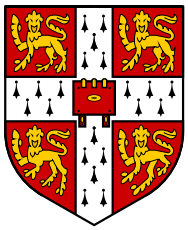
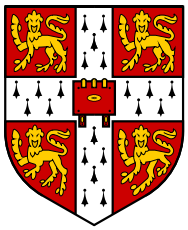


Spin Correlations in SUSY Simulations

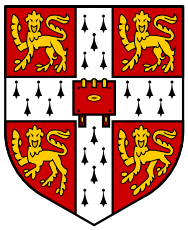
Peter Richardson
(Cavendish Laboratory and DAMTP, Cambridge)
hep-ph/0110108

- Why are Spin Correlations important?
- Spin Correlations in particle decays
- Examples
- Conclusions



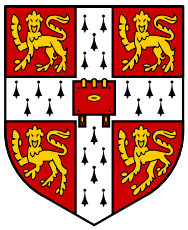
Spin Correlations

- In the production of heavy particles there are correlations between the production and decay of the particles due to their spin.
- These correlations are known to be important in tau production.
- Run II of the Tevatron should have sufficient statistics to measure the correlations in $\mathrm{t\bar{t}}$ production.
- These correlations are also important for $\mathrm{t\bar{t}}$ production at a future linear collider.



Spin Correlations in SUSY

- One potentially important application is in gaugino production at a future linear collider.
- Here both spin and polarization effects are important.
- This affects the distribution of the decay products.
- By measuring forward/backward asymmetries of the leptons produced in neutralino/chargino decay for different polarizations of the beams a number of observables can be constructed.
- This allows fundamental parameters of the gaugino sector to be measured.
- We can also test the consistency of the model.



Spin Correlations

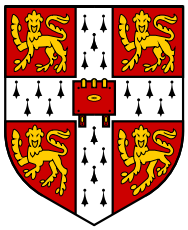
- There have been many studies of spin correlations
  - Gaugino Production in e^+e^- collisions
    G. Moortgat-Pick, *et.al*, Phys. Rev. D59 (1999) 015016, Eur. Phys. J. C7 (1999) 113, Eur. Phys. J. C9 (1999) 521, Eur. Phys. J. C18 (2000) 379.
  - Top production in hadron collisions
    A. Brandenburg Phys. Lett. B338 (1996) 626.

- Most of these studies used the same formalism.

- In the limit that the width of a particle is small relative to its mass we can neglect the width and keep the particles on mass shell.

- We can write the matrix element for a $2 \to 2$ process, including decays, as

$$\mathcal{M}_{\text{total}} = \frac{1}{m_a \Gamma_a} \frac{1}{m_b \Gamma_b} \sum_{\lambda_c \lambda_d} \mathcal{M}^{\lambda_c \lambda_d}_{ab \to cd} \mathcal{M}^{\lambda_c}_{c\ \text{decay}} \mathcal{M}^{\lambda_d}_{d\ \text{decay}}$$



Spin Correlations

- This means the amplitude squared can be written as

$$|\mathcal{M}_{\rm total}| = \frac{1}{m_a^2\Gamma_a^2}\frac{1}{m_b^2\Gamma_b^2}\rho^{\lambda_c\lambda_c'\lambda_d\lambda_d'}D_c^{\lambda_c\lambda_c'}D_d^{\lambda_d\lambda_d'}$$

  where

$$\rho_{\rm prod}^{\lambda_c\lambda_c'\lambda_d\lambda_d'} = \mathcal{M}_{ab\to cd}^{\lambda_c\lambda_d}\mathcal{M}_{ab\to cd}^{*\lambda_c'\lambda_d'},$$

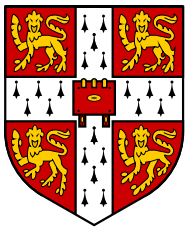
  is the unnormalized spin density matrix, and

$$D_c^{\lambda_c\lambda_c'} = \mathcal{M}_{c\ \rm decay}^{\lambda_c}\mathcal{M}_{c\ \rm decay}^{*\lambda_c'},$$

  the unnormalized decay matrix.

- Most authors then chose to evaluate these matrices analytically.

- However it will be easier for our purposes to evaluate the matrix elements using helicity amplitude techniques and then calculate the matrices numerically.



