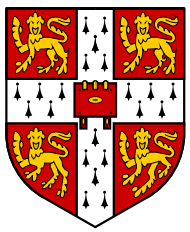

Tools for SUSY

Peter Richardson
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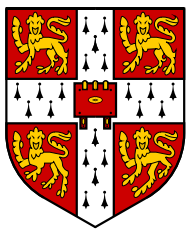
- Introduction to SUSY Tools
- RGE codes and Spectrum Generators
- Dark Matter and Cosmology Codes
- NLO Programs
- SUSY Event Generators
- Conclusions



Introduction to SUSY Tools

- There are many tools available for the study of SUSY.

- In general these tools can be divided into 4 classes:
  - RGE codes and spectrum generators;
  - Dark Matter and Cosmology Codes;
  - NLO cross section programs;
  - Event Generators.

- There is a lot of overlap between these classes:
  - Event Generators often include RGE codes and a spectrum generator;
  - Some of the RGE codes include dark matter and cosmology calculations.

RGE Codes and Spectrum Generators

- There are a number of programs available which use constrained models at a high energy scale in order to calculate the SUSY parameters at the electroweak scale.

- The main programs used to do this are:
  - ISAJET;
  - SOFTSUSY;
  - SUSPECT;
  - SUSYGEN;
  - SPYTHIA.

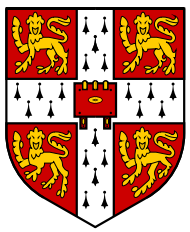
- A number of other private programs exist.

- The results of these programs are often quoted, however as they are not reproducible they are/should not be used for experimental analyses.

- SPYTHIA only has approximate analytically formulae.

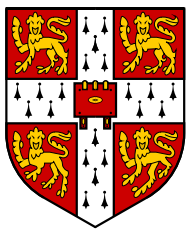
ISAJET

- ISAJET is the most commonly used of the RGE codes.
- ISAJET has 2-loop gauge beta functions and 1-loop soft masses.
- There are a large number of models implemented:
  - SUGRA;
  - GMSB;
  - AMSB;
  - Models with right handed neutrinos.
- In addition to calculating the spectrum also calculates the decays.
- Available from http://paige.home.cern.ch/paige/
- ISAJET has no finite corrections to the sparticle masses which means it is less accurate than SOFTSUSY and SUSPECT.



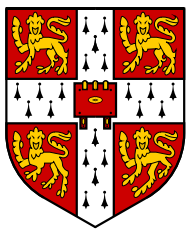
SOFTSUSY

- SOFTSUSY has 2-loop gauge beta functions and 1-loop soft masses (gaugino masses to 2-loop.)
- Finite corrections to sparticle masses.
- Supergravity, AMSB and GMSB models implemented.
- Interface to ISAJET for decay rate calculation.
- SOFTSUSY is available from http://allanach.home.cern.ch/allanach/softsusy.html



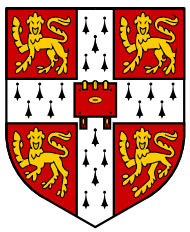
SUSPECT/SUSYGEN

- 2-loop gauge beta functions and 1-loop soft masses (gaugino masses to 2-loop.)
- Only Supergravity models implemented.
- Finite corrections to sparticle masses.
- SUSYGEN uses SUSPECT for its spectrum calculations.
- SUSPECT is available from http://www.lpm.univ-montp2.fr:7082/ kneur/suspect.html.



Higgs Masses and Decay Calculations

- While in general the RGE codes have a reasonably sophisticated calculation of the SUSY Higgs masses better programs exist
  - FeynHiggs.
    - Diagrammatic calculation of the masses of the neutral CP-even Higgs bosons in the MSSM at the two-loop level.
    - http://www-itp.physik.uni-karlsruhe.de/feynhiggs/
  - subhpole.
    - RGE improved one-loop effective potential.
    - http://gate.hep.anl.gov/cwagner/
- Also the NLO decay rates can be calculated using HDECAY.
- HDECAY is available from http://mspira.home.cern.ch/mspira/proglist.html



Dark Matter

- This is something I don't known much about.
- Again the problem seems to be that not many of the programs are publicly available.
- This has hindered comparisons between them.
- There are (at least) two public available programs
  - Micromegas (talk by G. Belanger in parallel session) http://wwwlapp.in2p3.fr/lapth/micromegas/
  - DARKSUSY http://www.physto.se/ edsjo/darksusy/download.html

NLO Cross Section Calculations

- There are a number of programs which calculate next-to-leading order cross sections.

- The major programs are
  - PROSPINO for squark/gluino production in hadron collisions.
  - Various NLO Higgs production calcualtions
  - These are available from http://mspira.home.cern.ch/mspira/proglist.html

