Durham University

Monte Carlo Event Generators

Lecture 2: Parton Showers and Perturbative QCD

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Plan

- Lecture 1: Introduction
  - Basic principles of event generation
  - Monte Carlo integration techniques
  - Matrix Elements
- Lecture 2: Parton Showers
  - Parton Shower Approach
  - Recent advances, CKKW and MC@NLO
- Lecture 3: Hadronization and Underlying Event
  - Hadronization Models
  - Underlying Event Modelling

Lecture 2

Today we will cover

- The Standard Parton Shower approach
  - Final-state showers
  - Initial-state showers.
- Merging Matrix Elements with Parton Showers
  - Traditional Approach
  - CKKW
  - MC@NLO
  - Where Next?

e^+e^- Annihilation to Jets

- In order to consider the basic idea of the parton shower let's start with the cross section for e^+e^- annihilation into three jets.
- The cross section can be written as

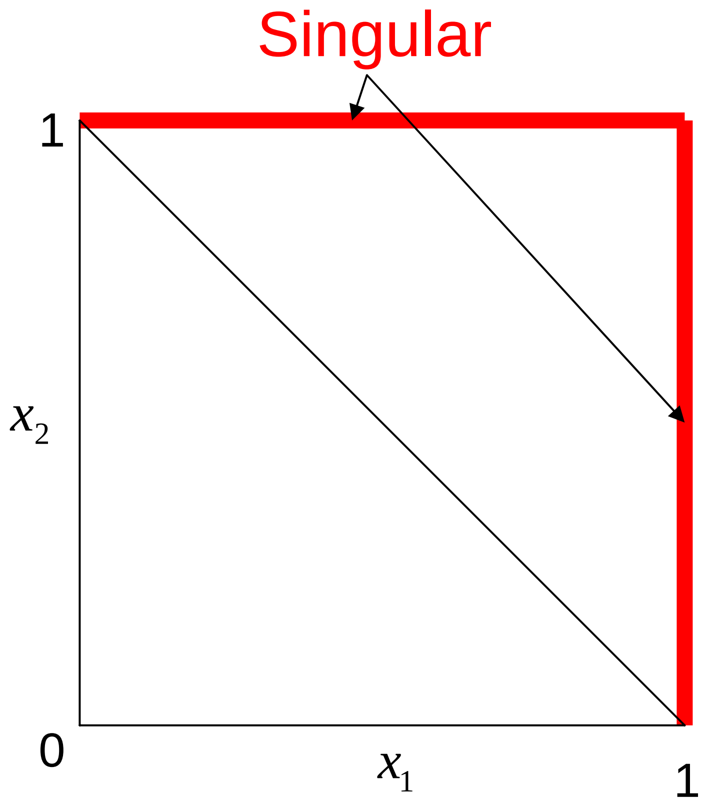
$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- where x_1 and x_2 are the momentum fractions of the quark and antiquark

$$x_i = 2 \times \frac{\text{Energy of } i}{\text{Centre of Mass Energy}}$$

- Singular as $x_1 \rightarrow 1$ or $x_2 \rightarrow 1$.
- Energy Conservation requires

$$x_1 + x_2 + x_g = 2$$



e^+e^- Annihilation to Jets

- So the matrix element is singular as $x_1 \to 1$, $x_2 \to 1$ or both.
- What does this mean physically?

$$p_{cm} = p_1 + p_2 + p_g$$
$$(p_{cm} - p_2)^2 = (p_1 + p_g)^2$$
$$E_{cm}^2 - 2E_{cm}E_2 = 2E_1E_3(1 - \cos\theta_{13})$$
$$(1 - x_2) = \frac{1}{2}x_1x_3(1 - \cos\theta_{13})$$

- So $x_2 \to 1$ means that 1 and 3 are collinear.
- Also $x_1 + x_2 + x_3 = 2$ means that $x_1 \to 1$ and $x_2 \to 1$ implies $x_g \to 0$, soft

QCD Radiation

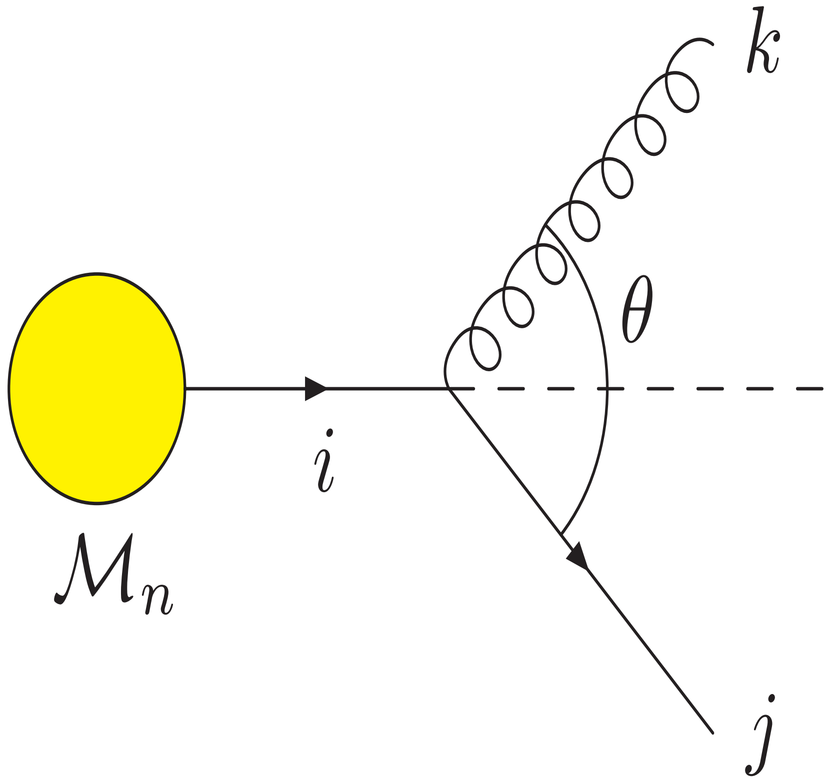
- It is impossible to calculate and integrate the matrix elements for large numbers of partons.
- Instead we treat the regions where the emission of QCD radiation is enhanced.
- This is soft and collinear radiation.

Collinear Singularities

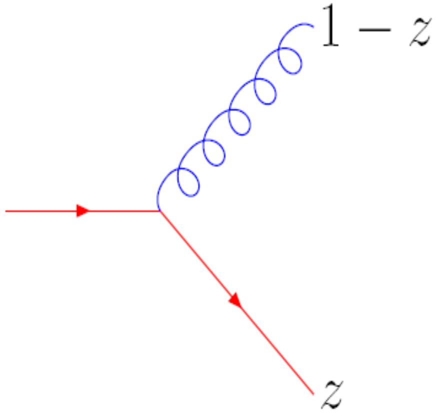
- In the collinear limit the cross section for a process factorizes

$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z)$$

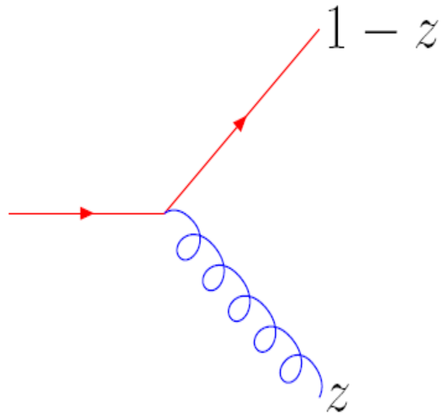
  - $P_{ji}(z)$ is the DGLAP splitting function.
- The splitting function only depends on the spin and flavours of the particles



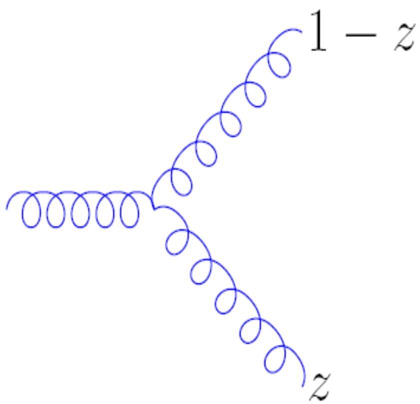
Splitting Functions



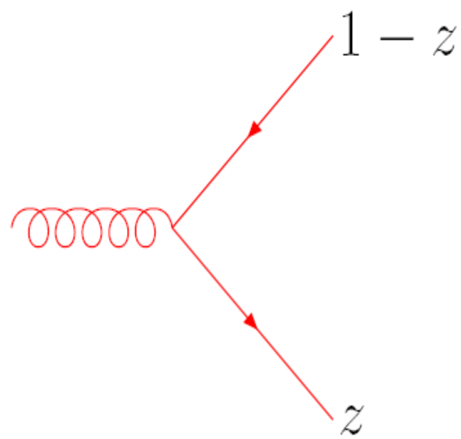
$$C_F \frac{1+z^2}{1-z}$$



$$C_F \frac{1+(1-z)^2}{z}$$



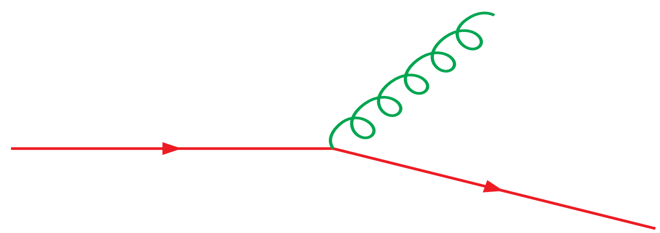
$$C_A\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)$$



$$T_R\left(z^2+(1-z)^2\right)$$

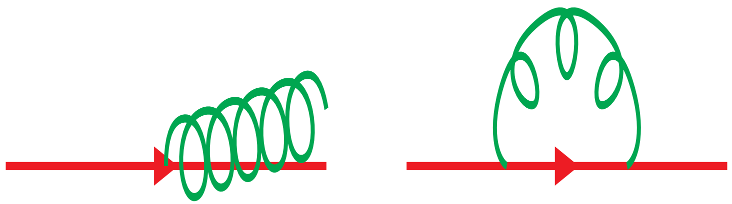
Collinear Singularities

- This expression is singular as $\theta \to 0$.
- What is a parton? (or what is the difference between a collinear pair and a parton)
- Introduce a resolution criterion, e.g. $k_T > Q_0$
- Combine the virtual corrections and unresolvable emission



Resolvable Emission

Finite



Unresolvable Emission

Finite

- Unitarity: Unresolved + Resolved =1

Monte Carlo Procedure

- Using this approach we can exponentiate the real emission piece.

$$\text{Unresolved} = 1 - \text{Resolved} \begin{array}{l} = 1 - \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z) \\ = \exp\left(- \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z) \right) \end{array}$$

- This gives the Sudakov form factor which is the probability of evolving between two scales and emitting no resolvable radiation.
- More strictly it is the probability of evolving from a high scale to the cut-off with no resolvable emission.

Numerical Procedure

Radioactive Decay

- Start with an isotope
- Work out when it decays by generating a random number $R \in [0,1]$ and solving

$$R = \exp[-t/\tau]$$

  where τ is its lifetime
- Generate another random number and use the branching ratios to find the decay mode.
- Generate the decay using the masses of the decay products and phase space.
- Repeat the process for any unstable decay products.
- This algorithm is actually used in Monte Carlo event generators to simulate particle decays.

