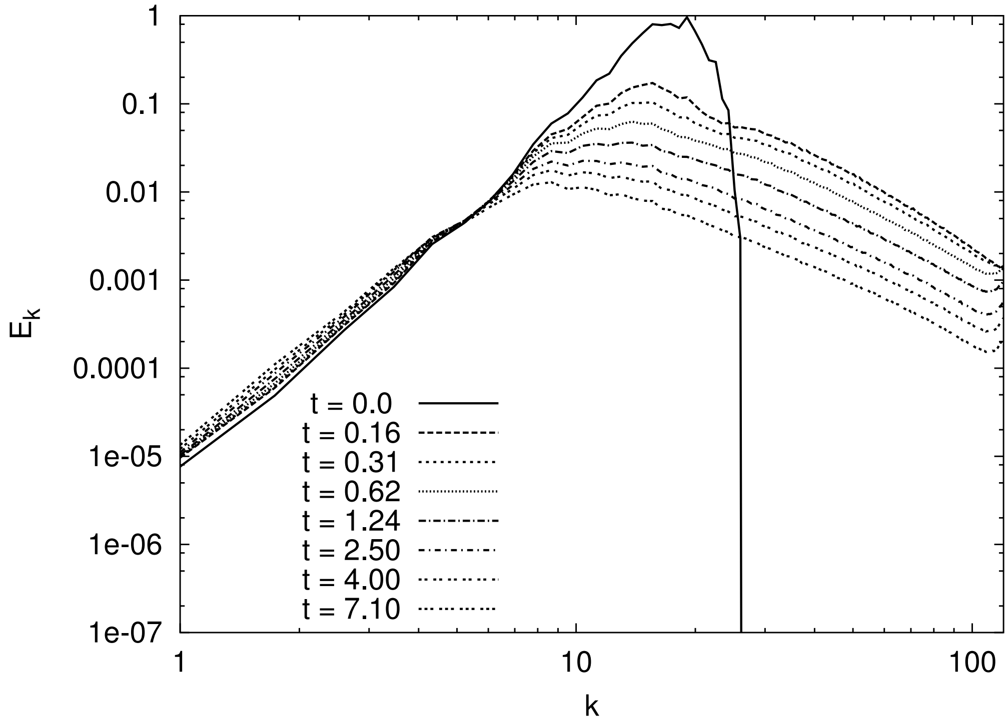


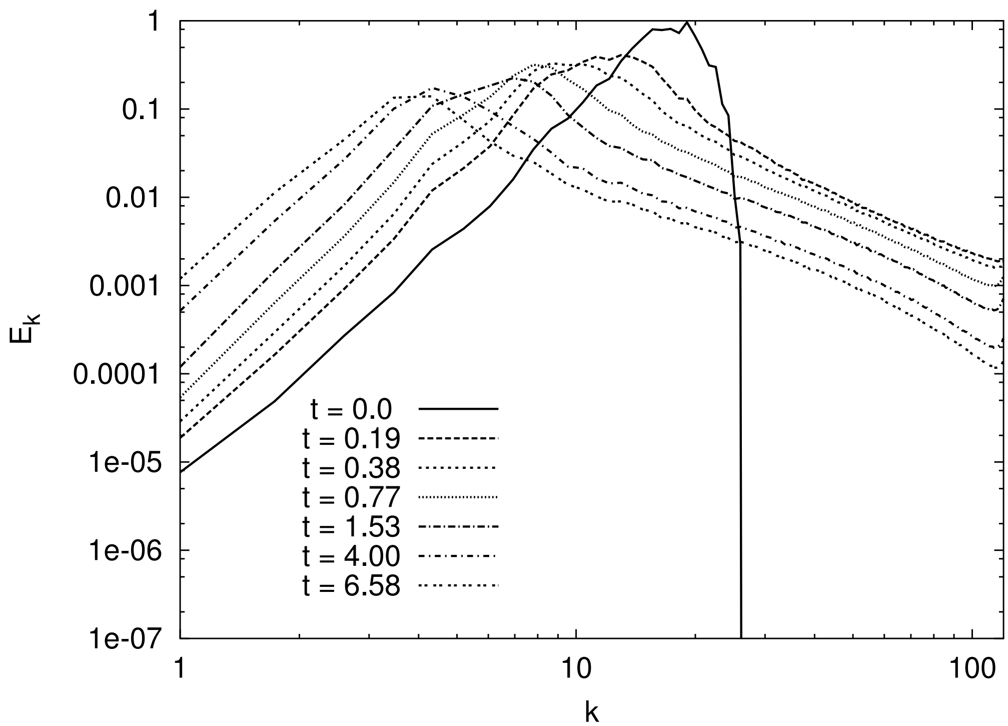
Turbulent regime