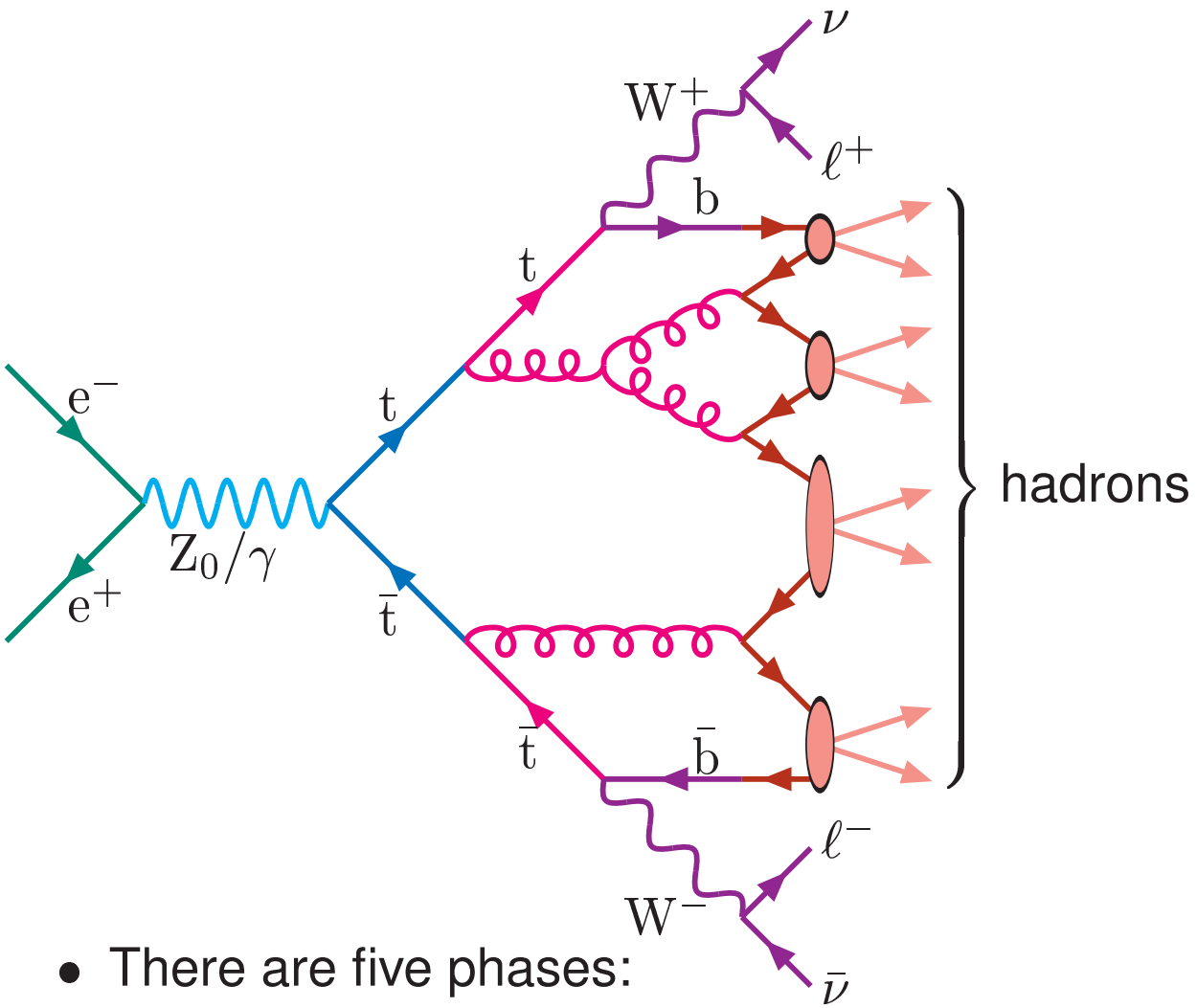
- Many other SUSY NLO calculations have been done.

- I'm not aware of any others which are publically available.

- Again this would be useful.

Monte Carlo Event Generators

- It's best to start by recalling the features of SM simulation.



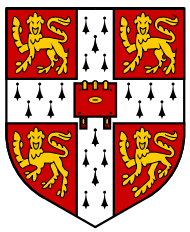
- There are five phases:
  - Hard Process;
  - Parton Shower;
  - Heavy particle decays;
  - Hadronization;
  - Hadron decays.

Monte Carlo Event Generators

- In general the main difference in SUSY simulations are the different:
  - Hard Production Processes;
  - Decays.
- The rest of the simulation is the same as for the Standard Model.

Monte Carlo Event Generators

- There are many SUSY event generators available.
- These event generators fall into two classes.
- General Purpose Event Generators
  - These programs were written for SM simulation but have been extended to include SUSY simulation.
  - There are three generators ISAJET, PYTHIA, HERWIG.
- SUSY Generators
  - These generators usually only handle the hard production and decay of the SUSY particles.
  - Usually they are interfaced to one of the general purpose event generators for the parton shower and hadronization phases.
  - Examples include SUSYGEN.
- I will briefly discuss the major event generators and their strengths and weaknesses.



Standard Model Simulation

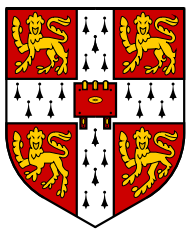
- Before considering the SUSY simulation it is useful to first consider the SM simulation.

- The major general purpose event generators differ in the treatment of perturbative QCD radiation and the modelling of the hadronization process.

- It is hard to decouple the effects of the parton shower and hadronization on experimental results.

Hadronization

- ISAJET uses the original independent fragmentation model of Feynman and Field.
- PYTHIA uses the Lund string model.
- HERWIG uses the cluster hadronization model.
- The independent fragmentation model cannot fit the LEP data.
- The cluster model gives good agreement with the LEP data on event shapes but doesn't fit the identified particle spectrum as well.
- The Lund string model give the best agreement with the LEP data.



Parton Shower

- ISAJET uses the original parton shower algorithm which only resum collinear logarithms.
- HERWIG uses the angular ordered parton shower algorithm which resums both soft and collinear singularities.
- PYTHIA use the collinear algorithm with an angular veto to try to reproduce the effect of the angular ordered shower.
- The collinear algorithm implemented in ISAJET does not give good agreement for angular ordering variables in hadron collisions or the string effect in e^+e^- collisions.
- HERWIG tends to give the best agreement for the hadron-hadron observables which are sensitive to soft effects.

