D-brane thermodynamics and cosmology

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Why considering brane cosmology/thermodynamics?

- Evidence that the Universe has emerged from a hot, dense phase

- String theory predicts that along with fundamental objects, strings, extended (p+1)-solitonic ($\tau_p \sim 1/g_c \sim 1/g_{op}^2$) objects exist: D-branes (string prejudice)

- Late, post-inflationary cosmology is OK, but little is known about the hottest epoch (of maximum curvature)

- The very early Universe history can be very different from the hot big-bang, as

  Extended objects can wrap over compact dimensions

  Branes have stringy internal structure

  They interact differently than particles

Outline

- Dual description of branes and correspondence principle:

  D- & Sugra- branes (G. Horowitz and J. Polchinski '96)

- Internal structure of branes, limit (Hagedorn) temperature

- Energy exchange (extremal or black) brane/bulk

- Stability of wrapped and unwrapped branes $\leftrightarrow$ dimension expansion

- Strings beyond the Hagedorn temperature

Branes as:

Gravity solution of

$$\mathcal{S} = \int d^{10}x\sqrt{g}\left[e^{-2\phi}(R + 4(\partial\phi)^2) - F^2_{(p+2)}/(2(p+2)!)\right]$$

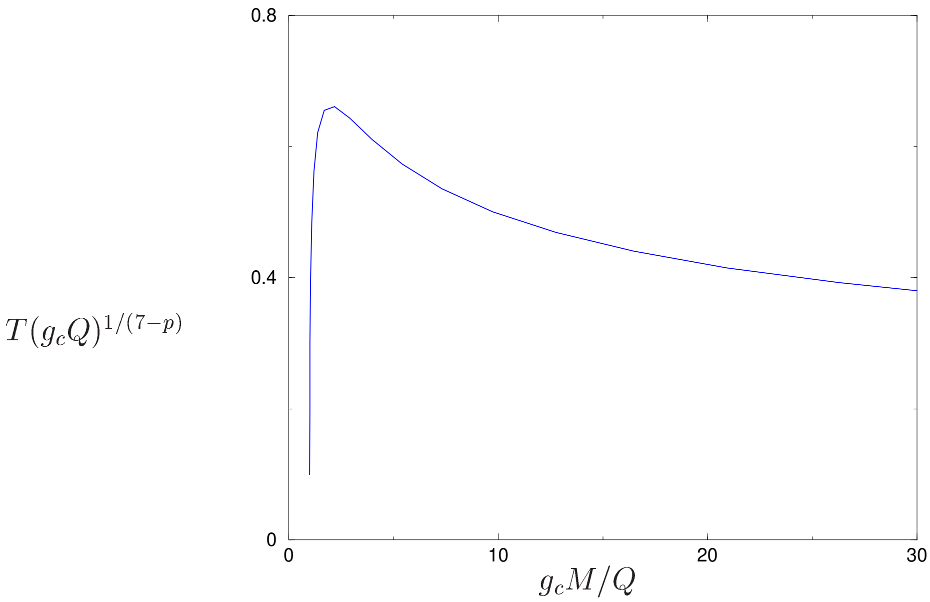
$$ds^2 = H^{-1/2}(-hdt^2 + dx^i dx_i) + H^{1/2}(dr^2/h + r^2 d\Omega^2_{8-p})$$

$$H(r) \equiv 1 + \sinh^2\alpha \left(\frac{r_0}{r}\right)^{7-p} \qquad h(r) \equiv 1 - \left(\frac{r_0}{r}\right)^{7-p}$$

