Braneworld Dynamics with the `BraneCode`®

Johannes Martin

(University of Toronto, CITA)

Project with:

- Gary Felder
- Andrei Frolov
- Lev Kofman
- Marco Peloso

Motivation

Braneworlds

☞ Motivated by Superstring and Supergravity theories

☞ Applications to:

Particle Physics

"Stringy" Models

- e.g. Hořava-Witten
- Interesting Phenomenology
- No Dynamics

Cosmology

Toy Models

- e.g. Randall-Sundrum
- 4d-Gravity
- Trivial Dynamics
- Stabilization?

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Cosmology

Our Toy Model

- Nontrivial Dynamics
- Geometric Inflation
- Dynamical Hierarchy
- Brane Collision

Toy Models

- e.g. Randall-Sundrum
- 4d-Gravity
- Trivial Dynamics
- Stabilization?

Action:

$$S = \int d^5x \sqrt{|g|} \left\{ \frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right\} - \sum_{i=1}^{2} \int_{b_i} d^4\xi \sqrt{|\gamma|} \left\{ [K]_i + U_i(\phi) \right\}$$

Gauge:

$$ds^2 = e^{2B(y,t)} (dy^2 - dt^2) + e^{2A(y,t)} d\vec{x}^2$$

Comoving y - coordinate:

- Branes located at $y = 0$ and $y = 1$
- Distance between branes: $D(t) = \int_0^1 e^{B(t,y)} dy$
- Proper time on either branes: $d\tau = e^B \big|_{y=0,1} dt$
- Induced 4d - Hubble parameter: $h = e^{-B} \big|_{y=0,1} \dot{A}$

Setup

E.O.M.:

$$\begin{aligned}
\ddot{A} - A'' + 3\dot{A}^2 - 3A'^2 &= \tfrac{2}{3}\,\mathrm{e}^{2B} V \\
\ddot{B} - B'' - 3\dot{A}^2 + 3A'^2 + \frac{\dot{\phi}^2}{2} - \frac{\phi'^2}{2} &= -\tfrac{1}{3}\mathrm{e}^{2B} V \\
\ddot{\phi} - \phi'' + 3\dot{A}\dot{\phi} - 3A'\phi' &= -\mathrm{e}^{2B}\frac{dV}{d\phi}
\end{aligned}$$

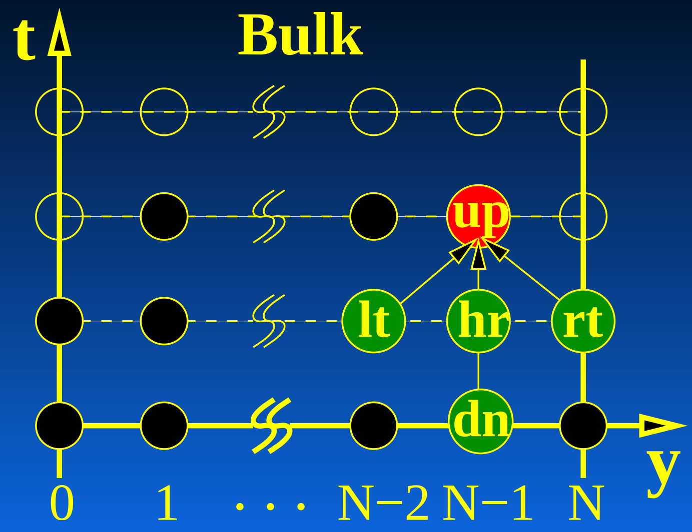
Two Constraint Equations

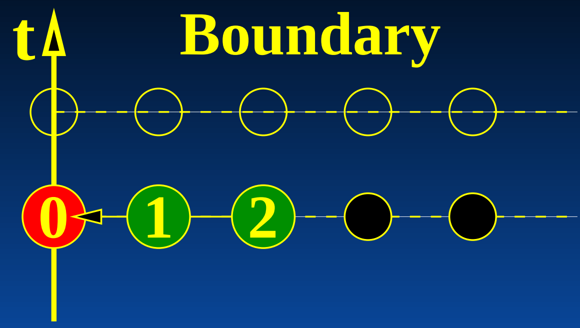
- Imposing initial conditions
- Check accuracy of numerical integration