Parton Shower

- Start with a parton at a high virtuality, Q, typical of the hard collision.
- Work out the scale of the next branching by generating a random number $R \in [0,1]$ and solving

$$R = \Delta(Q^2, q^2)$$

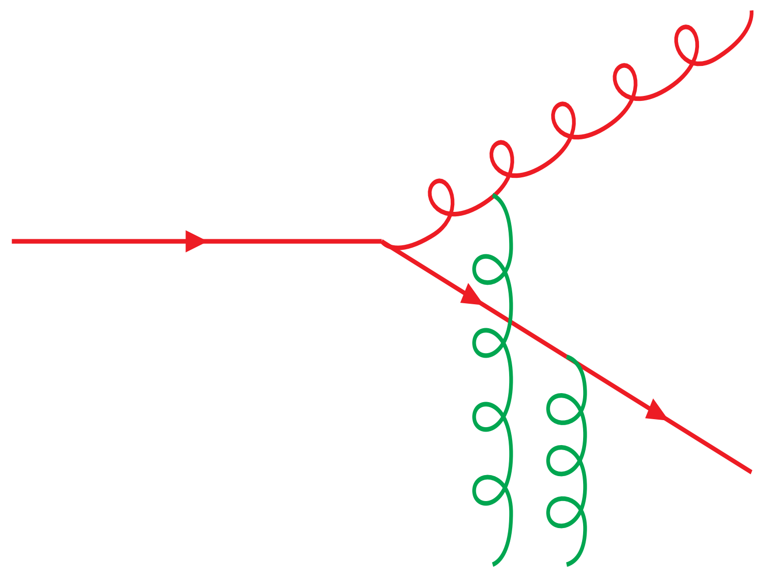
  where q is the scale of the next branching
- If there's no solution for q bigger than the cut-off stop.
- Otherwise workout the type of branching.
- Generate the momenta of the decay products using the splitting functions.
- Repeat the process for the partons produced in the branching.

Monte Carlo Procedure

- The key difference between the different Monte Carlo simulations is in the choice of the evolution variable.
- Evolution Scale
  - Virtuality, q^2
  - Transverse Momentum, k_T.
  - Angle, θ.
  - ….
- Energy fraction, z
  - Energy fraction
  - Light-cone momentum fraction
  - ….
- All are the same in the collinear limit.

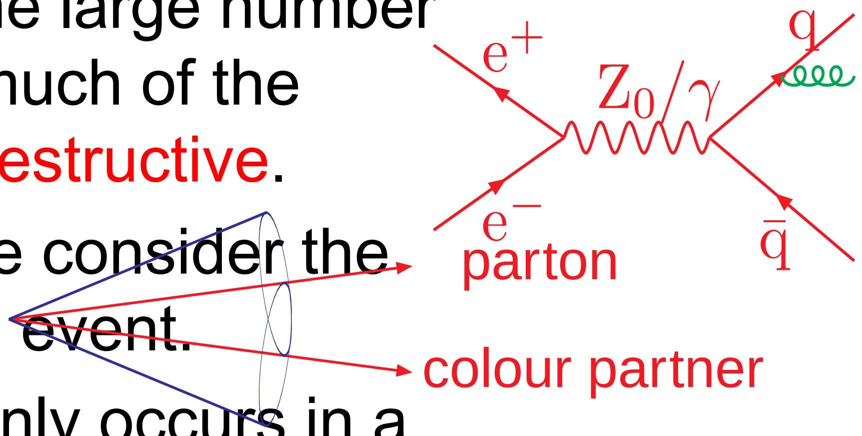
Soft Emission

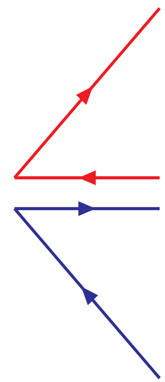
- We have only considered collinear emission. What about soft emission?
- In the soft limit the matrix element factorizes but at the amplitude level.
- Soft gluons come from all over the event.
- There is quantum interference between them.
- Does this spoil the parton shower picture?

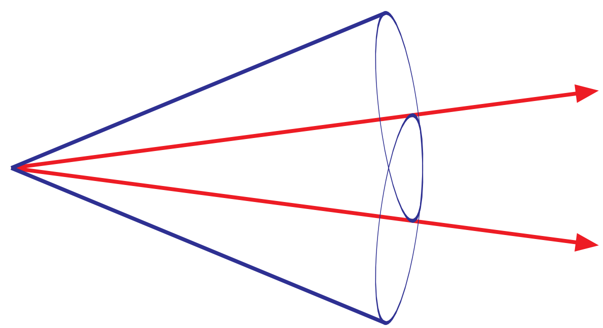


Angular Ordering

- There is a remarkable result that if we take the large number of colours limit much of the interference is destructive.
- In particular if we consider the colour flow in an event.
- QCD radiation only occurs in a cone up to the direction of the colour partner.
- The best choice of evolution variable is therefore an angular one.

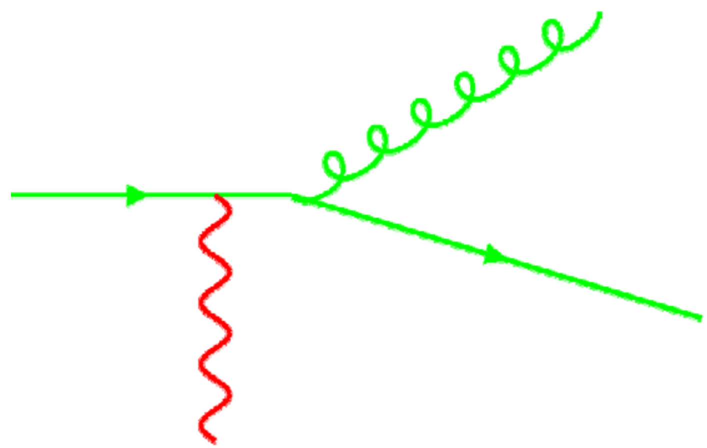






Colour Coherence

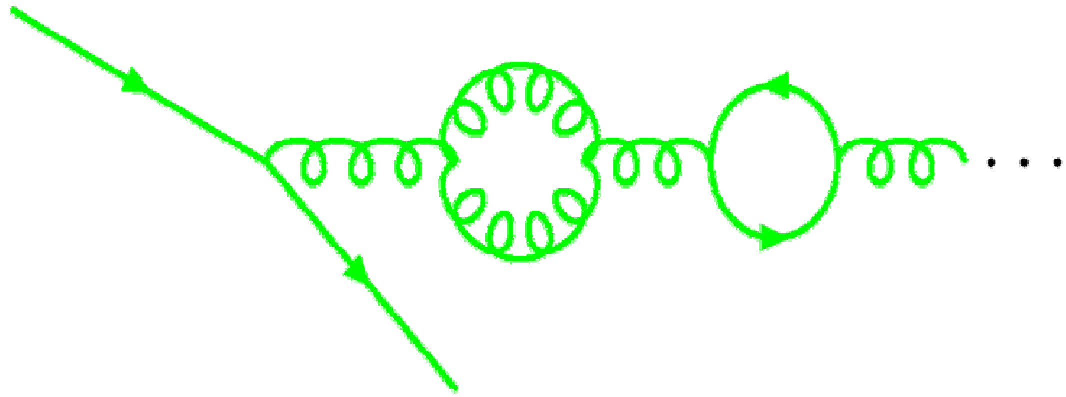
- Angular Ordering and Colour Coherence are often used interchangeably in talks etc..
- However there is a difference.
- Colour Coherence is the phenomena that a soft gluon can’t resolve a small angle pair of particles and so only sees the colour charge of the pair.



- Angular Ordering is a way of implementing colour coherence in parton shower simulations.

Running Coupling

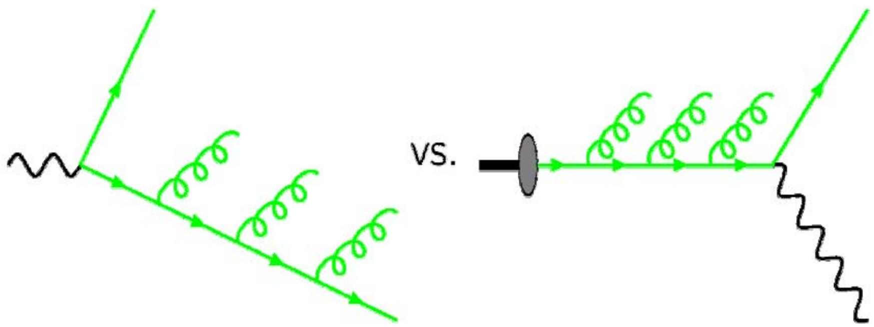
- It is often said that Monte Carlo event generators are leading-log.
- However they include many effects beyond leading log, e.g.
- Momentum Conservation
- Running Coupling Effects



  - Effect of summing higher orders is absorbed by replacing α_s with $\alpha_s(k_T^2)$.
  - Gives more soft gluons, but must avoid the Landau pole which makes the cut-off a physical parameter.

Initial-State Radiation

- In principle this is similar to final-state radiation.
- However in practice there is a complication
- For final-state radiation
  - One end of the evolution fixed, the scale of the hard collision.
- For initial-state radiation
  - Both ends of the evolution fixed, the hard collision and the incoming hadron



- Use a different approach based on the evolution equations.

Initial-State Radiation

- There are two options for the initial-state shower
- Forward Evolution
  - Start at the hadron with the distribution of partons given by the PDF.
  - Use the parton shower to evolve to the hard collision.
  - Reproduces the PDF by a Monte Carlo procedure.
  - Unlikely to give an interesting event at the end, so highly inefficient.
- Backward Evolution
  - Start at the hard collision and evolve backwards to the proton guided by the PDF.
  - Much more efficient in practice.

Initial-State Radiation

- The evolution equation for the PDF can be written as

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a\left(\frac{x}{z},Q^2\right) \frac{\alpha_s}{2\pi} P_{a\to bc}(z) \qquad \text{where} \quad t = \ln\left(\frac{Q^2}{\Lambda^2}\right)$$

- Or

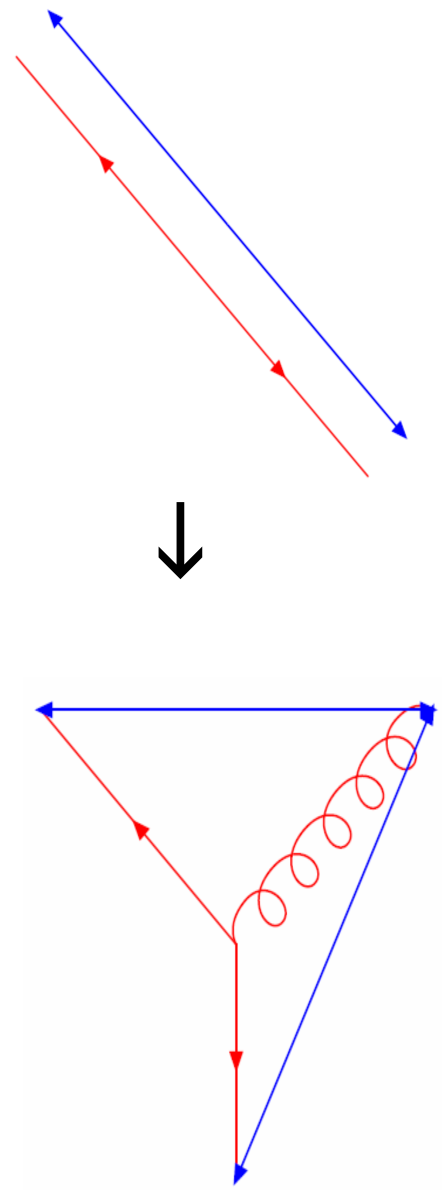
$$\mathrm{d}P_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \frac{x' f_a\left(x',Q^2\right)}{x f_b(x,Q^2)} \frac{\alpha_s}{2\pi} P_{a\to bc}(z) \;\; \text{where} \quad x' = \frac{x}{z}$$

- This can be written as a Sudakov form-factor for evolving backwards in time, i.e from the hard collision at high Q^2 to lower with

$$S = \exp\left(- \int \mathrm{d}P_b\right)$$

The Colour Dipole Model

- The standard parton shower approach starts from the collinear limit and makes changes to include soft gluon coherence.
- The Colour Dipole Model starts from the soft limit.
- Emission of soft gluons from the colour-anticolour dipole is universal.