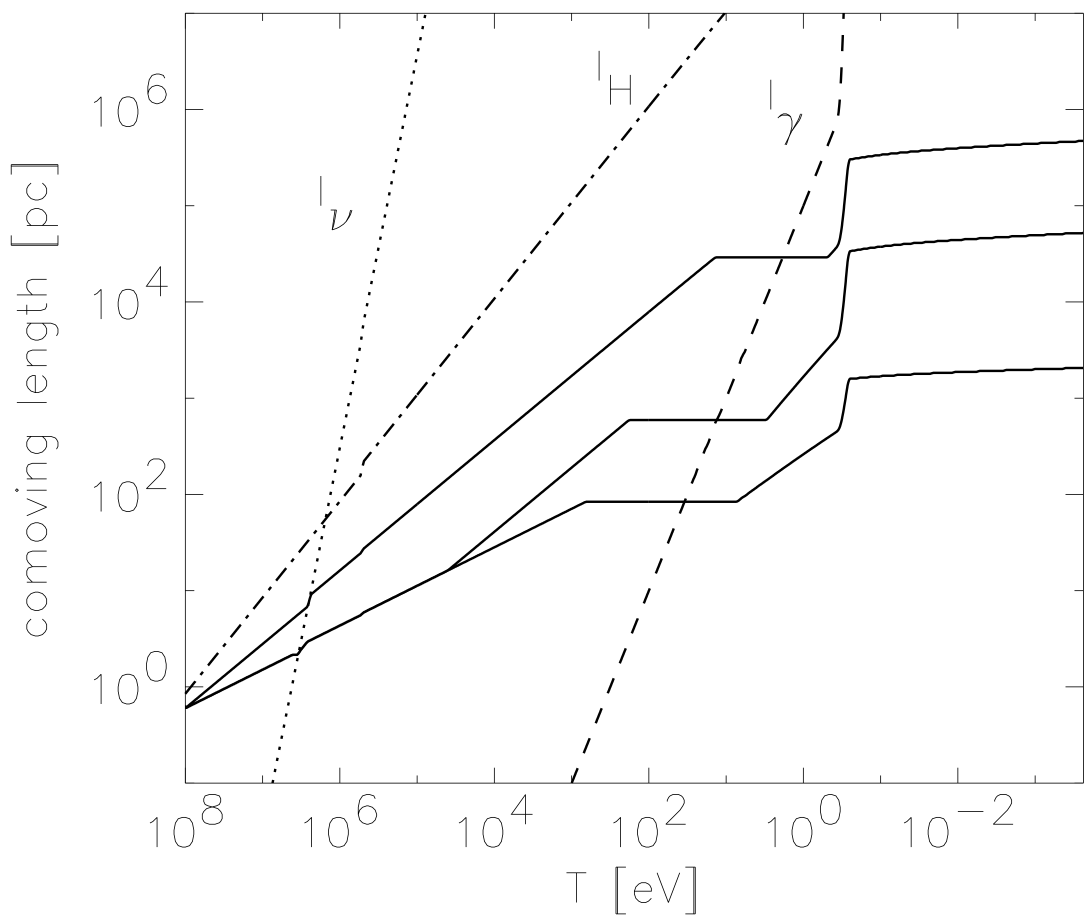


Viscous regime

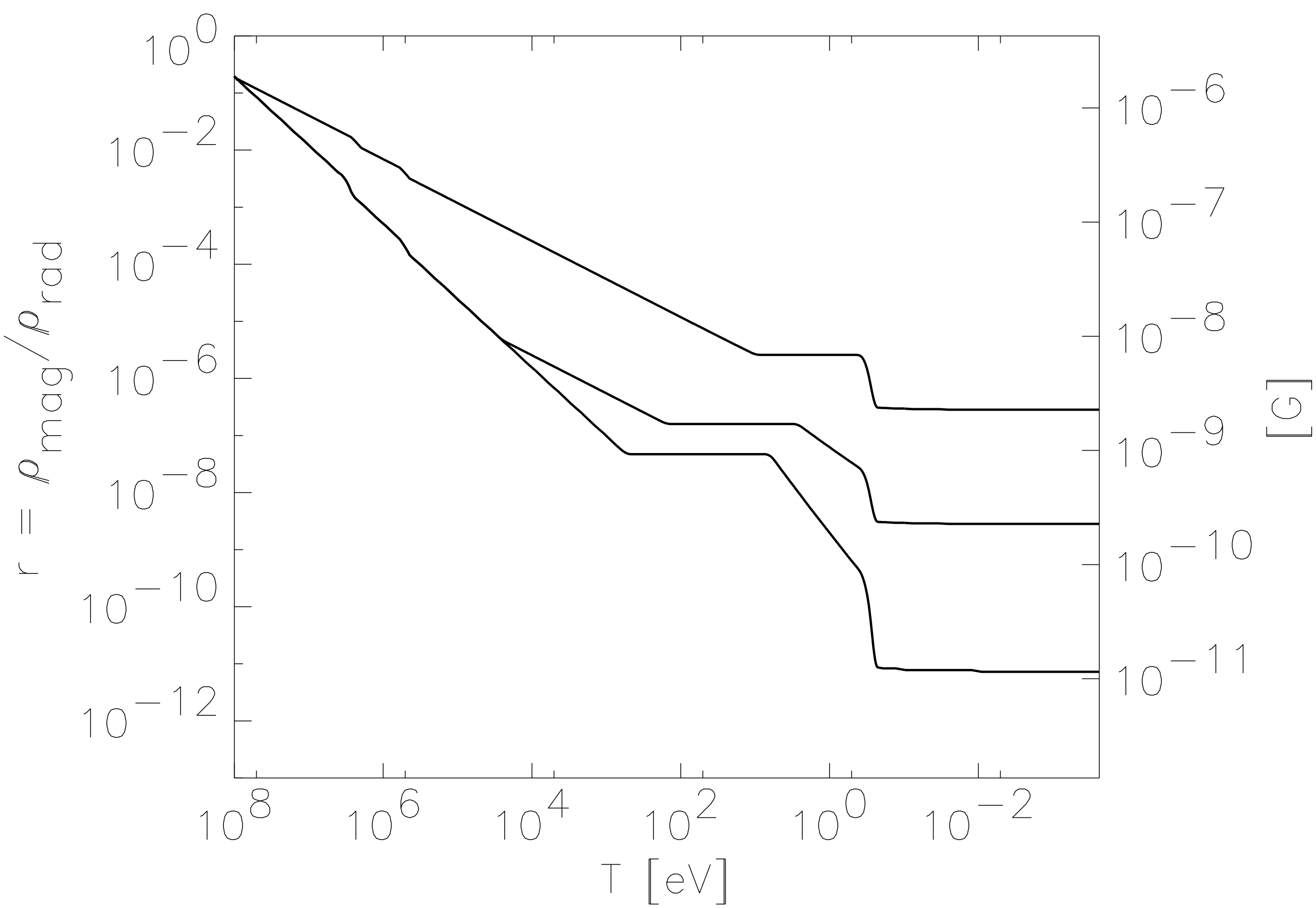