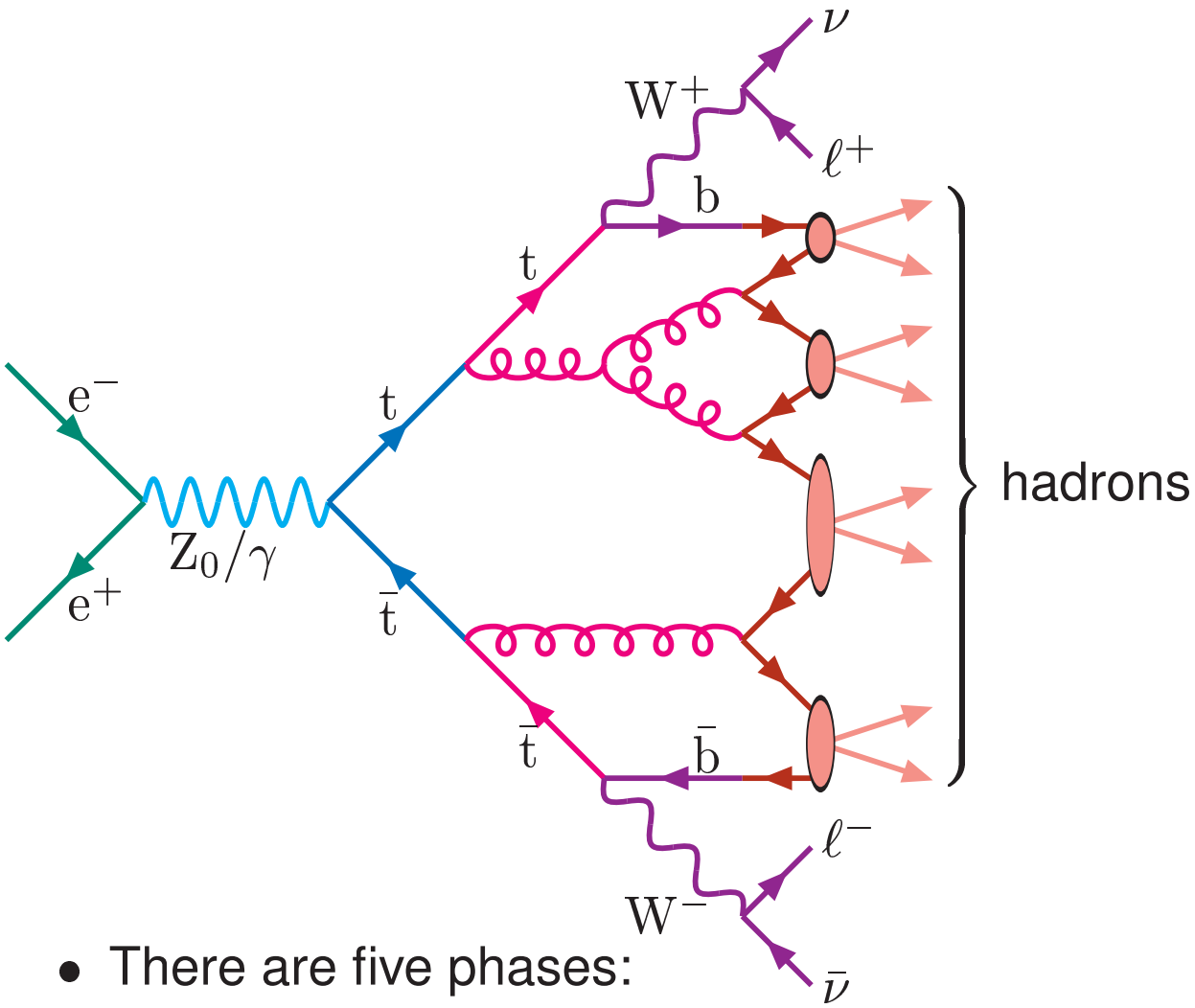
Spin Correlations

- These approaches require the production and decay of the particle to be generated at the same time.

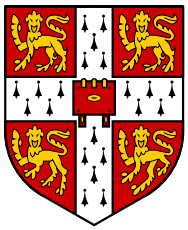
- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.

- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.

Monte Carlo Event Generation



- There are five phases:
  - Hard Process;
  - Parton Shower;
  - Heavy particle decays;
  - Hadronization;
  - Hadron decays.



