Neutrinoless Double Beta Decay Matrix Elements: Is there a convergence of QRPA results?

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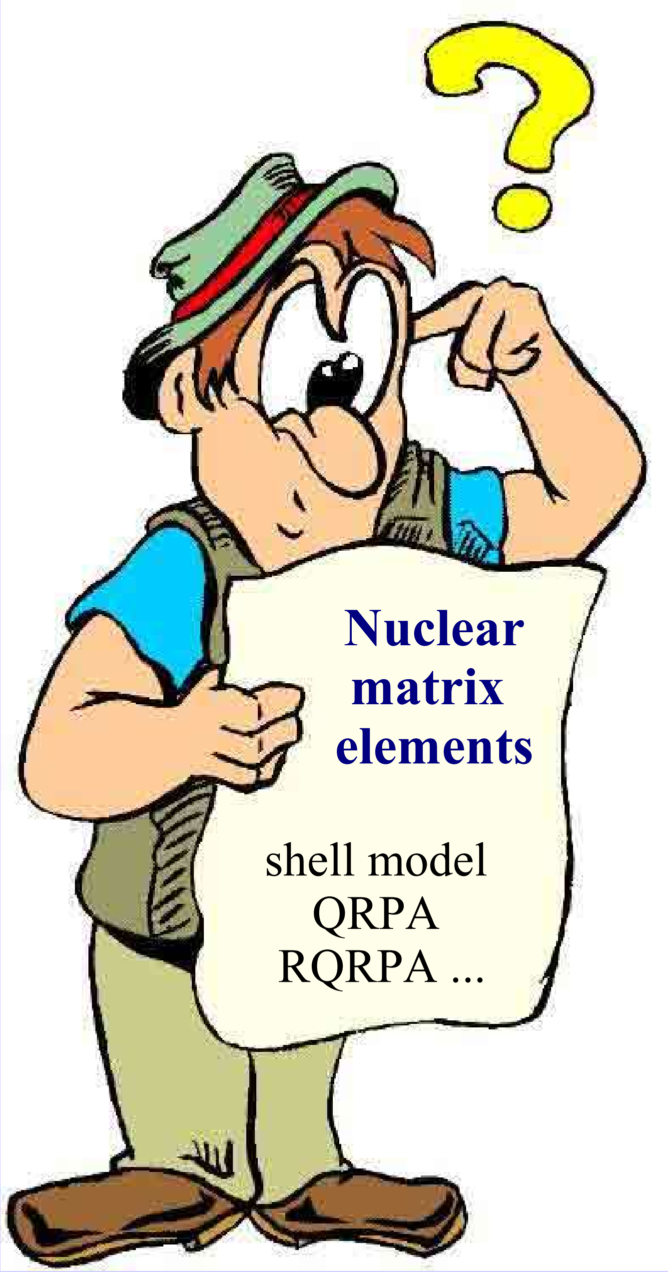
Status of the field: Experimentalists are confused



- What are the best $0\nu\beta\beta$-decay candidates?
- Which of future experiments has chance to observe $0\nu\beta\beta$-decay?
- Will the evidence of the $0\nu\beta\beta$-decay allow to deduce a valuable information about Majorana CP-phases?

Request for reliable NME

The evaluation of the $\beta\beta$-decay ME is a complex task:

- medium and heavy open-shell nuclei with a complicated nuclear structure
- the construction of a complete set of the states of intermediate states is needed
- many-body approximations, what are the limitations of nuclear structure approaches?
- what is the influence of the structure of the nucleus on the $0\nu\beta\beta$-decay matrix elements

