Holographic bounds on the UV cutoff scale in inflationary cosmology

Esko Keski-Vakkuri and Martin S. Sloth

JCAP08(2003)001 [hep-th/0306070]

Helsinki Institute of Physics

University of Helsinki

August 2003

Motivations

- Can we learn something about fundamental physics from inflation? (Trans-Planckian effects in the CMB)
- The Holographic principle is thought to be a generic feature of fund. phys.

$$\text{GSL}: \; dS_{total} = dS_{matter} + dS_{horizon} \geq 0$$

Matter system with arbitrary entropy could violate GSL

$\Rightarrow$ Bekenstein bound ... Covariant bound

$\boxed{\text{Holography}}$

Holography: bound on fund. d.o.f.

Cutoff determines d.o.f. of eff. field-th. of cosmic density pert.

$\Rightarrow$ Bound on cutoff from holography

D-bound

- De Sitter horizon is associated with entropy:

$$S_0 = \frac{1}{4}A_0 = 4\pi \frac{1}{H^2} \qquad \textit{Gibbons-Hawking}$$

- Consider freely falling matter system inside the horizon of some observer who will see it redshift away and convert into empty de Sitter space

- The entropy of the initial state is:

$$S = S_m + \frac{1}{4}A_c \longleftarrow \text{initial area}$$

while the entropy of the final state is: S_0

$$\text{GSL } (S \leq S_0) \Rightarrow \boxed{S_m \leq (A_0 - A_c)}$$

$\longrightarrow$ Natural to expect ϵ-dependent upper bound on entropy in cosmic perturbations during *slow-roll*

Entropy of cosmological perturbations

Evolution of cosmic pert. is well described by a free scalar field in de Sitter, initially in the Bunch-Davies Vacuum

- On Sub-horizon scales: $n_k = |\beta_k|^2 << 1 \quad \Rightarrow$
  fluctuations quantum. Entropy??

- On Super-horizon scales: $n_k = |\beta_k|^2 >> 1 \quad \Rightarrow$
  fluctuations classical $\rightarrow$ natural coarse graining $\rightarrow$
  non-zero entropy: $\quad S_k = (n_k + 1)\ln(n_k + 1) - n_k \ln n_k$

To find any constraints from holography, we need to evaluate the entropy of sub-horizon modes

$$\boxed{\textbf{Entanglement entropy}}$$

Entanglement entropy

The quantum field describing cosmic density perturbations is prepared on an initial vacuum state

$\longrightarrow$ Globally in the space-time the (fine-grained) entropy is zero:

The field starts in a pure state:

$$\tilde{\rho} = |0, in\rangle \langle 0, in|$$

and will always remain in it!

However, trace over d.o.f. outside the horizon radius r_0 and define a reduced density matrix:

$$\boxed{\tilde{\rho}_{in} = \mathrm{Tr}_{r>r_0} (\tilde{\rho})}$$

This describes the system within the horizon!!

- Now $\tilde{\rho}_{in}$ is a mixed state with an associated entropy:

$$S_{in} = -\mathrm{Tr} (\tilde{\rho}_{in} \ln \tilde{\rho}_{in})$$

- It has been shown that the entropy is

$$S_{in} \simeq 0.3 \, (r_0/\varepsilon)^2$$

ε is the UV cutoff (length) scale

[Scales like horizon area]

(Srednicki '93, Muller and Lousto '95)

Entanglement vs. coarse-grained entropy

★ Compare the entanglement entropy with coarse-grained entropy in the sub-horizon region

$\downarrow$

Giovannini and Gasperini ('93) suggested that the entropy per comoving mode is

$$S_k = 2r_k \longleftarrow \text{squeezing parameter}$$

For small squeezing

$$r_k \sim |\beta_k| = \frac{aH}{2k}$$

Hence, the entropy contained in the relativistic particles on sub-horizon scales ($n_k << 1$) is given by

$$S_{cg} = \frac{V}{\pi^2 a^3} \int_{aH}^{a\Lambda} dk k^2 \, \frac{aH}{2k} \simeq \frac{1}{3\pi} \left(\frac{\Lambda}{H}\right)^2 \qquad (V = \tfrac{4}{3}\pi H^{-3})$$

Agrees with the entanglement entropy up to an irr. numerical factor of order one

★ It is not evident why the coarse-graining process is valid, since it drops out information not manifestly associated with a region of space-time.