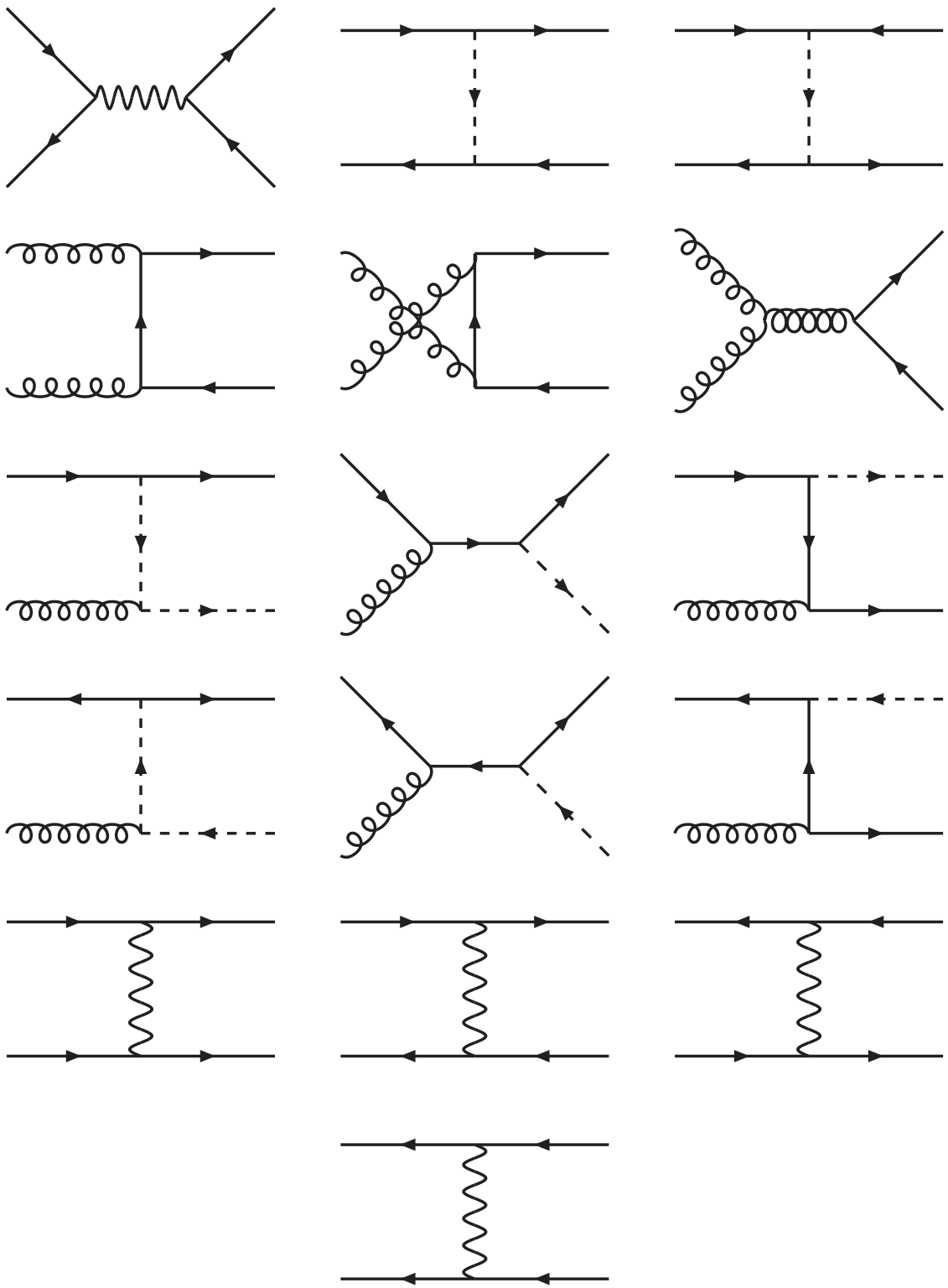
Helicity Amplitudes

- We will calculate all the cross sections and decays we will need as helicity amplitudes.
- The massive fermions will be defined as follows

$$u(p,\lambda) = \frac{1}{\sqrt{2p\cdot\ell}}(\not{p}+m)\,u_{-\lambda}(l),$$
$$v(p,\lambda) = -\frac{1}{\sqrt{2p\cdot\ell}}(\not{p}-m)\,u_{-\lambda}(l).$$

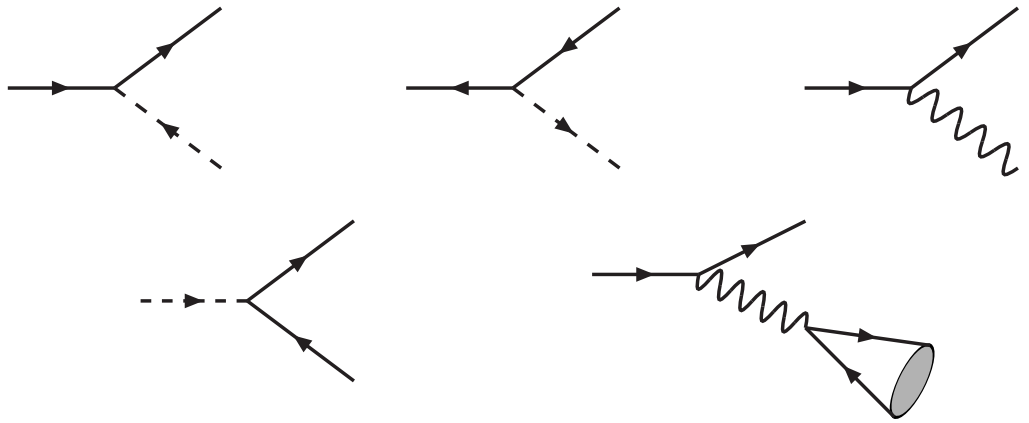
- Massive gauge bosons will not be included but replaced by their decay products.
- This enables us to include the correlations by calculating a small number of helicity amplitudes for the production and decay of the particles.

Production Feynman Diagrams

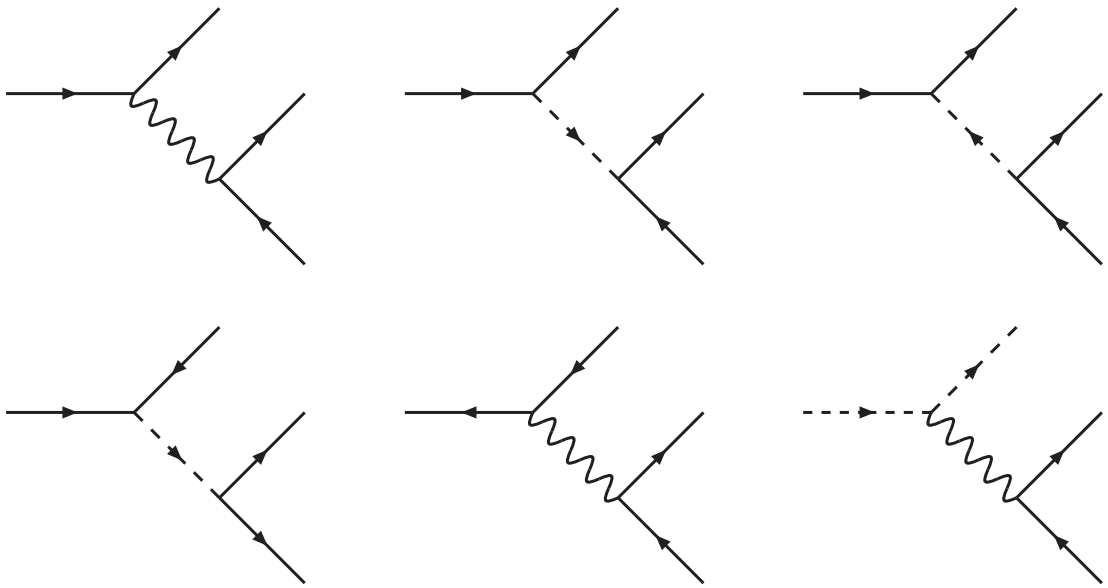


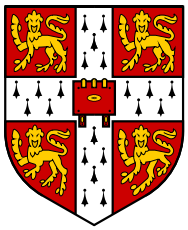
Decay Feynman Diagrams

- Two body decays



- Three body decays





Spin Correlations in the Decay of heavy particles

- The algorithm is as follows
- First the momenta of the particles in the hard collision are generated using

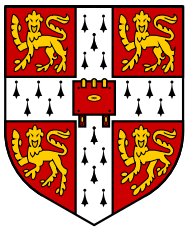
$$\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n}D^i_{\lambda_i\lambda_i'}$$

where $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.

