Polarization and Spin Correlations in Monte Carlo Simulations

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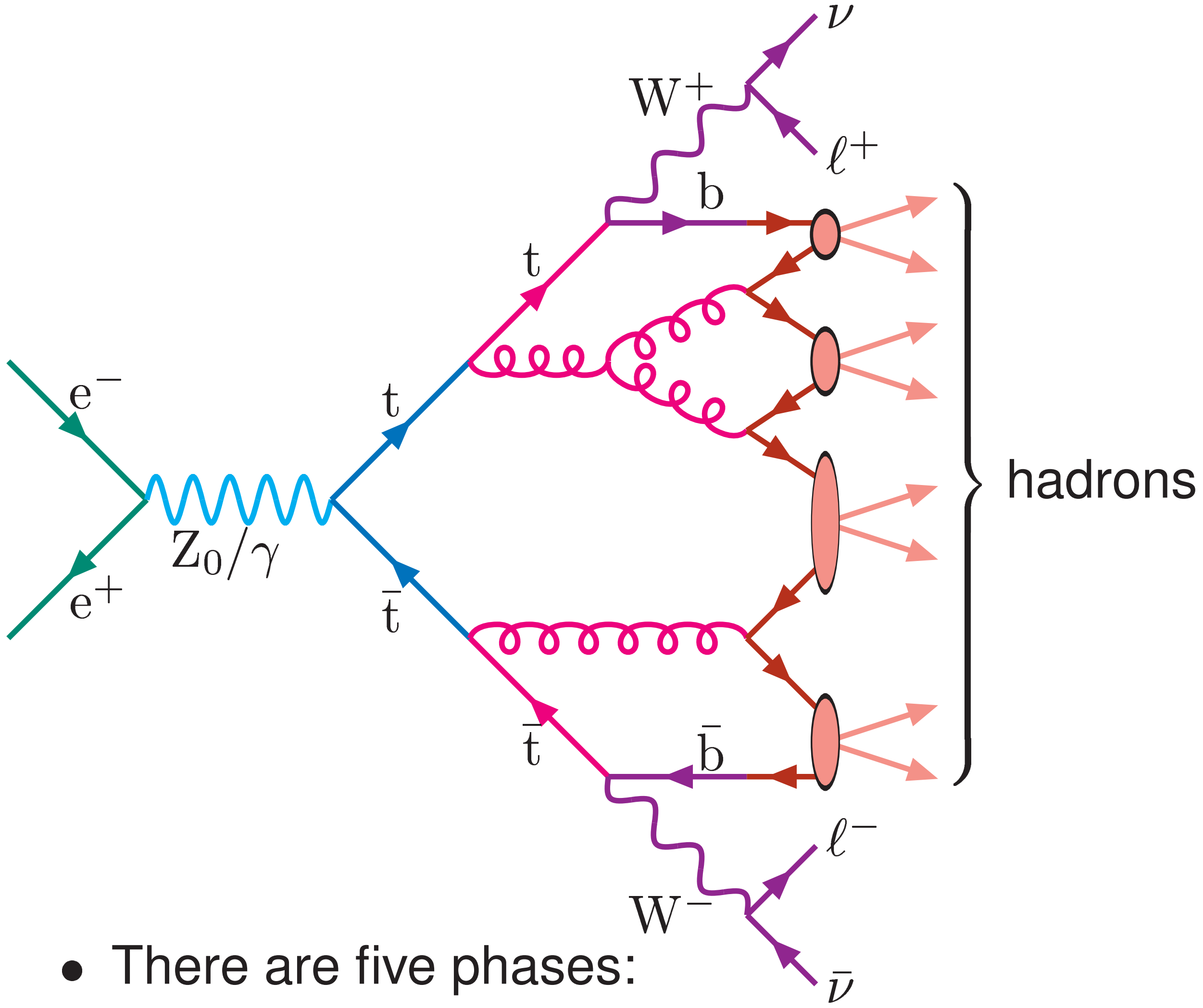
- Monte Carlo Generators
- Experience from LEP/SLD
- Polarization and Spin Correlations
- Different Techniques
- Conclusions

Monte Carlo Event Generators



- There are five phases:
  - Hard Process;
  - Parton Shower;
  - Heavy particle decays;
  - Hadronization;
  - Hadron decays.

Monte Carlo Event Generators

- There are many event generators available.
- These event generators fall into two classes.
- General Purpose Event Generators
  - These programs include a wide range of processes and simulate all the phases of an event.
  - *e.g.* ISAJET, PYTHIA, HERWIG.
- Parton Level Generators
  - These generators only handle the hard production.
  - Usually they are interfaced to one of the general purpose event generators for the parton shower and hadronization phases.
  - Usually give much better simulation of a given process.