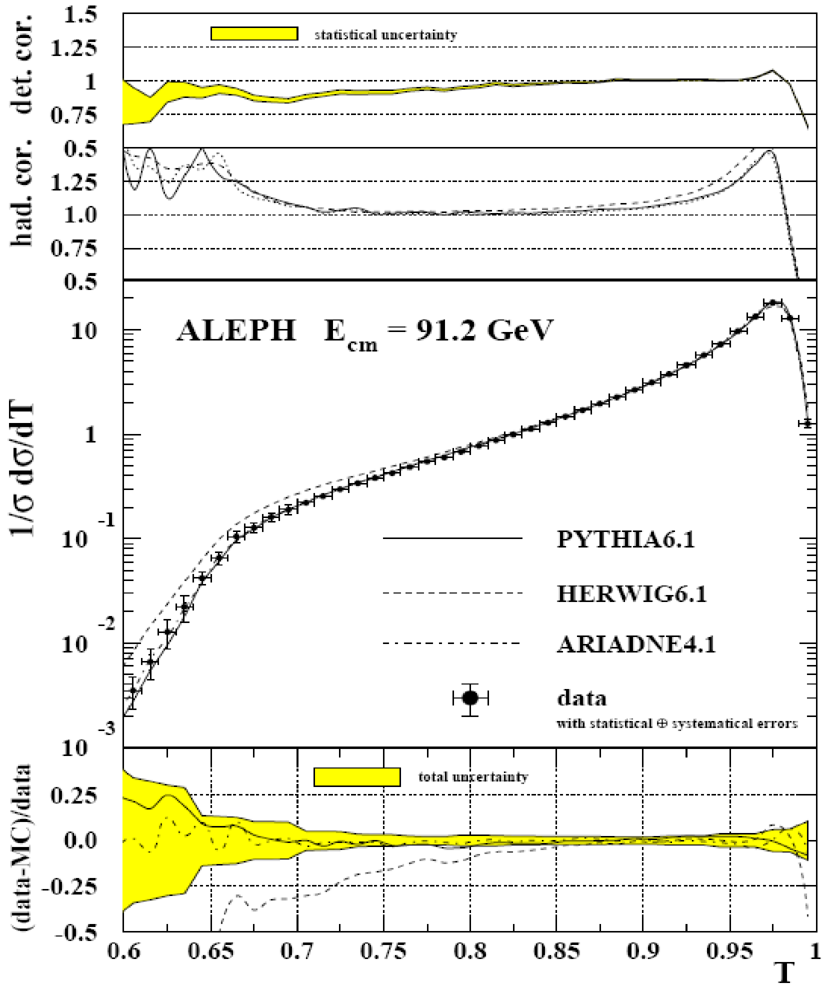
$$\mathrm{d}\sigma \approx \sigma_0 \frac{C_A}{2} \frac{\alpha_s(k_T)}{2\pi} \frac{\mathrm{d}k_T^2}{k_T^2} \mathrm{d}y \quad y = \text{rapidity} = \log \tan \frac{\theta}{2}$$

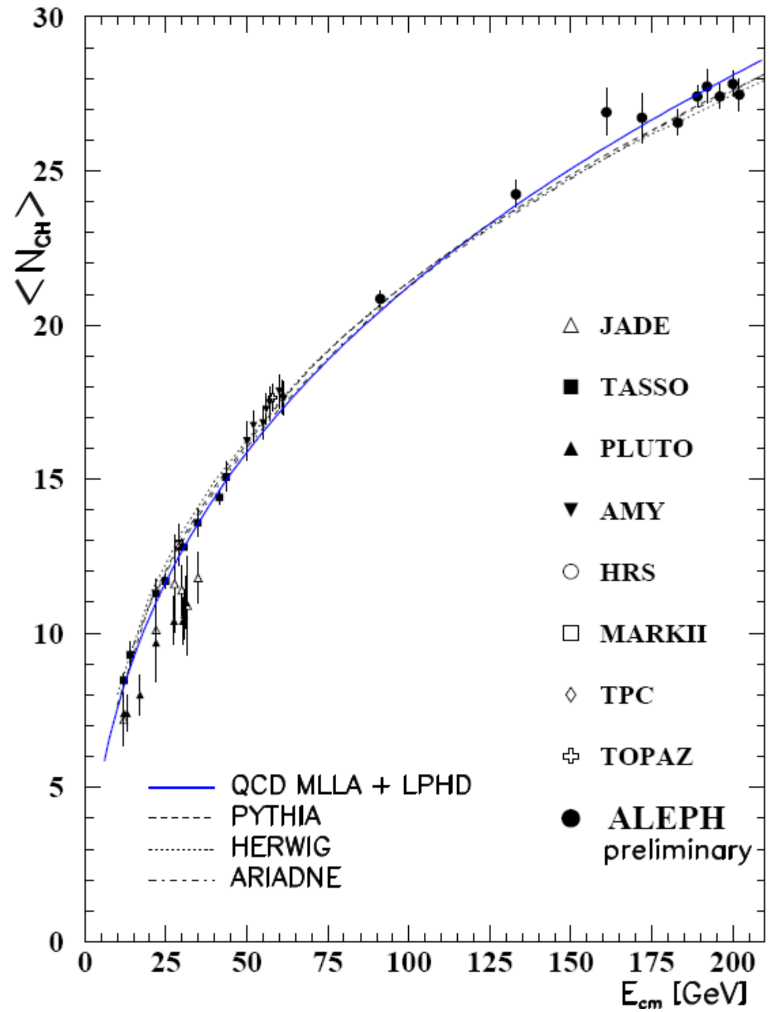
- After emitting a gluon, the colour dipole splits into two new dipoles

Parton Shower

- ISAJET uses the original parton shower algorithm which only resums collinear logarithms.
- PYTHIA uses the collinear algorithm with an angular veto to try to reproduce the effect of the angular ordered shower.
- HERWIG uses the angular ordered parton shower algorithm which resums both soft and collinear singularities.
- SHERPA uses the PYTHIA algorithm.
- ARIADNE uses the colour dipole model.

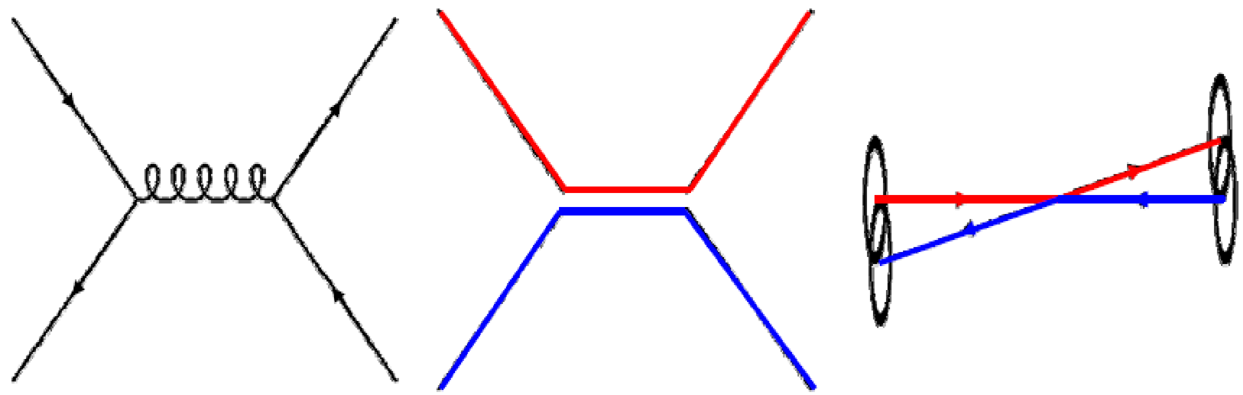
LEP Event Shapes





Hadron Collisions

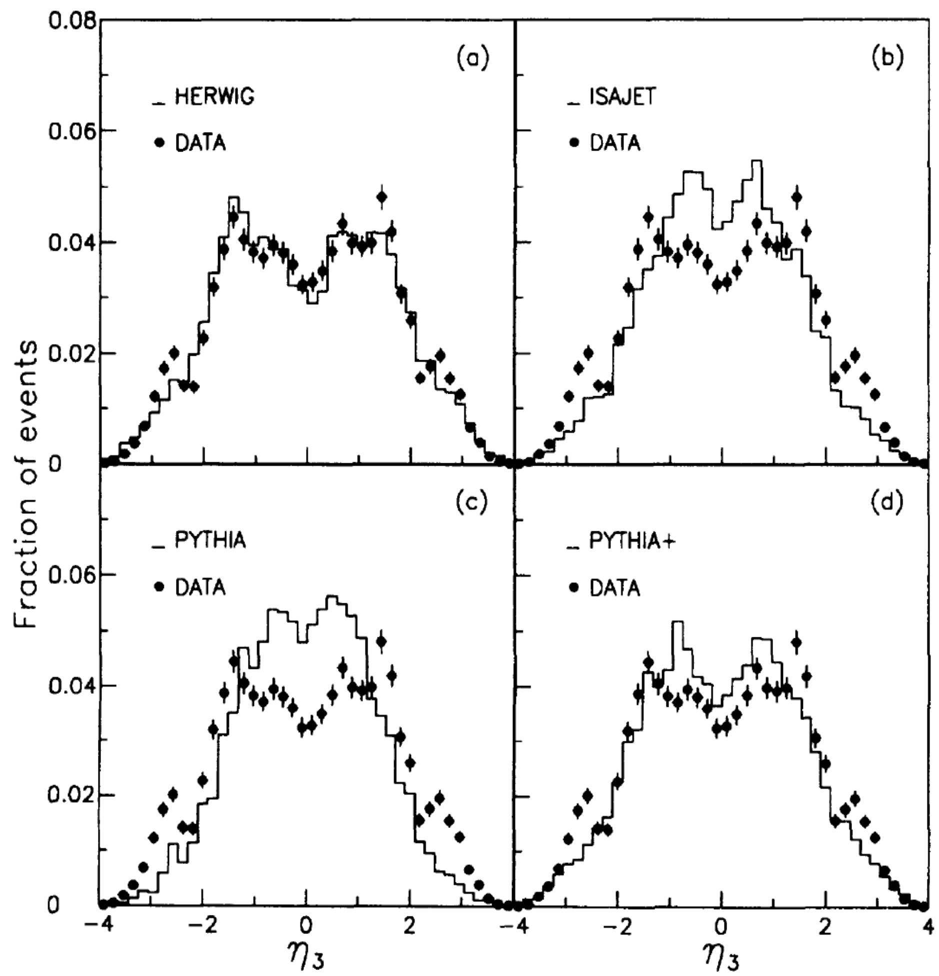
- The hard scattering sets up the initial conditions for the parton shower.
- Colour coherence is important here too.



- Each parton can only emit in a cone stretching to its colour partner.
- Essential to fit the Tevatron data.

