Baryogenesis from CP-violation at Electroweak tachyonic preheating

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The observed baryon asymmetry of the Universe can be expressed as [WMAP;2003]

$$\frac{n_B}{n_\gamma} \simeq 6.5 \times 10^{-10}$$

n_B, n_γ baryon and photon number density.

We study a model of baryogenesis, where inflation ends at the electroweak scale. Initially, $T = 0$. We assume that Higgs symmetry breaking is triggered by the end of inflation (see Jan Smit's talk). We study the baryon number changing processes in the electroweak sector under the influence of CP-violation during symmetry breaking.

$$\begin{aligned} B(t) - B(0) &= 3\langle[N_{cs}(t) - N_{cs}(0)]\rangle \\ &= 3\int_0^t dt \int dx^3 \langle \frac{1}{16\pi^2} \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right]\rangle \end{aligned}$$

Symmetry breaking as a quench.

We let the Higgs symmetry breaking happen through a quench of the Higgs potential

$$\begin{aligned} V(\phi) &= \mu_{\text{eff}}\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2 \\ \mu_{\text{eff}}^2 &= \mu^2,\ t < 0 \\ &= -\mu^2,\ t > 0 \end{aligned}$$

Initially, $\langle\phi\rangle = 0$. In the approximation $\lambda = 0$, field modes with $k < \mu$ grow exponentially. This is known as tachyonic preheating. Eventually, the quadratic term becomes non-negligible and the system thermalizes.

Large modes and thus particle numbers allow for classical treatment. The modes in this classical domain are the ones responsible for the baryon number violating processes.

Model

We perform classical real-time simulations on the lattice in the SU(2)-Higgs model with a CP-violating term.

$$S = -\int d^4x \left[\frac{1}{2g^2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D^\mu \phi \right.$$
$$-\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + V_0$$
$$\left. + \kappa\, \phi^\dagger \phi \mathrm{Tr}\, F^{\mu\nu} \tilde{F}_{\mu\nu} \right]$$

ϕ is the Higgs SU(2)-doublet.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$
$$D_\mu \phi = (\partial_\mu - i A_\mu)\phi, \;\; \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$
$$\frac{m_H}{m_W} = \sqrt{\frac{8\lambda}{g^2}}, \;\; k = 16\pi^2 \kappa m_W^2$$

We derive the classical equations of motion. They are rather complicated because of the κ-term and in particular implicit requiring us to solve them by iteration.

Observables

- Higgs expectation value

$$\langle \phi^\dagger \phi \rangle = \frac{1}{V} \int dx^3 \frac{\phi^\dagger \phi}{v^2/2}$$

- Chern-Simons number (with $N_{\mathrm{CS}}(0) = 0$)

$$N_{cs}(t) - N_{cs}(0) = \int_0^t dt \int dx^3 \frac{1}{16\pi^2} \mathrm{Tr}\, F^{\mu\nu} \tilde{F}_{\mu\nu}$$

- Higgs winding number:

$$\begin{aligned} N_W &= \frac{1}{24\pi^2} \sum_{x,ijk} \epsilon_{ijk} \mathrm{Tr} \left[\partial_i U_x U_x^\dagger \partial_j U_x U_x^\dagger \partial_k U_x U_x^\dagger \right] \\ U_x &= \frac{\Phi_x}{\sqrt{\frac{1}{2} \mathrm{Tr}\, \Phi_x^\dagger \Phi_x}}, \quad \Phi = (i\tau_2 \phi^*, \phi) \end{aligned}$$

N_W is integer, and in the ground states $N_{\mathrm{CS}} = N_W$, thereby minimizing the covariant derivative.

Initial conditions

$T = 0$ and $\langle\phi\rangle = 0$. The quantum vacuum has fluctuations. These will grow under the spinodal instability. We define

$$\langle \pi_{\mathbf{k}} \pi^{\dagger}_{\mathbf{k}} \rangle = (n_k + 1/2)\, \omega_k \quad , \quad \langle \phi_{\mathbf{k}} \phi^{\dagger}_{\mathbf{k}} \rangle = (n_k + 1/2)\, / \omega_k$$

$$\langle \pi_{\mathbf{k}} \phi^{\dagger}_{\mathbf{k}} \rangle = \tilde{n}_k + i/2$$

At some time during the transition, we perform a "resampling" reproducing the quantum correlation functions by classical ensembles of initial conditions. Quantum averages become averages over the ensemble. We generate realisations with probability distribution

$$P(\xi^{\pm}_{\mathbf{k}}) \propto \exp\left(-\frac{1}{2} \sum_{\mathbf{k}} \frac{|\xi^{+}_{\mathbf{k}}|^2}{(n_k + \frac{1}{2} + \tilde{n}_k)} + \frac{|\xi^{-}_{\mathbf{k}}|^2}{(n_k + \frac{1}{2} - \tilde{n}_k)} \right) \delta(\mathbf{G})$$

$$\xi^{\pm}_{\mathbf{k}} = \frac{1}{\sqrt{2\omega_k}} (\omega_k \phi_{\mathbf{k}} \pm \pi_{\mathbf{k}})$$

Since classical and quantum equations are identical for a quadratic potential, we can choose to "resample" at $t = 0$. We apply two schemes [Smit and Tranberg;2002], [Garcia-Bellido *et al.*;2002]