General purpose event generators

- The major general purpose event generators differ in the treatment of perturbative QCD radiation and the modelling of the hadronization process.

- It is hard to decouple the effects of the parton shower and hadronization on experimental results.

Hadronization

- ISAJET uses the original independent fragmentation model of Feynman and Field.
- PYTHIA uses the Lund string model.
- HERWIG uses the cluster hadronization model.
- The independent fragmentation model cannot fit the LEP data.
- The cluster model gives good agreement with the LEP data on event shapes but doesn't fit the identified particle spectrum as well.
- The Lund string model give the best agreement with the LEP data.

Parton Shower

- ISAJET uses the original parton shower algorithm which only resum collinear logarithms.
- HERWIG uses the angular ordered parton shower algorithm which resums both soft and collinear singularities.
- PYTHIA use the collinear algorithm with an angular veto to try to reproduce the effect of the angular ordered shower.
- The collinear algorithm implemented in ISAJET does not give good agreement for angular ordering variables in hadron collisions or the string effect in e^+e^- collisions.
- HERWIG tends to give the best agreement for the hadron-hadron observables which are sensitive to soft effects.

LEP/SLD Experience

- The experience from LEP/SLD is that the general purpose generators have mainly been used to generate the fragmentation.
- Usually specialized parton level generators were used to simulate the hard process.
- These often included:
  - Next-to-leading order electroweak corrections;
  - Resummation of initial state radiation effects;
  - All the diagrams for many body final states (*e.g.* four fermion generators.)
- There was a high level of theoretical sophistication in these codes.

Polarization and Spin Correlations

- The linear collider will need simulations with a similar level of accuracy.
- Polarization effects obviously need to be included.
- In many cases this is either already available, or can be done fairly easily.
- We also need full spin correlations in the final state:
  - We may want to study the correlations;
  - Correlations may affect signal and background in different ways;
  - Effects efficiency of the cuts applied.

Helicity Amplitude techniques

- Most of the existing precision generators use helicity amplitude techniques.
- Given this these generators either all ready include, or could be extended, to include beam polarization.
- Spin correlations automatically taken care of.
- In the main although these programs can include the effects many don't
- ERATO, EXCALIBUR, KORALW, WPHACT use helicity amplitudes but don't seem to have polarization effects.
- grc4f, WOPPER do have polarization effects.
- Work still needs to be done

Automatic Matrix Element Generators

- There are also programs available which automatically calculate matrix elements.

- These need to be interfaced to general purpose event generators for full simulation.
  - COMPHEP
    - Automatic calculation of cross sections using trace techniques.
    - Phase Space integration by VEGAS.
    - http://theory.sinp.msu.ru/comphep/
    - No polarization
  - AMEGIC++
    - Automatic calculation of cross sections using helicity amplitudes.
    - Phase Space integration using automatic multi-channel techniques.
    - Polarization possible
  - GRACE
    - Calculation of matrix elements using helicity techniques.
    - Phase Space integrates using BASES/SPRING
    - http://www-sc.kek.jp/minami/
    - Polarization possible

Other Approaches

- For most SM processes the calculation of the full final-state matrix element using helicity amplitude techniques is the best option.

- However there are some cases where we may want to use other techniques:
  - There are too many final state particles;
  - The final state is dominated by the decay of intermediate particles;
  - The full calculation hasn't been done.

- This is particularly important in the general purpose event generators where given the manpower available we can't do the best job for every thing.

Other Approaches

- Recently there has been a great deal of interest in polarization and spin correlation effects in SUSY.
- There are two approaches in use.
- These both start from the same point.
- We can write the matrix element for a $2 \rightarrow 2$ process, including decays, as

$$\mathcal{M}_{\mathrm{total}} = \frac{1}{m_a\Gamma_a}\frac{1}{m_b\Gamma_b}\sum_{\lambda_c\lambda_d}\mathcal{M}^{\lambda_c\lambda_d}_{ab\rightarrow cd}\mathcal{M}^{\lambda_c}_{c\ \mathrm{decay}}\mathcal{M}^{\lambda_d}_{d\ \mathrm{decay}}$$

- SUSYGEN uses this to compute the matrix element for an n-body final state.
- This matrix element is then integrated using a multichannel approach.
- This is fine for gaugino production in lepton collisions.

Spin Correlations

- We want a completely general way of implementing all spin correlations.
- The previous approach requires the production and decay of the particle to be generated at the same time.
- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.
- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.

