Cosmological structure problems of the pre-big-bang ← Venetiano - Gasperini etal and the ekpyrotic (cyclic) scenarios

Steinhardt - Turok etal

Jai-chan Hwang

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Perturbed world model

E. M. Lifshitz (46)

Notations:

Bardeen (88)

shear curvature GW

$$ds^2 = -(1+2\alpha)\,dt^2 - \chi_{,\alpha} dt dx^\alpha + a^2\left[g^{(3)}_{\alpha\beta}(1+2\varphi) + 2C_{\alpha\beta}\right] dx^\alpha dx^\beta$$

$$T^0_0 = -(\bar\mu + \delta\mu)\,, \quad T^0_\alpha = -\frac{1}{k}(\mu + p)\,\tilde v_{,\alpha}, \quad T^\alpha_\beta = (\bar p + \delta p)\,\delta^\alpha_\beta$$

velocity

Equation of motion:

Field-Shepley (68); Lukash (80); Mukhanov (85)

$$\boxed{\begin{aligned} v'' + \left(c_A^2 k^2 - \frac{z''}{z}\right) v = 0 \\ u'' + \left(c_A^2 k^2 - \frac{(1/\bar z)''}{(1/\bar z)}\right) u = 0 \end{aligned}}$$

where

$$v \equiv z\Phi, \quad u \equiv \frac{1}{8\pi G}\frac{a}{H\bar z}\Psi; \quad z \equiv a\sqrt{Q}, \quad \bar z \equiv c_A z$$

$$\begin{cases} \Phi = \varphi_{\tilde v}, & Q = \frac{\mu+p}{c_s^2 H^2} & c_A^2 \to c_s^2 & \text{(fluid)} \\ \Phi = \varphi_{\delta\phi} & Q = \frac{\dot\phi^2}{H^2} & c_A^2 \to 1 & \text{(field)} \\ \Phi = C^\alpha_\beta & Q = \frac{1}{8\pi G} & c_A^2 \to 1 & \text{(GW)} \end{cases}$$

$$\varphi_{\tilde v} \equiv \varphi - \frac{aH}{k}\tilde v, \quad \varphi_{\delta\phi} \equiv \varphi - \frac{H}{\dot\phi}\delta\phi, \quad \Psi \equiv \varphi_\chi \equiv \varphi - H\chi$$

Perturbed action:

Lukash (80); Mukhanov (88)

$$\delta^2 S = \frac{1}{2}\int a^3 Q\left(\dot\Phi^2 - c_A^2\frac{1}{a^2}\Phi^{|\alpha}\Phi_{,\alpha}\right) dt d^3x$$

Large-scale solution: $(z''/z \gg c_A^2 k^2$, etc)

LS

!

$$v \propto \mathcal{R} \;=\; \Phi(\mathbf{x},\eta) = C(\mathbf{x}) + 2\nabla^2 d(\mathbf{x})\int_0^\eta \frac{d\eta}{z^2}$$

compare !

$$u \propto \Phi_H \;=\; \Psi(\mathbf{x},\eta) = 4\pi G C(\mathbf{x})\frac{1}{x}\int_0^\eta \bar z^2 d\eta + d(\mathbf{x})\frac{1}{x}, \qquad x \equiv \frac{a}{H}$$

Generalized Gravity

- **$f(\phi, R)$ gravity:** CQG **7** 1613 (90); PRD **54** 1460 (96)

$$S = \int \sqrt{-g}\left[\frac{1}{2}f(\phi, R) - \frac{1}{2}\omega(\phi)\phi^{;a}\phi_{,a} - V(\phi) + L_c\right] d^4x$$