- "Just the half". Initial state is quantum vaccum in quadratic potential $V = V_0 + \mu^2 \phi^\dagger \phi$. Only unstable modes ($k < \mu$) are initialized.

$$\langle \pi_k \pi_k^\dagger \rangle = \frac{1}{2}\omega_k, \ \langle \phi_k \phi_k^\dagger \rangle = \frac{1}{2\omega_k}, \ \omega_k = \sqrt{k^2 + \mu^2}$$

- "Thermal". Bose-Einstein distribution at low temperature $T/m_H = 0.1$

$$\langle \pi_k \pi_k^\dagger \rangle = n_k^{BE} \omega_k, \ \langle \phi_k \phi_k^\dagger \rangle = n_k^{BE} / \omega_k$$

The initial gauge fields are set to zero, $A_i^a = 0$. This simplifies the Gauss constraint, from which we find the initial gauge momenta E_i^a. On a periodic lattice, total charge is zero.

Numerics

- We discretize the system on a space-time lattice.

- Choice of lattice spacing for N_W to be close to integers. $am_H = 0.35$.

- Choice of lattice size for sufficiently many unstable momentum modes. $Lm_H = 21$ gives around 50 unstable modes. The $\mathbf{n}$'th mode has wave vector $\mathbf{k} = 2\pi\mathbf{n}/L$.

- We deal with the implicit κ dependent part of the equations of motion by iteration up to computer accuracy. Good convergence at $a_t/a = 0.05$. 3-9 iterations required. 11 PC-years.

Initial bump

We can estimate the growth of the Chern-Simons number during the first Higgs roll-down. Higgs and gauge fields grow exponentially at early times:

$$\phi(t) = \phi_0 e^{\mu t}$$
$$A_i^a(t) = A_0 e^{c\mu t}$$

We consider homogeneous fields and treat κ-term as perturbation. We can integrate the equations of motion untill $\phi^\dagger \phi = v^2/2$ (at t_{nl}) and find:

$$\frac{\langle N_{\mathrm{CS}} \rangle}{(L m_W)^3} = k \frac{4\sqrt{2}}{(16\pi^2)^2 (1+c)^2} \left(\frac{m_H}{m_W} \right)^3 \left(\frac{\langle B^2 \rangle_{t_{nl}}}{m_H^4} \right)$$

We read off $\langle B^2 \rangle_{t_{nl}}$ and c from the simulations and find

$$\frac{\langle N_{\mathrm{CS}} \rangle}{(L m_W)^3} = 1.0 \times 10^{-6} k \left(\frac{m_H}{m_W} \right)^3$$

Wash-out

We need the rate of sphaleron transitions after baryogenesis to be small enough not to wash out the generated asymmetry. The standard criterion is

$$\frac{v}{T} > 1, \; v = 246\text{GeV}$$

First estimate from distributing energy on available degrees of freedom

$$V_0 = \frac{m_H^4}{16\lambda} = \frac{\pi^2}{30} g_* T^4$$

$$\frac{T}{m_H} = \left(\frac{30}{g^* \pi^2 16\lambda} \right)^{\frac{1}{4}}$$

With $v = 246\text{GeV}$ and $m_H = 114\text{GeV}$ we find

$$\frac{v}{T} \simeq 3.7, \; (g^* = 10)$$

$$\frac{v}{T} \simeq 5.5, \; (g^* = 80)$$

So we should be safe.

However, our system is not yet in equilibrium and our particles are not massless. In [Skullerud *et al.*;2003] we define and calculate particle numbers and find effective temperatures and chemical potentials in this scenario. We find

$$\frac{T}{m_H} \simeq 0.4, \ \frac{\mu}{m_H} \simeq 1.0$$

Distribution is Bose-Einstein with large chemical potentials. Sphaleron activity is larger than temperature suggests.

In full theory, W and Higgs particles decay. W width is $\Gamma_W \simeq 3\text{GeV}$. At time t, suppression by

$$\simeq e^{-\frac{3}{114}m_H t}$$

No wash-out seen on time-scales $m_H t \simeq 100 - 500$.

Conclusions

Introducing CP-violation in a scenario of out-of-equilibrium Higgs symmetry breaking indeed results in a baryon asymmetry. In the linear regime the asymmetry is given by:

$$\frac{n_B}{n_\gamma} \simeq 8 \times 10^{-3} \kappa m_W^2, \quad \frac{m_H}{m_W} = 1$$
$$\frac{n_B}{n_\gamma} \simeq 8 \times 10^{-3} \kappa m_W^2, \quad \frac{m_H}{m_W} = \sqrt{2}$$

We therefore require

$$\rightarrow \quad \kappa \simeq 8 \times 10^{-8}/m_W^2 = 1 \times 10^{-5}/1\mathrm{TeV}^2$$

We can predict the "initial bump", which scales in the expected linear way with κ.

The result appears to be insensitive to the initial conditions scheme. The dependence on m_H/m_W also seems to be weak, as opposed to the analogous model in 1+1 dimensions, [Smit,Tranberg;2002].

For related work, see [Garcia-Bellido *et al.*:1999], [Krauss,Trodden:1999], [Rajantie *et al.*:2000], [Cornwall *et al.*:2001], [Copeland *et al.*:2002], [Garcia-Bellido *et al.*:2003].