$0\nu\beta\beta$-decay matrix elements

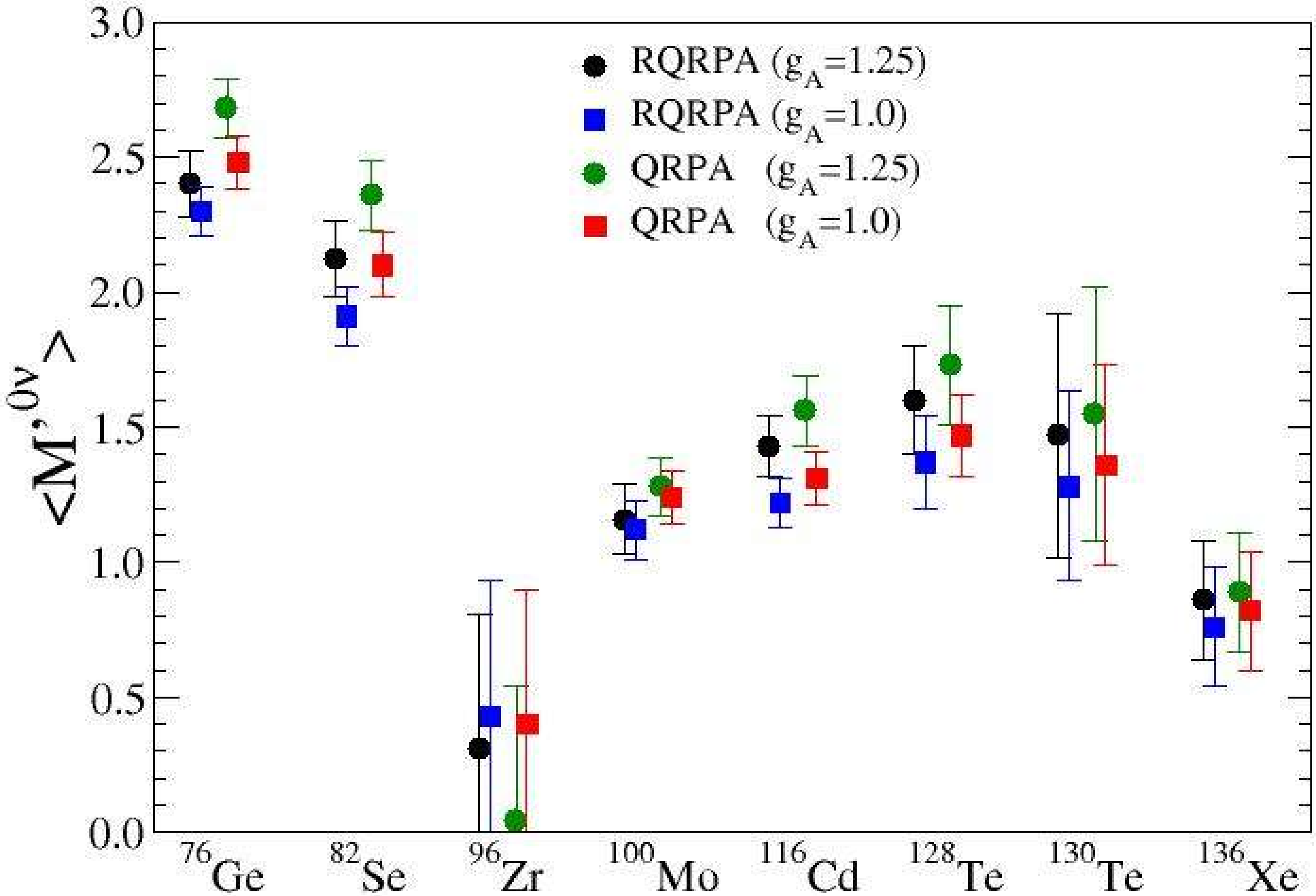
Bratislava-Caltech-Tuebingen

Neutrinoless Double Beta Decay Matrix Elements

**Tuebingen
Bratislava
Caltech
Collaboration**

**g_{pp} fixed to
$2\nu\beta\beta$-decay
half-life**

**The most
reliable
QRPA/RQRPA
NME**



**V. Rodin, A. Faessler, F. Simkovic, P. Vogel, PRC 68 (2003) 044303;
nucl-th/0503063 and submitted**

Sources of uncertainties in the $0\nu\beta\beta$-decay Nuclear Matrix Elements

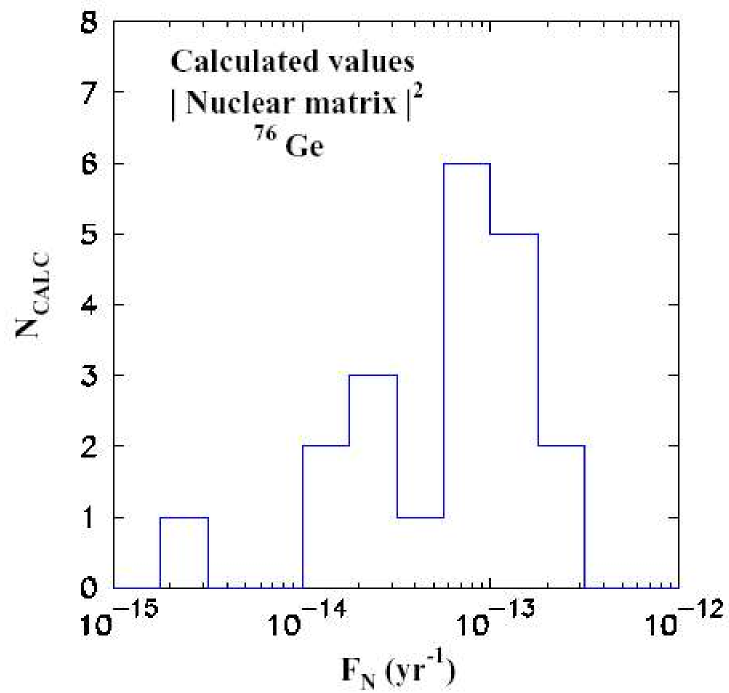
Bratislava-Caltech-Tuebingen

Please, no!
Do not put different NME calculations on the same level!

System	$G_1^{(0\nu)} \times 10^{14}$	N.M.E.	N.M.E. (this work)	$\langle m_\nu \rangle$max
^{48}Ca	6.43	1.08–2.38		8.70–19.0
^{76}Ge	0.63	2.98–4.33	3.33	0.30–0.43
^{82}Se	2.73	2.53–3.98	3.44	4.73–7.44
^{96}Zr	5.70	2.74	3.55	19.1–24.7
^{100}Mo	4.57	0.77–4.67	2.97	2.18–13.2
^{116}Cd	4.68	1.09–3.46	3.75	2.37–8.18
^{128}Te	0.16	2.51–4.58		9.51–17.4
^{130}Te	4.14	2.10–3.59	3.49	1.87–3.20
^{136}Xe	4.37	1.61–1.90	4.64	0.79–2.29

Civitarese, Suhonen, Nucl. Phys. A (2003) 867

J. Bahcall, H. Murayama, C Pena-Garay, Phys.Rev.D 70 (2004) 033012



$0\nu\beta\beta$-decay matrix element calculation

$0\nu\beta\beta$-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0,Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \; ,$$

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \langle f| - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

NME = sum of Fermi, Gamow-Teller and tensor contributions

$$\Omega = \tau^+\tau^+ \frac{(-h_F + h_{GT}\sigma_{12} - h_T S_{12})}{q(q + E_J^m - (E^i_{g.s.} + E^f_{g.s})/2)} \; , \quad \sigma_{12} = \vec{\sigma_1}\cdot\vec{\sigma_2} \; , \quad S_{12} = 3\vec{\sigma_1}\cdot\vec{q}\vec{\sigma_2}\cdot\vec{q} - \sigma_{12}$$

Neutrino potential

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2)qdq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Form-factors induced PS included

$$\begin{aligned} h_F &= g_V^2(q^2) \\ h_{GT} &= g_A^2\left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \\ h_T &= g_A^2\left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right] \end{aligned}$$

partial NME

$$M_{K=F,GT,T} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\}$$
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$
$$\times \langle 0_f^+ || \widetilde{[c_{p'}^+ \tilde{c}_{n'}]_J} || J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_J || 0_i^+\rangle$$

two-nucleon s.r.c.

$$f(r_{12}) = 1 - e^{-\gamma_1 r_{12}^2}(1 - \gamma_2 r_{12}^2)$$
$$\gamma_1 = 1.1 \; fm^2, \qquad \gamma_2 = 0.68 \; fm^2$$

List of reasons, why RPA-like 0νββ-decay NME might be different

Quasiparticle mean field:
pp,nn pairing (pn pairing)

Many-body approximations:
QRPA, RQRPA, SRQRPA

NN-force
schematic, realistic (Bonn, ...)

The size of the model space

p-h interaction (g_{ph}~1):
fixed to GT resonance

p-p interaction (g_{pp}):
g_{pp}=1, fixed to β-decay,
fixed to ββ-decay

Two-nucleon s.r.c. (~50%):
Jastrow function

Finite size of the nucleon:
form-factors, 10% effect

Higher order terms of n.c.:
induced PS, weak magnetism

The closure approximation

The overlap factor:
(BCS overlap?)

i

The axial-vector coupling:
g_A=1.0 or g_A=1.25

Nuclear shape:
spherical, not deformed yet

QRPA and RQRPA Nuclear Matrix Elements

Ref.	Method	r_0 [fm]	g_{pp}	g_A	$M'^{0\nu}$ ^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
					Without higher order terms of nucleon current Differences understandable								
EVZ-88	QRPA	1.1	β	1.0	3.0	2.3	1.9	3.5		4.0	3.5	1.6	
MBK-89	QRPA	1.1	β	1.00	2.84	2.65		1.69		2.91	2.36	1.08	4.17
				1.25	3.84	3.59		1.94		3.93	3.18	1.45	5.57
T-91	QRPA	1.1	1.0	1.0	2.86	2.59		3.14		2.55	2.22	1.27	3.47
				1.25	3.97	3.60		4.30		3.53	3.07	1.74	4.80
SKF-91	QRPA	1.1	1.0	1.00	3.37	2.72				3.17	2.94	1.71	
				1.25	4.55	3.71				4.24	3.95	2.31	
PSVF-96	QRPA	1.1	1.0	1.25	3.04	2.23	2.41	1.09	0.94	2.48	2.33	1.55	
SPVF-99	RQRPA	1.1	1.0	1.00	4.05	3.82	2.24	4.58	2.86	3.38	2.87	1.20	5.15
				1.25	3.60	3.40	1.99	4.12	2.58	2.96	2.50	1.02	4.51
present	QRPA	1.1	$2\nu\beta\beta$	1.25	3.35	2.95		1.83		2.32	1.98	1.30	3.10