Cases

Minimally coupled scalar field	$L = \frac{1}{16\pi G}R - \frac{1}{2}\phi^{;a}\phi_{,a} - V(\phi)$
Nonminimally coupled scalar field	$L = \frac{1}{2}\left(\frac{1}{8\pi G} - \xi\phi^2\right)R - \frac{1}{2}\phi^{;a}\phi_{,a} - V(\phi)$
Brans-Dicke theory	$L = \frac{1}{16\pi}\left(\phi R - \omega\frac{\phi^{;a}\phi_{,a}}{\phi}\right)$
Induced gravity	$L = \frac{1}{2}\epsilon\phi^2 R - \frac{1}{2}\phi^{;a}\phi_{,a} - \frac{1}{4}\lambda(\phi^2 - v^2)^2$
R^2 gravity	$L = \frac{1}{16\pi G}\left(R - \frac{R^2}{6M^2}\right)$
Low-energy string theory	$L = \frac{1}{2}e^{-\phi}\left(R + \phi^{;a}\phi_{,a}\right)$

- **Stringy corrections:** PRD **61** 043511 (00); PRD **64** 103504 (01)

$$\begin{aligned} L_c &= -\frac{1}{2}\alpha'\lambda\xi(\phi)[c_1(R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2) \\ &\quad + c_2 G^{ab}\phi_{,a}\phi_{,b} + c_3\Box\phi\phi^{;a}\phi_{,a} + c_4(\phi^{;a}\phi_{,a})^2] \end{aligned}$$

- **Axion coupling:** PRD **61** 084026 (00)

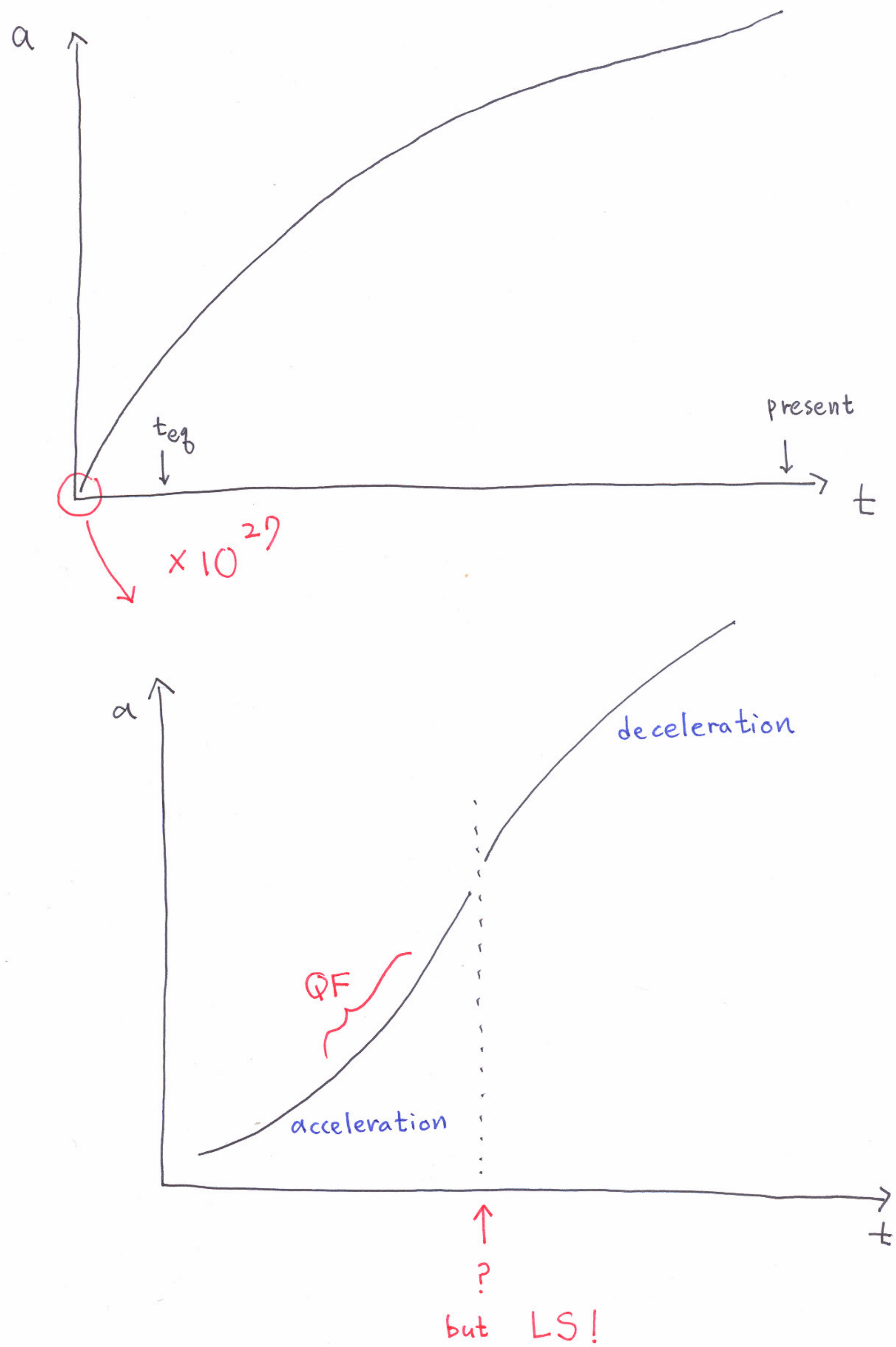
$$L_c = \frac{1}{8}\nu(\phi)\left(\eta^{abcd}R_{ab}{}^{ef}R_{cdef}\right)$$