★ However, the agreement with entanglement entropy adds credibility to it

Entropy bounds

- The cosmological perturbations on sub-horizon scales is a relativistic fluid with energy density

$$\rho = \frac{\tilde{\varepsilon}}{2} 3H^2 M_p^2 \quad , \qquad \tilde{\varepsilon} << 1$$

- For a closed system, matter would dilute (redshift) away and the horizon would change by

$$\boxed{\Delta A = A_f - A_0 = \frac{\tilde{\varepsilon}}{2} A_f}$$

A_0 is the horizon area in the initial de Sitter space with matter (ρ) include and

$$A_f = 4\pi H^{-2}$$

is the horizon area of the final pure de Sitter space

$\boxed{\textbf{D-bound}}$

$$\Downarrow$$

$$\boxed{S \leq \frac{1}{4} \frac{\Delta A}{l_p^2} = 4\pi^2 \tilde{\epsilon} \frac{M_p^2}{H^2}}$$

Indirect bound

- During slow-roll the horizon changes by (in one e-fold)

$$\Delta A \;\simeq\; \frac{\epsilon}{2} A \;=\; 4\pi\epsilon\, \frac{1}{H^2}$$

Thus, during slow-roll the entropy that exits the horizon during one e-fold is limited by

$$\boxed{\Delta S_{exit} \;\leq\; \frac{1}{4}\frac{\Delta A}{l_p^2} \;=\; 8\pi^2\epsilon\frac{M_p^2}{H^2}}$$

Applications

★ Consider the coarse-grained or entanglement entropy

$$S \simeq C\left(\frac{\Lambda}{H}\right)^2 \qquad C = \text{ const. of order one}$$

$\longrightarrow$ Direct bound is trivially satisfied

(a hint that we are not doing too much coarse-graining)

★ Indirect bound is more interesting

$$\Delta S_{exit} \;=\; \frac{1}{\pi}\frac{\Lambda^2}{H^2}$$

Indirect bound $\Rightarrow$ $\boxed{\Lambda \lesssim \sqrt{8\pi^3\epsilon}M_p}$

(Compare to Albrecht and Kaloper '02: $\Lambda \leq \epsilon^{1/3}\left(HM_p^2\right)^{1/3}$

which in our approach corresponds to a coarse-graining

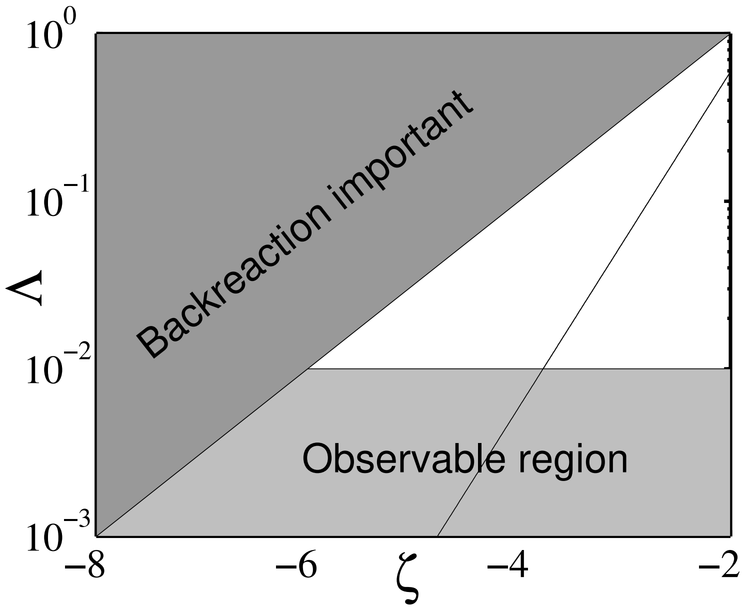
with entropy per mode approximated by a constant)

★ If the modes are initially taken to be in the α-vac.

(Goldstein and Lowe '02) one finds:

$$\Lambda \;\lesssim\; 6\pi^2\epsilon M_p$$

Figure



$\epsilon = 10^{\zeta}$ vs. the cutoff scale Λ.

- For $H \sim 10^{-4} M_p$, the range in which trans-Planckian effects are observable are indicated.
- If $H/\Lambda > 10^{-2}$, the trans-Planckian effects are observable (Bergström et al. '02).
- Region excluded in standard case is indicated by the dark-gray region. The region below the additional line is the region allowed by the bound obtained from the α-vacua scenario.

Conclusions

- Entanglement entropy seems to be useful in understanding the entropy of perturbations modes
- The redshifting of modes $\Leftrightarrow$ flux of entropy, leads to entropy bounds on the cosmological perturbations on sub-horizon scales (Direct and Indirect)
- The bounds are potentially interesting if the vacuum of perturbations is non-standard (α-vac.)
- Also specific models of trans-Planckian cosmology could be interesting in this context