Tachyonic electroweak transition, CP violation and baryogenesis

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1. Electroweak baryogenesis from CP violation in the (Extended) Standard Model not impossible if electroweak transition took place at zero temperature

2. Tachyonic electroweak transition
   electroweak quench
   classical approximation

The Extended Standard Model (ESM)

$$
\begin{aligned}
\mathcal{L}_{\mathsf{F}} &= -\bar{\Psi} D \Psi + \mathcal{L}_{\mathsf{M}} \\
D &= \gamma^\mu D_\mu + m P_{\mathsf{R}} + m^\dagger P_{\mathsf{L}} \\
m &= \Phi Y \\
\mathcal{L}_{\mathsf{M}} &= -\frac{1}{2} \Psi_\nu^T C^\dagger M P_{\mathsf{R}} \Psi_\nu + \text{h.c.}
\end{aligned}
$$

standard form chosen for the Yukawa coupling matrices in which, for quarks,

$$
Y \to \begin{pmatrix} \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} & \\ & V \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix} V^\dagger \end{pmatrix}
$$

and for leptons

$$
Y \to \begin{pmatrix} V^{(\nu)} \begin{pmatrix} y_{\nu_e} & & \\ & y_{\nu_\mu} & \\ & & y_{\nu_\tau} \end{pmatrix} V^{(\nu)\dagger} & \\ & \begin{pmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{pmatrix} \end{pmatrix}
$$

with a diagonal Majorana mass matrix

$$M = \begin{pmatrix} M_{\nu_e} & 0 & 0 \\ 0 & M_{\nu_\mu} & 0 \\ 0 & 0 & M_{\nu_\tau} \end{pmatrix}$$

(and positive eigenvalues of Y and M)

The Cabbibo-Kobayashi-Maskawa (CKM) matrix V can parametrized as

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -S_{13}e^{i\delta} & 0 & C_{13} \end{pmatrix}$$
$$\times \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with the CP-violating phase δ, and

$$C_{13} = \cos(\theta_{23}), \quad S_{13} = \sin(\theta_{23}), \quad \text{etc.}$$

The analog for the leptons, $V^{(\nu)}$, has *three* CP-violating phases

Sakharov's criteria: B, C, and CP violation are met by the Standard Model (SM) and its extension including neutrino masses (ESM)

- B violation ends after electroweak transition

- measured CP violation in K and B systems is compatible with CKM matrix

$\Rightarrow$ seek to explain B generation by (E)SM physics during electroweak transition

Standard lore: not possible

- with current bounds on the Higgs mass, the finite-temperature electroweak transition is too slow for providing the required non-equilibrium conditions
-

$$\begin{aligned}\text{CP-viol.} &\propto (y_u - y_c)(y_c - y_t)(y_t - y_u)\\ &\quad (y_d - y_s)(y_s - y_b)(y_b - y_u) \times J\\ &= O(10^{-9} \times 10^{-5})\end{aligned}$$

$$\begin{aligned}J &= \pm \mathrm{Im}(V_{ij} V_{kl} V_{il}^* V_{kj}^*)\\ &= C_{12} C_{23} C_{13}^2 S_{12} S_{23} S_{13} \sin\delta\\ &\approx 3 \times 10^{-5}\end{aligned}$$

But the measured CP violation contradicts this estimate ...

Take a closer look in the framework of effective CP-violation

integrate out fermions $\Rightarrow$

$$\mathcal{L}_{CP} = \theta\text{-terms} + \kappa\, \phi^\dagger \phi\, \mathrm{tr}(F\tilde{F}) + \cdots$$

$\cdots$ contributions of increasing mass-dimension

θ-terms like $\theta\,\mathrm{tr}(F\tilde{F})$ very small

$$\kappa = \frac{\delta_{CP}}{\Lambda^2} = ?$$

Consider first quark contribution

Shaposhnikov '86,'87: $\kappa \neq 0$ in 14th order perturbation
Rubakov & Shaposhnikov '96: $m_j = y_j \langle \phi \rangle$,
$\Lambda = T$ (temperature),

$$\begin{aligned}\delta_{CP} &\propto J \times \prod (m_j^2 - m_k^2)/T^2 \\ &= O(10^{-20}) \quad \text{at } T = O(100) \text{ GeV}\end{aligned}$$