- One of the outgoing partons is selected at random and a spin density matrix calculated.

$$\begin{aligned}\rho_{\lambda_j\lambda_j'} &= \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\prod_{i\neq j}D^i_{\lambda_i\lambda_i'} \\ &\quad \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1'\ldots\lambda_j'\ldots\lambda_n'}\end{aligned}$$

In the initial step we take $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.
The normalization is chosen so that $\mathrm{Tr}\rho=1$.



Spin Correlations in the Decay of heavy particles

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

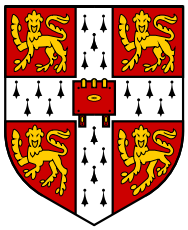
$$\rho_{\lambda_0\lambda_0'}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n}D^i_{\lambda_i\lambda_i'}$$

  where $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.

- One of the particles is selected and a spin density matrix calculated

$$\rho_{\lambda_j\lambda_j'} = \frac{1}{N_{D\rho}}\rho_{\lambda_0\lambda_0'}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_j'\ldots\lambda_n'}\prod_{i\neq j}D^i_{\lambda_i\lambda_i'}$$

  In the initial step we take $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.
  The normalization is chosen so that $\mathrm{Tr}\rho=1$.



Spin Correlations in the Decay of heavy particles

- These steps are repeated until a stable particle is selected. The decay matrix of this particle is set to $\delta_{\lambda_i\lambda_i'}$ and another particle from the same decay selected to be decayed next.

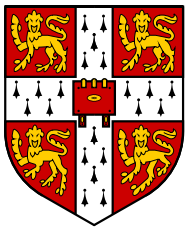
- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda_0'} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n} D^i_{\lambda_i\lambda_i'}$$

  The normalization is chosen so that $\mathrm{Tr}D = 1$.

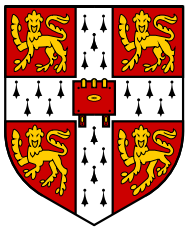
- We then select another particle from the decay which produced this particle and repeat the procedure and decay it.

- Eventually the hard process is reached and another particle selected. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.



Examples

- Neutralino production in squark decay.
- Neutralino production in $\mathrm{e}^+\mathrm{e}^-$ collisions.
- Chargino production in $\mathrm{e}^+\mathrm{e}^-$ collisions.



Neutralino Production in squark decay

- In most SUSY models the production of electroweak gauginos and sleptons at the LHC is dominated by production in squark decays rather than direct production.

- In many models the decay chain

$$\tilde{\mathrm{q}}_{\boldsymbol{L}} \to \mathrm{q}\tilde{\boldsymbol{\chi}}_{\boldsymbol{2}}^{\boldsymbol{0}} \to \mathrm{q}\tilde{\boldsymbol{\ell}}_{\boldsymbol{R}}^{\boldsymbol{\pm}}\boldsymbol{\ell}^{\mp} \to \mathrm{q}\boldsymbol{\ell}^{+}\boldsymbol{\ell}^{-}\tilde{\boldsymbol{\chi}}_{\boldsymbol{1}}^{\boldsymbol{0}}$$

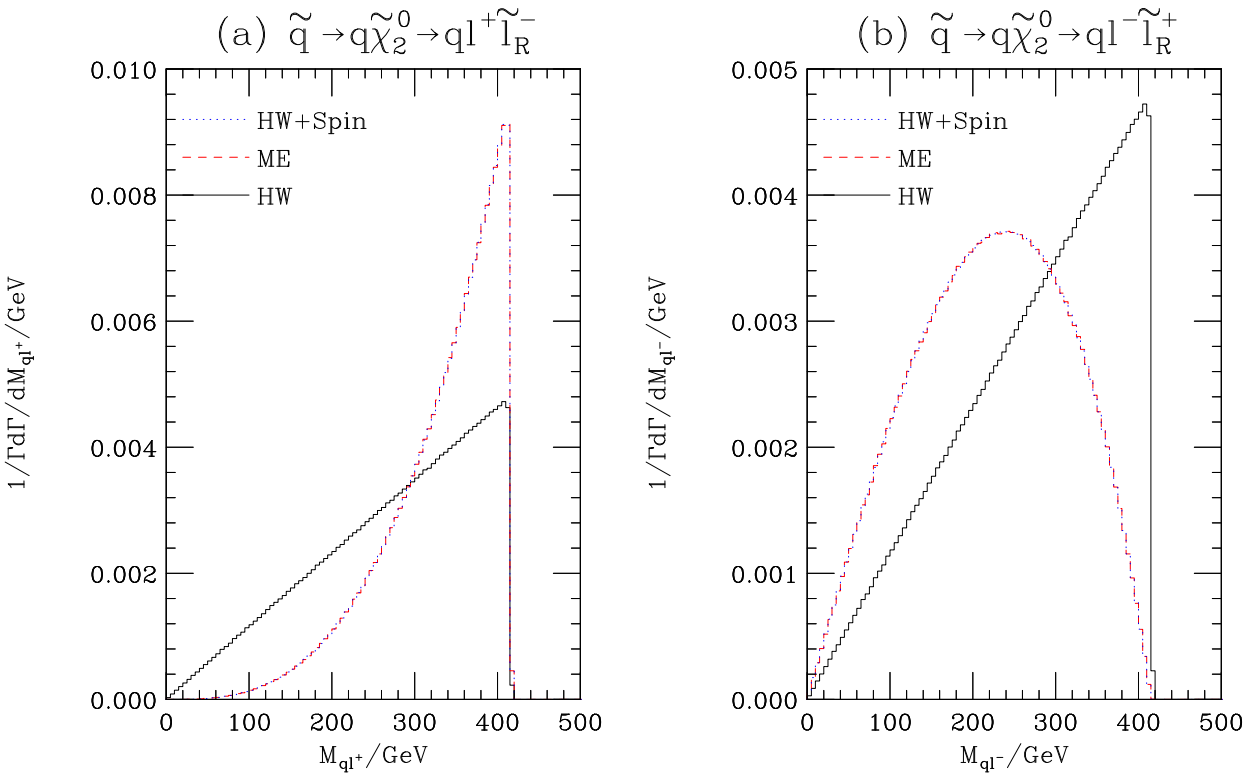
  is important as by measuring edges in the $\ell^+\ell^-$, $\mathrm{q}\ell$ and $\mathrm{q}\ell^+\ell^-$ distributions the masses of $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\ell}_R$ and $\tilde{\mathrm{q}}$ can be measured.

