The electroweak phase diagram at finite net lepton number density

Antti Gynther*

Department of physical sciences,
Theoretical physics division,
University of Helsinki

at Cosmo -03
Ambleside, 25.8.2003

*e-mail: antti.gynther@helsinki.fi

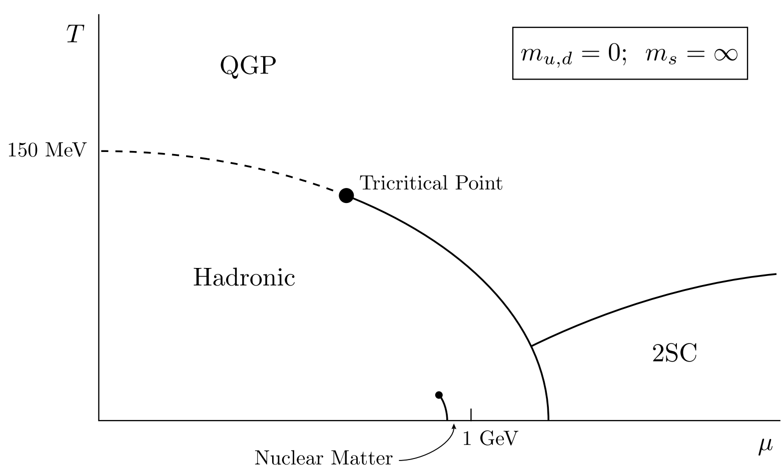
Motivation for finite μ_L

Cosmology:

- $n_\nu - n_{\bar{\nu}}$ unknown, upper limits from BBN and CMB.
- Leptogenesis of ΔB, . . .

Theory:

- Standard model describes Nature $\Rightarrow$ all its properties are interesting.
- Comparison to QCD at finite μ_B.



Electroweak thermodynamics

Thermodynamics $\leftrightarrow$ partition function (free energy):

$$\mathcal{Z} = \mathrm{Tr}\exp\left[-\beta\left(H - \sum_i \mu_i N_i\right)\right]$$

Electroweak theory: three conserved global charges

$$X_i = \frac{1}{n_f}B - L_i,$$

$\Rightarrow$ leptonic chemical potentials μ_{L_i}.

In addition two conserved gauge charges.

- corresponding chemical potentials must be fixed to ensure neutrality.

In general, there might also be an external hypercharge magnetic field $\boldsymbol{B}_Y$ related to the $U(1)$ gauge group.

$$\Rightarrow \mathcal{Z} = \mathcal{Z}(T, \mu_{L_i}, \boldsymbol{B}_Y)$$

Set now $\boldsymbol{B}_Y = 0$ and $\mu_{L_i} = \mu \; \forall \, i = 1 \ldots n_f$.

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\varphi \; \mathrm{e}^{-S[\varphi] + \mu \int_0^\beta \mathrm{d}\tau (L-B)} \quad ? \\ \text{where } L &= \sum_{i=1}^{n_f} L_i \end{aligned}$$

with the (euclidean) electroweak action. The chemical potentials related to the gauge charges can be absorbed to the action.

Dimensional reduction

Problems in resolving the thermodynamics:

- Infrared divergences in perturbative calculations
  - expansion parameter for static bosonic degrees of freedom becomes large.
- Monte Carlo studies of the full electroweak theory difficult and tedious.
  - chiral fermions, multiple scales, ...

Solution: combine both methods!

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\varphi \, \mathrm{e}^{-S+\mu_i N_i} \\ &\stackrel{\text{pert.}}{\approx} \underbrace{\int \mathcal{D}\Phi \mathcal{D}A_i^a \mathcal{D}B_i \, \mathrm{e}^{-\int \mathrm{d}^3x \, \mathcal{L}_{\text{eff}}}}_{\text{solve numerically}} \end{aligned}$$

The effective theory = the most general theory for the modes in question respecting the desired symmetries:

$$\mathcal{L}_{\text{eff}} = (D_i\Phi)^\dagger D_i\Phi + m_3^2\Phi^\dagger\Phi + \lambda_3(\Phi^\dagger\Phi)^2 + \frac{1}{4}F_{ij}F_{ij} + \frac{1}{4}G^a_{ij}G^a_{ij},$$

with some (perturbatively calculated) relation between the parameters of this theory and the physical variables

$$\left\{ \begin{array}{l} T \\ m_H \\ \mu \\ \vdots \end{array} \right\} \overset{?}{\Longleftrightarrow} \left\{ \begin{array}{l} m_3^2 \\ \lambda_3 \\ g_3^2 \\ g_3'^2 \end{array} \right\}.$$

The task:

Solve the PD of $\mathcal{L}_{\text{eff}}$ and map that to a PD in terms of T, μ_L, m_H!