Boundary Conditions:

$$\begin{aligned}
A'|_{y=0,1} = B'|_{y=0,1} &= \mp\tfrac{1}{6}\,\mathrm{e}^{B}\,U\big|_{y=0,1} \\
\phi'|_{y=0,1} &= \pm\tfrac{1}{2}\,\mathrm{e}^{B}\,\frac{dU}{d\phi}\bigg|_{y=0,1}
\end{aligned}$$

BraneCode®





Bulk Equations

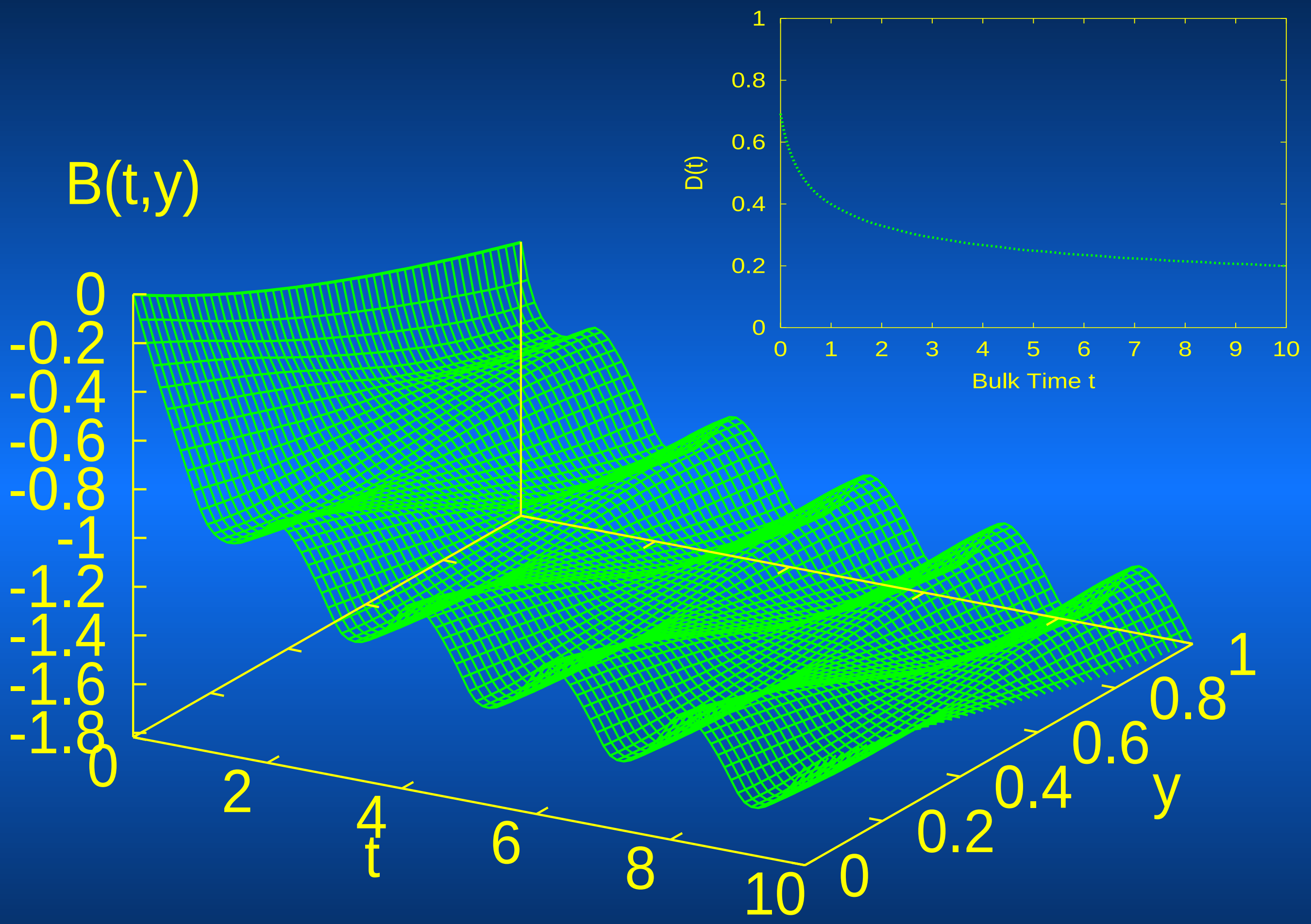
- ☞ Initial values given on first two grid lines
- ☞ Time evolution of $N-1$ grid sites

