Higher Order Moments of Momentum Spectra

Nick Brook

University of Bristol

Ian Skillicorn

UNIVERSITY of GLASGOW

- Introduction
- MLLA predictions
- Comparisons with e^+e^- & DIS data

Fragmentation Functions

D(ξ,Q,Λ) not calculable in pQCD

BUT know how to evolve in Q^2

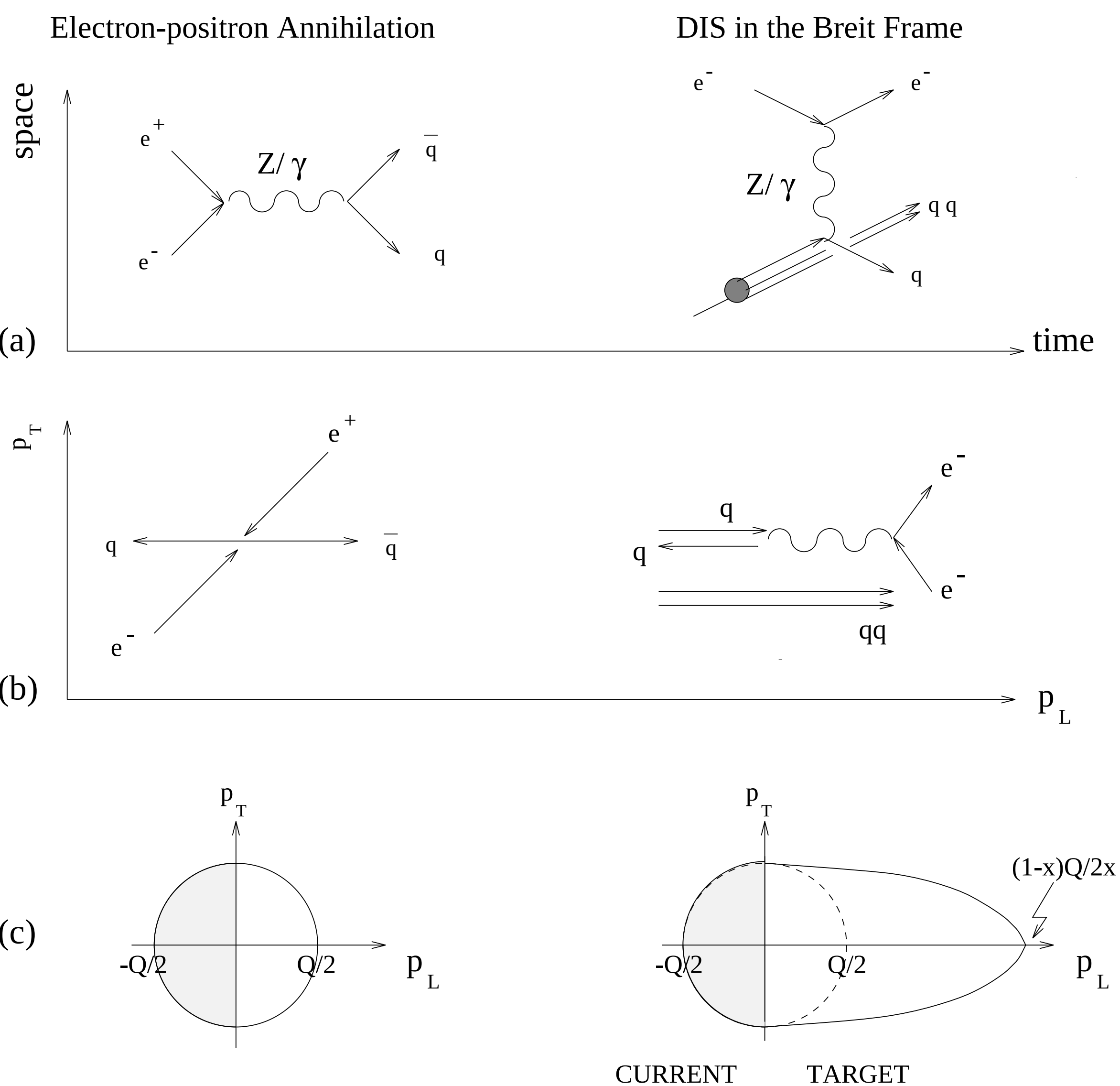
$$x_p = p/p_{\max} = 2p/Q \qquad \xi = \log\left(1/x_p\right)$$