Spin Correlations

- HERWIG already includes correlations in and between jets using the algorithm of Collins and Knowles

  J. C. Collins, Nucl. Phys. B304, (1988) 794.

  I. G. Knowles, Nucl. Phys. B304, (1988) 767.

  I. G. Knowles, Nucl. Phys. B310, (1988) 571.

  I. G. Knowles, Comput. Phys. Commum. 58 (1990) 271.

- A basically identical algorithm can be used for the decays.

  P. Richardson, JHEP 0111:029, 2001.

- Using the factorization of the matrix element we can write

$$|\mathcal{M}_{\mathrm{total}}|^2 = \frac{1}{m_a^2\Gamma_a^2}\frac{1}{m_b^2\Gamma_b^2}\rho^{\lambda_c\lambda'_c\lambda_d\lambda'_d} D_c^{\lambda_c\lambda'_c} D_d^{\lambda_d\lambda'_d}$$

where

$$\rho_{\mathrm{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda'_d} = \mathcal{M}^{\lambda_c\lambda_d}_{ab\to cd}\mathcal{M}^{*\lambda'_c\lambda'_d}_{ab\to cd},$$

is the unnormalized spin density matrix, and

$$D_c^{\lambda_c\lambda'_c} = \mathcal{M}^{\lambda_c}_{c\ \mathrm{decay}}\mathcal{M}^{*\lambda'_c}_{c\ \mathrm{decay}},$$

the unnormalized decay matrix.

Spin Correlations

- Most analytically studies then calculate these matrices analytically, but in a Monte Carlo approach it is easier to use helicity amplitude techniques as with SUSYGEN.
- Instead of presenting the algorithm I will simply give an example
- Let's consider the processes $e^+e^- \to t\bar{t}$ followed by semi-leptonic decay of the top quarks.
- First we generate the momenta of the top quarks $e^+e^- \to t\bar{t}$ using the polarized cross-section

$$\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda_1\ldots\lambda_n}$$

Spin Correlations

- One of the outgoing quarks is selected at random and a spin density matrix calculated.

$$
\begin{aligned}
\rho_{\lambda_j\lambda'_j} &= \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j} D^i_{\lambda_i\lambda'_i} \\
&\quad \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}
\end{aligned}
$$

In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- This normalization is therefore the same as the matrix element used in the previous step.

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

$$
\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda_1\ldots\lambda_n}
$$

Spin Correlations

- In principle we could now continue down the decay chain, but here all the particles produced are stable.

- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda'_0} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda'_1\ldots\lambda'_n}\prod_{i=1,n} D^i_{\lambda_i\lambda'_i}$$

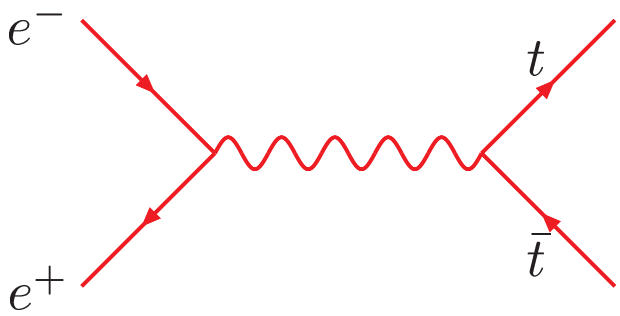
  The normalization is chosen so that $\mathrm{Tr}D = 1$.

- We then select another particle from the hard process. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.

- In this case this means we decay the second quark.

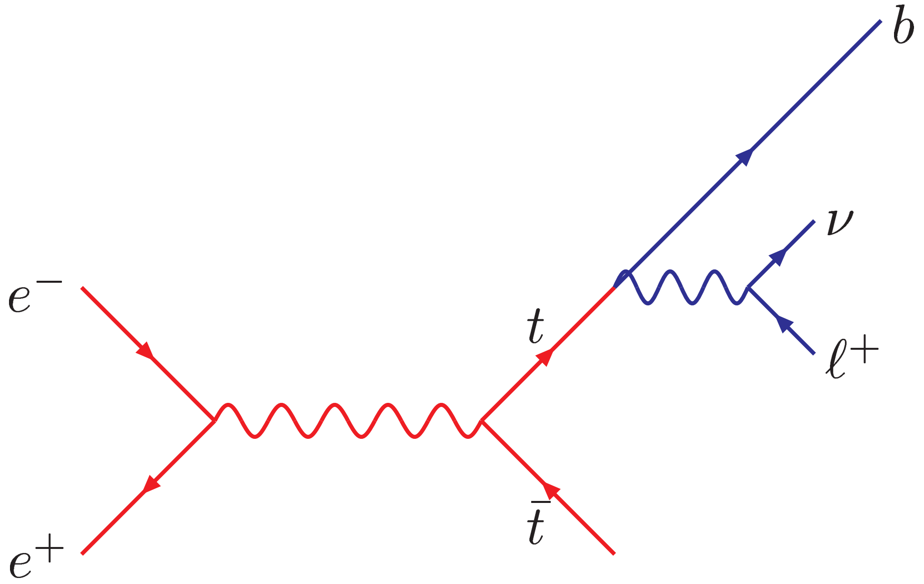
- As before the normalization of the spin density matrix cancels the matrix element used for the previous step.