- **Tachyonic coupling:** PRD **66** 084009 (02)

$$S = \int d^4x\sqrt{-g}\frac{1}{2}f(R, \phi, X), \quad X \equiv \frac{1}{2}\phi^{;a}\phi_{,a}$$

$\Rightarrow$ u, v equations and solutions available in the same forms.

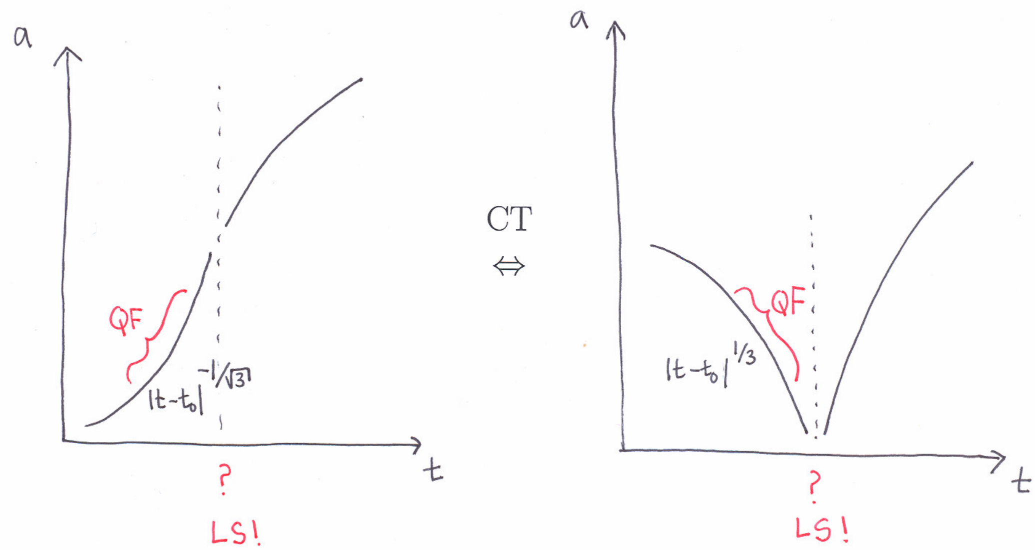
Inflation



Pre-big bang ← string theory low-energy effective action

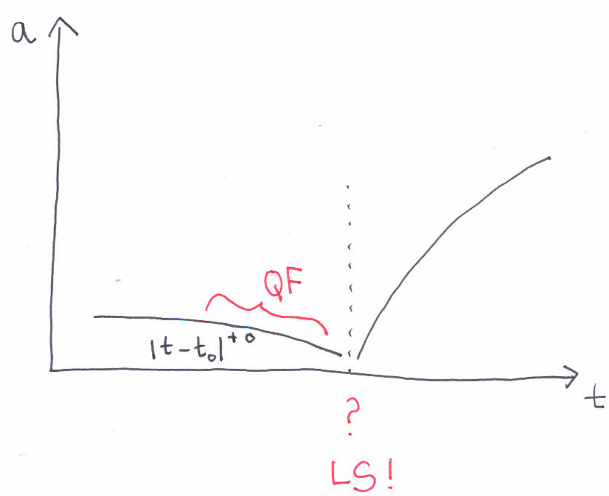
Veneziano (91)

String frame　　　　Einstein frame



Ekpyrotic (cyclic) ← brane cosmology ← string/M-program

Khoury-Steinhardt-Turok et al (01-03)



Quantum Generation

Action:

$$\delta^2 S = \frac{1}{2}\int a^3 Q\left(\dot{\Phi}^2 - c_A^2 \frac{1}{a^2}\Phi^{|\alpha}\Phi_{,\alpha}\right) dt d^3x$$

Quantization:

$$[\hat{\Phi}(\mathbf{x},t), \dot{\hat{\Phi}}(\mathbf{x}',t)] = \frac{i}{a^3 Q}\delta^3(\mathbf{x}-\mathbf{x}')$$

Mode function solution, assuming $a\sqrt{Q} \propto |\eta|^q$: & c_A = constant

$$\Phi_{\mathbf{k}}(\eta) = \frac{\sqrt{\pi|\eta|}}{2a\sqrt{Q}}\left[c_1(k)H_\nu^{(1)}(c_A k|\eta|) + c_2(k)H_\nu^{(2)}(c_A k|\eta|)\right]$$

$$\nu \equiv \frac{1}{2} - q, \quad |c_2(k)|^2 - |c_1(k)|^2 = 1$$

Power spectra:

1. In the large scale limit: CQG **15** 1387, 1401 (98)

$$\mathcal{P}_{\hat{\Phi}}^{1/2}(\mathbf{k},\eta) = \frac{H}{2\pi}\frac{\Gamma(\nu)}{\Gamma(3/2)}\frac{1}{aH|\eta|}\left(\frac{k|\eta|}{2}\right)^{3/2-\nu}\frac{1}{c_A^\nu\sqrt{Q}}$$