For e^+e^-: $Q = \sqrt{s}$

For DIS: Q is the virtuality of the exchanged photon and p is measured in the current region of the Breit frame (where p_{max} is Q/2)

The Breit Frame

`Brickwall' frame



Phase space for e^+e^- annihilation evolves with $Q/2 = \sqrt{s}/2$

Current hemisphere of Breit frame evolves as Q/2

Current region ≡ e^+e^- annihilation

Moments of ξ dist[bn]

MLLA theory assumes energy spectra - expt measure momenta

Khoze, Lupia & Ochs suggested (Phys Lett B386 1996 451) the cut-off in evolution can be related to an effective mass, m_{eff}, of the hadrons

MLLA predictions for the Q evolution of moments (Dokshitzer et al, Int J Mod Phys A7 1992, 1875.)

Fong & Webber suggested region around peak can be described by a distorted Gaussian

Extract moments from fits to MLLA theoretical spectra and data in a consistent manner

(PLB 479 2000 173 & PLB 497 2001 55)

$$D(\xi, Q, \Lambda) \propto \exp\left[\tfrac{1}{8}k - \tfrac{1}{2}s\delta - \tfrac{1}{4}(2+k)\delta^2 + \tfrac{1}{6}s\delta^3 + \tfrac{1}{24}k\delta^4\right]$$

$$\text{where} \quad \delta = \frac{(\xi - \bar{\xi})}{\sigma}$$

Scaled Momentum Distributions, ξ_p

Following the approach of Khoze et al., can relate a cut-off, Q_0, in the parton evolution to the mass of hadrons, m_{eff}, and the MLLA spectra to expt. spectra:

$$\frac{1}{N}\frac{dn_h}{d\xi_p} \propto \frac{p_h}{E_h}\overline{D}(\xi, Q, \Lambda)$$

where

$$\xi = \log\left(\frac{Q}{\sqrt{Q^2 e^{-2\xi_p} + Q_0^2}}\right) \quad \& \quad E_h = \sqrt{p_h^2 + Q_0^2}$$