Evolution of magnetic coherence length



Evolution of magnetic energy density



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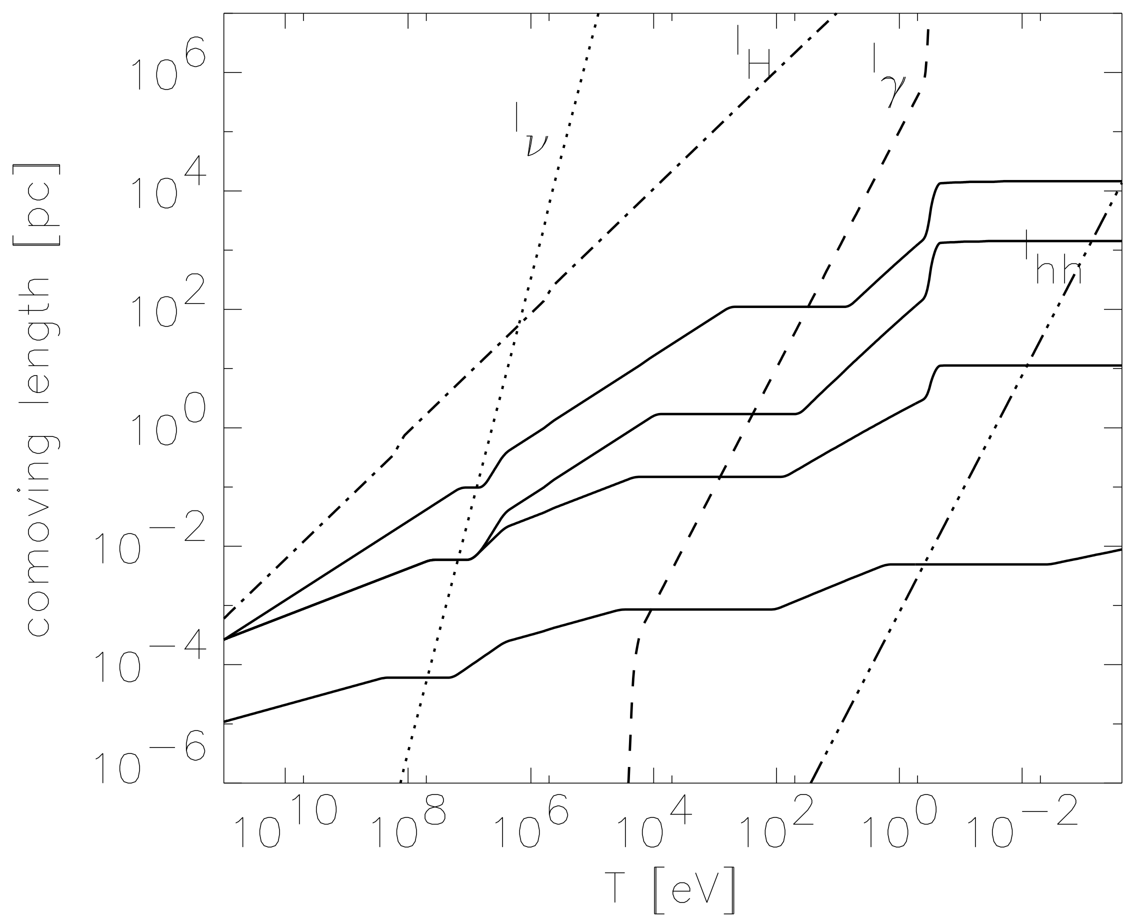