Hadron Collisions

- Distributions of the pseudorapidity of the third jet.
- Only described by
  - HERWIG which has complete treatment of colour coherence.
  - PYTHIA+ has partial
- PRD50, 5562, CDF (1994)

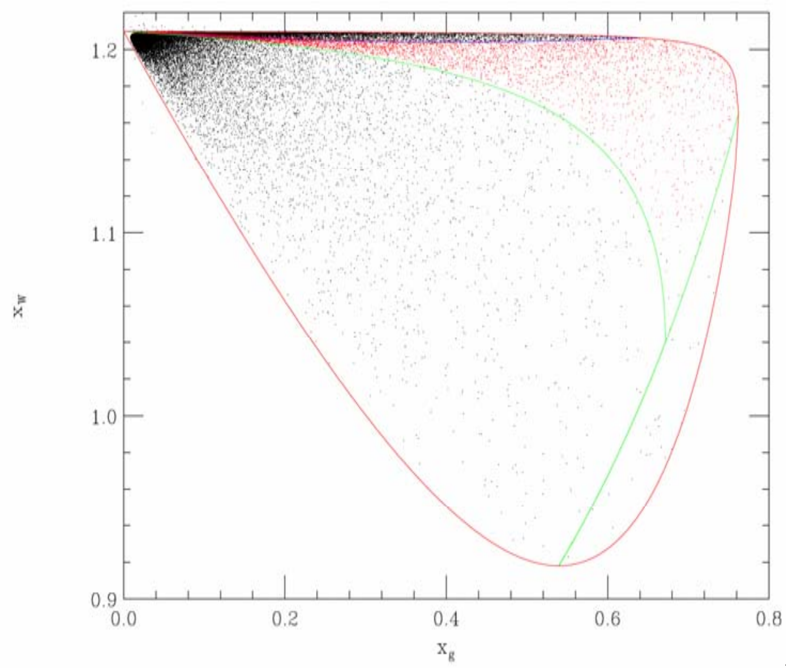


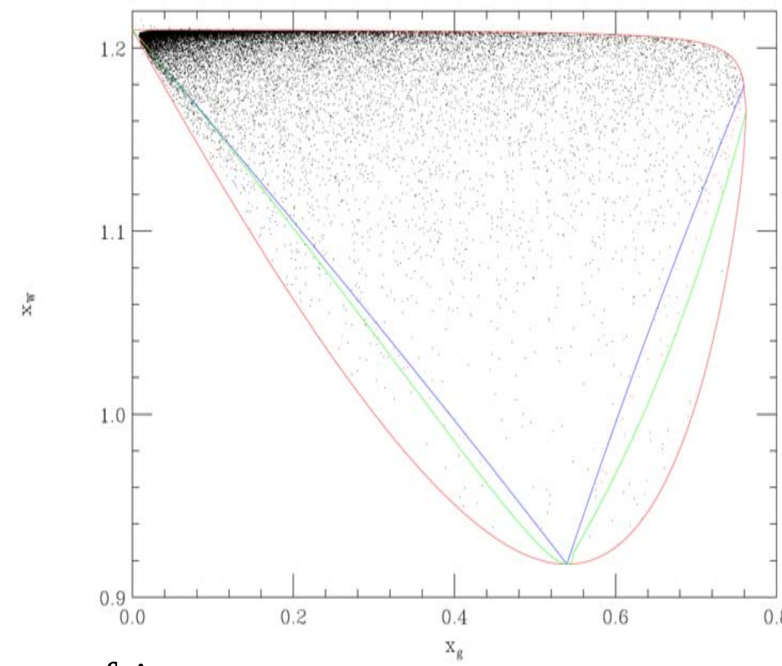
Recent Progress

- In the parton shower per se there have been two recent advances.

1) New Herwig++ shower
   - Based on massive splitting functions.
   - Better treatment of radiation from heavy quarks.
   - More Lorentz invariant.
2) New PYTHIA p_T ordered shower
   - Order shower in p_T, should be coherent.
   - Easier to include new underlying event models.
   - Easier to match to matrix elements

Herwig++ for $t \to bW^+g$

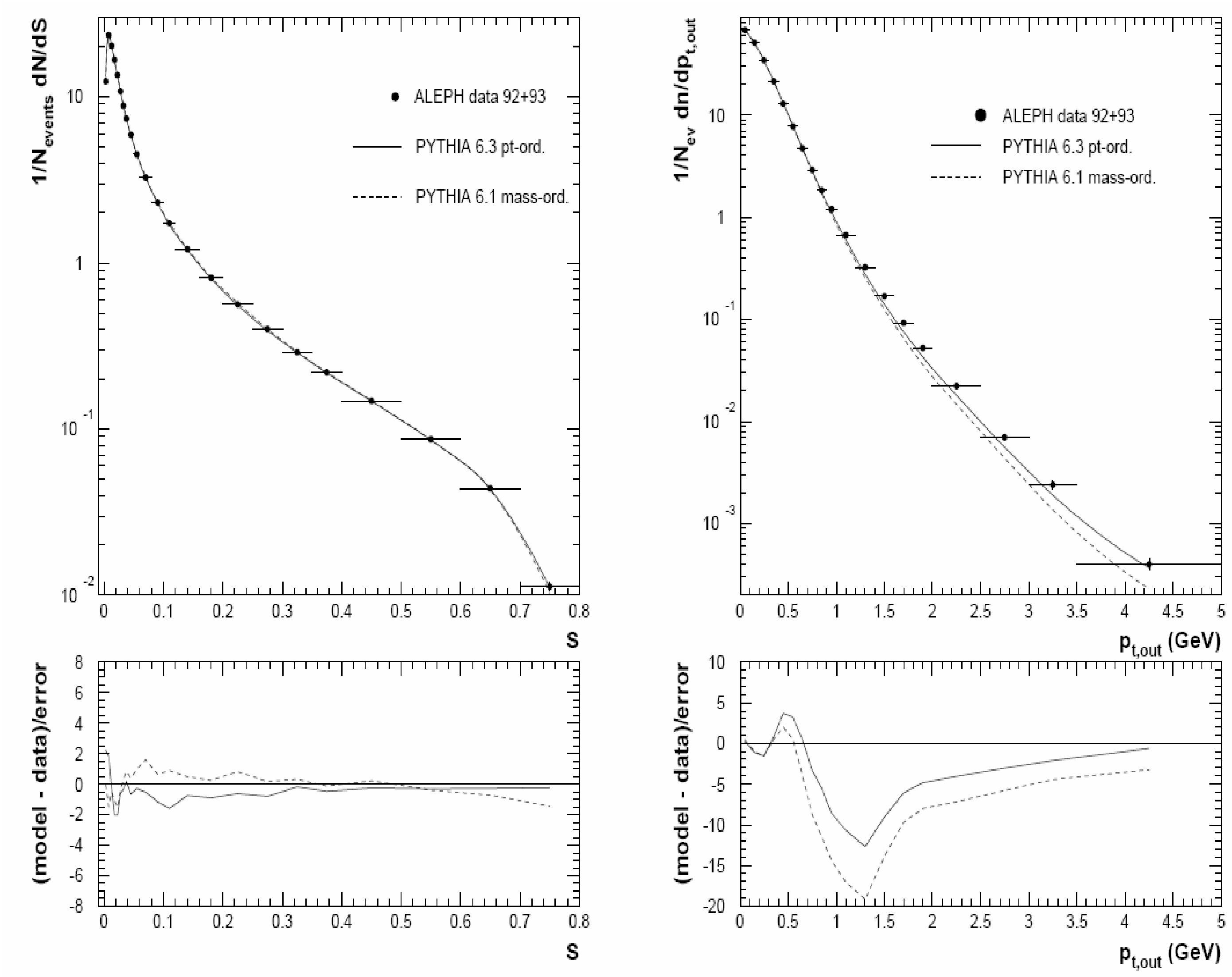




$$x_i = 2 \times \frac{\text{Energy of i}}{\text{Mass of Top Quark}}$$

- Based on the formalism of S. Gieseke, P. Stephens and B.R. Webber, JHEP 0312:045,2003.
- Improvement on the previous FORTRAN version.

PYTHIA p_T ordered Shower



Hard Jet Radiation

- The parton shower is designed to simulate soft and collinear radiation.
- While this is the bulk of the emission we are often interested in the radiation of a hard jet.
- This is not something the parton shower should be able to do, although it often does better than we except.
- If you are looking at hard radiation HERWIG/PYTHIA will often get it wrong.

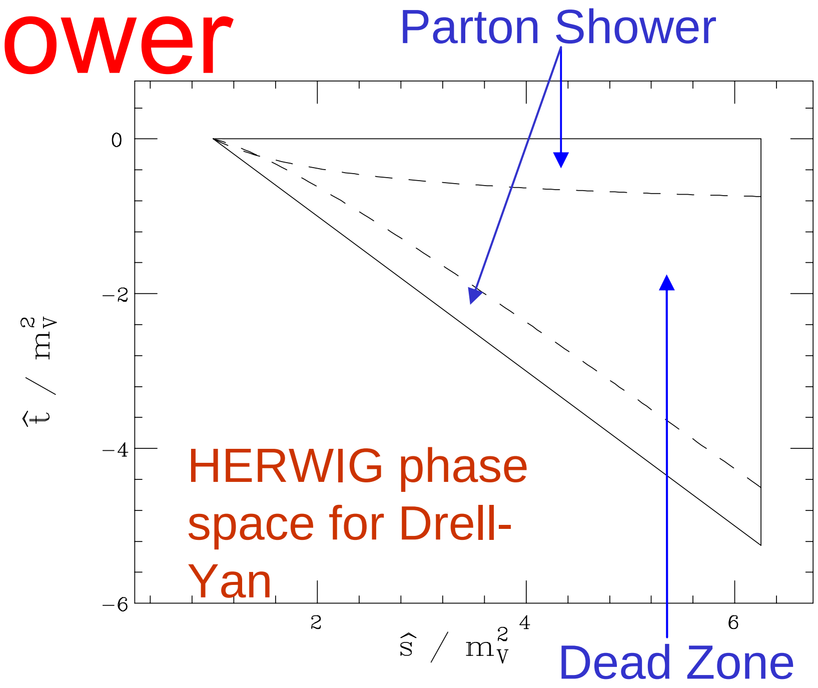
Hard Jet Radiation

- Given this failure of the approximations this is an obvious area to make improvements in the shower and has a long history.
- You will often here this called
  - Matrix Element matching.
  - Matrix Element corrections.
  - Merging matrix elements and parton shower
  - MC@NLO
- I will discuss all of these and where the different ideas are useful.

Hard Jet Radiation: General Idea

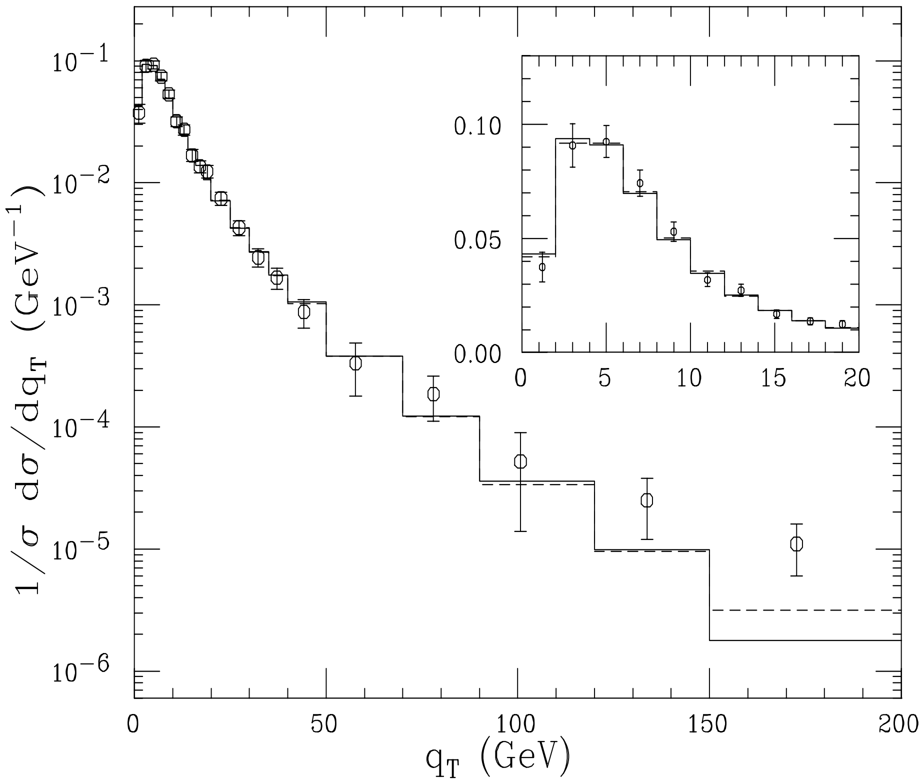
- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high p_T jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating a few high p_T jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, i.e.
  - ME gives hard emission
  - PS gives soft/collinear emission
  - Smooth matching between the two.
  - No double counting of radiation.