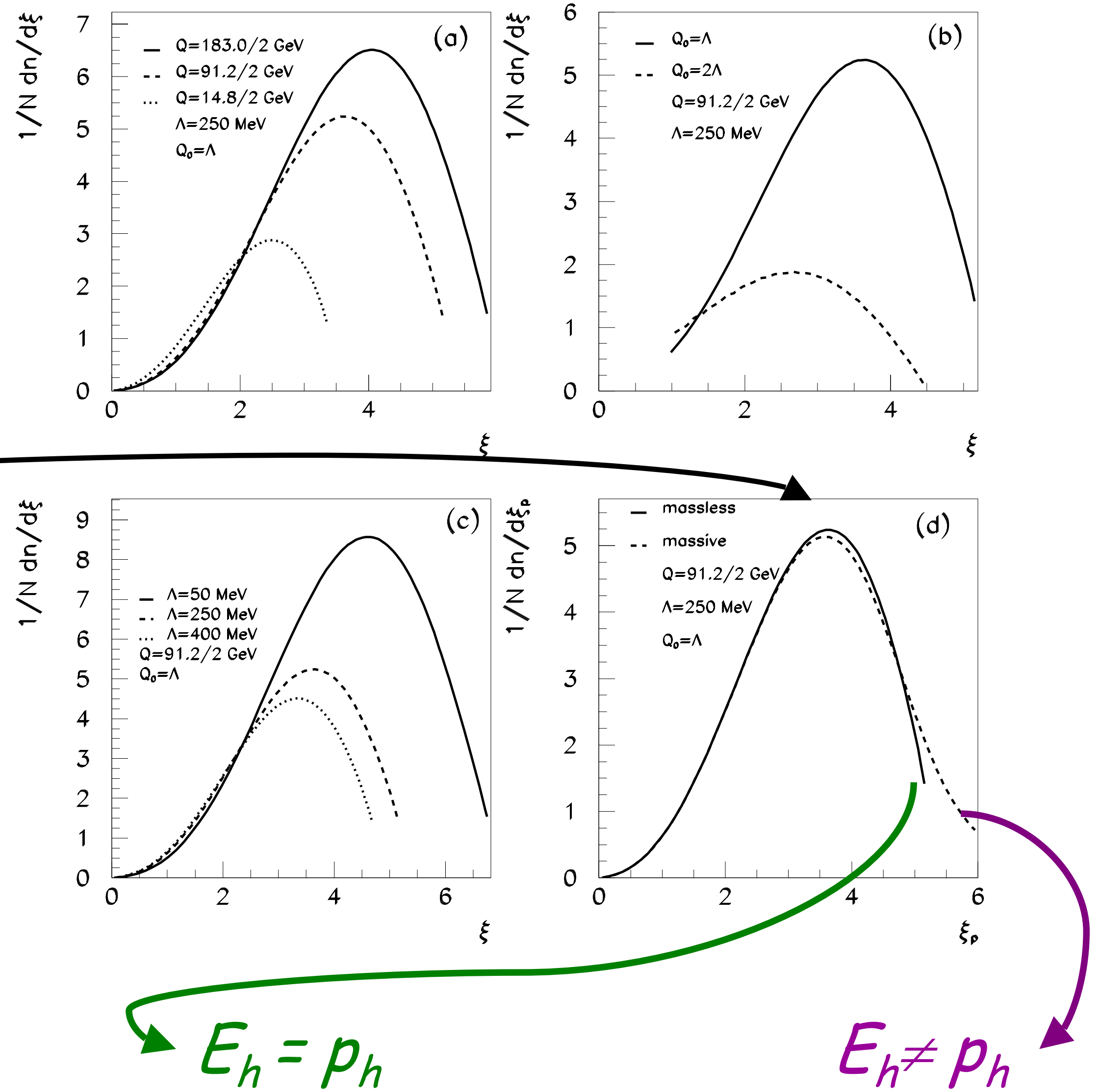
MLLA Spectra

- Typical hump back shape with dependence on energy, Q_0 & Λ

Introduce m_{eff}

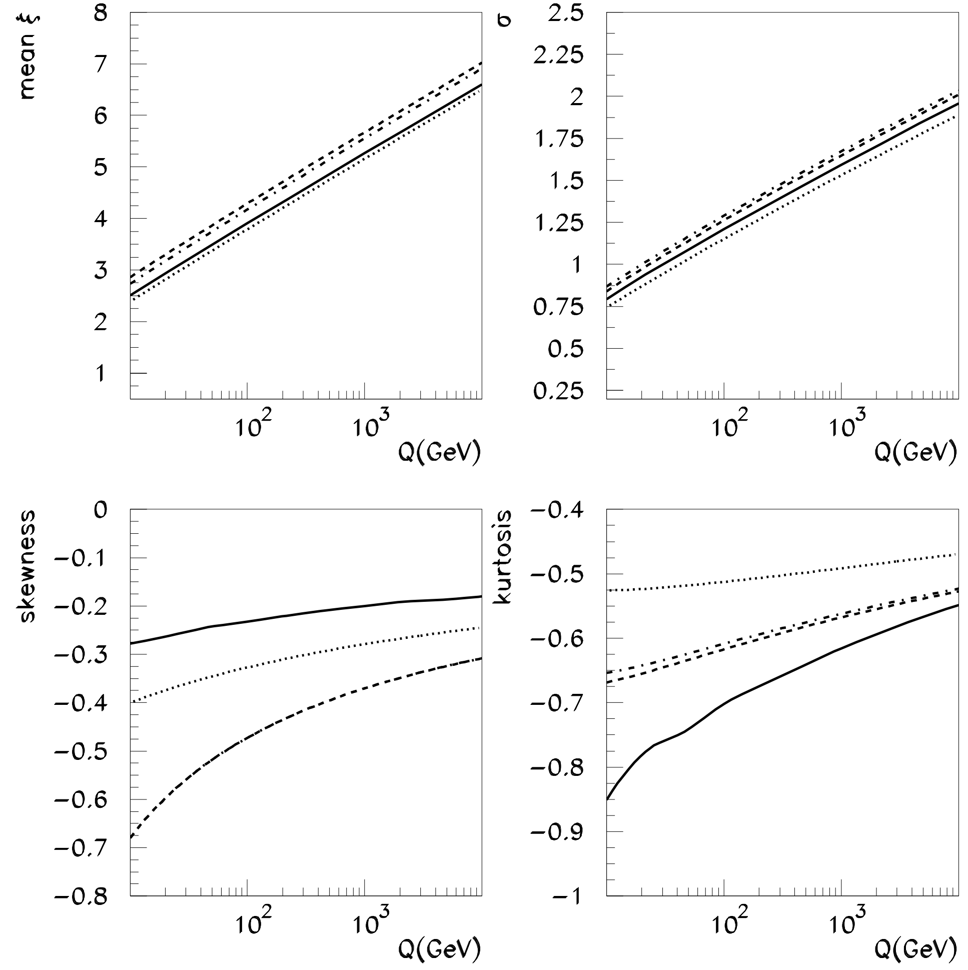
- Changes in RHS side of distorted Gaussian by introducing m_{eff} term (i.e. momentum getting smaller)