Zero temperature?
L.L. Salcedo: *) gradient expansion in increasing number of (gauge-covariant) derivatives

$$\mathcal{L}_{CP} = \frac{1}{48\pi^2}\,\epsilon^{\kappa\lambda\mu\nu}\,\mathrm{tr}(N_{12}F_{\kappa\lambda}F_{\mu\nu}) + \cdots$$

where N_{12} may have arbitrary dependence on the Higgs field and fermion-Higgs couplings

Actually, Salcedo calculates terms of the form $\mathrm{tr}(N_{123}\,D_\kappa m\,D_\lambda m\,F_{\mu\nu})$, $\mathrm{tr}(N_{1234}\,D_\kappa m \cdots D_\nu m)$, which imply the above by partial integration

The N's are complicated homogeneous functions of m and $m^\dagger$, which becomes explicit upon diagonalization:

$$m = \Phi Y,\;\; \Phi = \rho U \to \text{eigenvalues}\;\; m_j = \rho y_j$$

ρ is the radial component of the Higgs field ($\rho^2 = \phi^\dagger\phi$)

the effect of U is to transform to the unitary gauge:

$$W_\mu = U^\dagger A_\mu U + iU^\dagger\partial_\mu U, \quad W_{\mu\nu} = U^\dagger F_{\mu\nu} U$$

*) L.L. Salcedo, hep-th/0012166,0012174

the effect of the CKM matrix in Y is the usual mixing

$$W_\mu \to \tilde{W}_\mu \equiv \begin{pmatrix} W_\mu^3 & W_\mu^+ V \\ W_\mu^- V^\dagger & -W_\mu^3 \end{pmatrix}$$

and similar for $\tilde{W}_{\mu\nu}$

Example:*)

$$\begin{aligned} & -i\epsilon^{\kappa\lambda\mu\nu}\,\mathrm{tr}(N^{(0)}_{123}\,D_\kappa m\,D_\lambda m^\dagger\,F_{\mu\nu}) \\ = {} & \epsilon^{\kappa\lambda\mu\nu}\,\mathrm{Im}\sum_{jkl} N^{(0)}_{jkl}\,(\rho^{-1}\partial_\kappa\rho - i\tilde{W}_\kappa)_{jk}\,\rho^2 y_k^2 \\ \times {} & (\rho^{-1}\partial_\lambda\rho + i\tilde{W}_\lambda)_{kl}\left(\tilde{W}_{\mu\nu}\right)_{lk} \end{aligned}$$

with

$$N^{(0)}_{jkl}\,\rho^2 y_k^2 = 2y_k^2\,\frac{2y_j^4 - y_j^2 y_k^2 - 2y_j^2 y_l^2 - y_k^2 y_l^2 + 2y_l^4}{(y_j^2 - y_k^2)(y_l^2 - y_k^2)(y_j^2 - y_l^2)}$$

Note:

- *no* dependence on overall scale of y's

*) gauge fields limited to SU(2)

Sadly, the imaginary part = 0 (only two V's, need four for J). Same holds for complete quark-contribution to $\mathcal{L}_{CP}$ to fourth order in the derivative expansion. We expect a non-zero result in sixth order.

With non-zero Majorana mass M, the leptonic contribution will be non-vanishing already in fourth order, with complicated dependence on ρ and M, since only two $V^{(\nu)}$'s are needed for a non-zero phase-invariant $J^{(\nu)}$.

Conclude (zero T):
- strength of CP violation determined by J, and not $\prod y$'s

The form $\kappa\, \phi^\dagger \phi\, \mathrm{tr}(F\tilde{F}) + \cdots$ appears too simple; may need full-fledged non-equilibrium treatment

The above suggests to investigate the possibility that the electroweak transition took place at zero temperature

Tachyonic electroweak transition

Basic idea: in Higgs potential

$$\mu^2_{\text{eff}}\,\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

μ^2_{eff} changes sign form positive to negative and ends up at today's value $-\mu^2 = -m^2_H/2$

How come this sign change?