Matching Matrix Elements and Parton Shower

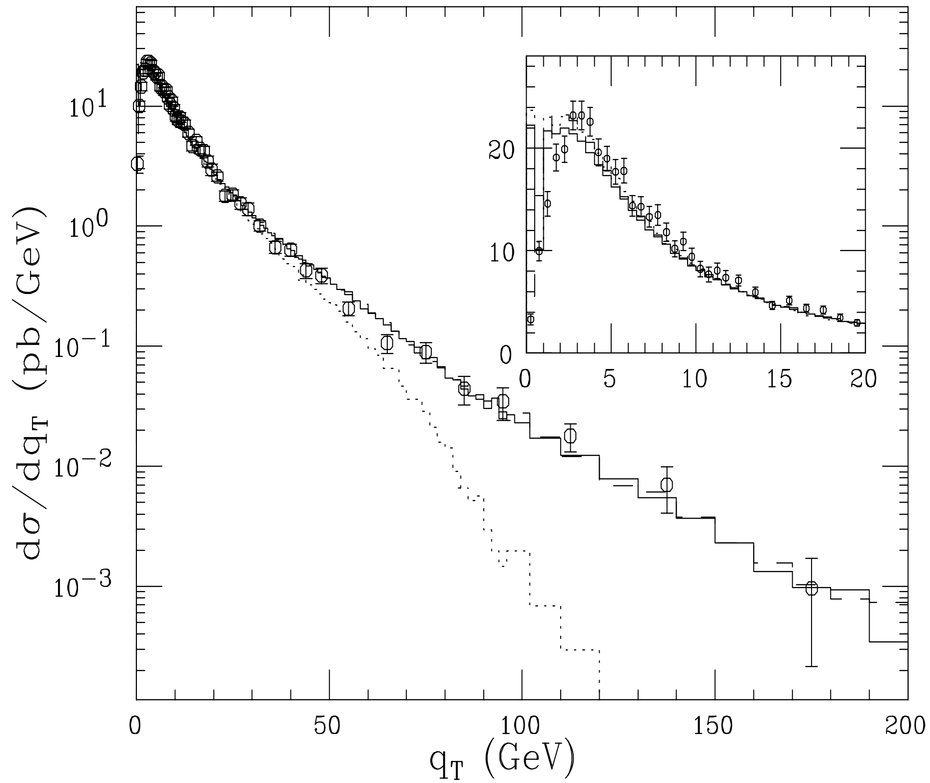


- The oldest approaches are usually called matching matrix elements and parton showers or the matrix element correction.
- Slightly different for HERWIG and PYTHIA.
- In HERWIG
  - Use the leading order matrix element to fill the dead zone.
  - Correct the parton shower to get the leading order matrix element in the already filled region.
- PYTHIA fills the full phase space so only the second step is needed.

Matrix Element Corrections



W q_T distribution from D0



Z q_T distribution from CDF

G. Corcella and M. Seymour, Nucl.Phys.B565:227-244,2000.

Matrix Element Corrections

- There was a lot of work for both HERWIG and PYTHIA. The corrections for
  - e^+e^- to hadrons
  - DIS
  - Drell-Yan
  - Top Decay
  - Higgs Production

  were included.
- There are problems with this
  - Only the hardest emission was correctly described
  - The leading order normalization was retained.

Recent Progress

- In the last few years there has been a lot of work addressing both of these problems.
- Two types of approach have emerged

1) NLO Simulation
   - NLO normalization of the cross section
   - Gets the hardest emission correct
2) Multi-Jet Leading Order
   - Still leading order.
   - Gets many hard emissions correct.

NLO Simulation

- There has been a lot of work on NLO Monte Carlo simulations.
- Only the MC@NLO approach of Frixione, Nason and Webber has been shown to work in practice.
- Although an alternative approach by Nason looks promising and a paper with results for Z pairs appeared last week.

MC@NLO

- MC@NLO was designed to have the following features.
  - The output is a set of fully exclusive events.
  - The total rate is accurate to NLO
  - NLO results for observables are recovered when expanded in α_s.
  - Hard emissions are treated as in NLO calculations.
  - Soft/Collinear emission are treated as in the parton shower.
  - The matching between hard emission and the parton shower is smooth.
  - MC hadronization models are used.

Basic Idea

- The basic idea of MC@NLO is
  - Work out the shower approximation for the real emission.
  - Subtract it from the real emission from
  - Add it to the virtual piece.
- This cancels the singularities and avoids double counting.
- It's a lot more complicated than it sounds.

Toy Model

- I will start with Bryan Webber's toy model to explain MC@NLO to discuss the key features of NLO, MC and the matching.
- Consider a system which can radiate photons with energy with energy x with

$$0 \leq x \leq x_s \leq 1$$

where x_s is the energy of the system before radiation.

- After radiation the energy of the system $x_s' = x_s - x$
- Further radiation is possible but photons don't radiate.

Toy Model

- Calculating an observable at NLO gives

$$\langle O \rangle = \lim_{\varepsilon \to 0} \int_0^1 dx x^{-2\varepsilon} O(x) \left[\left(\frac{d\sigma}{dx} \right)_B + \left(\frac{d\sigma}{dx} \right)_V + \left(\frac{d\sigma}{dx} \right)_R \right]$$

where the Born, Virtual and Real contributions are

$$\left(\frac{d\sigma}{dx} \right)_B = B\delta(x) \quad \left(\frac{d\sigma}{dx} \right)_V = \alpha \left(\frac{B}{2\varepsilon} + V \right) \delta(x) \quad \left(\frac{d\sigma}{dx} \right)_R = \alpha \frac{R(x)}{x}$$

α is the coupling constant and $\lim_{x \to 0} R(x) = B$

Toy Model

- In a subtraction method the real contribution is written as

$$\langle O \rangle_R = \alpha B O(0) \int_0^1 dx \frac{1}{x^{1+2\varepsilon}} + \alpha \int_0^1 dx \frac{O(x)R(x) - BO(0)}{x^{1+2\varepsilon}}$$

- The second integral is finite so we can set $\varepsilon = 0$

$$\langle O \rangle_R = -\alpha \frac{B}{2\varepsilon} O(0) + \alpha \int_0^1 dx \frac{O(x)R(x) - BO(0)}{x}$$

- The NLO prediction is therefore

$$\langle O \rangle_{sub} = \int_0^1 dx \left[\alpha O(x) \frac{R(x)}{x} + O(0) \left(B + \alpha V - \alpha \frac{B}{x} \right) \right]$$

Toy Monte Carlo

- In a MC treatment the system can emit many photons with the probability controlled by the Sudakov form factor, defined here as

$$\Delta(x_1, x_2) = \exp\left[-\alpha \int_{x_1}^{x_2} dx \frac{Q(x)}{x} \right]$$

where $Q(x)$ is a monotonic function which has

$$0 \le Q(x) \le 1 \qquad \lim_{x \to 0} Q(x) = 1 \qquad \lim_{x \to 1} Q(x) = 0$$

- $\Delta(x_1, x_2)$ is the probability that no photon can be emitted with energy x such that $x_1 \le x \le x_2$.

Toy MC@NLO

- We want to interface NLO to MC. Naïve first try
  - $O(0) \Rightarrow$ start MC with 0 real emissions: F_{MC}^{0}
  - $O(x) \Rightarrow$ start MC with 1 real emission at x: $F_{MC}^{1}(x)$
- So that the overall generating functional is

$$\int_0^1 dx \left[F_{MC}^{0} \left(B + \alpha V - \frac{\alpha B}{x} \right) + F_{MC}^{1}(x) \frac{\alpha R(x)}{x} \right]$$

- This is wrong because MC with no emissions will generate emission with NLO distribution

$$\left(\frac{d\sigma}{dx} \right)_{MC} = \alpha B \frac{Q(x)}{x}$$

Toy MC@NLO

- We must subtract this from the second term

$$F_{MC@NLO} = \int_0^1 dx \left[F_{MC}^0 \left(B + \alpha V + \frac{\alpha B(Q(x)-1)}{x} \right) + F_{MC}^1(x) \frac{\alpha (R(x) - BQ(x))}{x} \right]$$

- This prescription has many good features:
  - The added and subtracted terms are equal to $O(\alpha)$
  - The coefficients of F_{MC}^0 and F_{MC}^1 are separately finite.
  - The resummation of large logs is the same as for the Monte Carlo renormalized to the correct NLO cross section.

  However some events may have negative weight.

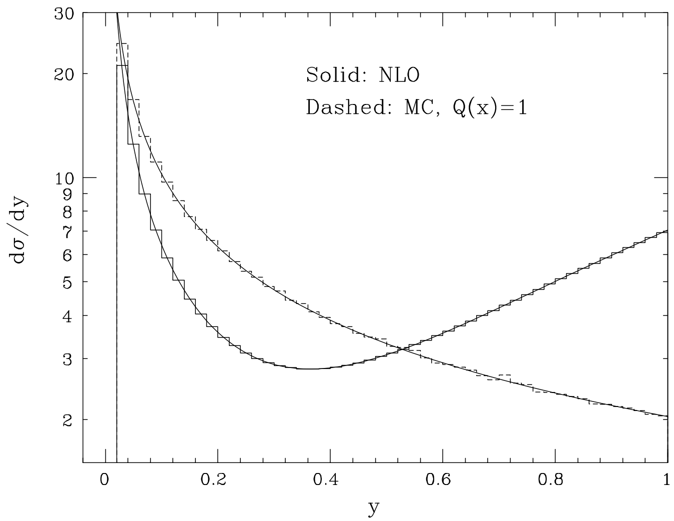
Toy MC@NLO Observables

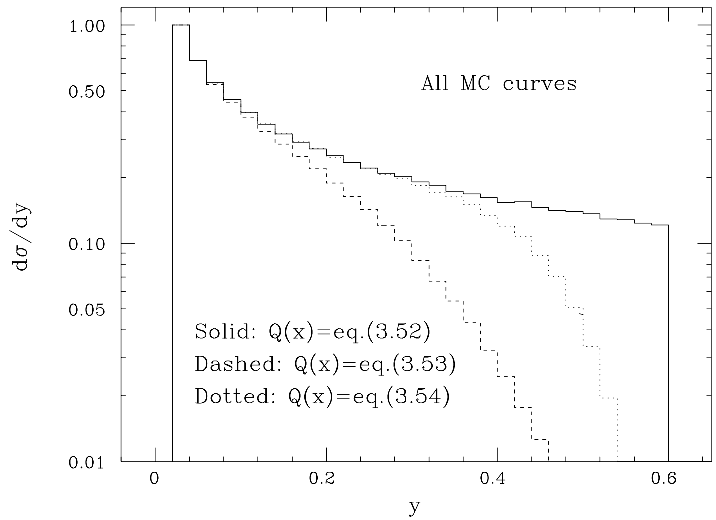
- As an example of an “exclusive” observable consider the energy y of the hardest photon in each event.
- As an “inclusive” observable consider the fully inclusive distributions of photon energies, z
- Toy model results shown are for