ISAJET

- ISAJET is the oldest of the SUSY generators.
- Pros
  - There is a built in RGE code with a large number of models.
  - Calculation of the decay rates.
  - Includes decay matrix elements for the SUSY decays.
  - Has beam polarization effects in lepton collisions, but no spin correlations.
- Cons
  - ISAJET is known to give a poor description of Standard Model processes.
  - R-parity violation can only be added by hand.
- http://paige.home.cern.ch/paige/

PYTHIA

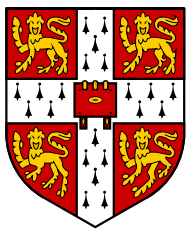
- Pros
  - PYTHIA gives the best description of SM physics.
  - Wide range of processes implemented.
  - Polarization in some processes.
  - Calculation of the decay rates.
  - Lepton number violating R-parity violating processes
- Cons
  - SUSY spectrum calculation makes use of approximate formula, not as good as ISAJET.
  - No spin correlations.
- ttp://www.thep.lu.se/tf2/staff/torbjorn/Pythia.html

SUSYGEN

- Pros
  - Now simulates lepton-lepton, lepton-hadron and hadron-hadron collisions.
  - Includes RGE code.
  - Polarization and spin correlations.
  - R-parity violation implemented
  - Calculation of decay rates.
  - Intefraced to PYTHIA for parton shower and hadronization.
- Cons
  - Can't simulate the background.
- http://lyoinfo.in2p3.fr/susygen/susygen3.html

HERWIG

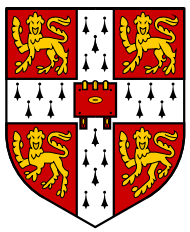
- Cons
  - No built in RGE code or decay rate calculation, uses an interface to ISAJET.
- Pros
  - The interface to ISAJET gives access to all the models in ISAJET, and can also add R-parity violating decays.
  - Has beam polarization effects in lepton collisions.
  - Also includes treatment of spin correlation effects in ALL SUSY production processes and decays.
  - Option of either matrix elements or the full spin correlations for SUSY decays.
  - Good treatment of R-parity violating production and decay processes.
- http://hepwww.rl.ac.uk/theory/seymour/herwig/



Automatic Matrix Element Generators

- There are also programs available which automatically calculate matrix elements.

- These need to be interfaced to general purpose event generators for full simulation.
  - COMPHEP
    - Automatic calculation of cross sections using trace techniques.
    - Phase Space integration by VEGAS.
    - http://theory.sinp.msu.ru/comphep/
  - AMEGIC++
    - Automatic calculation of cross sections using helicity amplitudes.
    - Phase Space integration using automatic multi-channel techniques.
    - Web page soon, talk by R. Kuhn in parallel session.
  - MadGraph
    - Calculation of matrix elements using helicity techniques.
    - No phase space integration provided.
    - http://pheno.physics.wisc.edu/Software/MadGraph



Les Houches Accord

- There are many parton-level generators available.
- These need to be interfaced to a general-purpose event generator for parton showers and hadronization.
- This has typically been difficult.
- At the Les Houches workshop there was an agreement on a common standard for the interface between the two.
- Should allow easier interface of COMPHEP/AMEGIC etc to event generators.
- We will discuss this in the parallel session.

Event Generation

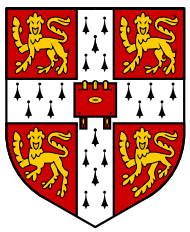
- While we want to simulate everything as well as possible the most important things vary with machine.

- Tevatron
  - Simulation of the SM background is the most important.
  - HERWIG or PYTHIA should be used for background simulation.
  - Also use higher order and n-body matrix elements where available.
  - Any of the major generators for signal simulation.

Event Generation

- LHC
  - In the initial discovery phase the same considerations apply as with the Tevatron.
  - When there is sufficient data hard cuts can be applied to remove SM background.
  - SUSY combinatoric background are most important in measurement studies.
  - Need accurate simulation for the SUSY signal.
  - Any of the major event generators of SUSY signal and backgrounds.
  - HERWIG or PYTHIA for SM background simulation.

- Linear Collider
  - Studies use detailed properties of the event and polarization and spin correlation effects to measure the parameters of the model.
  - Need accurate simulation of the SUSY signals.
  - Eventually this will need NLO calculations.
  - The best event generators for the signal are SUSYGEN and HERWIG.
  - PYTHIA or HERWIG for background simulations.



Spin Correlations

- Recently the major improvements in SUSY simulation have been in the inclusion of polarization and spin correlation effects.
- There are two approaches in use.
- These both start from the same point.
- We can write the matrix element for a $2 \to 2$ process, including decays, as

$$\mathcal{M}_{\rm total} = \frac{1}{m_a\Gamma_a}\frac{1}{m_b\Gamma_b}\sum_{\lambda_c\lambda_d}\mathcal{M}^{\lambda_c\lambda_d}_{ab\to cd}\mathcal{M}^{\lambda_c}_{c\ \rm decay}\mathcal{M}^{\lambda_d}_{d\ \rm decay}$$

- SUSYGEN uses this to compute the matrix element for an n-body final state.
- This matrix element is then integrated using a multichannel approach.
- This is fine for gaugino production in lepton collisions.

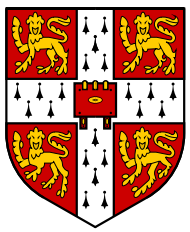
Spin Correlations

- We want a completely general way of implementing all spin correlations.

- The previous approach requires the production and decay of the particle to be generated at the same time.

- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.

- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.



Spin Correlations

- HERWIG already includes correlations in and between jets using the algorithm of Collins and Knowles

  J. C. Collins, Nucl. Phys. B304, (1988) 794.

  I. G. Knowles, Nucl. Phys. B304, (1988) 767.

  I. G. Knowles, Nucl. Phys. B310, (1988) 571.

  I. G. Knowles, Comput. Phys. Commum. 58 (1990) 271.

- A basically identical algorithm can be used for the decays.