- Here the spin correlation algorithm must calculate the spin density matrix for $\tilde{\mathrm{q}}_L \to \mathrm{q}\tilde{\chi}_2^0$ and then use this to perform the decay $\tilde{\chi}_2^0 \to \tilde{\ell}_R^{\pm}\ell^{\mp}$.

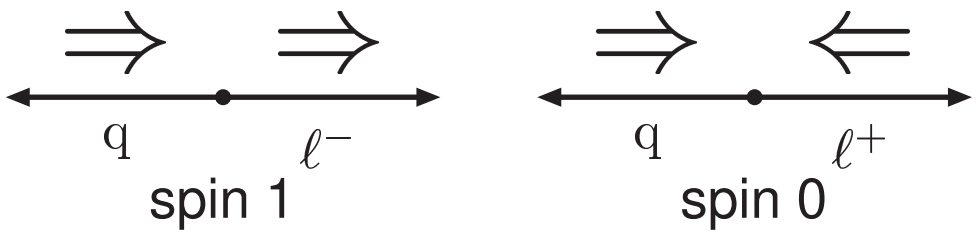
- The effects are most visible in the $\mathrm{q}\ell$ mass distribution.

- Used LHC SUGRA point 5 $M_0 = 200\ \mathrm{GeV}$, $M_{1/2} = 300\ \mathrm{GeV}$, $A_0 = 300\ \mathrm{GeV}$, $\tan\beta = 2.1$ and $\mathrm{sgn}\,\mu+$.

Neutralino Production in squark decay

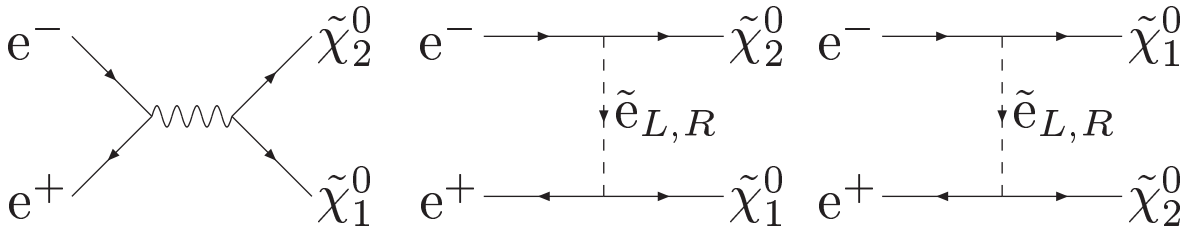


- The quark will always be left-handed and the lepton right-handed

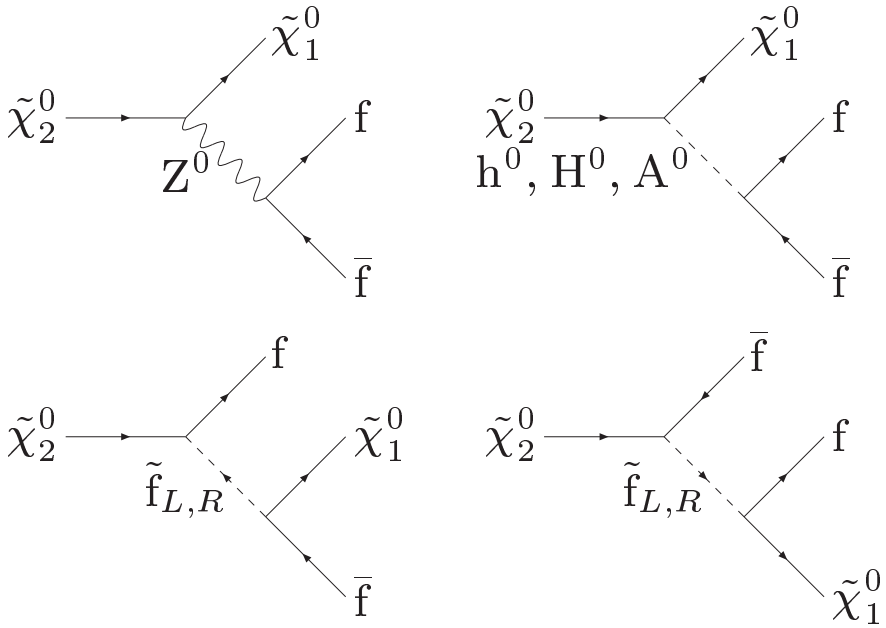


- As we can't experimentally distinguish quarks and antiquarks the average is observed.