- No longer displays usual truncation at large ξ



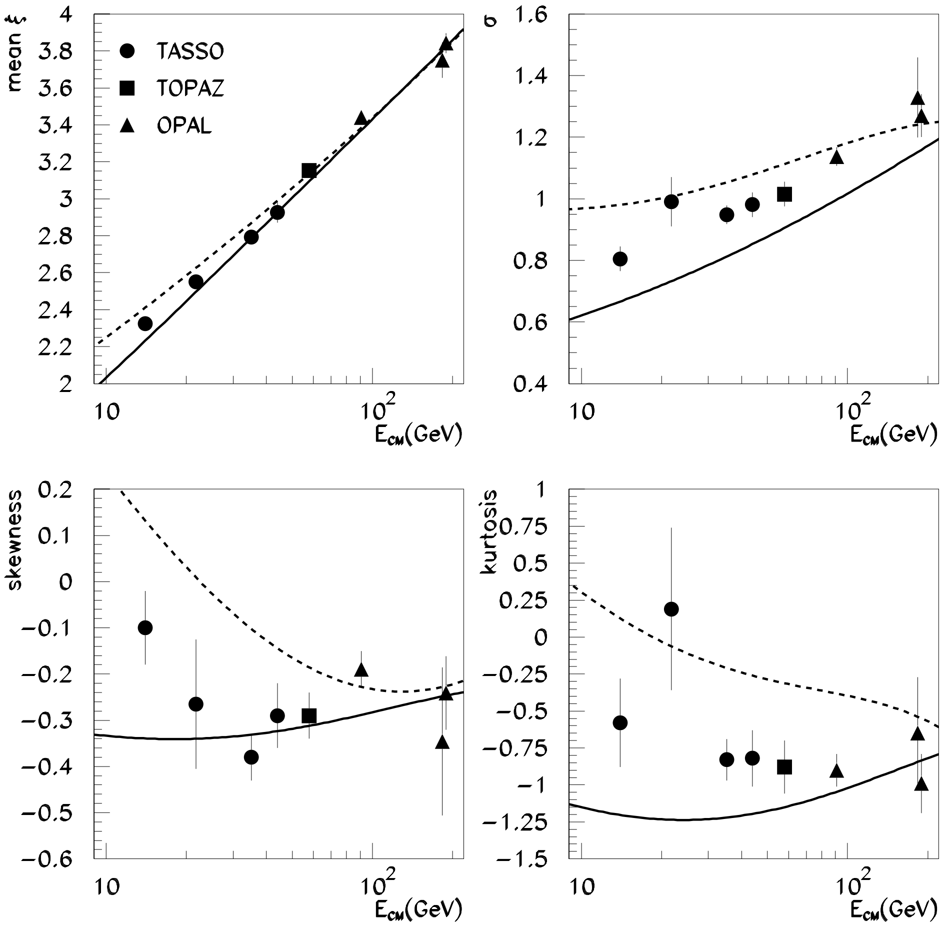
MLLA Moments

- Marked difference between Dokshitzer et al & fit to MLLA spectra (calculating moments over whole range of spectra give same answer as analytic calculation)
- Important moments are calculated/extracted in consistent manner!!
- Different assumption in for small ξ account for difference between F&W and Dokshitzer et al ??
- As Q↑ all predictions converge



— fit to MLLA spectra; ···· Dokshitzer et al analytic; - · - · Fong & Webber analytic