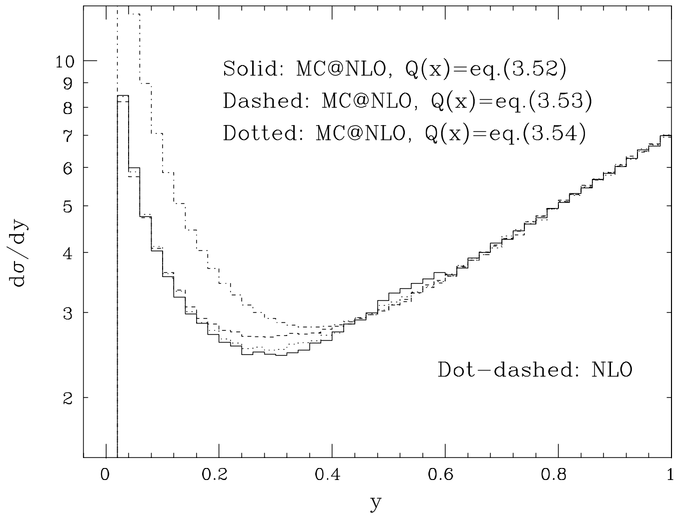
$$\alpha = 0.3, \qquad B = 2, \qquad V = 1,$$

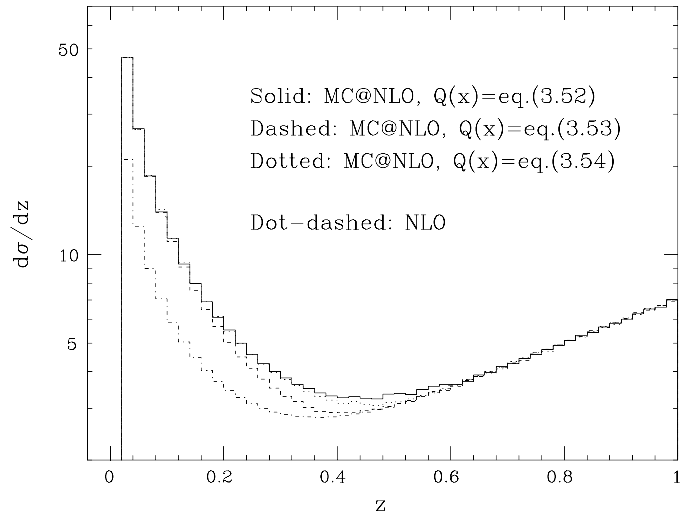
$$R(x) = B + x\left(1 + \frac{x}{2} + 20x^2\right)$$

Toy MC@NLO Observables









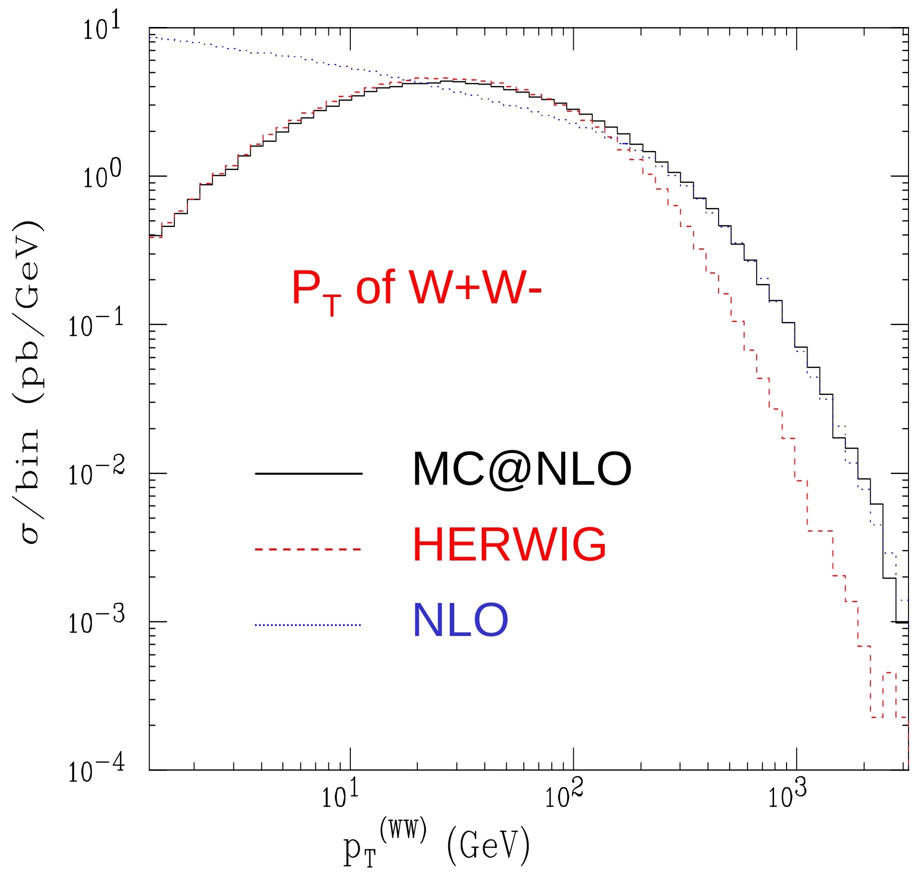
Real QCD

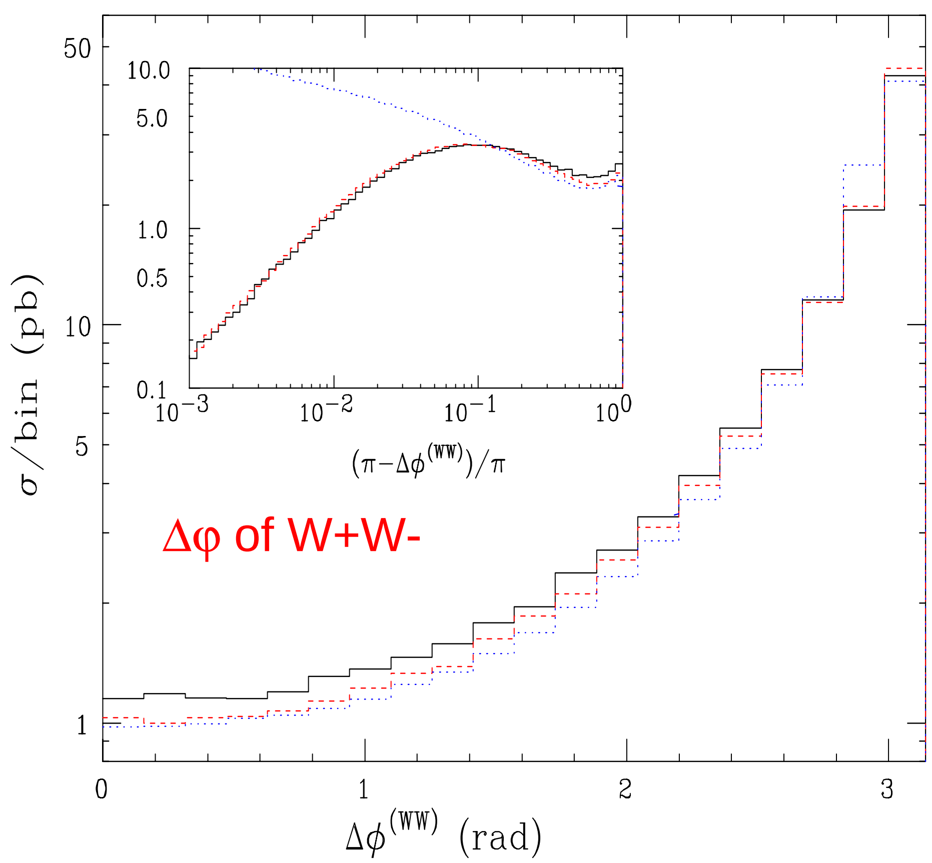
- For normal QCD the principle is the same we subtract the shower approximation to the real emission and add it to the virtual piece.
- This cancels the singularities and avoids double counting.
- It’s a lot more complicated.

Problems

- For each new process the shower approximation must be worked out, which is often complicated.
- While the general approach works for any shower it has to be worked out for a specific case.
- So for MC@NLO only works with the HERWIG shower algorithm.
- It could be worked out for PYTHIA or Herwig++ but this remains to be done.

W^+W^- Observables

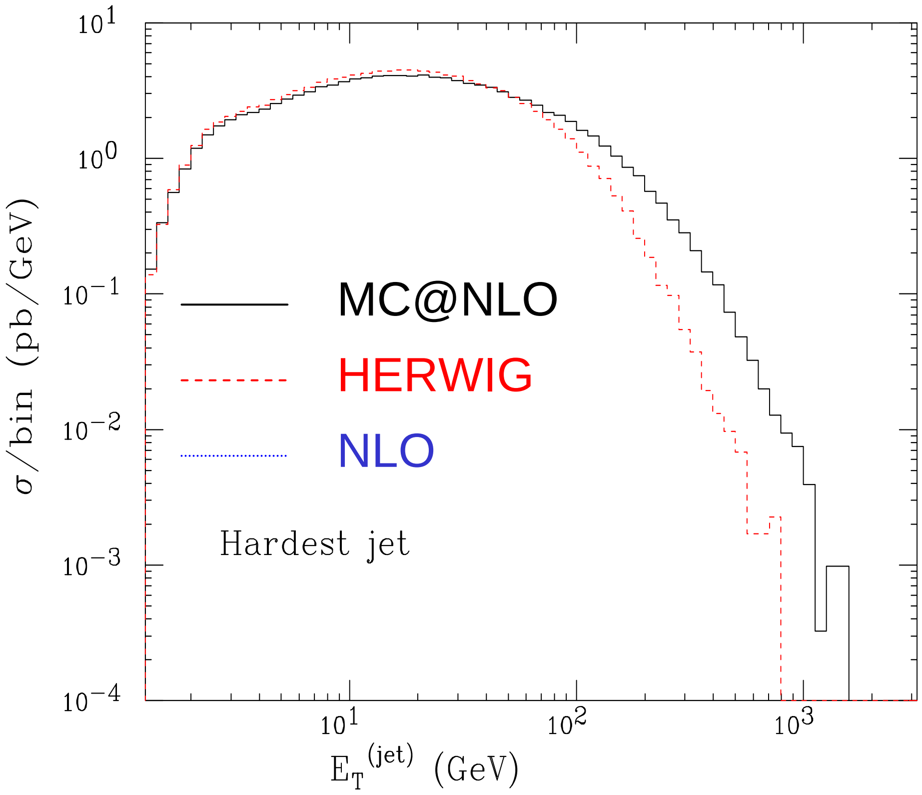


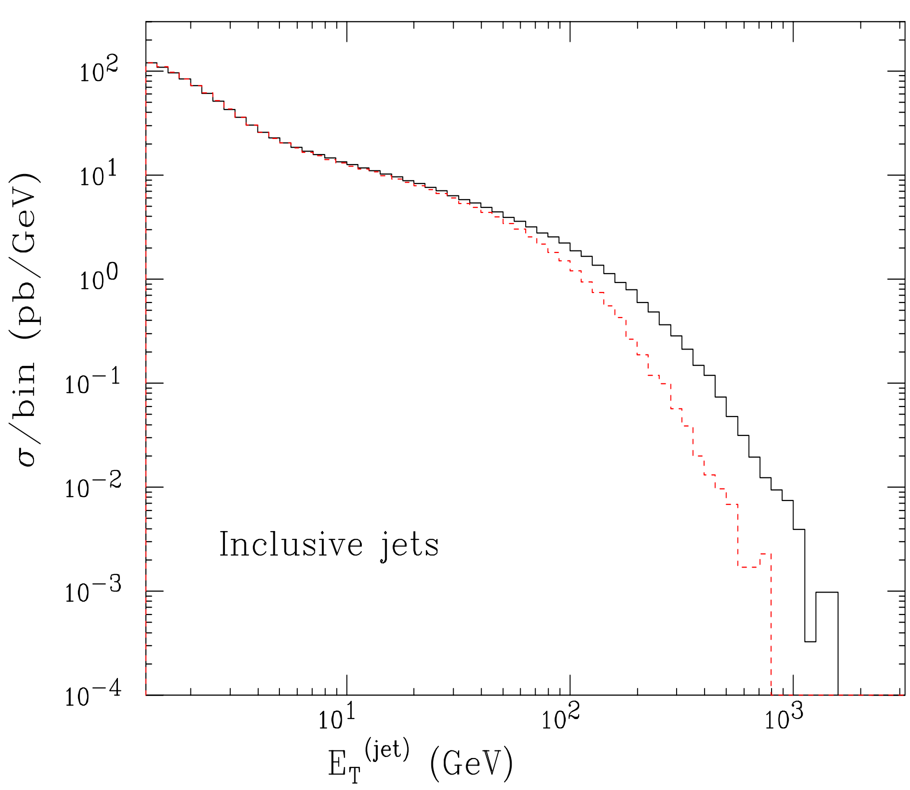


MC@NLO gives the correct high P_T result and soft resummation.