- For $\nu = 0$ we have additional $2\ln(c_A k|\eta|)$ factor
- For $\hat{\Phi} = \hat{C}^\alpha_\beta$ we need additional $\sqrt{2}$ factor
- General vacuum dependences:

$$|c_2(\mathbf{k}) - c_1(\mathbf{k})|, \quad \sqrt{\frac{1}{2}\sum_\ell |c_{\ell 2}(\mathbf{k}) - c_{\ell 1}(\mathbf{k})|^2}$$

2. Identify

$$\mathcal{P}_{\hat{\Phi}} \equiv \mathcal{P}_\Phi$$

3. The growing modes of Φ are conserved in the large scale

Spectral indices:

$$n_S - 1, \; n_T \equiv \frac{\partial \ln \mathcal{P}_\Phi}{\partial \ln k}$$

2dFGRS

CMB and LSS observations: $n_s = 0.97 \pm 0.03$ without running

WMAP data: $n_S = 0.93^{+0.0\cancel{3}4}_{-0.0\cancel{3}5}$ Spergel, etal, astro-ph/0302209

Inflation vs. other scenarios

Power-law expansion:

Einstein gravity: Lucchin-Matarresse (85)

$$a \propto |t|^p, \quad V = -\frac{p(1-3p)}{8\pi G} e^{-\sqrt{16\pi G/p}\phi}$$

Power spectra: Stewart-Lyth (93)

$$\boxed{\begin{aligned} n_S - 1 &\simeq \frac{2}{1-p} \simeq n_T \\ \mathcal{P}_{C_{\alpha\beta}}^{1/2} &= (2/\sqrt{p})\mathcal{P}_{\varphi_{\delta\phi}}^{1/2} \end{aligned}}$$

Power-law inflation: $p \gg 1$

$$n_S - 1 \simeq 0 \simeq n_T, \quad \mathcal{P}_{C_{\alpha\beta}} \ll \mathcal{P}_{\varphi_{\delta\phi}}$$

Harrison-Zel'dovich spectrum!