  P. Richardson, JHEP 0111:029, 2001.

- Using the factorization of the matrix element we can write

$$|\mathcal{M}_{\rm total}|^2 = \frac{1}{m_a^2\Gamma_a^2}\frac{1}{m_b^2\Gamma_b^2}\rho^{\lambda_c\lambda_c'\lambda_d\lambda_d'}D_c^{\lambda_c\lambda_c'}D_d^{\lambda_d\lambda_d'}$$

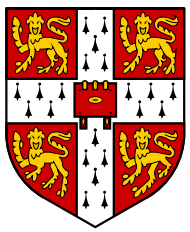
  where

$$\rho_{\rm prod}^{\lambda_c\lambda_c'\lambda_d\lambda_d'} = \mathcal{M}_{ab\to cd}^{\lambda_c\lambda_d}\mathcal{M}_{ab\to cd}^{*\lambda_c'\lambda_d'},$$

  is the unnormalized spin density matrix, and

$$D_c^{\lambda_c\lambda_c'} = \mathcal{M}_{c\ \rm decay}^{\lambda_c}\mathcal{M}_{c\ \rm decay}^{*\lambda_c'},$$

  the unnormalized decay matrix.



Spin Correlations

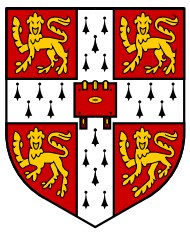
- Most analytically studies then calculate these matrices analytically, but in a Monte Carlo approach it is easier to use helicity amplitude techniques as with SUSYGEN.

- Instead of presenting the algorithm I will simply give an example

- Let's consider the processes $e^+e^- \to t\bar{t}$ followed by semi-leptonic decay of the top quarks.

- First we generate the momenta of the top quarks $e^+e^- \to t\bar{t}$ using the polarized cross-section

$$\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1\ldots\lambda_n}$$



Spin Correlations

- One of the outgoing quarks is selected at random and a spin density matrix calculated.

$$\rho_{\lambda_j\lambda'_j} = \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j} D^i_{\lambda_i\lambda'_i}$$
$$\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}$$

  In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
  The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- This normalization is therefore the same as the matrix element used in the previous step.

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

$$\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda_1\ldots\lambda_n}$$

Spin Correlations

- In principle we could now continue down the decay chain, but here all the particles produced are stable.

- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda_0'} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n} D^i_{\lambda_i\lambda_i'}$$

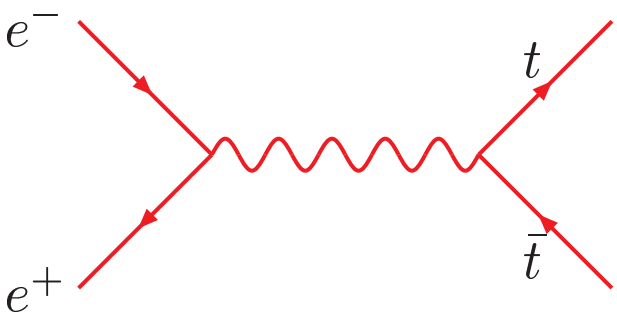
  The normalization is chosen so that $\mathrm{Tr}\,D = 1$.

- We then select another particle from the hard process. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.

- In this case this means we decay the second quark.

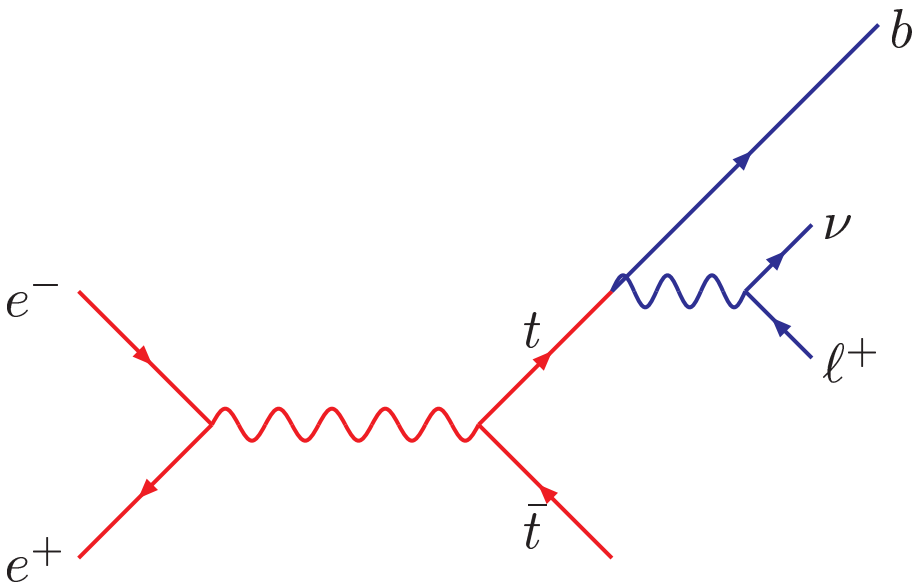
- As before the normalization of the spin density matrix cancels the matrix element used for the previous step.