Boundary Conditions

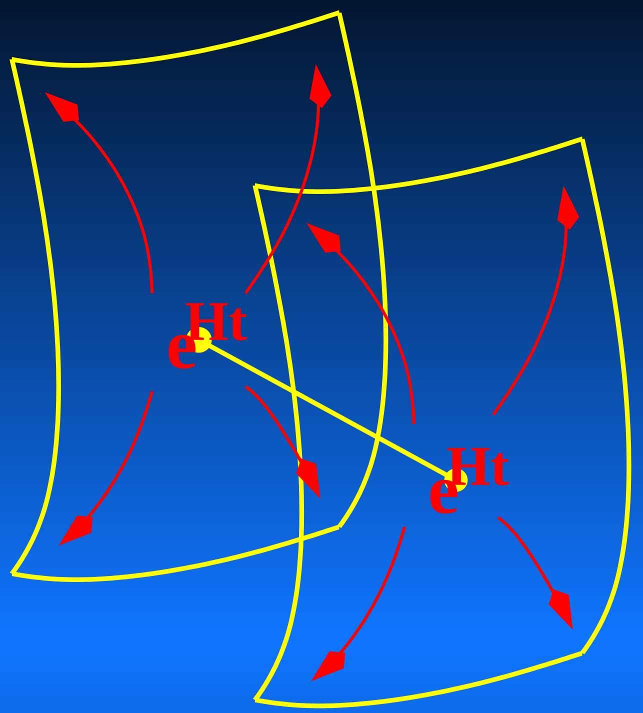
- ☞ Determine field values at the position of the branes

☞ Second order accurate differencing scheme

☞ Output:
- Fields: $A(t,y)$, $B(t,y)$, $\phi(t,y)$
- Invariants: R, $C^{MNOP}C_{MNOP}$
- Induced quantities: h_i, τ_i


B(t,y)
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
0
2
4
6
8
10
t
0
0.2
0.4
0.6
0.8
1
y
D(t)
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
Bulk Time t