S. Frixione and B.R. Webber JHEP 0206(2002) 029, hep-ph/0204244, hep-ph/0309186

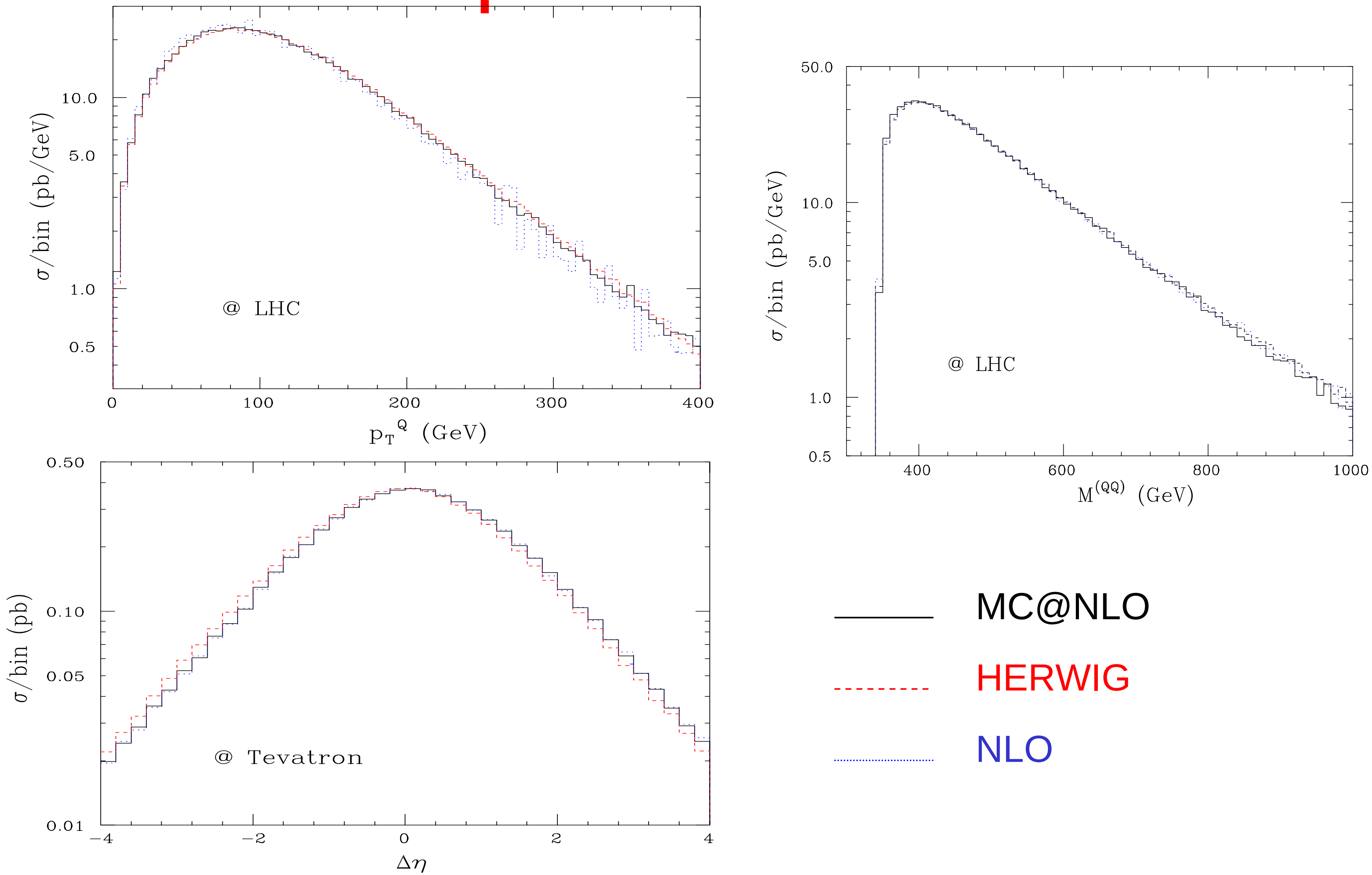
W^+W^- Jet Observables





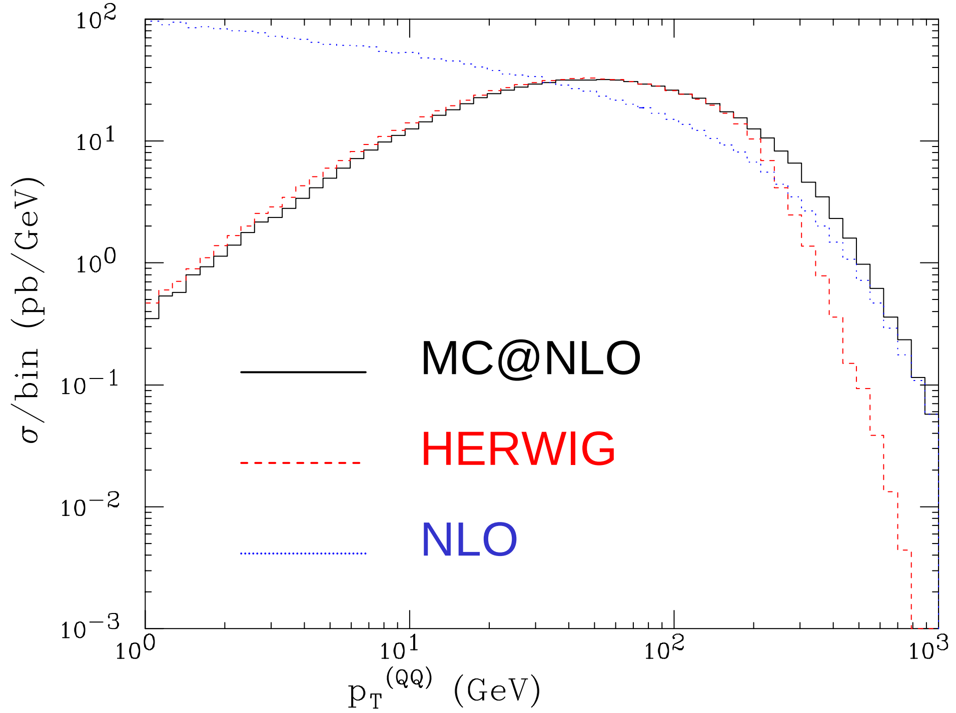
S. Frixione and B.R. Webber JHEP 0206(2002) 029, hep-ph/0204244, hep-ph/0309186

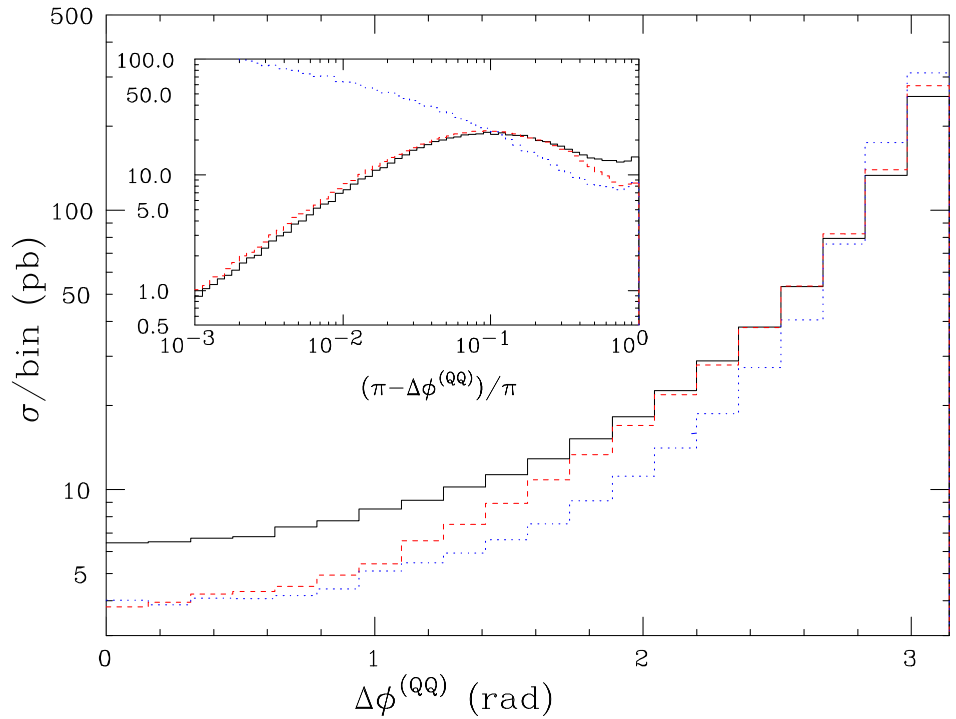
Top Production



S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.