Top pair Production



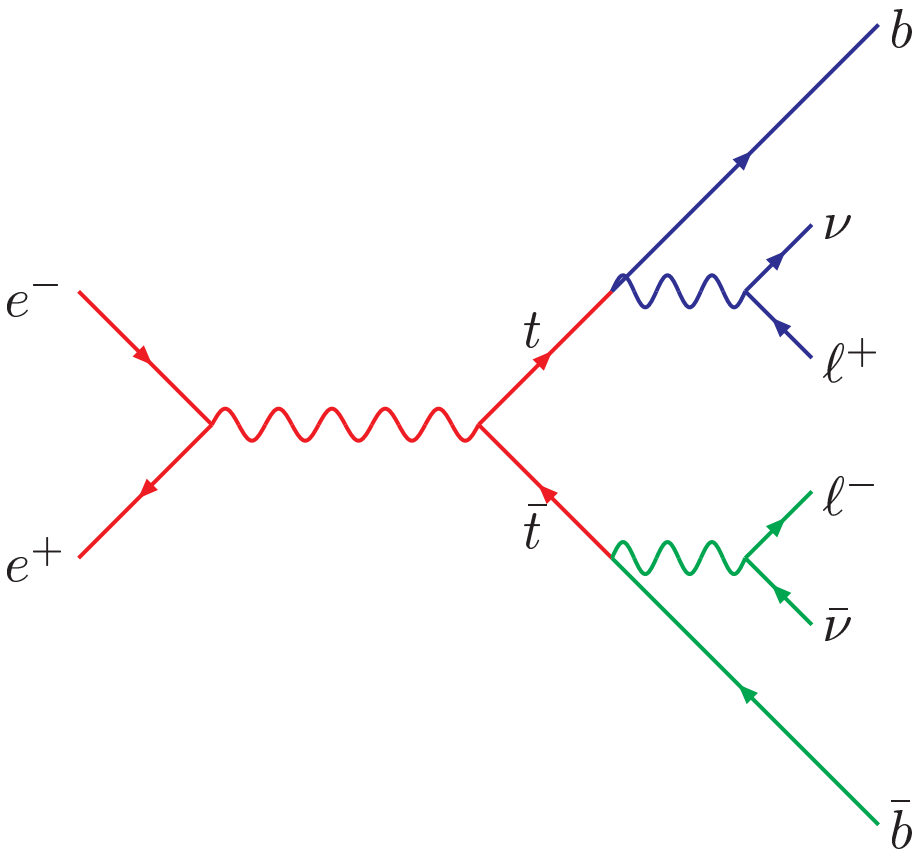
$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{e^+e^-\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*e^+e^-\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t\lambda_{\bar{t}}}$$

Top pair Production

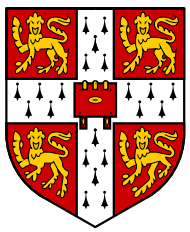


$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{e^+e^-\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*e^+e^-\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}} \mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}$$

Top pair Production



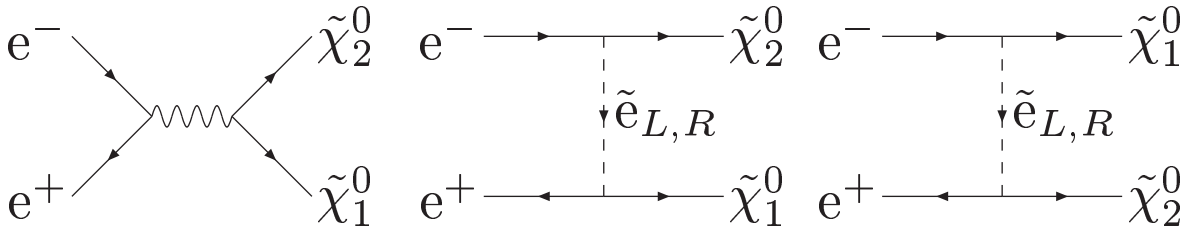
$$|\mathcal{M}|^2 = \rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}^{e^+e^-\to t\bar{t}}_{\kappa_1\kappa_2;\lambda_t\lambda_{\bar{t}}}\mathcal{M}^{*e^+e^-\to t\bar{t}}_{\kappa_1'\kappa_2';\lambda_t'\lambda_{\bar{t}}'}$$
$$\mathcal{M}^{t\to b\ell\nu}_{\lambda_t}\mathcal{M}^{*t\to b\ell\nu}_{\lambda_t'}$$
$$\mathcal{M}^{\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}}\mathcal{M}^{*\bar{t}\to\bar{b}\ell\nu}_{\lambda_{\bar{t}}'}$$



Spin Correlations

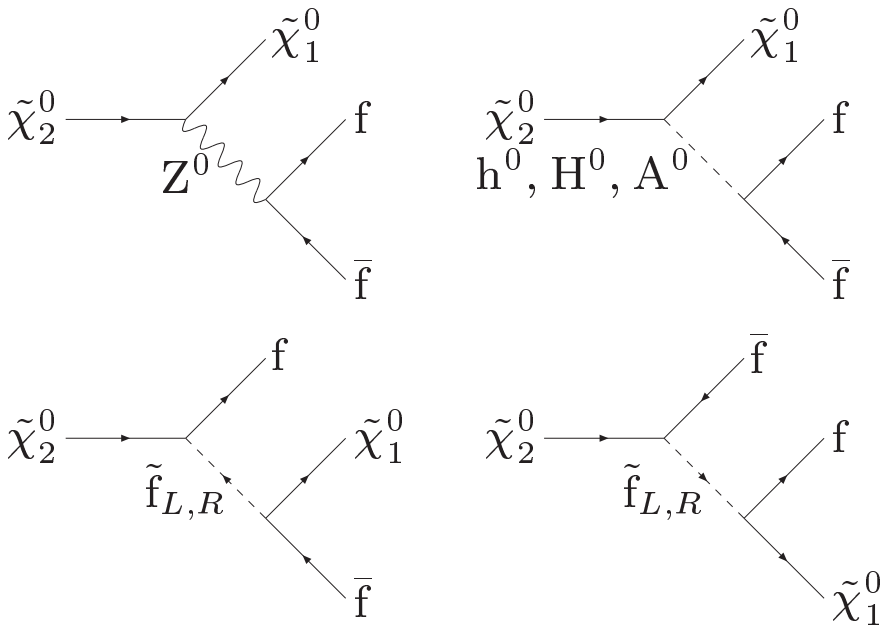
- So this procedure gives the full matrix element, but allows us to generate the event step by step.
- Examples
- Neutralino production in $\mathrm{e}^+\mathrm{e}^-$ collisions.
- Top quark pair production in $\mathrm{e}^+\mathrm{e}^-$ collisions.