Top pair Production



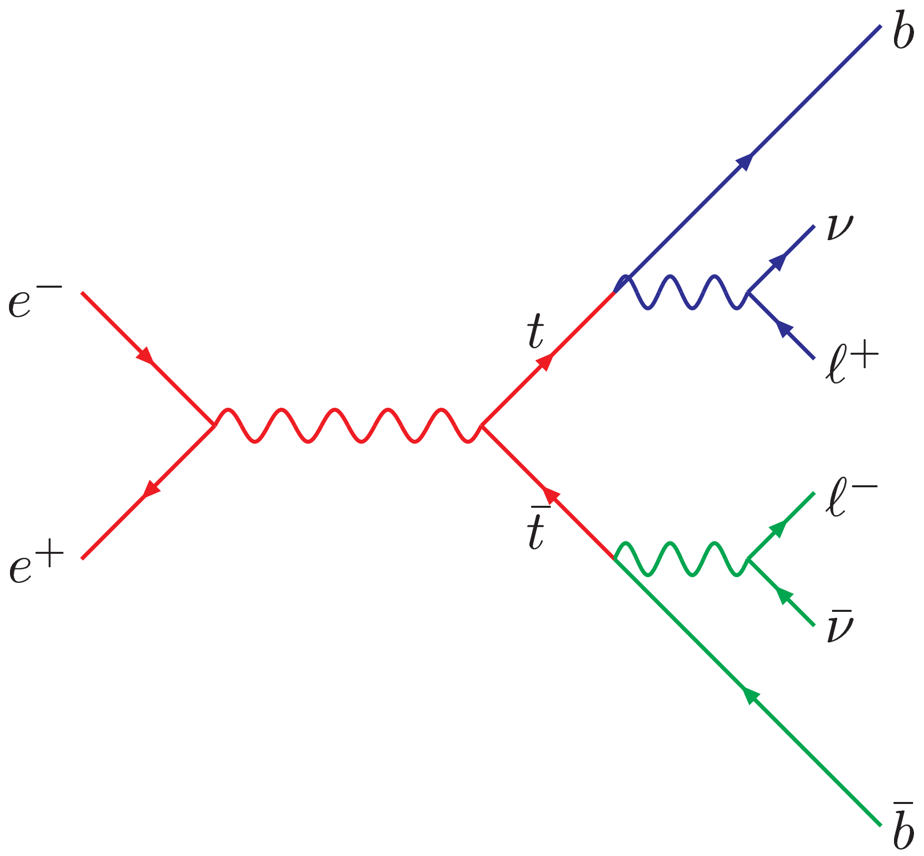
$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}^{e^+e^-\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*e^+e^-\to t\bar{t}}_{\kappa'_1\kappa'_2;\lambda_t\lambda_{\bar{t}}}$$

Top pair Production



$$
\begin{aligned}
|\mathcal{M}|^2 \quad = \quad & \rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}^{e^+e^-\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*e^+e^-\to t\bar{t}}_{\kappa'_1\kappa'_2;\lambda'_t\lambda_{\bar{t}}} \\
& \mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda'_t}
\end{aligned}
$$

Top pair Production

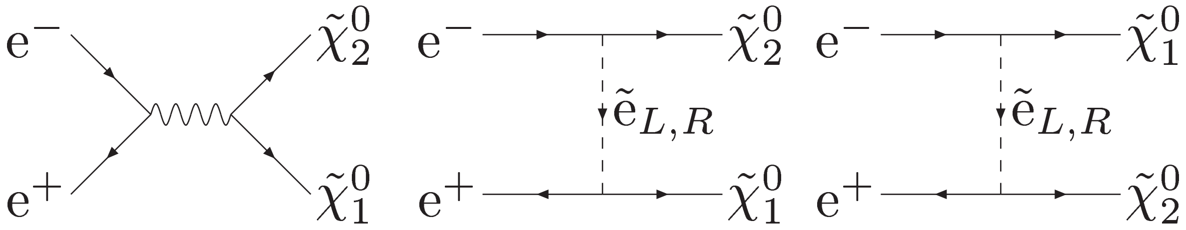


$$
\begin{aligned}
|\mathcal{M}|^2 \quad = \quad & \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{e^+e^-\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*e^+e^-\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}'} \\
& \mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'} \\
& \mathcal{M}^{\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}}\mathcal{M}^{*\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}'}
\end{aligned}
$$