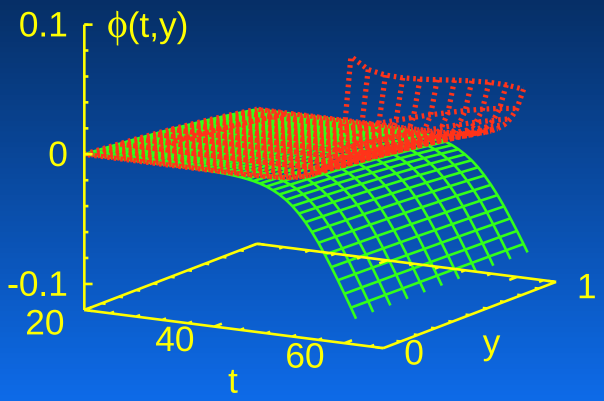
Stationary Solutions

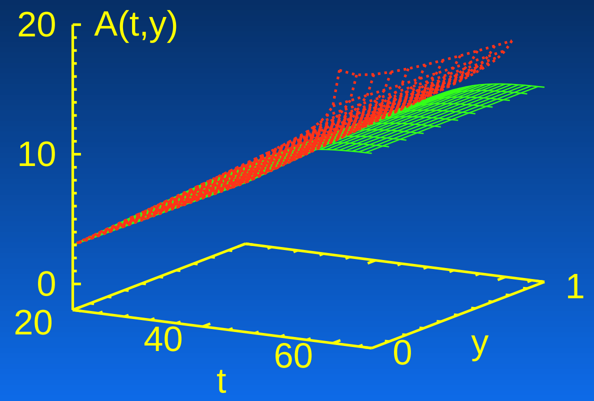


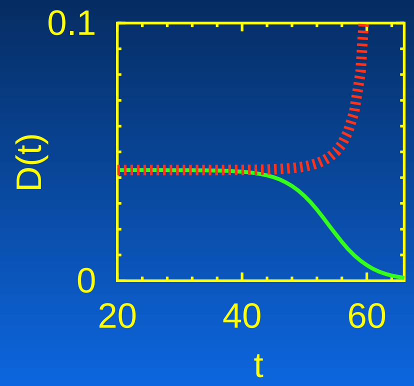
$$
\begin{aligned}
ds^2 &= e^{2B(y)}\left\{dy^2 - dt^2 + e^{2Ht}d\vec{x}^2\right\} \\
\phi &= \phi(y) \\
V &= \tfrac{1}{2}m^2\phi^2 + \Lambda \\
U_i &= \zeta_i(\phi - \nu_i)^2 + \lambda_i
\end{aligned}
$$

- Controllable initial conditions:
  - $A(t, y) = B(y) + Ht$
  - $B(t, y) = B(y)$
  - $\phi(t, y) = \phi(y)$
- Goldberger-Wise potentials
- $\boxed{a = e^{Ht}, D = const}$ **Stable solution ?**

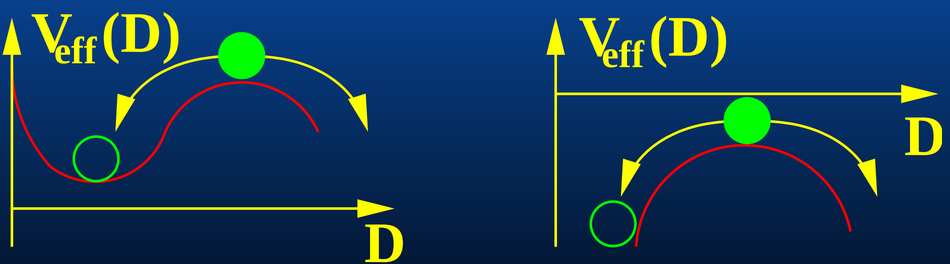
Tachyonic Instability







Physical Instability !

- ☞ Analysis of perturbations around stationary background
- ☞ $\boxed{m^2_{\text{eff}} \leq -4H^2 + m_0^2(\phi')}$ [A. Frolov, L. Kofman '03]
- ☞ 

Given: $\boxed{V(\phi), U_i(\phi)}$

Number of stationary solutions ?

- ☞ Nonlinear boundary value problem
- ☞ Two solutions possible

Transition ?

Dynamical Transition

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Number of stationary solutions ?