EVZ-88: Engel, Vogel, Zirnbauer, 37 (1988) 731
MBK-89: Muto, Bender, Klapdor, Z. Phys. A 334 (1989) 187
T-91: Tomoda, Rept. Prog. Phys. 54 (1991) 53
SKF-91: Suhonen, Khadkikhar, Faessler, NPA 535 (1991) 509
PSVF-96: Pantis, Simkovic, Vergados, Faessler, PRC 53 (1996) 695
SPVF-99: Simkovic, Pantis, Vergados, Faessler, PRC 60 (1999) 055502

Questions addressed

Ref.	Method	r_0 [fm]	g_{pp}	g_A	$M'^{0\nu}$ ^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
					Without higher order terms of nucleon current								
					Results discussed								
SK-01	QRPA	?	$2\nu\beta\beta$	?	4.45	5.60	4.16	5.37	3.99	4.84	4.73	1.69	
					1.71	4.71	2.75	3.81	2.85	3.43	3.77	1.35	
	RQRPA	?	$2\nu\beta\beta$	?	3.74	4.30	3.01	4.36	3.61	4.29	4.55	1.57	
					1.87	2.70	2.72	3.40	3.39	2.83	3.00	1.02	
AS_{WS}-98	QRPA	1.1	β	1.00	3.98	3.69	2.88		2.21		4.62	2.49	
				1.25	5.30	4.93	3.85		2.93		6.15	3.34	
AS_{AWS}-98	QRPA	1.1	β	1.00	4.85	3.61	3.70		3.97		3.81	2.15	
				1.25	6.44	4.82	3.96		5.25		5.05	2.84	
CS-03	QRPA	?	β	1.25	3.33	3.44	3.55	2.97	3.75		3.49	4.64	

small m.s.

Why large difference?

(large) m.s.

The difference between AS and CS calculations not known

Why so large?

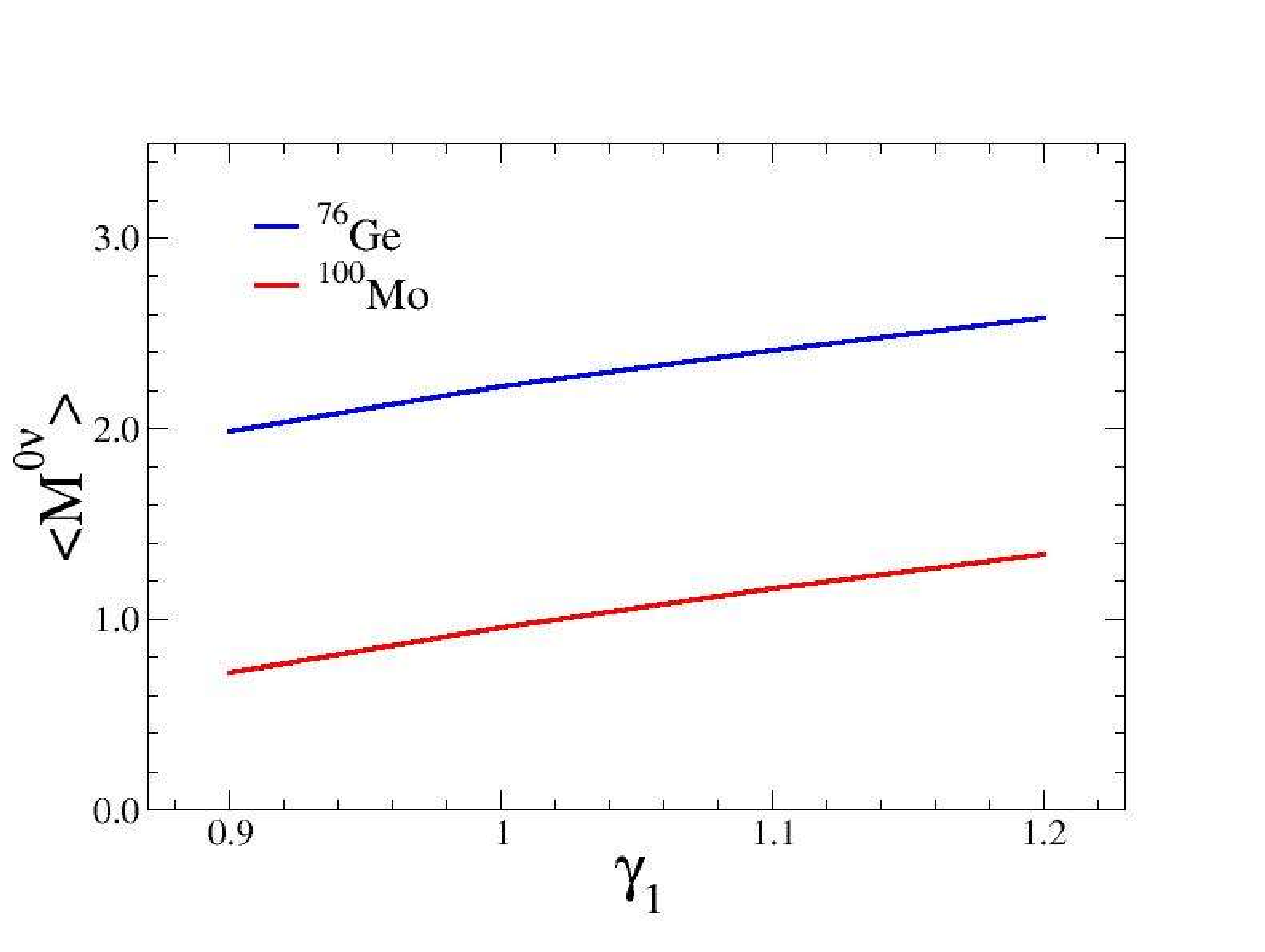
SK: Stoica, Klapdor-Kleingrothaus. PRC 63 (2001) 064304; NPA 694 (2001) 269
AS: Aunola, Suhonen, NPA 643 (1998) 207
CS: Civitarese, Suhonen, NPA 729 (2003) 867

If g_{pp}=1, there is a strong dependence on the size of model space.
If g_{pp} fixed to 2νββ-half-life, there is only slight dependence on the size of m.s.

Ref.	Method	r_0 [fm]	g_{pp}	g_A	$M'^{0\nu}$								
					^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
					With higher order terms of nucleon current								
SPVF-99	RQRPA	1.1	1.0	1.25	2.80	2.64	1.49	3.21	2.05	2.17	1.80	0.66	3.33
present	QRPA	1.1	$2\nu\beta\beta$	1.00	2.48	2.10	0.40	1.24	1.31	1.47	1.36	0.94	1.96
				1.25	2.68	2.36	0.04	1.28	1.56	1.73	1.55	1.03	2.25
	RQRPA	1.1	$2\nu\beta\beta$	1.00	2.30	2.91	0.43	1.12	1.22	1.37	1.28	0.90	1.79
				1.25	2.40	2.12	0.31	1.16	1.43	1.60	1.47	0.98	2.05

SPVF: Simkovic, Pantis, Vergados, Faessler, PRC 60 (1999) 055502
present: Rodin, Faessler, Simkovic, Vogel, nucl-th/0503063

Effect of the two-nucleon s.r.c.