Neutralino Production in e^+e^- collisions



- As the lightest neutralino is stable in R-parity conserving SUSY models the SUSY final state which requires the lowest centre-of-mass energy to produce it may be $\tilde{\chi}_2^0\tilde{\chi}_1^0$.

- The $\tilde{\chi}_2^0$ then decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\bar{f}$.



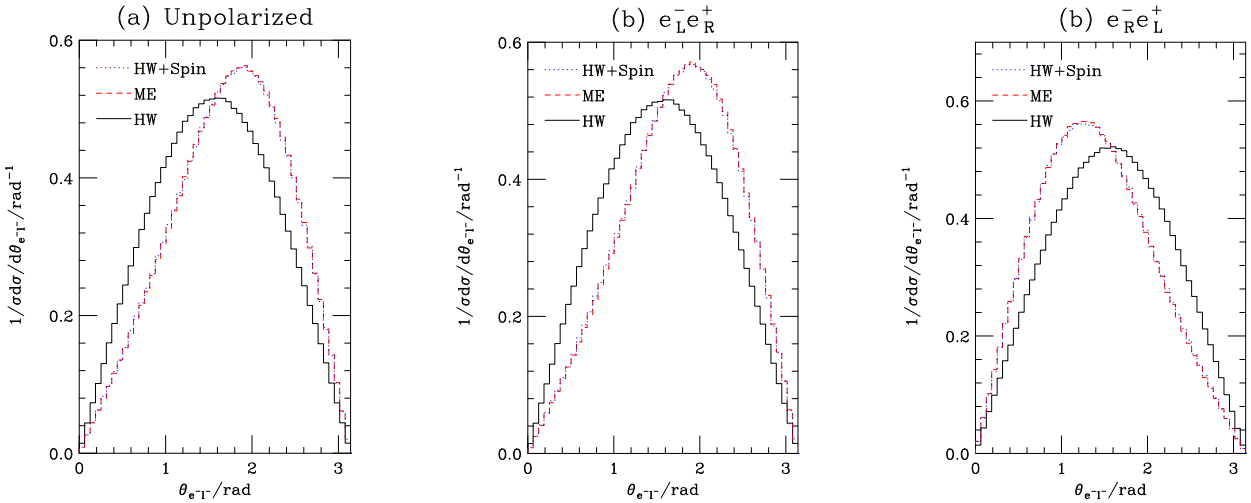
- The leptonic decay modes are the most important.

Neutralino Production in e^+e^- collisions

- Here the neutralino is produced in a $2 \to 2$ process, with the possibility of polarized beams so the calculation of the spin density matrix is more complicated.

- The decay of the neutralino, via a 2 or 3 body decay, is then performed using this decay matrix.

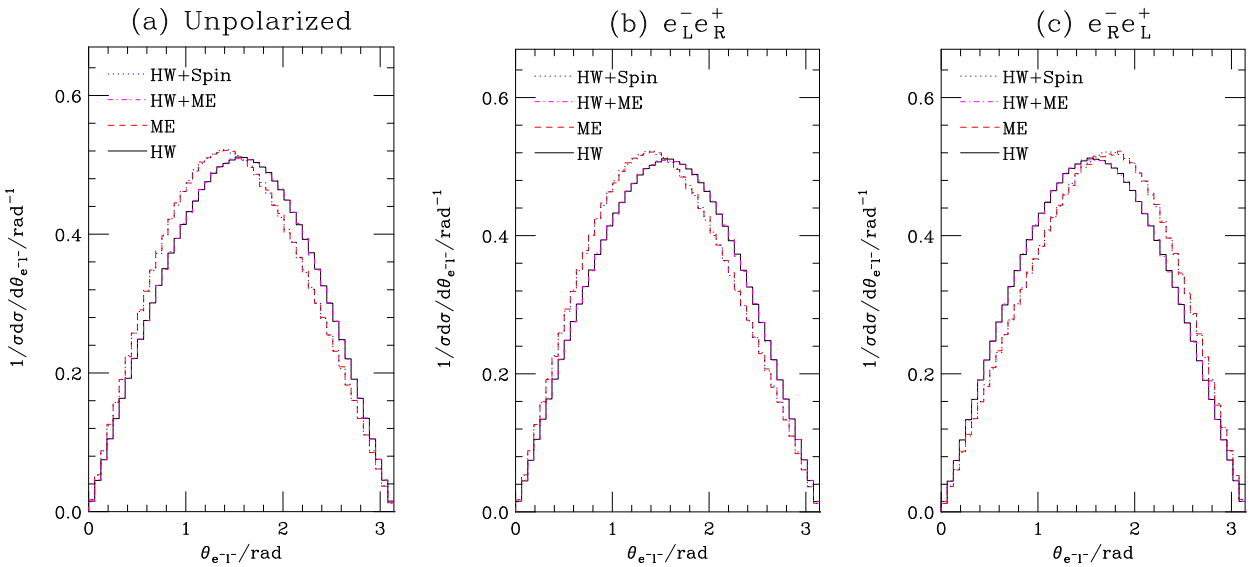
- First consider the two body decay $\tilde{\chi}_2^0 \to \tilde{\ell}_R^+ \ell^-$ and study the angle between the lepton and the beam, again using SUGRA point 5.