Neutralino Production in e^+e^- collisions



- This is a more complicated example.
- As the lightest neutralino is stable in R-parity conserving SUSY models the SUSY final state which requires the lowest centre-of-mass energy to produce it may be $\tilde{\chi}_2^0\tilde{\chi}_1^0$.
- The $\tilde{\chi}_2^0$ then decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\bar{f}$.



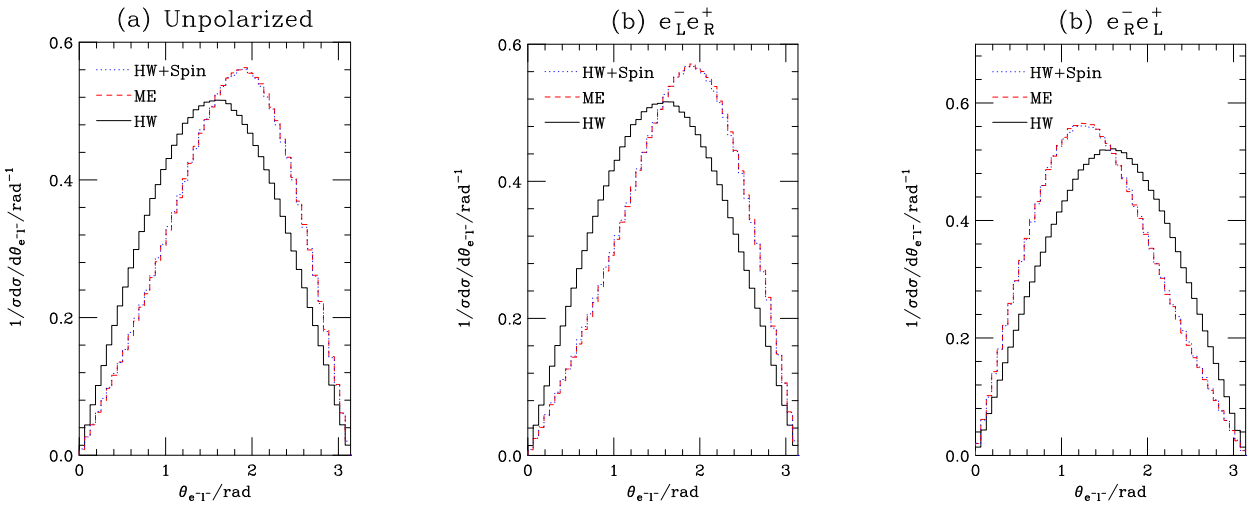
- The leptonic decay modes are the most important.

Neutralino Production in e^+e^- collisions

- Here the neutralino is produced in a $2 \to 2$ process, with the possibility of polarized beams so the calculation of the spin density matrix is more complicated.

- The decay of the neutralino, via a 2 or 3 body decay, is then performed using this decay matrix.

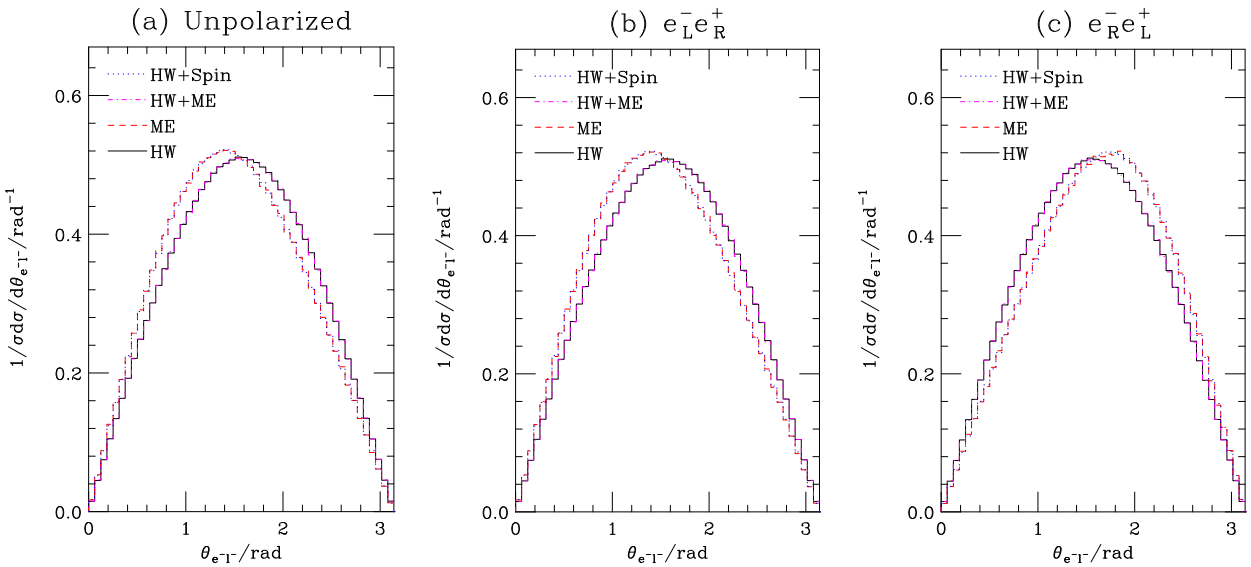
- First consider the two body decay $\tilde{\chi}_2^0 \to \tilde{\ell}_R^+ \ell^-$ and study the angle between the lepton and the beam, again using SUGRA point 5.