Jastrow function:

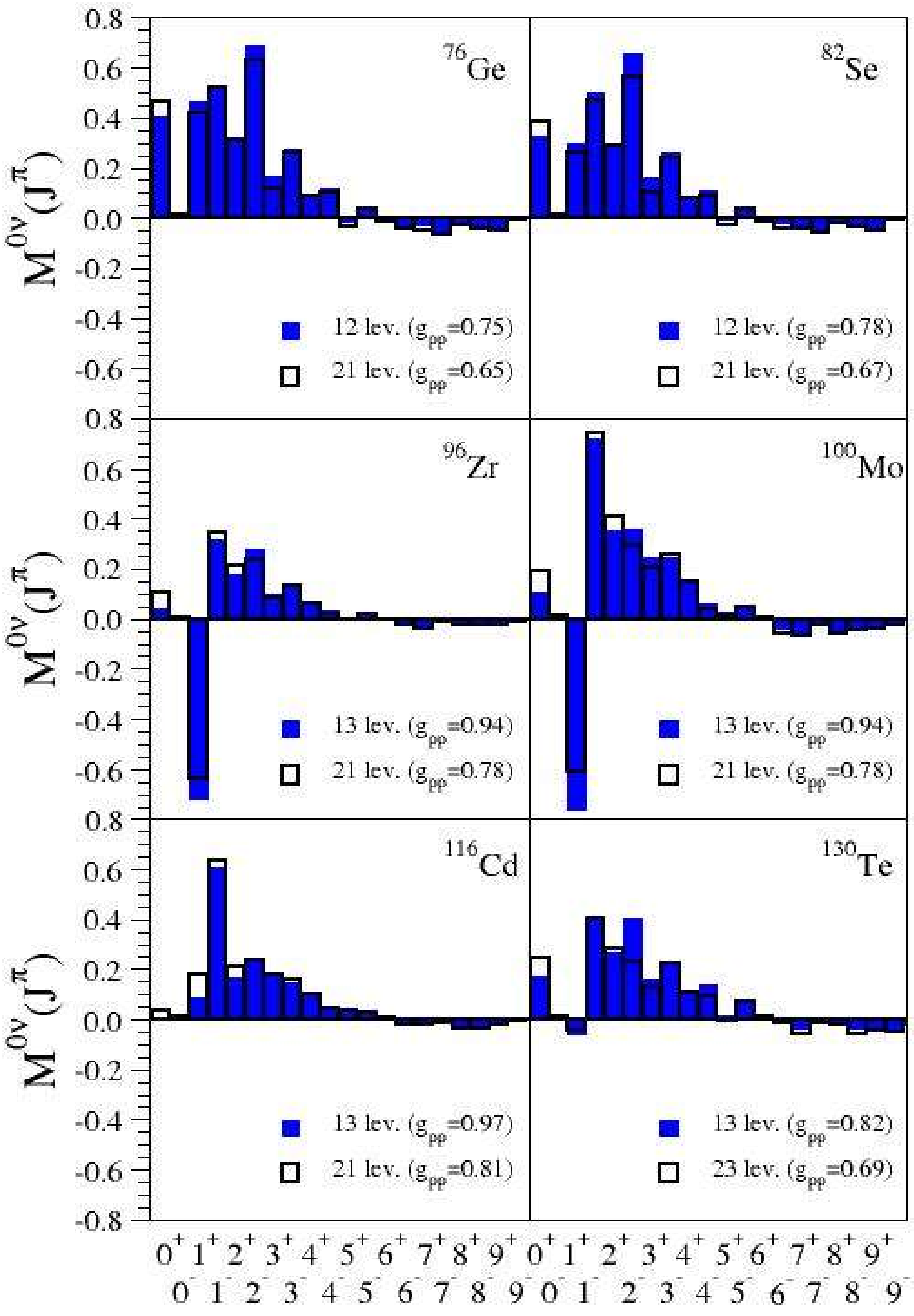
$$f(r_{12}) = 1 - e^{-\gamma_1 r_{12}^2}(1 - \gamma_2 r_{12}^2)$$

$$\gamma_1 = 1.1\ fm^2, \quad \gamma_2 = 0.68\ fm^2$$

Multipole decomposition of NME

1^+ sensitive to g_{pp}, might be large

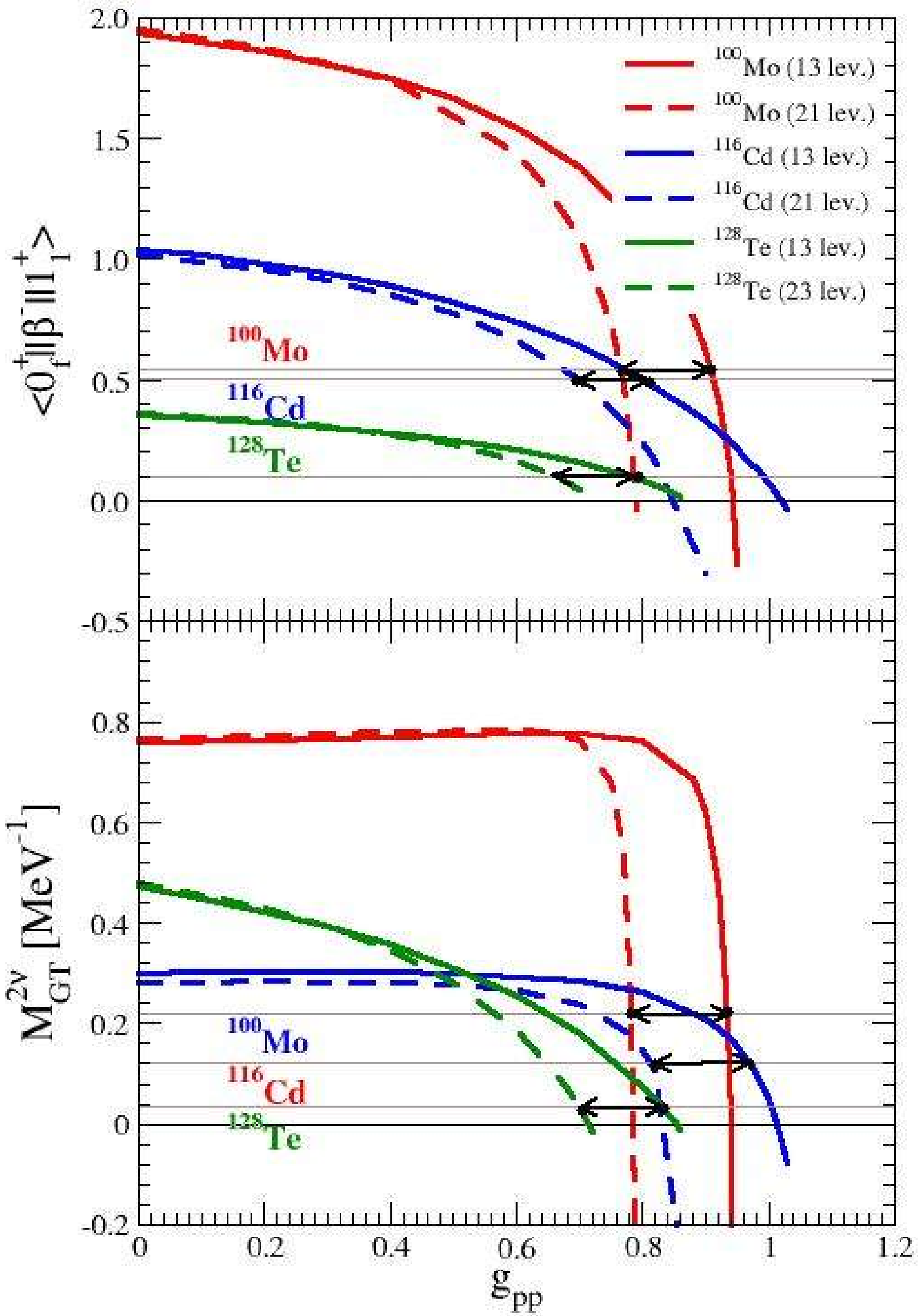
many multipolarities large
$<q_\nu> \sim 100$ MeV