Neutralino Production in e^+e^- collisions

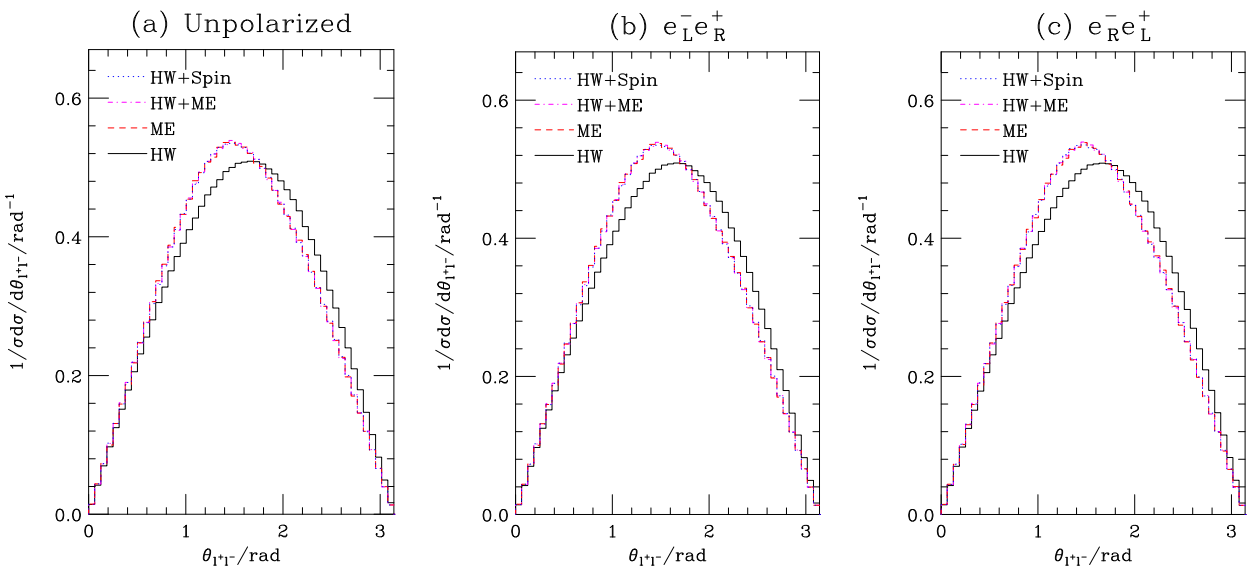
- We can also consider the three body decay $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$ using the SUSY parameters $M_0 = 210$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$, $M_1 = 450$ GeV, $M_2 = 350$ GeV, $M_3 = 350$ GeV.

- Study the angle between the lepton and the beam,



Neutralino Production in e^+e^- collisions

- and between the lepton and antilepton.

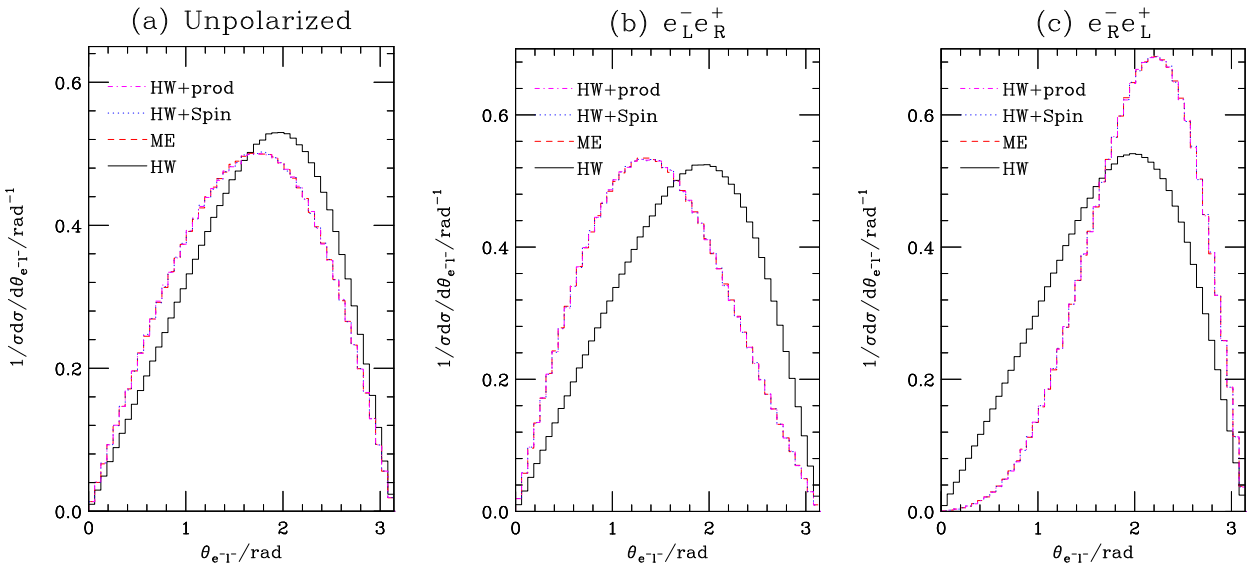


Top pair production in e^+e^- collisions

- In this example in addition to the calculation of the spin density matrix for the production we also need to calculate the decay matrix for the first decay and use this in performing the second decay.