Pre-big bang scenario (original frame):

$f = e^{-\phi}R$, $\omega = -e^{-\phi}$, $V = 0$: $\to a \propto |t - t_0|^{-1/\sqrt{3}}$

$$n_S - 1 \simeq 3 \simeq n_T, \quad \mathcal{P}_{C_{\alpha\beta}} = 12\mathcal{P}_{\varphi_{\delta\phi}}$$

Too blue; fails observationally! Astropart. Phys. **8** 201 (98)

Pre-big bang scenario (Einstein frame): $p = \frac{1}{3}$

$$n_S - 1 \simeq 3 \simeq n_T, \quad \mathcal{P}_{C_{\alpha\beta}} = 12\mathcal{P}_{\varphi_{\delta\phi}}$$

Observable results are conformally invariant! Brustein etal (95)

Ekpyrotic (cyclic) scenario: $0 < p \ll 1$

$$n_S - 1 \simeq 2 \simeq n_T, \quad \mathcal{P}_{C_{\alpha\beta}} \gg \mathcal{P}_{\varphi_{\delta\phi}}$$

Too blue and no scalar perturbation generated. Lyth (02), PRD 65 063514 (02)

Singular bounce: perturbation theory breaks down. Lyth (02) Brandenberger-Finelli (02)

Troubles in ekpyrotic (cyclic) scenario

- **Singular bounce:**

  In collapsing phase the d-mode unambiguously becomes singular

  $\Rightarrow$ perturbation theory breaks down! Bardeen (80); Lyth (02)

  Whether the singular bounce is possible is unclear

- **Nonsingular-smooth bounce:**

  nonlinear transition is likely to occur during the collapsing phase

- **Nonsingular-smooth bounce + valid linear theory:**

  Large-scale conditions ($z''/z \gg c_A^2 k^2$) well met during the bounce.

  (i) using the known matching conditions (see below) twice

  (ii) using the exact asymptotic solution

  $\Rightarrow \Phi = C$ mode survives after the bounce! PRD **65** 124010 (02)

- **Khoury et al.** resort to a strange (unjustified) matching conditions to recover the d mode of Ψ (which happens to have a $n_S \simeq 1$ spectrum) as the growing solution after the bounce.

$$\epsilon_m = \epsilon_0 D(\eta) + \epsilon_2 E(\eta), \quad \epsilon_m = \delta_v$$
$$\epsilon_0, \quad \epsilon_2 \quad \text{continuous}$$

  The known gauge-invariant matching variables: ApJ **382** 363 (91)

$$\varphi_\chi (= \Phi_H), \quad \varphi_\delta (= \zeta_{\text{Bardeen}})$$

- **Why is d-mode of Ψ not interesting?**

  d-mode of Ψ diverges more strongly than the one of Φ, and

  "this **overstates** the physical strength of the singularity"

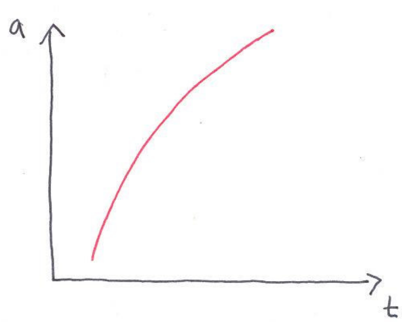
  Bardeen (80)

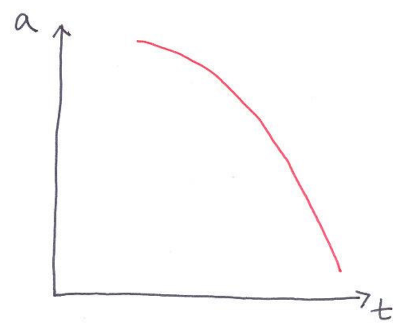
  Thus, d-mode of Ψ exaggerates the perturbation compared with the d-mode of Φ.