Spin Correlations

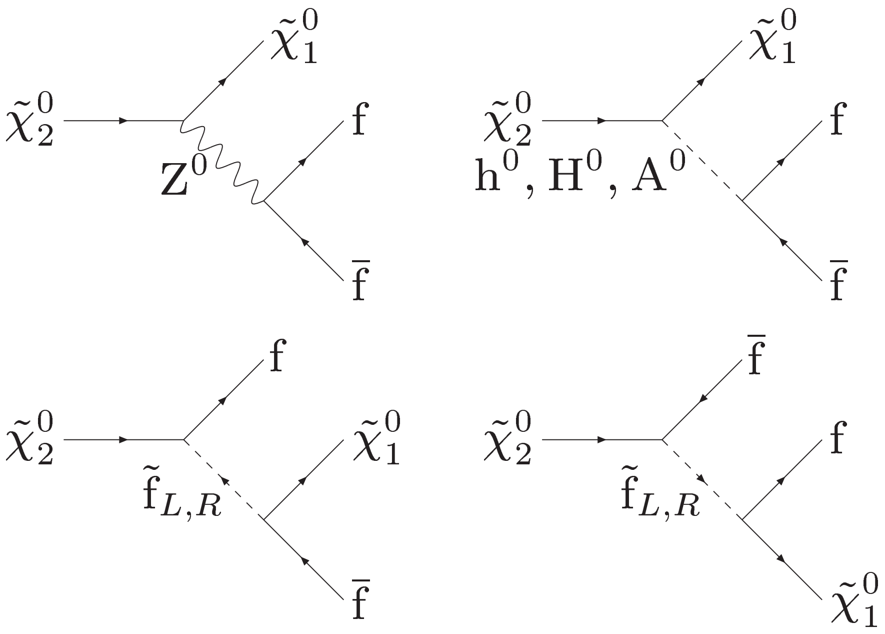
- So this procedure gives the full matrix element, but allows us to generate the event step by step.
- Examples
- Neutralino production in $\mathrm{e}^+\mathrm{e}^-$ collisions.
- Top quark pair production in $\mathrm{e}^+\mathrm{e}^-$ collisions.

Neutralino Production in e^+e^- collisions



- As the lightest neutralino is stable in R-parity conserving SUSY models the SUSY final state which requires the lowest centre-of-mass energy to produce it may be $\tilde{\chi}^0_2\tilde{\chi}^0_1$.

- The $\tilde{\chi}^0_2$ then decays $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 f\bar{f}$.



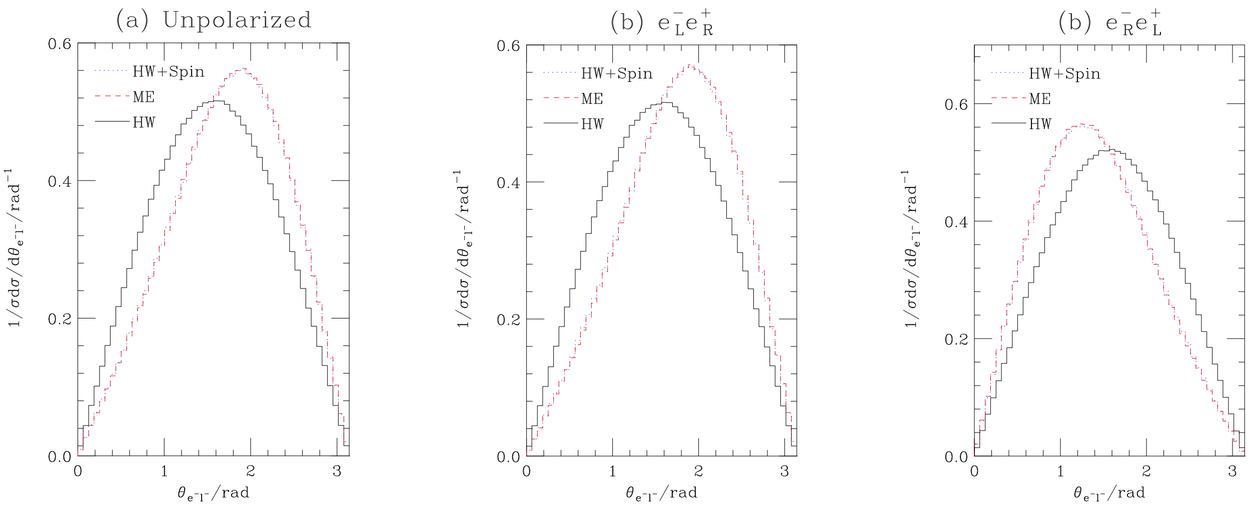
- The leptonic decay modes are the most important.