Results

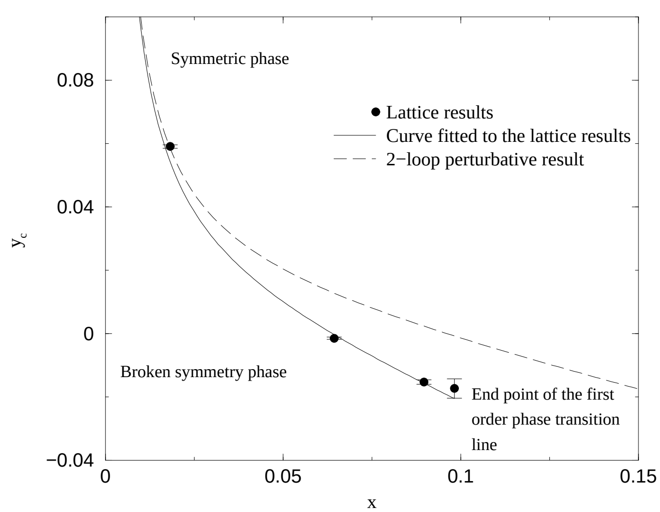
Define dimensionless parameters

$$x \equiv \frac{\lambda_3}{g_3^2} \quad y \equiv \frac{m_3^2(g_3^2)}{g_3^4} \quad z \equiv \frac{g_3'^2}{g_3^2}$$

and let the dimensions be given by g_3^2 $\left(\left[g_3^2\right] = 1\,\mathrm{GeV}\right)$

$\Rightarrow$ The phase diagram in terms of x and y:

Kajantie *et al*, Karsch *et al*



No phase transition for $x \gtrsim 0.1$!

Perturbative methods unreliable at large x.

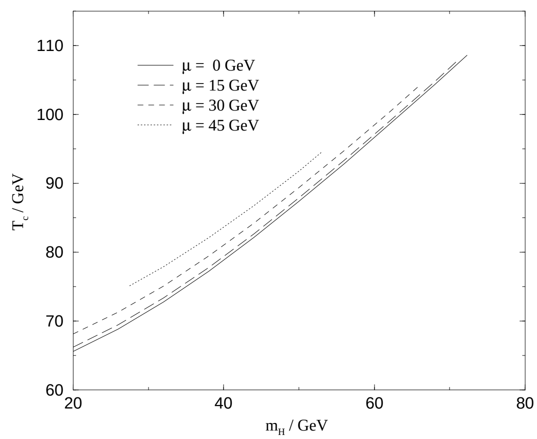
Matching to physical variables:

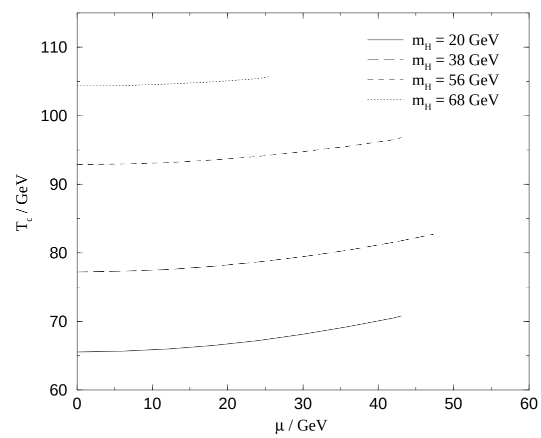
$$\begin{aligned} x(T,\mu,m_H) &\approx \frac{m_H^2}{8m_W^2} + \frac{1}{g^2}\frac{96}{1331}\frac{\mu^2}{T^2} \\ y(T,\mu,m_H) &\approx -\frac{m_H^2}{2g^4T^2} + \frac{1}{16g^4}\left[g^2\frac{m_H^2}{m_W^2} + 3g^2 + g'^2 \right. \\ &\quad \left. +4g_Y^2\left(1+\frac{1}{3\pi^2}\frac{\mu^2}{T^2}\right)\right] - \frac{1}{g^4}\frac{16}{121}\frac{\mu^2}{T^2} \end{aligned}$$

For fixed T and m_H, increasing μ leads to increasing x and decreasing y.

The phase diagram in terms of T, μ and m_H:

AG, PRD68:016001, 2003
(hep-ph/0303019)





Remarks: for a fixed m_H, increasing μ leads to

- increasing T_c
- a weaker transition

No phase transition for sufficiently large μ for any m_H!
(only a crossover transition)

Understanding all this

In general, finite μ for the global charges $\Rightarrow$ finite μ for the gauge charges

$\Longrightarrow$ finite μ for the scalar field

Consequences:

1. Increased "tendency" to condense (Bose condensation)

   $\Rightarrow$ Higher temperature needed to restore the symmetry (to melt the condensate)

2. Interactions with temporal components of the gauge fields is altered.

   $\Rightarrow$ Additional interactions in the scalar sector changes the nature of the phase transition.

How far can we go?

1. Integration over the nonstatic modes
   - can be performed for arbitrary $\mu/(\pi T)$ to a desired order in g^2
   - reduction consistent only if $\mu/(\pi T) \lesssim 1$
2. Integration over the adjoint scalars
   - some of the couplings of the effective theory proportional to $\mu/(\pi T)$ $\Rightarrow$ must require $\mu/(\pi T) \lesssim g$
   - impossible to perform the reduction to a certain order in g^2 for arbitrary $\mu/(\pi T)$

Conclusions

- Dimensional reduction of the electroweak theory is performed at finite lepton number density.

- The electroweak phase diagram is solved in terms of (T, μ_L, m_H).

- It is seen that T_c grows with μ_L and the transition becomes weaker.

  - For sufficiently large μ_L there is no phase transition for any value of m_H, just a crossover transition.

- Methods used are not applicable for large chemical potentials.