$n_s \simeq 5$!

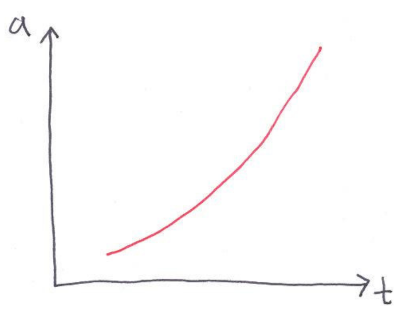
$\mathrm{w} \equiv P/\mu =$ constant, flat:

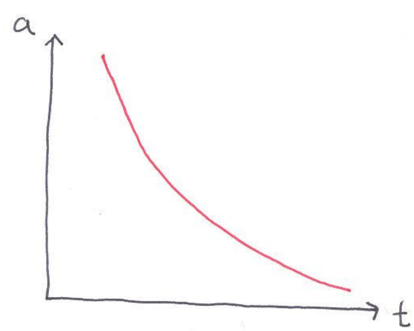
- $\underline{\mathrm{w} > -\frac{1}{3}} \quad \to \quad a \propto |t|^p,\ p < 1$:



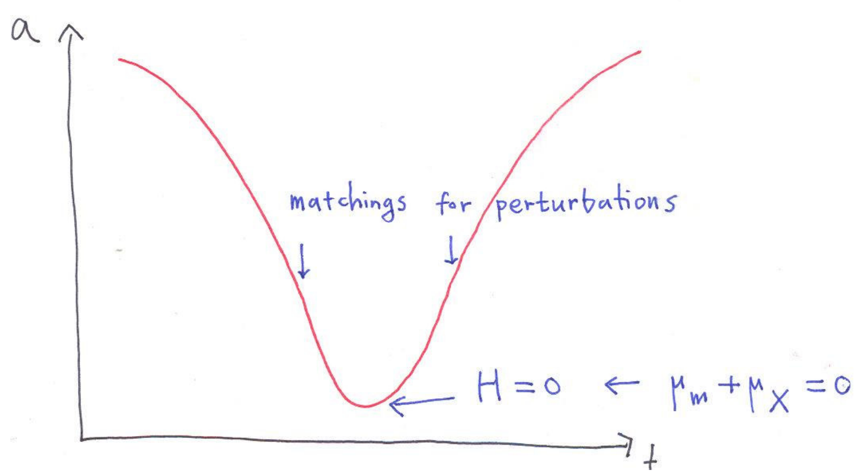


- $\underline{\mathrm{w} < -\frac{1}{3}} \quad \to \quad a \propto |t|^p,\ p > 1$:





- **Additional exotic matter**, $\mu_X \propto -a^{-6}$, say: PRD **65** 124010 (02)



Conclusions

- Quantum fluctuations $\Rightarrow$ classical seed fluctuations
  $\Rightarrow$ CMB anisotropy, large-scale structure
- Generated seeds depend sensitively on the expansion/contraction rate during the quantum generation stage

- Inflation has enough parameter space for successful structure formation.
  In fact, there are "too many" successful specific realizations.

- Pre-big bang and Ekpyrotic (cyclic) scenarios fail observationally!
  - In fact, no mechanism for singular bounce is known.
  - Perturbation breaks down inevitably near singularity.
  - In case the linear perturbation remains valid under a nonsingular smooth bounce
    $\Rightarrow$ generated spectra are too blue with negligible amplitude for Ekpyrotic (cuclic) scenario.
- Thus, either the scenarios fail to produce the LSS
  or we cannot rely on the linear perturbation theory.

- Additional fields (axion, moduli, curvaton) could save the situations with some unnatural tunings, of course.