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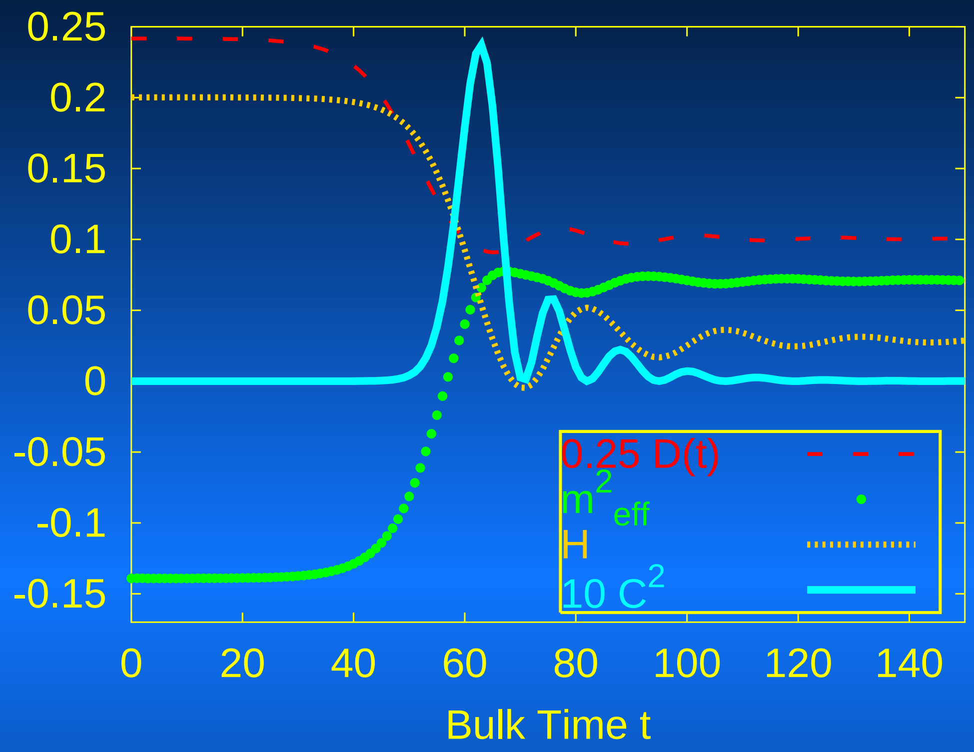
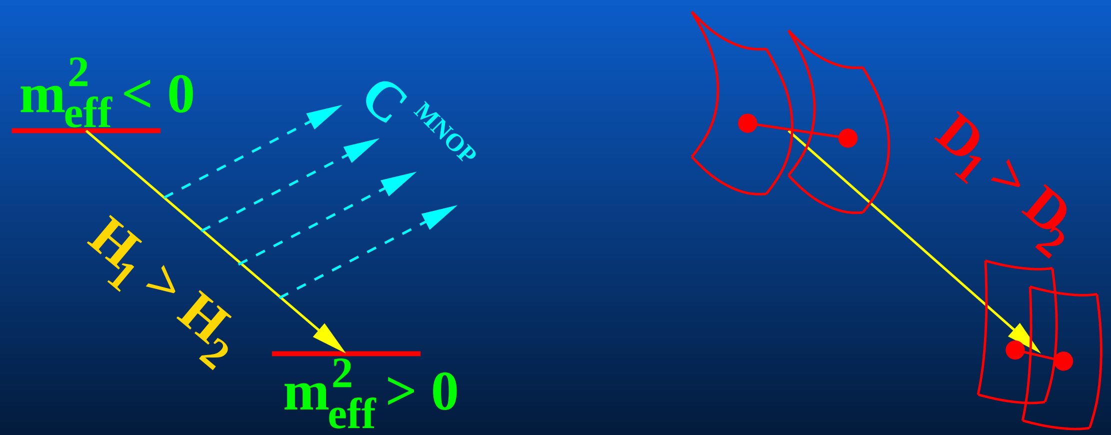
Transition ?





☞ Nonlinear reconfiguration

☞ Transition towards flatter branes

Moduli Approximation?