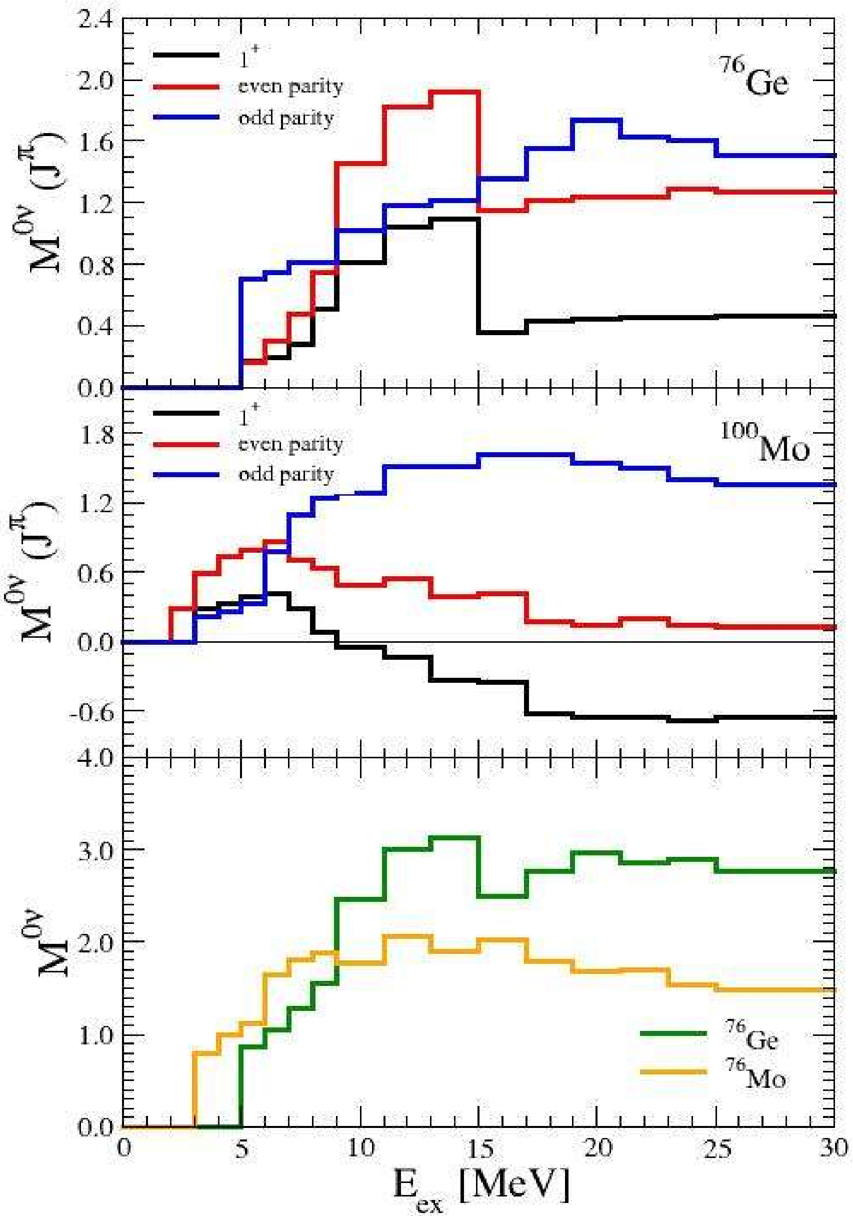
Fixing of g_{pp} to
- single β-decay (*)
- double β-decay (**)

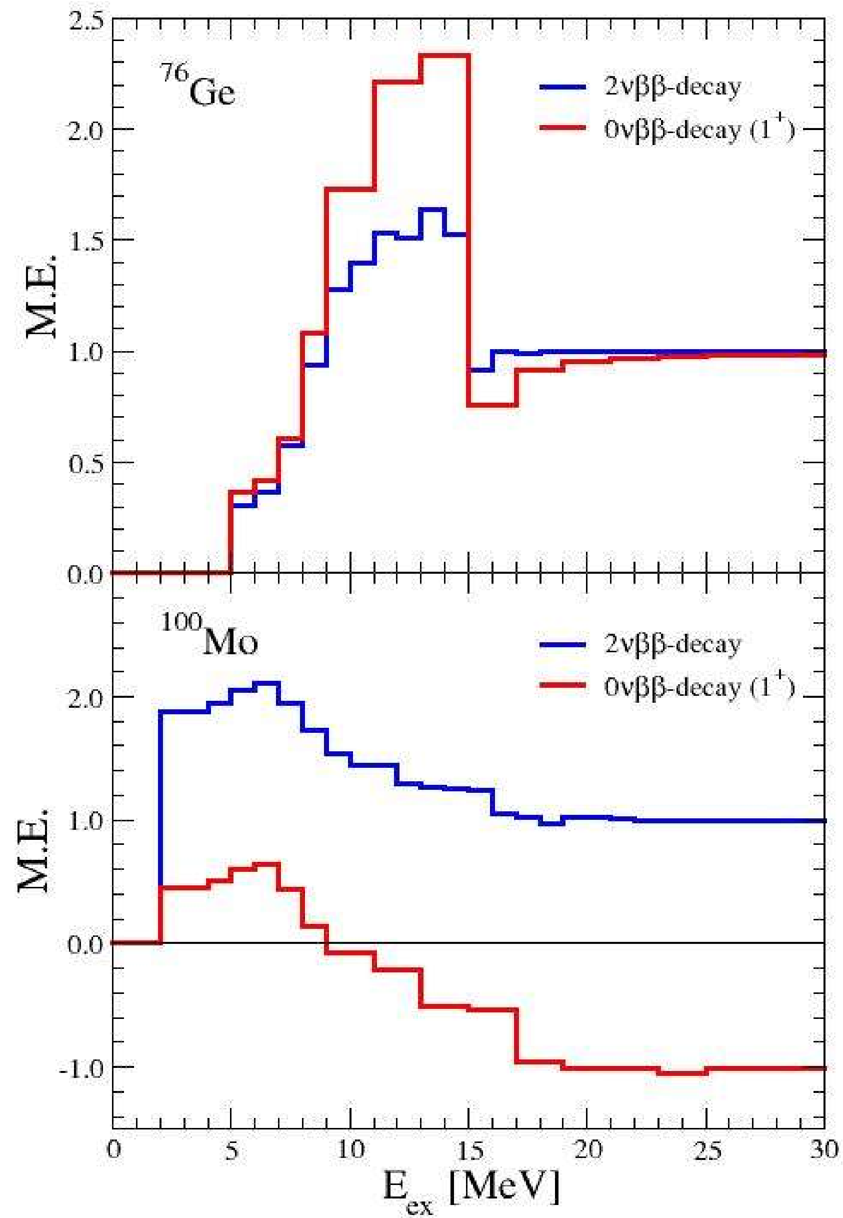
(*) restricted to ^{100}Mo, ^{116}Cd, ^{128}Te
(the g.s. of (A,Z+1) nucleus is 1^+-state)