Neutralino Production in $\mathrm{e}^+\mathrm{e}^-$ collisions

- Here the neutralino is produced in a $2 \to 2$ process, with the possibility of polarized beams so the calculation of the spin density matrix is more complicated.

- The decay of the neutralino, via a 2 or 3 body decay, is then performed using this decay matrix.

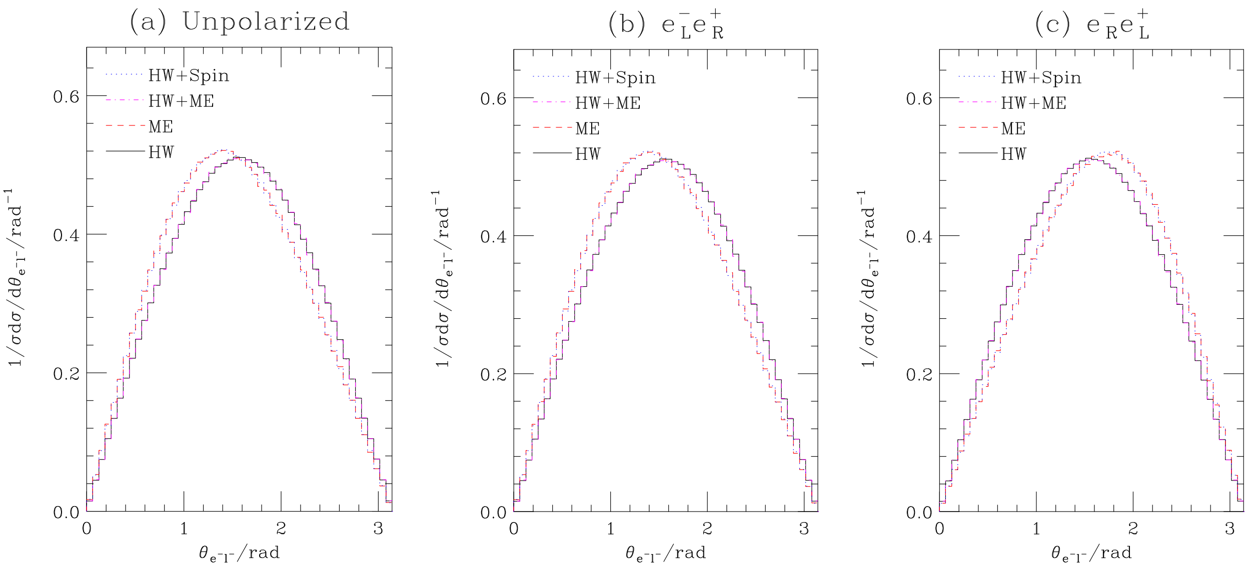
- First consider the two body decay $\tilde{\chi}^0_2 \to \tilde{\ell}^+_R \ell^-$ and study the angle between the lepton and the beam, again using SUGRA point 5.



Neutralino Production in e^+e^- collisions

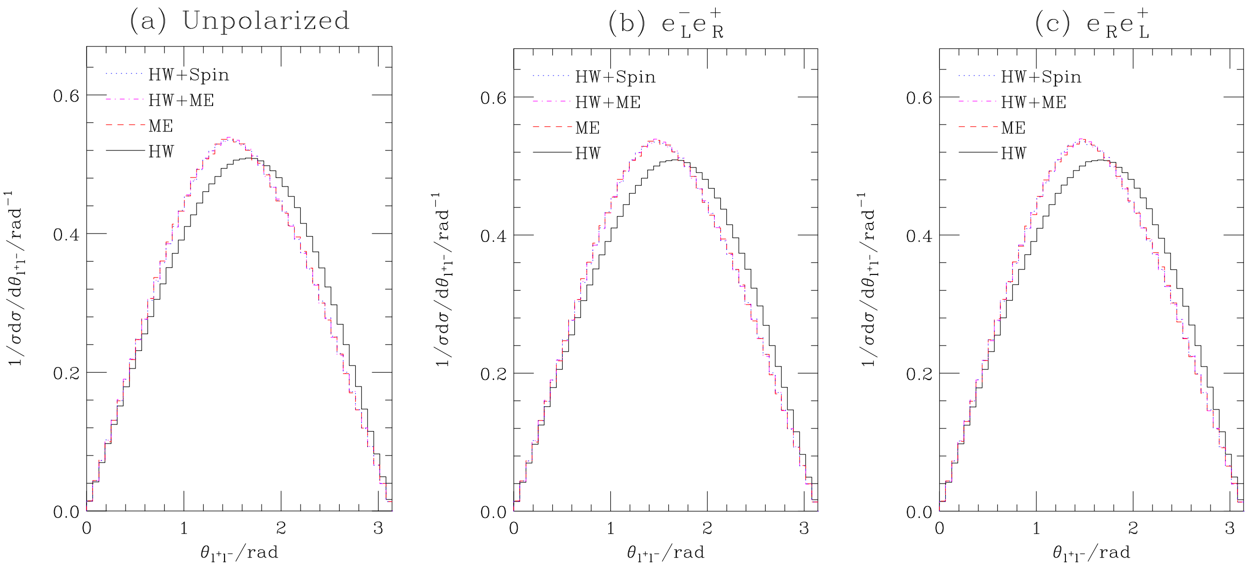
- We can also consider the three body decay $\tilde{\chi}^0_2 \to \ell^+\ell^-\tilde{\chi}^0_1$ using the SUSY parameters $M_0 = 210$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$, $M_1 = 450$ GeV, $M_2 = 350$ GeV, $M_3 = 350$ GeV.