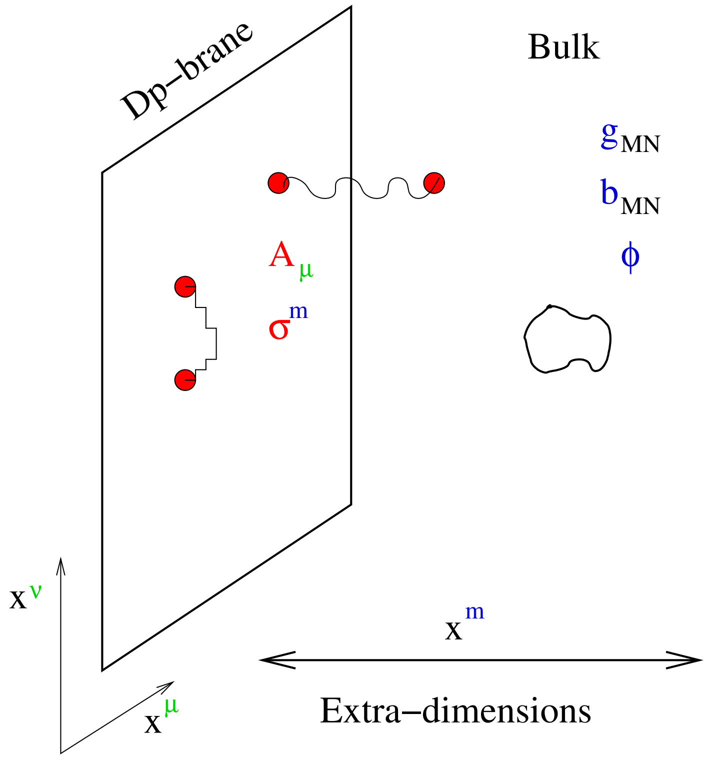
$$e^{\phi} = H^{(3-p)/4} \qquad F = \epsilon_{\mu_0\ldots\mu_{p+2}} Q$$

$$M = \frac{r_0^{7-p} V_{\|}}{G_N}\left(\cosh 2\alpha + \frac{9-p}{7-p}\right)$$

$$S = \frac{r_0^{8-p} V_{\|}}{G_N}\cosh\alpha \qquad Q = \frac{1}{g_c}\left(\frac{r_0}{\sqrt{\alpha'}}\right)^{7-p}\sinh 2\alpha$$



These Sugra branes are the same as the stringy Dp-branes



Open strings $\leftrightarrow$ Chan-Paton factors •: gauge degrees of freedom

A_μ gauge field, σ^m brane fluctuation (branon)

Closed strings $\leftrightarrow$ gravitation

$$M = N\tau_p V_\| + N^2 T^{p+1} V_\|$$

$$S = N^2 T^p V_\| \qquad Q = N$$

Proof: They both

- Preserve 1/2 Susy (if extremal, $r_0^{\frac{7-p}{2}} \propto e^{-\alpha} \to 0$, $S, T = 0$)
- Carry RR-charge (under (p+2) F-form)

Gravity picture valid at

low coupling $g_c \ll 1$ (at $g_c \sim 1$ the 11^{th} dim. opens up)

low curvature ($g_c Q \gg 1$)

Field theory picture at low coupling $g_{op}^2 N \ll 1$

Correspondence principle: pictures match when horizon curvature $\sim \alpha'^{-1}$ ($\alpha'/r_0^2 \sim \cosh\alpha$)

No need for correspondence in the case $p = 3$, in both pictures

$$S \propto E^{3/4} \quad (c_V > 0) \qquad S \propto E^{5/4} \quad (c_V < 0)$$

The $c_V < 0$ branch $\sim$ stack of brane and antibranes

Black brane case (U. Danielsson, A. Güijosa and M. Kruczenski, '01)

$$M = (N + \bar{N})\tau_p V_{\|} + N^2 T^{p+1} V_{\|} + \bar{N}^2 \bar{T}^{p+1} V_{\|}$$

$$S = N^2 T^p V_{\|} + \bar{N}^2 \bar{T}^p V_{\|} \qquad Q = N - \bar{N}$$

and

$$\frac{2}{T_{FT}} = \frac{1}{T} + \frac{1}{\bar{T}}$$

Tachyon condenses (no $b\bar{b}$ instability) as

$$\delta F = -4\tau_p (\delta t)^2 + N\beta^{1-p} (\delta t)^2$$

for $T > \frac{\alpha'^{1/2}}{(g_c N)^{\frac{1}{p-1}}} \implies$ no low coupling description

Asymptotic string states $d(E)$

$$d(E) = \frac{e^{\beta_{Hag} E}}{E^a}$$

microcanonical ensemble analysis shows that energy ends up mostly in very long strings

Limit temperature in (open) string theory $T_{Hag} \sim \sqrt{\alpha'}$ ($c_V \to \infty$ for $T \to T_{hag}$), closed strings are not limiting and give energy to limiting ones (at string scale density)