There is a large overlap between allowed values of g_{pp} of both approaches



The importance of transitions through higher-lying states of (A,Z+1) nucleus





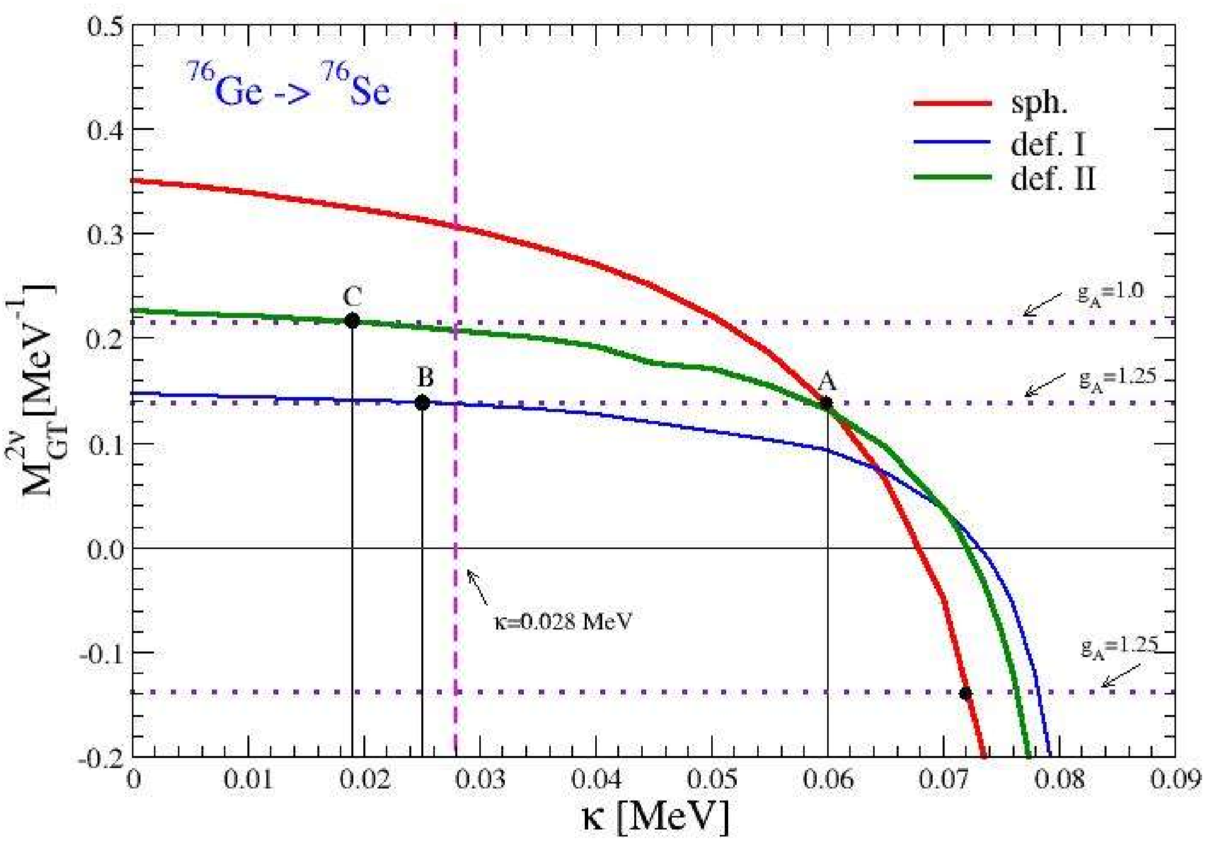
M_{GT} negative is disfavored

$$1/T_{1/2} = G_{01} |M_{GT}|^2 \Rightarrow M_{GT} > 0 \text{ or } M_{GT} < 0$$

M_{GT} is disfavored:

- **disagreement with systematic study of single beta decay Homma et al. PRC 54 (1996) 2972**
- **The lowest EC- transition (A,Z+1) -> (A,Z) too large**
- **If Pauli exclusion principle fully taken into account M_{GT} negative appears for too large value of g_{pp} Simkovic et al, PRC 61 (2000) 044319**

M_{GT} positive and M_{GT} negative



Which $0\nu\beta\beta$-decay NME to consider?

If the authors do not specify what choices they made, and do not discuss the dependence of their result on the particular choice they made, their result should not be taken on the same footing as those where these points are carefully explained!

May 2005: Recommendation

The most carefully calculated (reliable) QRPA/RQRPA $0\nu\beta\beta$-decay NME are:

Rodin, Faessler, Simkovic, Vogel, nucl-th/0503063

Should not be put on the same level with other QRPA/RQRPA calculations

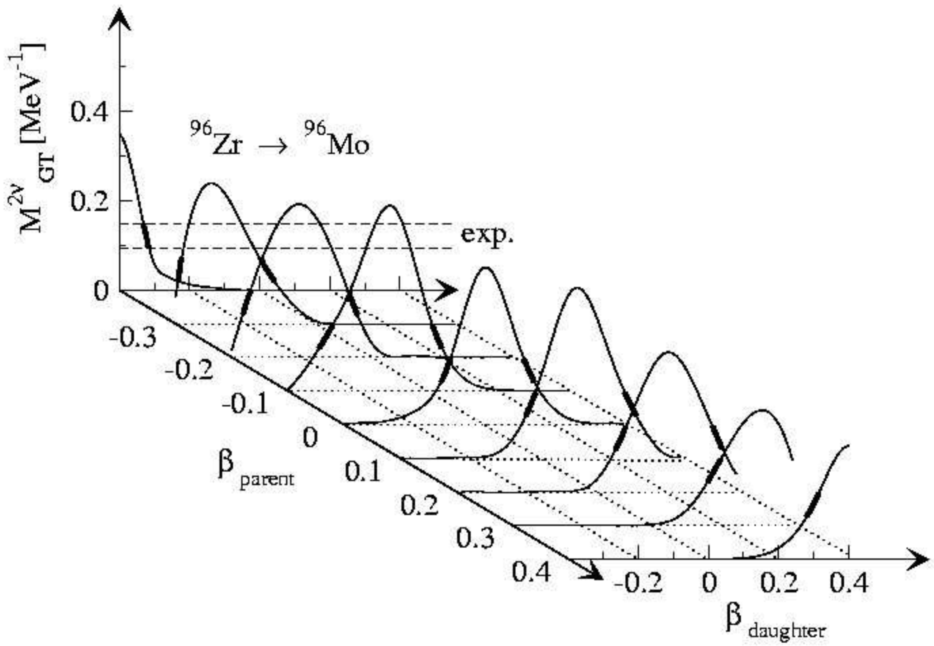
Nuclear Deformation
and
Two-Neutrino DBD