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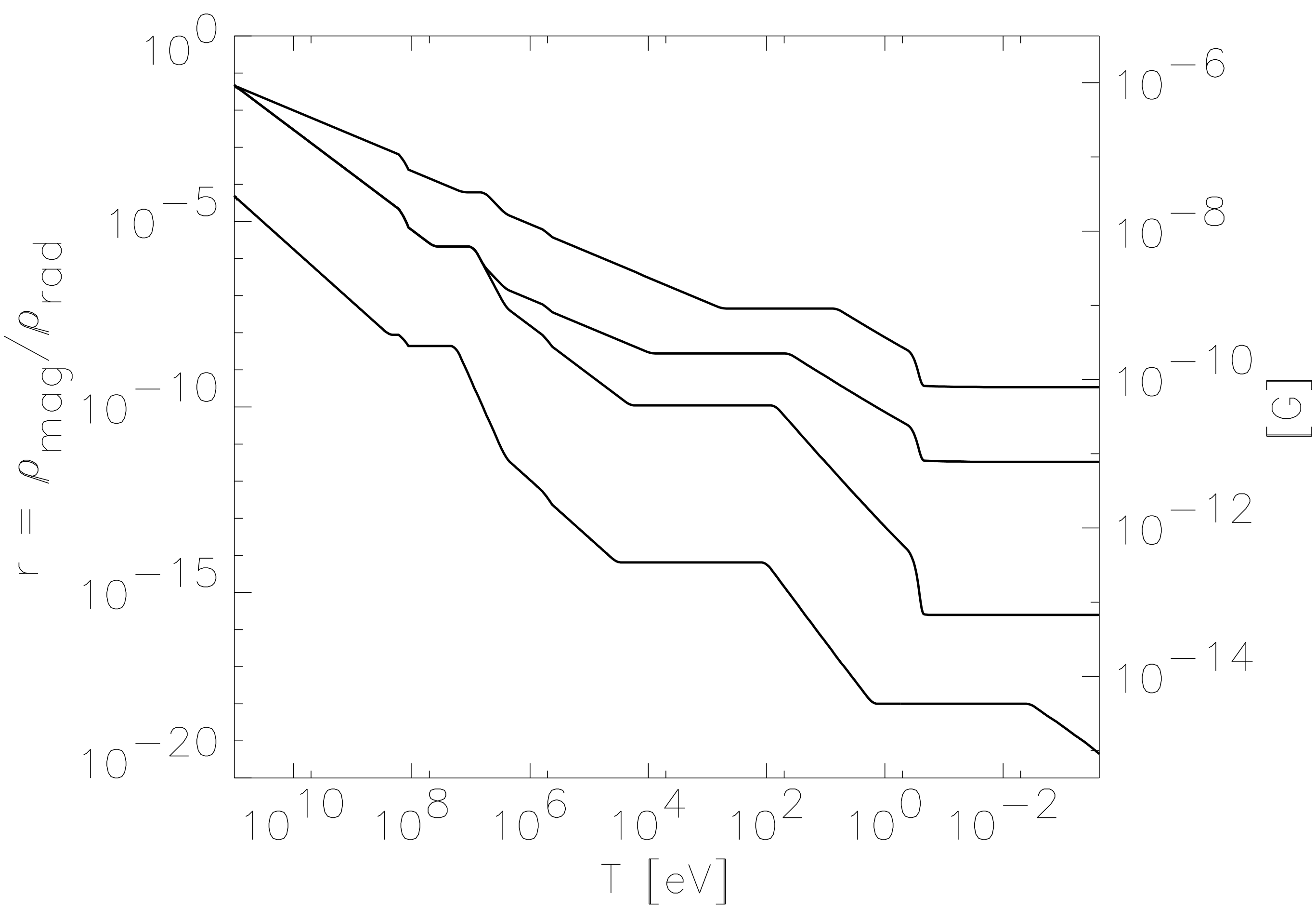
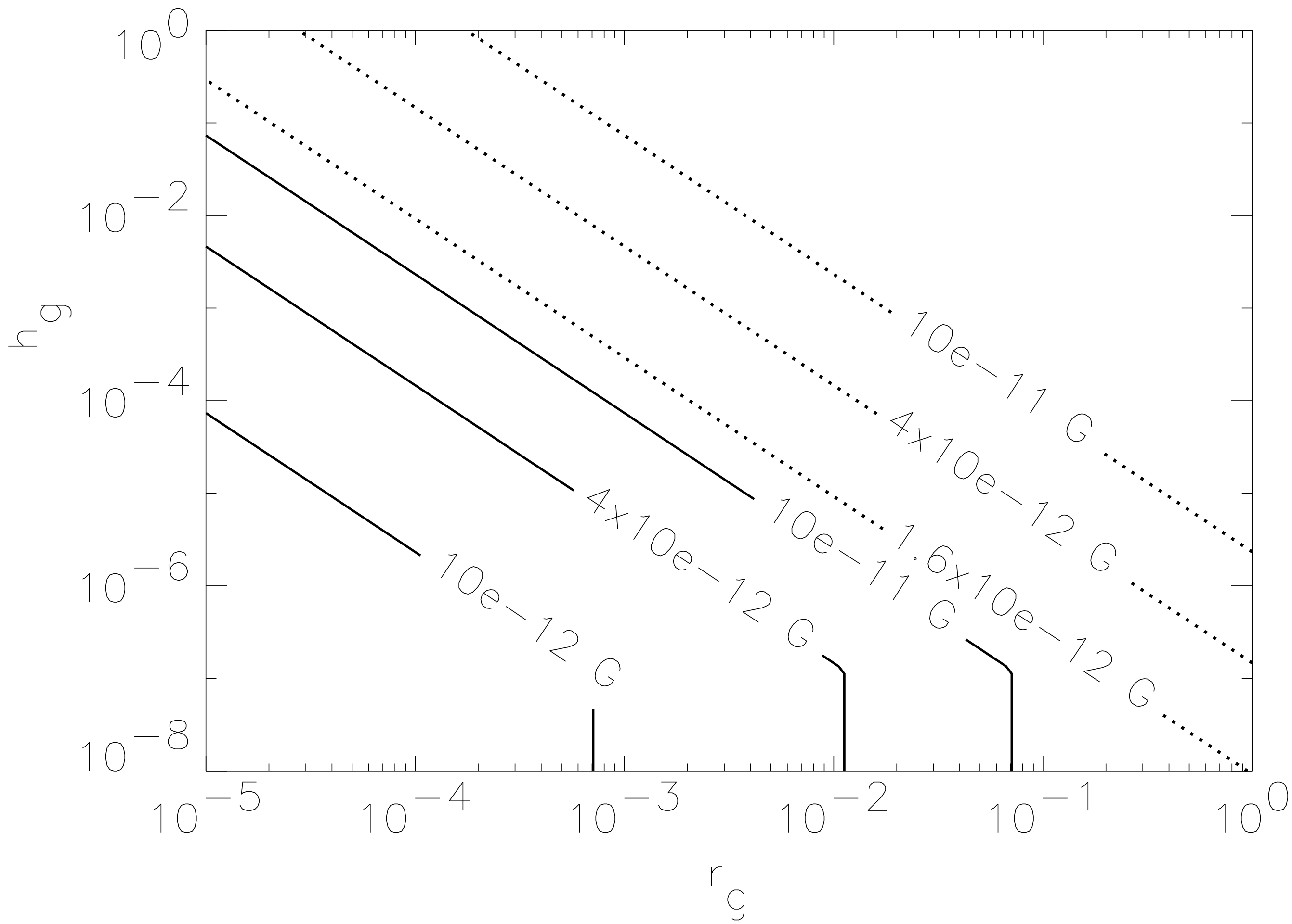


Final magnetic field strength



$T_g = 100\,\text{MeV}, n = 3$

Summary

- Magnetic fields in the early universe evolve not only by $B \propto T^2$
- PMF are not damped by ambipolar diffusion in the tight coupling regime
- Helicity plays a crucial role in turbulent MHD (inverse cascade)
- Cluster fields could be of primordial origin if $B \sim 10^{-11}$ G (Dolag, *et al.* A&A. **348** (1999) 351)