- Study the angle between the lepton and the beam,



Neutralino Production in $\mathrm{e}^{+}\mathrm{e}^{-}$ collisions

- and between the lepton and antilepton.

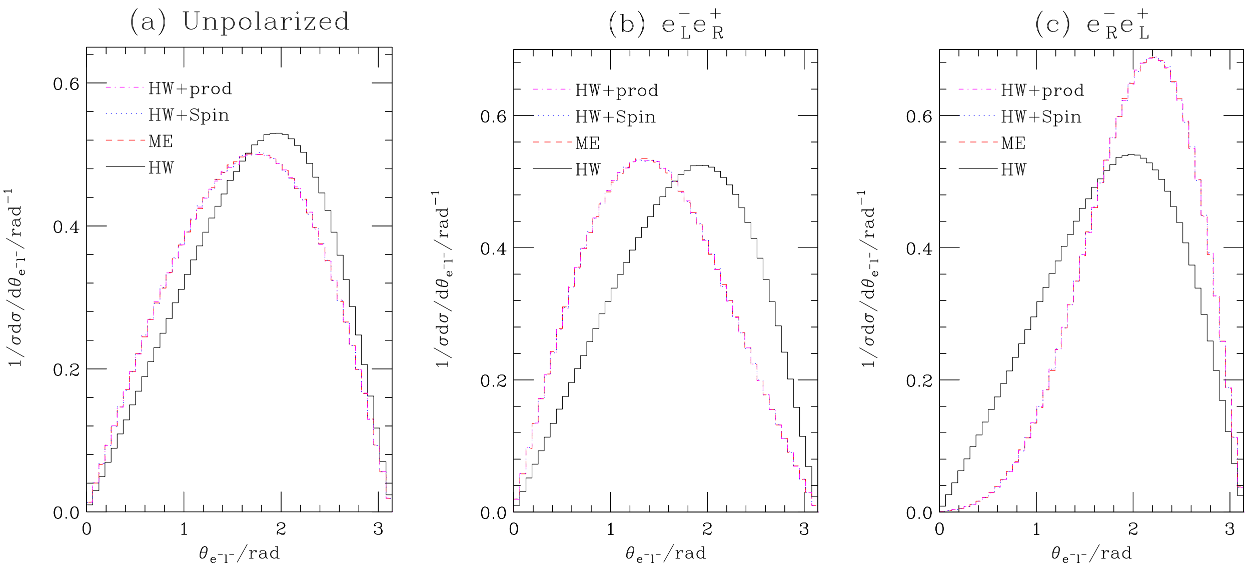


Top pair production in e^+e^- collisions

- In this example in addition to the calculation of the spin density matrix for the production we also need to calculate the decay matrix for the first decay and use this in performing the second decay.