Neutralino Production in e^+e^- collisions

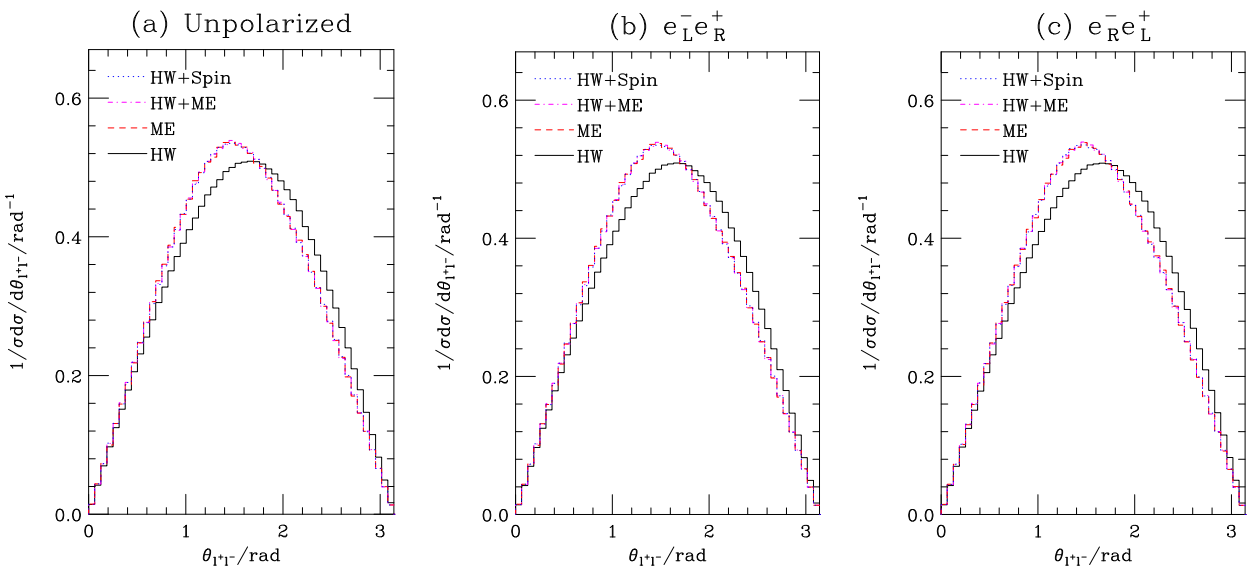
- We can also consider the three body decay $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$ using the SUSY parameters $M_0 = 210$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$, $M_1 = 450$ GeV, $M_2 = 350$ GeV, $M_3 = 350$ GeV.

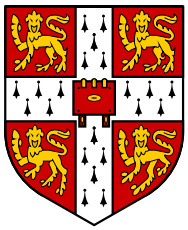
- Study the angle between the lepton and the beam,



Neutralino Production in e^+e^- collisions

- and between the lepton and antilepton.





Chargino production in e^+e^- collisions

- In a Monte Carlo simulation we often want to include the widths of the particles.
- Can't factorized the matrix element as before.
- We have two options
  - Use off-shell momenta and on-shell mass

$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p}+m+\frac{\not{l}}{2p\cdot l}(m^2-p^2)$$

  - Use off-shell momenta and off-shell mass

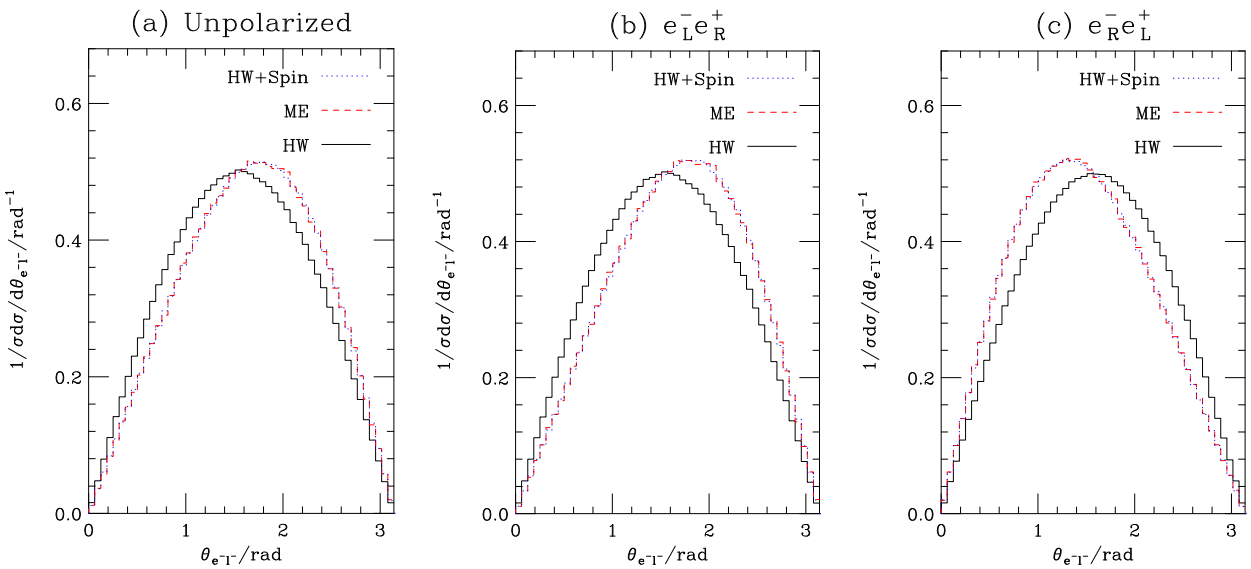
$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p}+m_{\rm off},$$

- easiest to study effects in $\tilde{\chi}_1^\pm\tilde{\chi}_2^\mp$ production.
- At point we studied the $\tilde{\chi}_1^\pm$ width is 73 MeV and the $\tilde{\chi}_2^\pm$ width is 2.9 GeV. We also considered ten times the $\tilde{\chi}_2^\pm$ width. The $\tilde{\chi}_1^\pm$ mass is 127.8 GeV and the $\tilde{\chi}_2^\pm$ mass is 357.6 GeV.