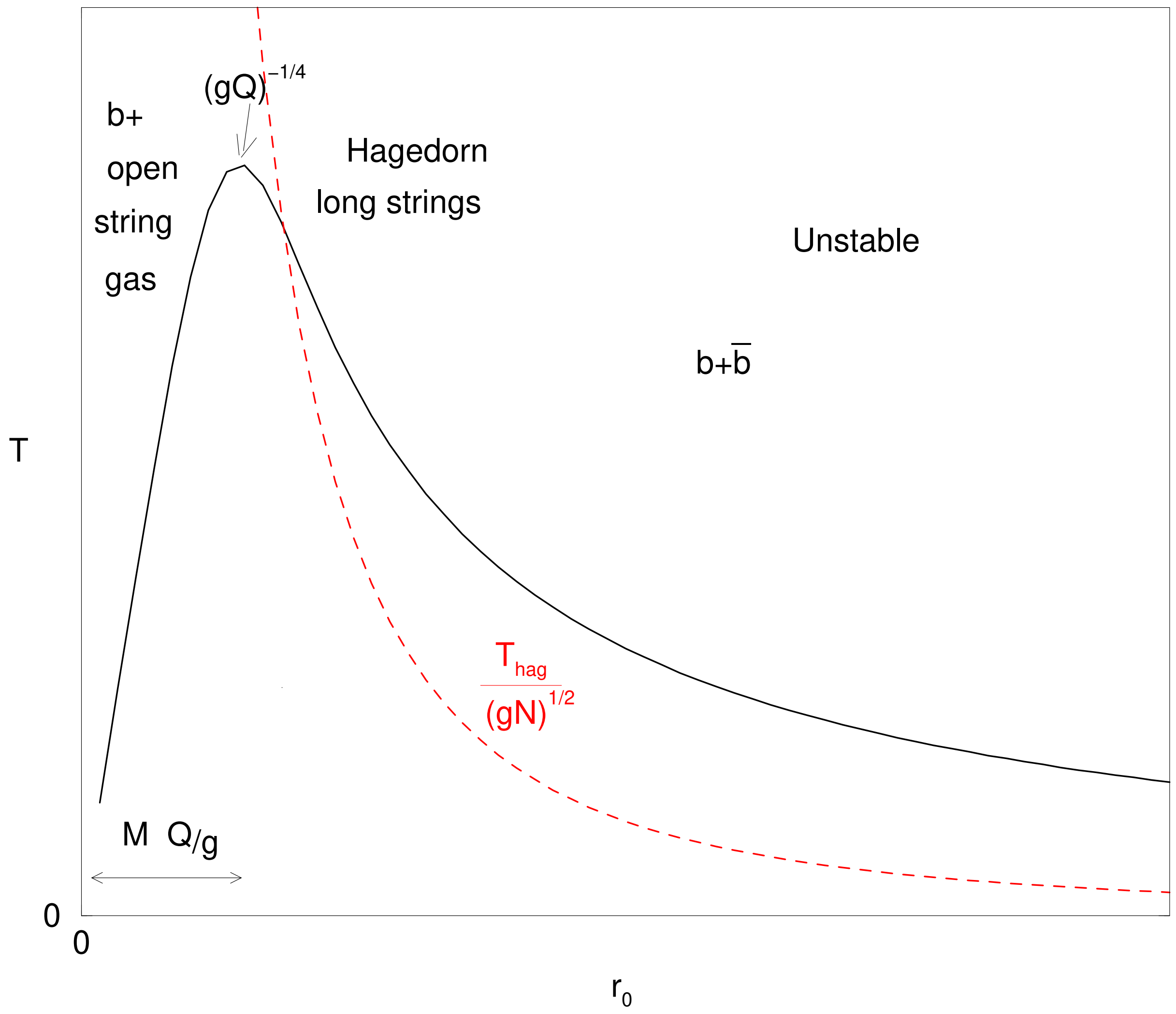
$$\frac{\partial S_{tot}}{\partial E_L} = \beta_L - \beta_{NL} < 0$$

Among L-systems, smaller a (higher p) $\implies$ higher energy

(S. Abel, J. Barbon, I. Kogan and E. Rabinovici, '99)

Also in the Hagedorn regime tachyon can condense, provided enough energy density is available (K. Hotta, '03)

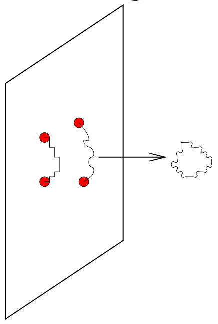
$$\frac{E}{V_{\parallel}} \gtrsim \frac{\exp(V_{\perp} \tau_p)}{V_{\perp}}$$