- Radion treated as light effective
  4d - scalar field
- $\psi(\tau) \propto \ln(\frac{\tau}{\tau_c})$
- $a(\tau) \propto \tau^{1/3}$

Brane Collision

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Universal Attractor

☞ Solutions y - independent

☞ Potentials unimportant: $e^{2B}V \longrightarrow 0$

☞ Analytic solution: 5D - Kasner-like solution + Scalar field

$$\boxed{\begin{aligned} ds^2 &= -d\tau^2 + \tau^{2p_y} dy^2 + \sum_{i=1}^{3} \tau^{2p_i} dx_i^2 \\ \phi &= q \ln \tau \\ 1 &= p_y + p_1 + p_2 + p_3 \\ 1 - q^2 &= p_y^2 + p_1^2 + p_2^2 + p_3^2 \end{aligned}}$$

[V.Belinskiy, I.Khalatnikov ’73]

Brane Collision

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Strong 5d Gravity Regime

- $p_1 = p_2 = p_3$ (3-branes)
- $p_1 < \frac{1}{3}$
- Induced metric:
  $ds^2 = -d\tau^2 + (\tau_c - \tau)^{2p_1} d\vec{x}^2$
- $D(t) \propto (\tau_c - \tau)^{p_y}$

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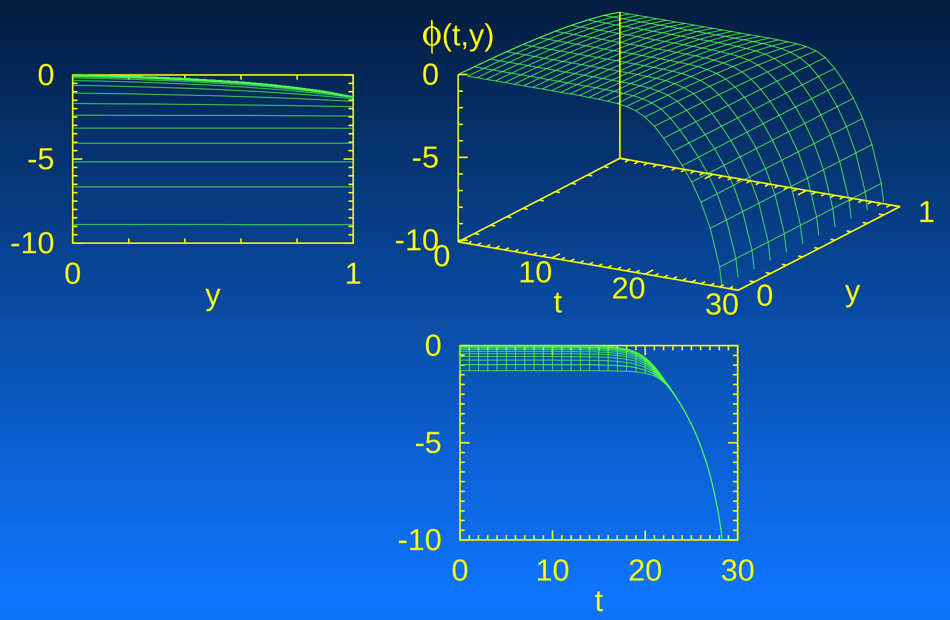
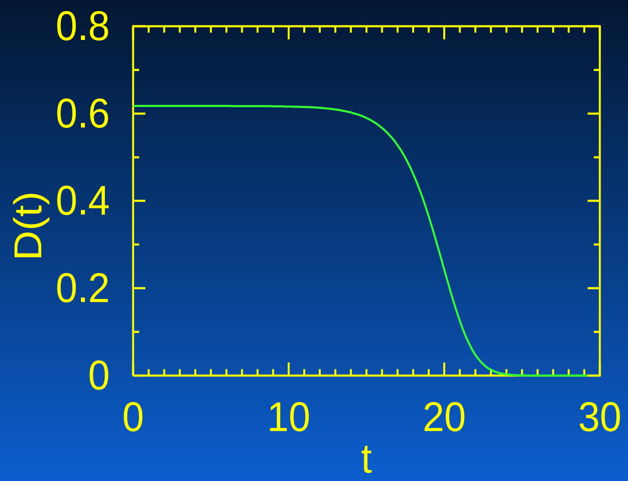
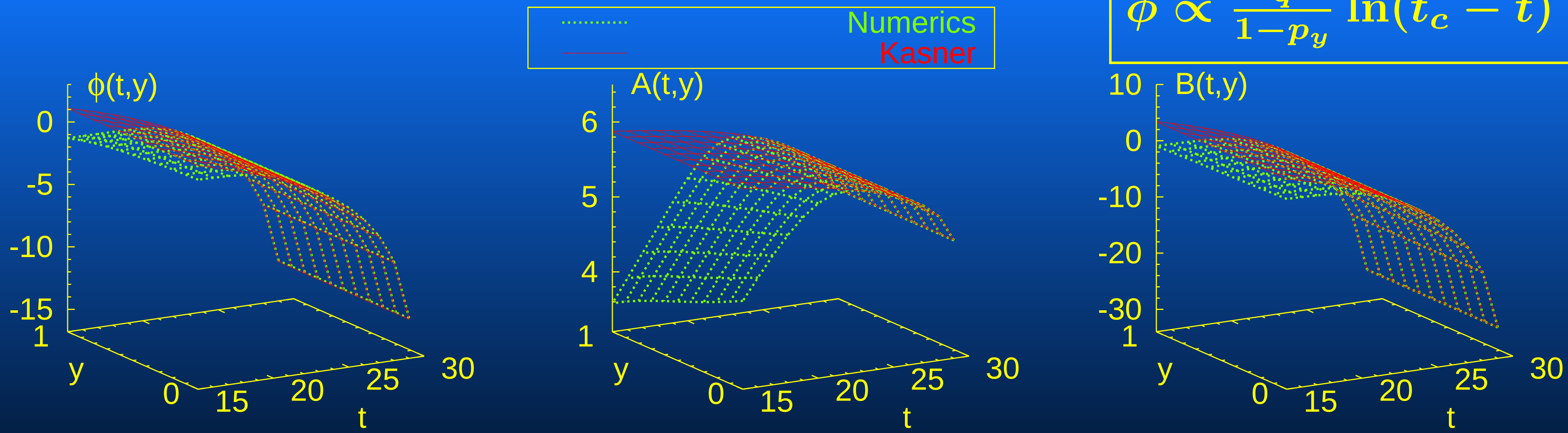
Brane Collision