Moments of Spectra & e^+e^- Data



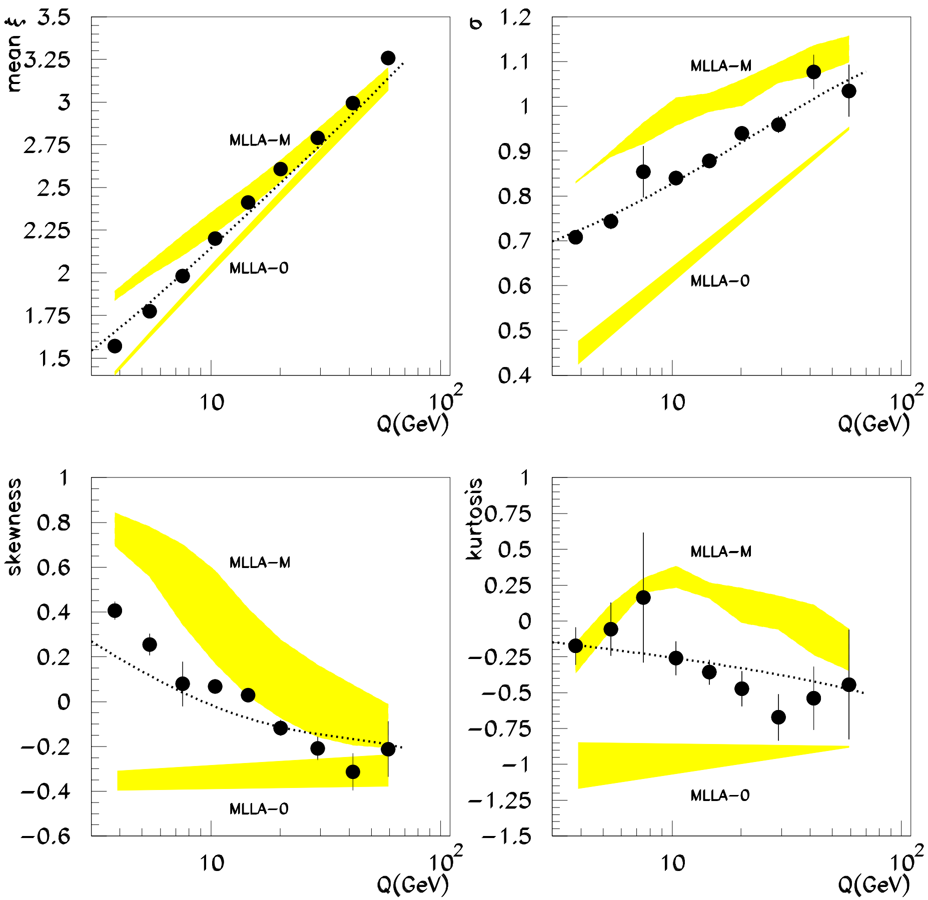
Moments extracted from fit of distorted Gaussian to data BUT also to theory

- MLLA-0 gives reasonable description of mean & skewness
- MLLA-0: σ is smaller & more platykurtic than data
- MLLA-M gives poorer description of all varaibles at low E_{CM}
- MLLA-M approaches MLLA-0 & data at high E_{CM}

— MLLA-0 (no m_{eff} term - $Q_0 = \Lambda$)

--- MLLA-M (with m_{eff} term - $m_{eff} = Q_0 = \Lambda$)

Moments of Spectra & DIS Data



.... $Q_0 = \Lambda \neq m_{eff}$

Moments extracted from fit of distorted Gaussian to data & theory curves

- m_{eff} allowed to be free parameter in fits to data
- MLLA-M & MLLA-0 bracket the data. At high Q both approach the data
- Fit to DIS data Λ=280 MeV & m_{eff} = 230 MeV. Reasonable description of all variables except mean (and peak position)

$$\xi_{\max} = \bar{\xi} + \frac{\sigma}{s}\left(1 - \left(1 + s^2\right)^{1/2}\right)$$

Summary

- Care needs to be taken in choosing the range of ξ when comparing data & theory
- Wide discrepancy in higher order moments (skewness & kurtosis) between MLLA-M & MLLA-0 and data. Predictions converge as Q↑
- Limiting spectra preferred over truncated spectra ($Q_0 \neq \Lambda$) in comparison with data
- m_{eff} has a large effect at all but highest Q
- MLLA gives reasonable agreement with higher moments of data for fitted values of Λ and m_{eff}