Chargino production in e^+e^- collisions

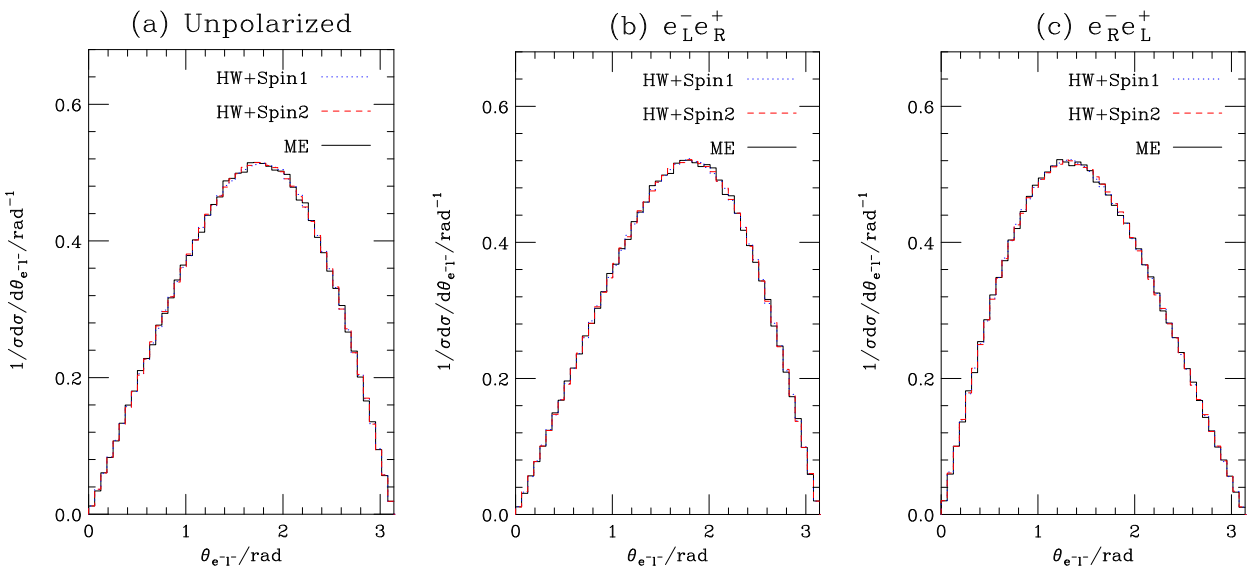
- Look at the angle between the beam and the lepton produced in decay of the $\tilde{\chi}_2^+$ via W exchange.

- No width

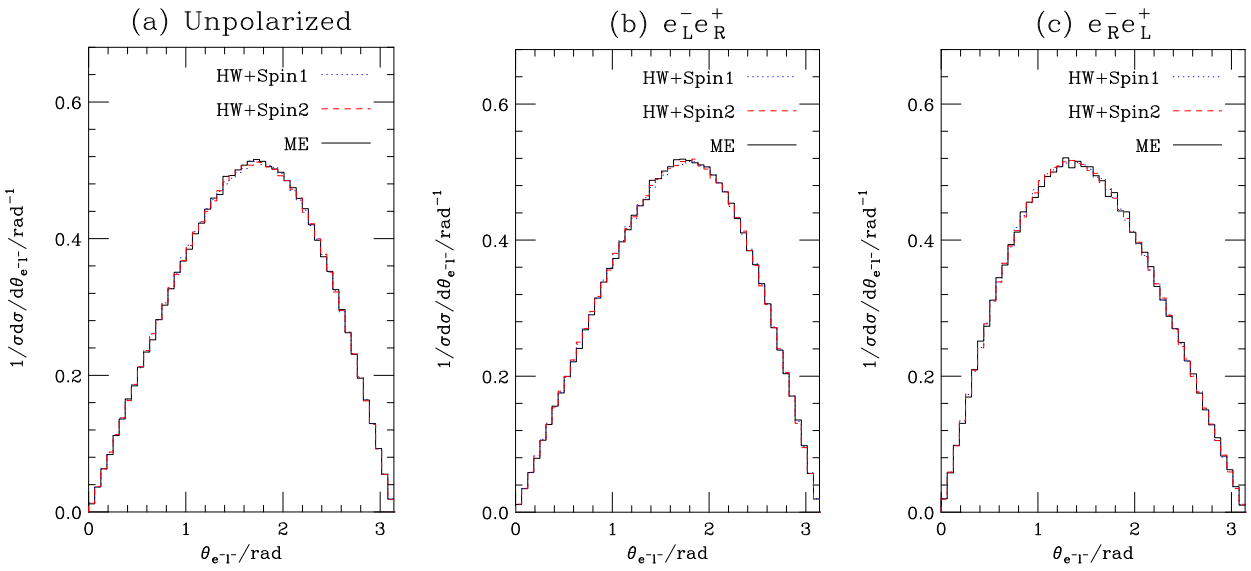


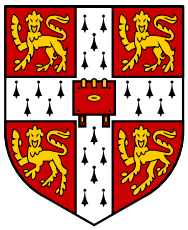
Chargino production in e^+e^- collisions

- Physical width



- Ten times physical width





Conclusions

- We have shown that it's possible to include spin correlations in a Monte Carlo event generator
- The algorithm:
  - allows production and decay to be generated as separate steps;
  - has a complexity which only grows linearly with the number of particles.
- The results are in good agreement with matrix element calculations.
- This is available in the most recent version of HERWIG, *i.e.* 6.4.
- These correlations are important for the study of SUSY.