$(gQ)^{-1/4}$
b+
open
string
gas
Hagedorn
long strings
Unstable
$b+\bar{b}$
T
$\frac{T_{hag}}{(gN)^{1/2}}$
M Q/g
0
0
r_0

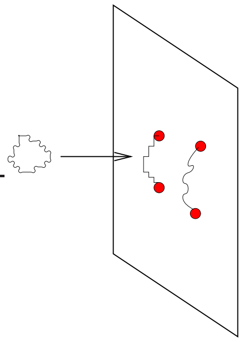
Energy trade

At sub-Hagedorn temperature field theory analysis



$$\frac{1}{\rho_{br}}\frac{d\rho_{br}}{dt} \sim \frac{T^{p+\delta}}{M_{Pl}^{p+\delta-1}}$$

$$\frac{1}{\rho_{bk}}\frac{d\rho_{bk}}{dt} \sim \frac{E^{p+\delta}}{M_{Pl}^{p+\delta-1}} \times (ER)^{-\delta} \sim \frac{E^{p}}{\left(M_{Pl}^{(p)}\right)^{p-1}}$$



(N. Arkani-Hamed, S. Dimopoulos and G. Dvali, '98)

The bulk (only gravitational interaction) is not likely to be in thermal equilibrium: $T_h < M_{Pl}$ (as $g_c < 1$), it will expand with

$$H \sim \frac{\rho_{bk}^{1/2}}{M_{Pl}^{4}}$$

In Hagedorn regime open strings soak up the available energy but in sub-Hagedorn $\rightarrow$ leaking (moreover $\Gamma_{bk\rightarrow br} < H$)

Anyway if not in thermal equilibrium...

Brane gas cosmology

(R. Brandenberger and C. Vafa, '91)

Branes wrapped around compact dimensions prevent expansion unless they meet and annihilate (possible in $d < 2p + 1$)

(A. Kaya and T. Rador, '03)

Black branes are unstable if wrapped over big tori ($R > 1/T_{Haw}$)

(R. Gregory and R. Laflamme, '94; Danielsson et al.)

$$\frac{\partial S}{\partial R} < 0$$

A black brane eventually decays into lower dimensional brane

$$S_p \propto M^{\frac{8-p}{7-p}} (RV_{p-1})^{-\frac{1}{7-p}} \rightarrow S_{p-1} \propto M^{\frac{9-p}{8-p}} V_{p-1}^{-\frac{1}{8-p}}$$

or black holes, but stable if wrapped on small tori ($R \lesssim T_{Haw}$)

Extremal branes (wr. & uw.) are stable at high T ($T > 1/R$)

At low T wrapped branes are unstable if wrapped $R < 1/T$, as

$$S_w = N(NR)L^{p-1}T^p \qquad vs. \qquad S_{uw} = N^2 L^{p-1} T^{p-1}$$

but if $R, L < 1/T$ then $S_{uw} > S_w$

Need of a small compact volume, expansion, and that our dimensions be free from windings

Conclusions (?)

- In early Universe Hagedorn phase matters (strings $\neq$ p.cles)
- Relevance of extended objects: wrapping

but... more problems than results

- Necessity of small dims. from the beginning or homogeneously wrapped extremal brane
- Analysis of the annihilation rate of $D-p$ branes is missing
- We can't follow the brane decay
- Stability radius $R \sim R_{sch}$
- Our brane (if we don't live in the bulk) must not prevent expansion

What's beyond Hagedorn phase? (See next slide, if time left)

The Hagedorn transition $\leftrightarrow$ disappearance of open strings

Thermodynamic equilibrium $\rightarrow X^0$ euclidean, compact, β-periodic

In string theory X^0 acquires a potential due to vortex condensation : vortexes are

$T < T_{Hag}$	$T \sim T_{Hag}$
irrelevant	condense

KT phase transition on the 2-D world-sheet

Vortexes break periodicity of X^0 and the potential dynamically localize X^0 (I. Kogan, '87; M. Maggiore, '02)

Cosine potential coincide with tachyon deformation of the world sheet

$$\delta S_2 = \int d^2 z \; t(X) e^{i\frac{R}{\alpha'}(X^0_L - X^0_R)} + h.c.$$

$$\downarrow$$

array of D-Branes at $X^0 = i2\pi(n + 1/2)$

D-branes at imaginary time have been shown to be a closed string background , not the vacuum, time-dependent

(D. Gaiotto, N. Itzhaki and L. Rastelli, '03)

(Very) Indirect confirmation that T_{Hag} is limiting for open string systems