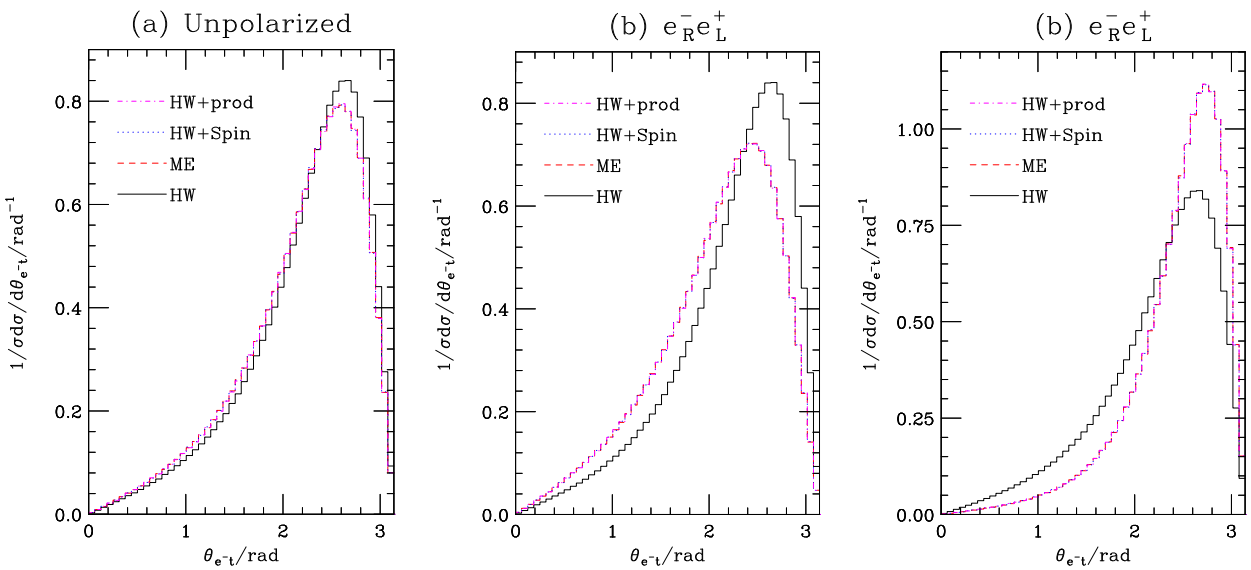
- Study the semi-leptonic decays of both top quarks and look at the correlations.

- Between the lepton produced in the antitop decay and the beam.

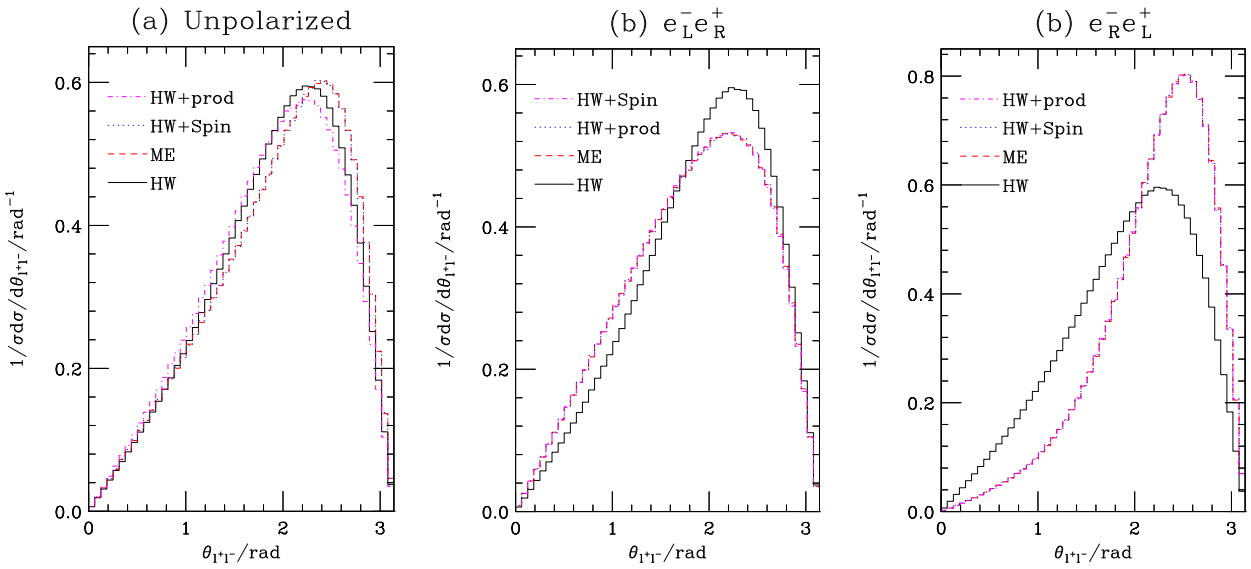


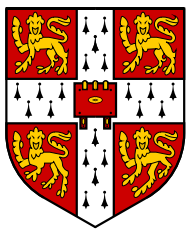
Top pair production in e^+e^- collisions

- Between the lepton and the top quark.



- Between the lepton and antilepton.

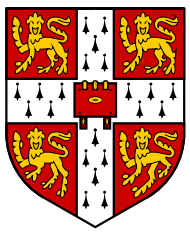




Top pair production in e^+e^- collisions

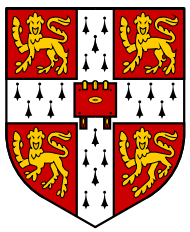
- The correlation between the lepton and the beam or top quark is only affected by the production mechanism.

- The correlation between the lepton and antilepton is also effected by the decay of the first quark.



The Future

- The major experiments are moving to C++ for their software.
- Therefore the major event generators are changing as well.
- There are already some event generators in C++.
  e.g. APACIC++
  http://www.physik.tu-dresden.de/itp/members/kuhn/cpc/cpc.html
- PYTHIA and HERWIG will also be rewritten.
- They will both be in the PYTHIA7 framework.
  http://www.thep.lu.se/ leif/Pythia7/Welcome.html
- This should make it easier for users to switch between the generators and use different physics modules from them.
- Talk in the parallel session by A. Ribon.



Conclusions

- The field of SUSY event generator has become relatively mature.
- There are a number of programs available which give sophisicated modelling of SUSY events.
- Many RGE codes exist and there have been many comparisons. It would be helpful if more of these were publically available.
- The same applies to dark matter codes and NLO programs.
- The move to C++ continues.