Bratislava-Tuebinge- Madrid

Nuclear Deformation

Nucl.	Exp. I	Exp. II	Theor. I	Theor. II
^{48}Ca	0.00	0.101	0.00	0.00
^{48}Ti	+0.17	0.269	-0.01	0.00
^{76}Ge	+0.09	0.26	0.16	0.14
^{76}Se	+0.16	0.31	-0.24	-0.24
^{82}Se	+0.10	0.19	0.13	0.15
^{82}Kr		0.20	0.12	0.07
^{96}Zr		0.081	0.22	0.22
^{96}Mo	+0.07	0.17	0.17	0.08
^{100}Mo	+0.14	0.23	0.25	0.24
^{100}Ru	+0.14	0.22	0.19	0.16
^{116}Cd	+0.11	0.19	-0.26	-0.24
^{116}Sn	+0.04	0.11	0.00	0.00
^{128}Te	+0.01	0.14	-0.00	0.00
^{128}Xe		0.18	0.16	0.14
^{130}Te	+0.03	0.12	0.03	0.00
^{130}Xe		0.17	0.13	-0.11
^{136}Xe		0.09	0.00	0.00
^{136}Ba		0.12	0.00	0.00
^{150}Nd	+0.37	0.28	0.22	0.24
^{150}Sm	+0.23	0.19	0.18	0.21

$$\beta = \sqrt{\frac{\pi}{5}}\frac{Q_p}{Zr_c^2}$$



Exp. I (nuclear reorientation method)
Exp. II (based on measured E2 trans.)
Theor. I (Rel. mean field theory)
Theor. II (Microsc.-Macrosc. Model of Moeller and Nix)

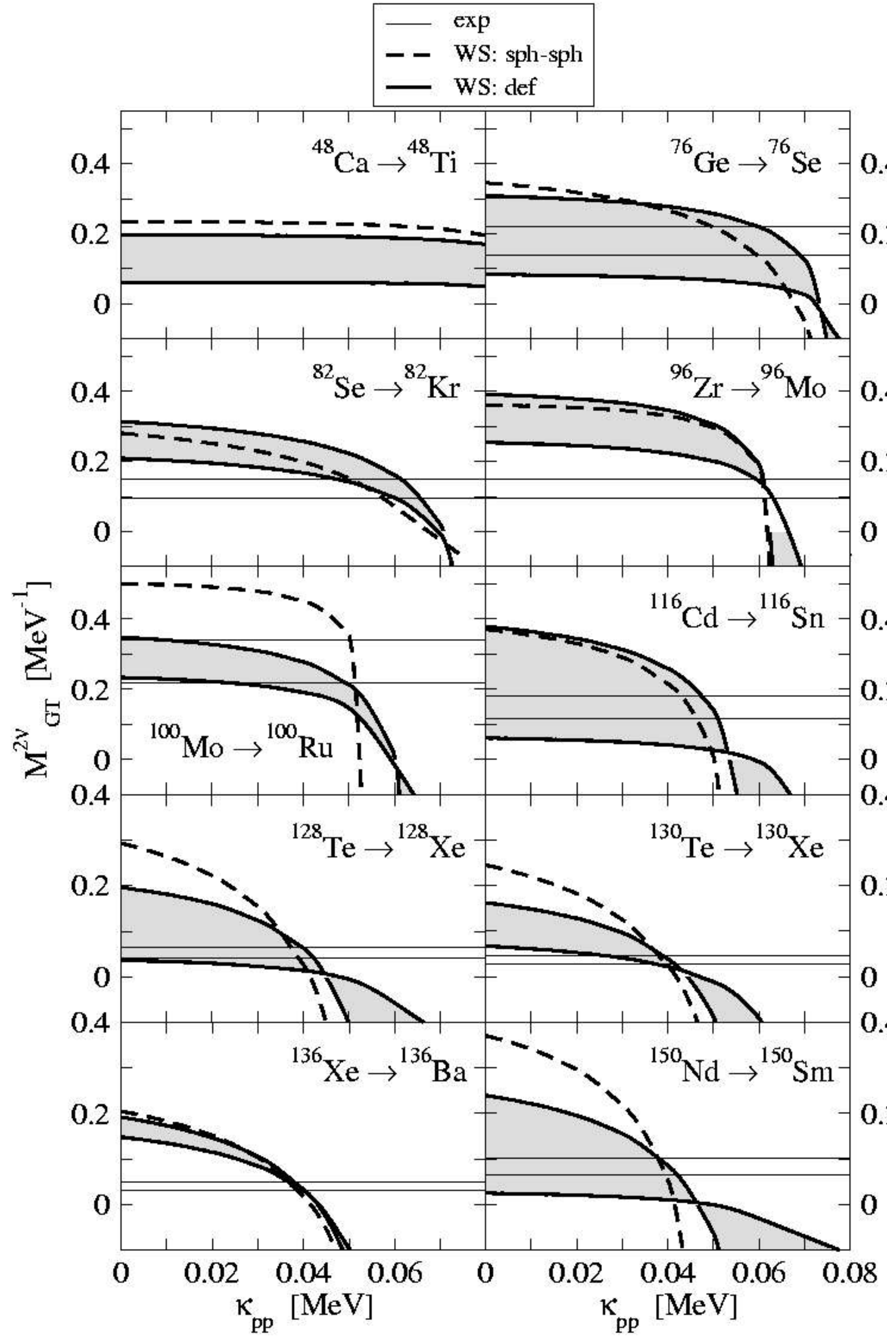
New suppression mechanism

The suppression of the NME depends on the relative deformation of initial and final nuclei.