Suppose a non-minimal coupling to the scalar curvature,

$$\mu^2_{\text{eff}} = \xi R - \mu^2$$

With $\xi = O(1/6)$, $\mu = O(100)$ GeV, μ^2_{eff} changes sign when $R = O(10^4)$ GeV^2. This could happen in a preheating phase after inflation, matter-dominated by the inflaton. Then the Hubble rate $H = O(100)$ GeV, so the transition will be delayed to perhaps $H = O(10-1)$ GeV

Problem: at the transition the energy density $3m^2_{\text{P}}H^2$ is far above the electroweak scale *) and the subsequent expansion will dilute the baryon asymmetry generated during the transition

*) unless $\xi = O(10^{32})$!

other possibility: *)
Inflation ends at the electroweak scale, with coupling to the inflaton (σ) given by

$$\mu_{\rm eff}^2 = \mu_\phi^2 - \lambda_{\sigma\phi}\,\sigma^2 \quad \sigma:\ 0 \to \langle 0|\sigma|0\rangle$$

Low-scale inflation has been investigated in [a]; [b] contains a detailed model with above coupling; [c] investigates (also numerically) the transition assuming a certain inflaton speed $\dot\sigma$

Problem: present horizon left Hubble scale during inflation only $N \approx 29$ e-folds before the end of inflation, which easily leads to a spectral index $n_S < 0.94$ for the suitable models in [a,b] **), in possible conflict with WMAP

*) This is very similar to the scenarios proposed in Krauss & Trodden hep-ph/9902420; García-Bellido et al. hep-ph/9902449
[a] Germán et al. hep-ph/0103243
[b] Copeland et al. hep-ph/0103231
[c] García-Bellido et al. hep-ph/0208228, 0304285
**) observation thanks to B. van Tent

Avoiding committing ourselves to a detailed model, consider the quench:

$$\mu_{\rm eff}^2 = +\mu^2, \quad t < 0$$
$$= -\mu^2, \quad t > 0$$

- presumably this rapid-transition limit leads to the largest possible baryon asymmetry in the tachyonic electroweak scenario, which is useful for deciding if it is viable

- leads to classical approximation

As first test we used the conventional

$$\Delta\mathcal{L}_{CP} = \kappa\,\phi^\dagger\phi\,\mathrm{tr}(F\tilde{F})$$

in numerical simulations of the gauge-Higgs sector of the SM

and

$$\Delta\mathcal{L}_{CP} = \kappa\,\phi^\dagger\phi\,\epsilon^{\mu\nu}\,F_{\mu\nu}/2$$

for the analog abelian-Higgs model in 1+1 dimensions

Quantum quench and classical approximation

Quench at $t = 0$ (gaussian approx.):

$$\hat{\phi}_{\mathbf{p}} = \frac{1}{\sqrt{2\omega_p^+}} \left(\hat{a}_{\mathbf{p}} e^{-i\omega_p^+ t} + \hat{a}^\dagger_{-\mathbf{p}} e^{i\omega_p^+ t} \right), \quad t < 0$$

$$= \hat{\alpha}_{\mathbf{p}} e^{-i\omega_p^- t} + \hat{\beta}_{\mathbf{p}} e^{i\omega_p^- t}, \qquad t > 0$$

$$\omega_p^\pm = \sqrt{\pm\mu^2 + p^2}$$

match at $t = 0$

unstable modes $p < \mu$ grow exponentially:

$$\omega_p^- = i|\omega_p^-|, \qquad |\omega_p^-| = \sqrt{\mu^2 - p^2}$$

$$\phi_{\mathbf{p}} \to \alpha_{\mathbf{p}}\, e^{|\omega_p^-| t}, \qquad \pi_{\mathbf{p}} \to |\omega_p^-|\, \alpha_{\mathbf{p}}\, e^{|\omega_p^-| t} \approx |\omega_p^-| \phi_{\mathbf{p}}$$

classical behavior for $|\omega_p^-| t \gg 1$