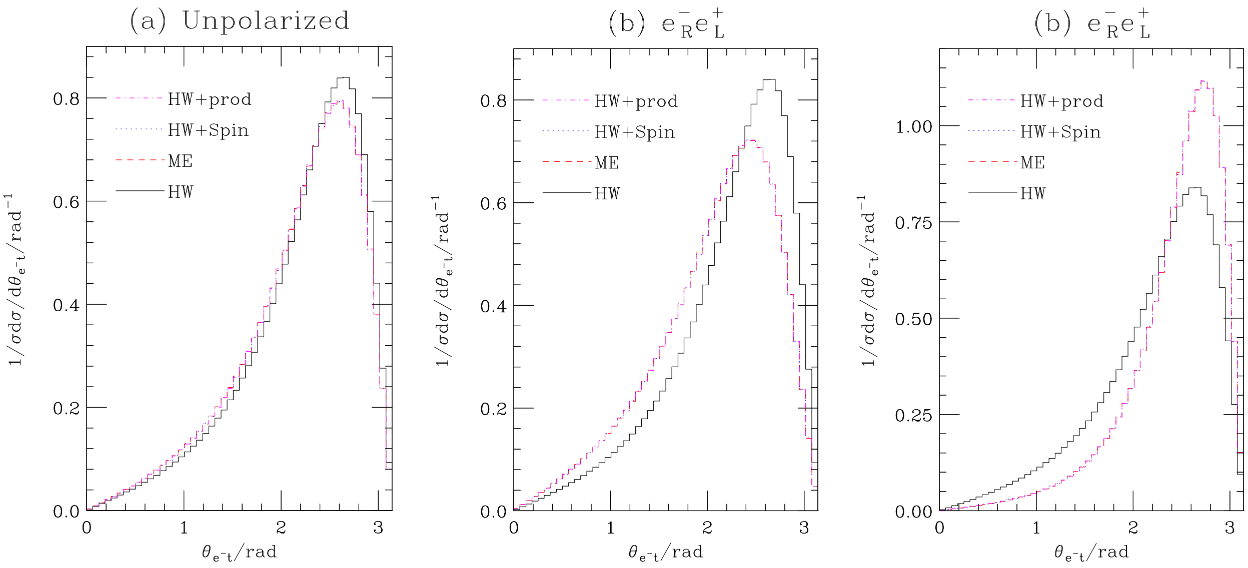
- Study the semi-leptonic decays of both top quarks and look at the correlations.

- Between the lepton produced in the antitop decay and the beam.

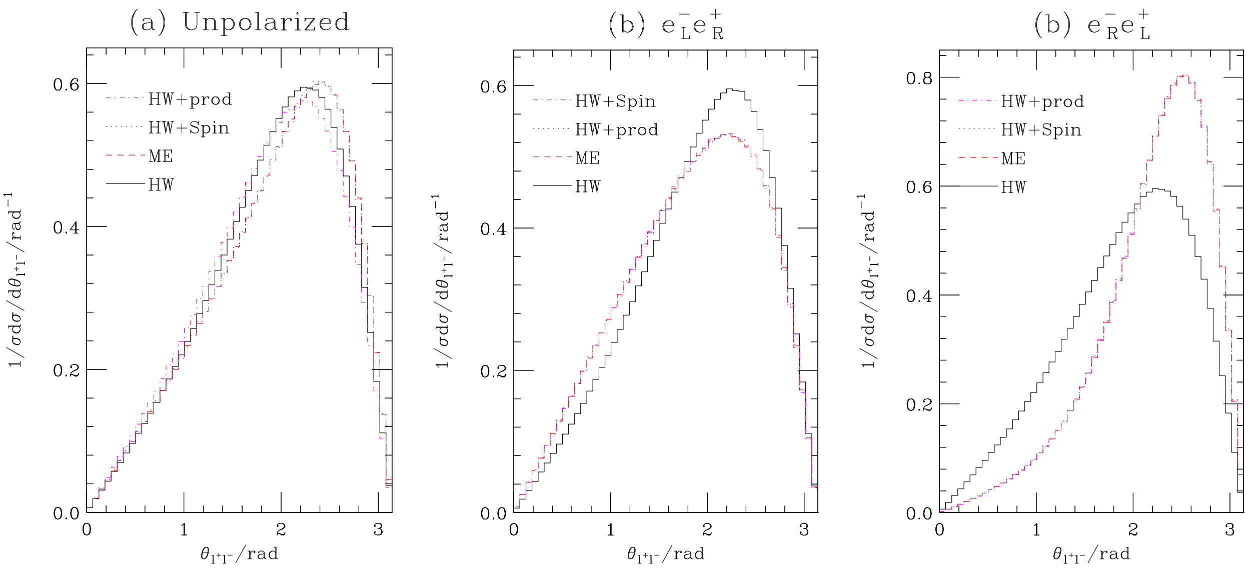


Top pair production in e^+e^- collisions

- Between the lepton and the top quark.



- Between the lepton and antilepton.



Top pair production in e^+e^- collisions

- The correlation between the lepton and the beam or top quark is only affected by the production mechanism.

- The correlation between the lepton and antilepton is also effected by the decay of the first quark.

Conclusions

- There are techniques available to give a sophisticated theoretical description of polarization and spin correlations.
- Helicity amplitudes for most SM processes.
- Spin density and Decay matrix techniques for SUSY signals.
- More work remains to be done.