Simkovic, Pacearescu, Faessler, NPA 733 (2004) 321

Systematic study of the deformation effect on the two-neutrino DBD NME within deformed QRPA

Alvarez, Sarriguren, Moya de Guerra, Pacearescu, Faessler, Simkovic, PRC 70 (2004) 321



Sterile Neutrinos
and
Neutrinoless Double Beta Decay

Bratislava-Dubna-Prague-Tuebingen

Sterile neutrinos in 0νββ-decay

3 light neutrinos ($m_i \leq$ 1eV) and one heavy sterile neutrino ($m_h \geq$1 keV)

half-life

$$[T_{1/2}^{0\nu}]^{-1} = G_{01} \left| \frac{\langle m_\nu \rangle_{ee}}{m_e} M_\nu^{light} + U_{eh}^2 \frac{m_h}{m_e} M^{0\nu}(m_h) \right|^2 .$$

Nuclear matrix element

$$M^{0\nu}(m_h) = \langle H_F(m_h, r_{12}) \mathbf{1} + H_{GT}(m_h, r_{12}) \sigma_{12} + H_T(m_h, r_{12}) \mathbf{S}_{12}$$

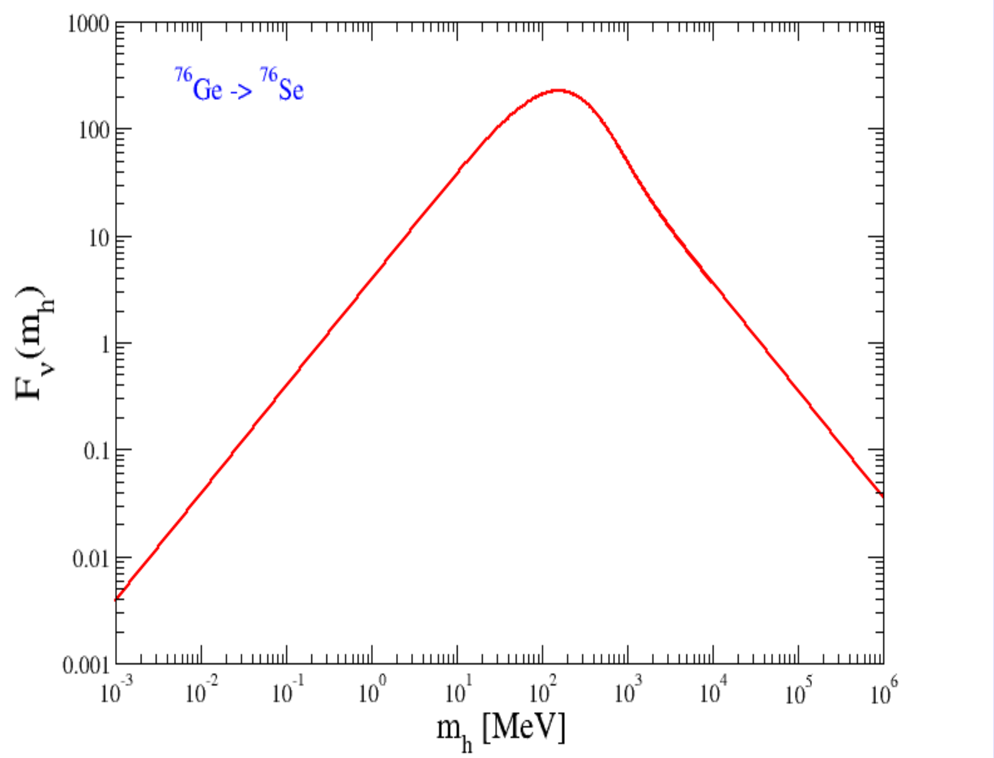
Neutrino potential

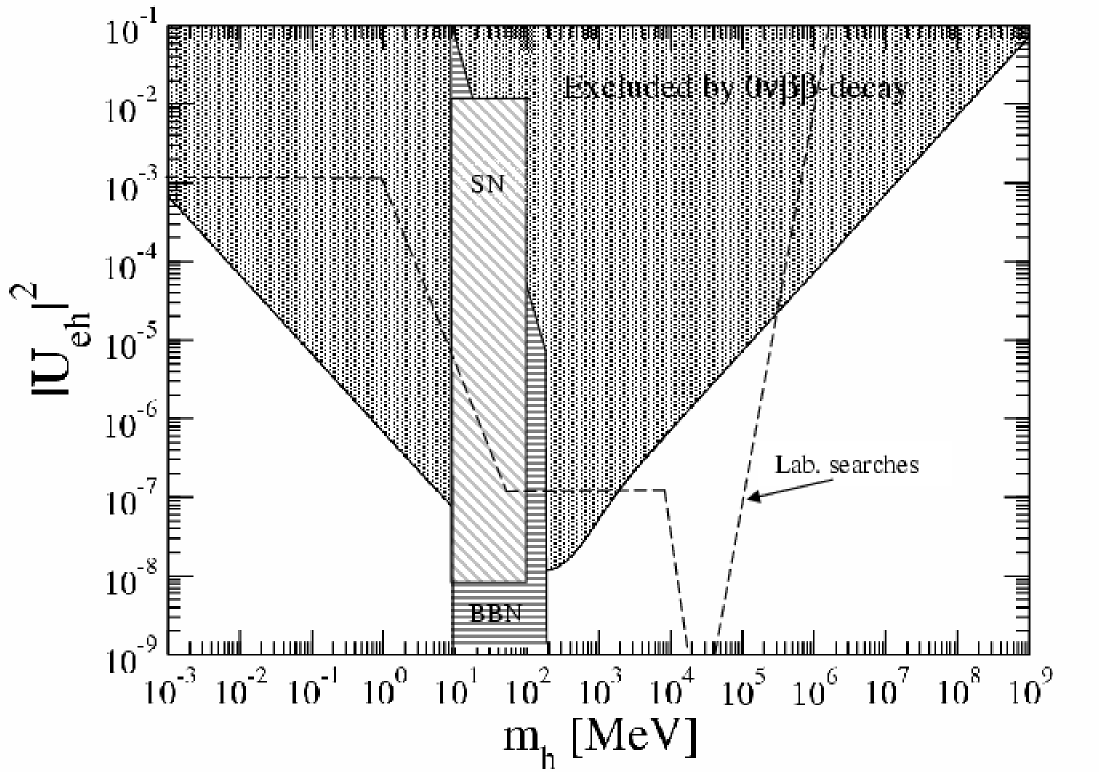
$$H_{K=F,GT}(m_h, r_{12}) = \frac{2}{\pi g_A^2} \frac{R}{r_{12}} \int_0^\infty \frac{\sin(q r_{12})}{\sqrt{q^2 + m_h^2} \left(\sqrt{q^2 + m_h^2} + E_J^m - (E_{g.s.}^i + E_{g.s.}^f)/2 \right)} h_K(q^2)\, q\, dq.$$

Sterile neutrino mass dependence

$$F_\nu(m_h) = \frac{m_h}{m_e} M^{0\nu}(m_h)$$

Constraint on sterile ν mixing $|U_{eh}|^2$





$$F_\nu(m_h) = \frac{m_h}{m_e} M^{0\nu}(m_h)$$

$$|U_{eh}|^2 \le \frac{1}{|F_\nu(m_h)|} \frac{1}{\sqrt{T_{1/2}^{0\nu-exp} G_{01}}},$$

Benes, Faessler, Simkovic, Kovalenko, PRD 71 (2005) 077901

Conclusions/Summary

- **There is a convergence of the QRPA results!**
- **New 0νββ-decay NME have been presented: the best choice (may 2005)**
- **We understand the differences with previous calculations of the 0νββ-decay NME except Stoica/Klapdor-Kleingrothaus results**
- **We offered clear arguments why to fix g_{pp} to double beta decay half-life instead of to the single beta decay of the ground state of the intermediate nucleus**
- **It is important to study the effect of deformation on the 0νββ-decay NME**
- **The 0νββ-decay offers a strong constraints on mixing of sterile neutrino**