$$A \propto \frac{1}{3}\ln(t_c - t)$$
$$B \propto \frac{p_y}{1-p_y}\ln(t_c - t)$$
$$\phi \propto \frac{q}{1-p_y}\ln(t_c - t)$$



Results

- ☞ `BraneCode`® for the Study of the Nonlinear System
- ☞ De-Sitter Branes Generically Unstable
- ☞ Dynamical Evolution Towards Flatter Branes
- ☞ Universal Attractor of Colliding Branes
- ☞ New Mechanism for the Generation of Perturbation

Perturbations:

$$\begin{aligned} ds^2 &= e^{2B(y)}\left\{\left(1+2\Phi(t,y,\vec{x})\right)dy^2+\left(1+2\Psi(t,y,\vec{x})\right)\left[-dt^2+e^{2Ht}d\vec{x}^2\right]\right\} \\ y_i &= \begin{cases} 0+\xi_0(t,\vec{x}) \\ 1+\xi_1(t,\vec{x}) \end{cases} \\ \delta\phi & \end{aligned}$$

E.O.M.:

$$\begin{aligned} \Psi &= -\tfrac{1}{2}\Phi \\ \xi_i &= 0 \\ Y &\equiv e^{2B}\Phi \\ Y &= \sum_m Q_m(\vec{x},t)Y_m(y) \qquad {}^{(4)}\Box Q_m = m^2 Q_m \\ \Rightarrow & \quad \boxed{-(g(y)Y_m')' + f(y)Y_m = (m^2+4H^2)g(y)Y_m \qquad Y_m'\big|_{y=0,1}=0} \end{aligned}$$

Mass Bound:

$$\boxed{m^2 \leq -4H^2 + \frac{2}{3}\frac{\int_0^1 e^{-B}dy}{\int_0^1 \frac{e^{-B}}{\phi'^2}dy}}$$