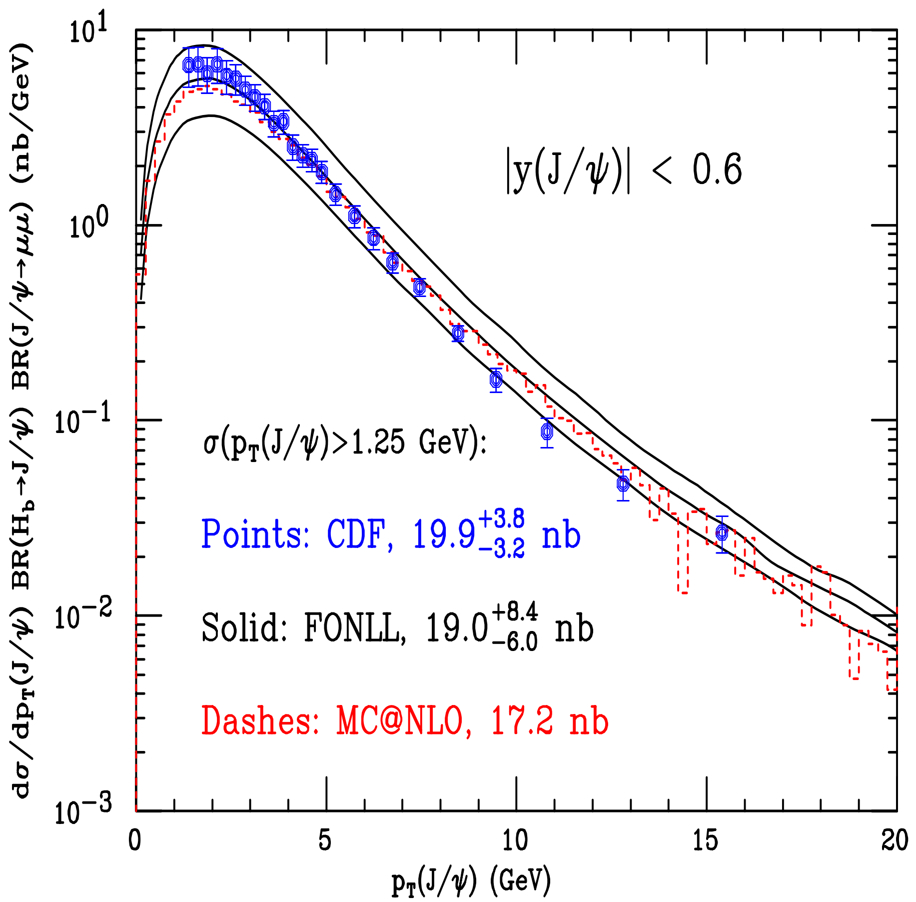
Top Production at the LHC

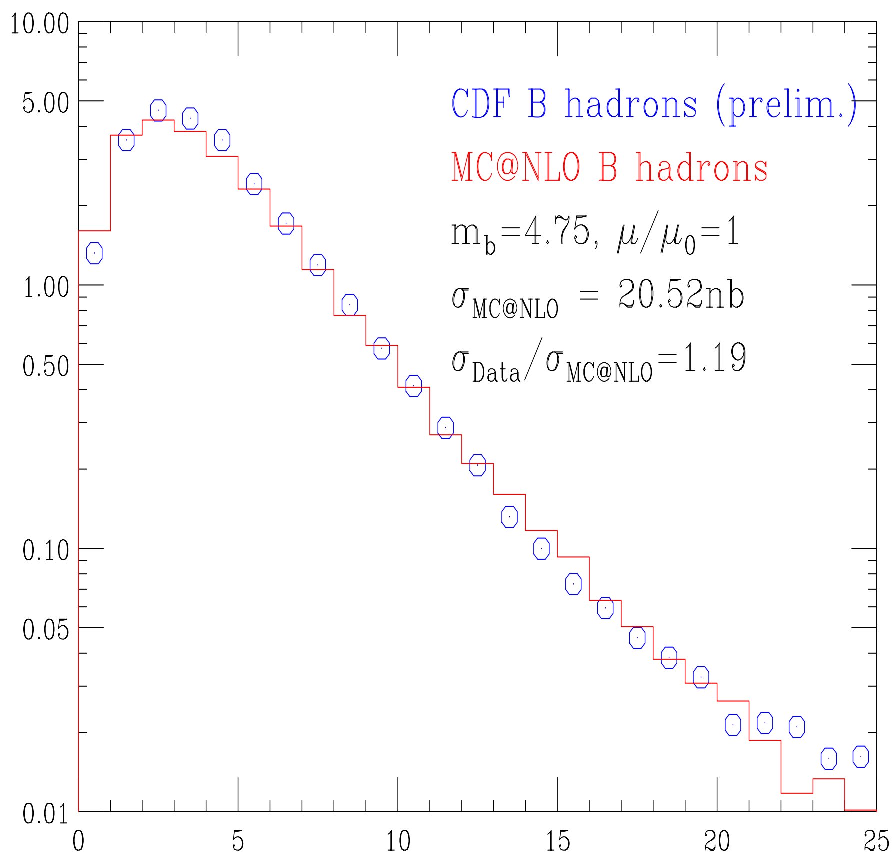




S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.

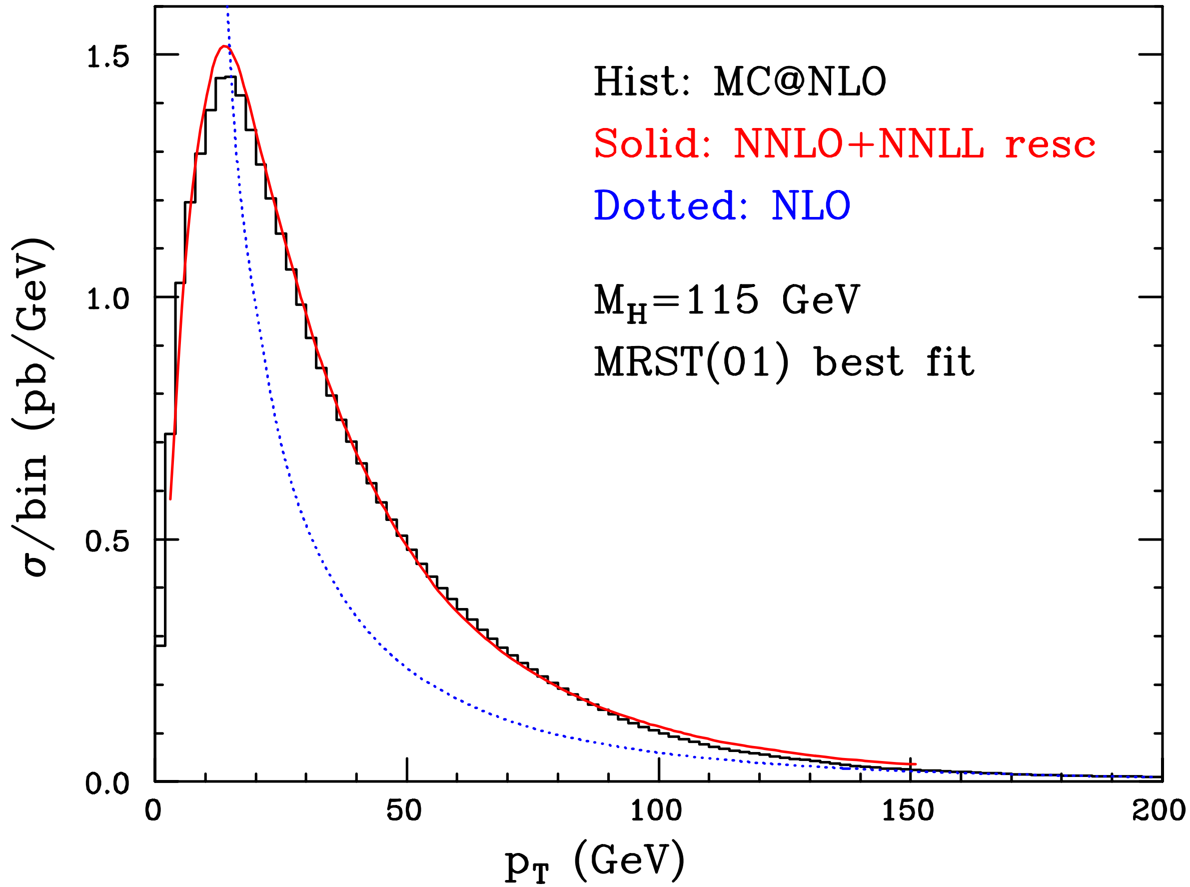
B Production at the Tevatron





S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.

Higgs Production at LHC



S. Frixione and B.R. Webber JHEP 0206(2002) 029, hep-ph/0204244, hep-ph/0309186

NLO Simulation

- So far MC@NLO is the only implementation of a NLO Monte Carlo simulation.
- Recently there have been some ideas by Paulo Nason JHEP 0411:040,2004 and recent results.
- In this approach there are no negative weights but more terms would be exponentiated beyond leading log.

Multi-Jet Leading Order

- While the NLO approach is good for one hard additional jet and the overall normalization it cannot be used to give many jets.
- Therefore to simulate these processes use matching at leading order to get many hard emissions correct.
- I will briefly review the general idea behind this approach and then show some results.

CKKW Procedure

- Catani, Krauss, Kuhn and Webber JHEP 0111:063,2001.
- In order to match the ME and PS we need to separate the phase space:
- One region contains the soft/collinear region and is filled by the PS;
- The other is filled by the matrix element.
- In these approaches the phase space is separated using in k_T-type jet algorithm.

Durham Jet Algorithm

- For all final-state particles compute the resolution variables

$$d_{kB} \approx E_k^2 \theta_{kB}^2 \approx k_{\perp kB}^2 \qquad \theta_{kB}^2 \to 0$$

$$d_{kl} \approx \min(E_k^2, E_l^2)\theta_{kl}^2 \approx k_{\perp kl}^2 \qquad \theta_{kl}^2 \to 0$$

- The smallest of these is selected. If d_{kl} is the smallest the two particles are merged. If d_{kB} is the smallest the particle is merged with the beam.
- This procedure is repeated until the minimum value is above some stopping parameter d_{cut} .
- The remaining particles and pseudo-particles are then the hard jets.

CKKW Procedure

- Radiation above a cut-off value of the jet measure is simulated by the matrix element and radiation below the cut-off by the parton shower.

1) Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{N} \sigma_k}$$

where σ_n is the *n*-jet matrix element evaluated at resolution d_{ini} using d_{ini} as the scale for the PDFs and α_S, *n* is the number of jets

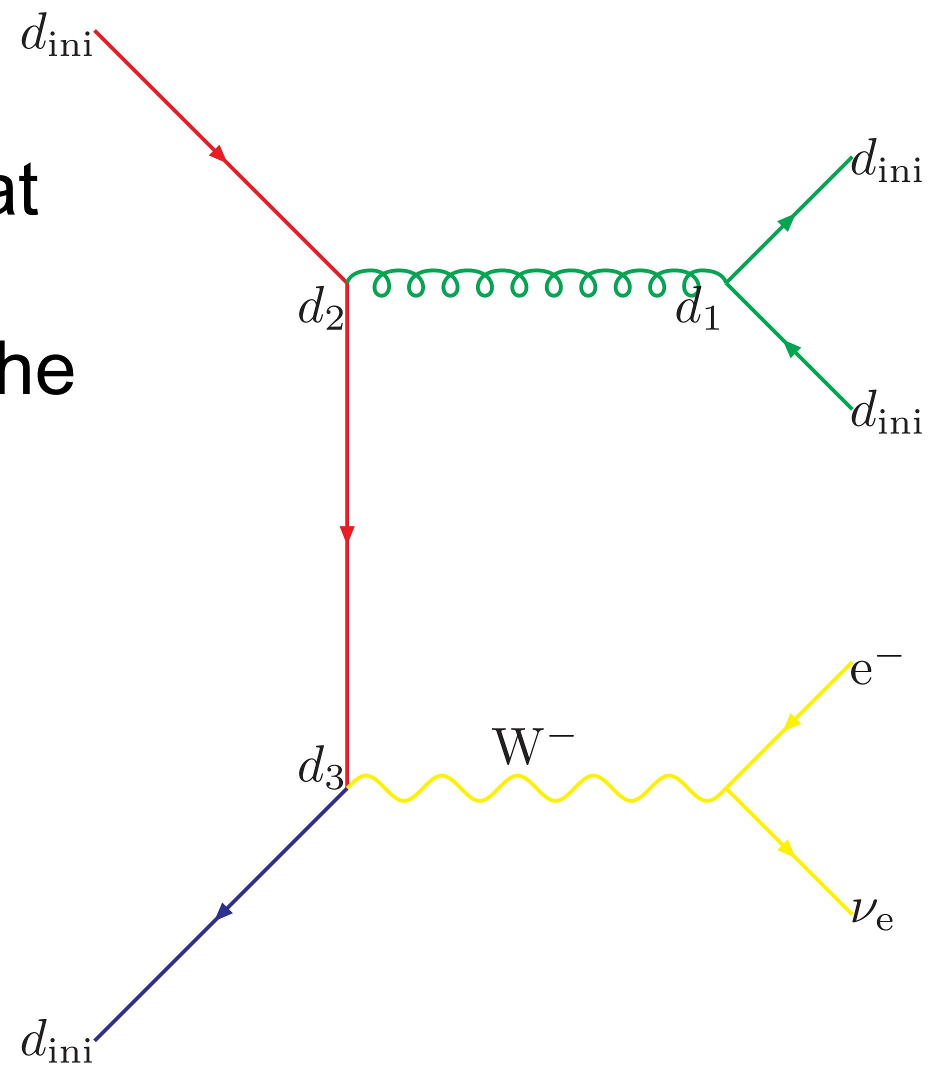
2) Distribute the jet momenta according the ME.

CKKW Procedure

3) Cluster the partons to determine the values at which 1,2,..*n*-jets are resolved. These give the nodal scales for a tree diagram.

4) Apply a coupling constant reweighting.

$$\frac{\alpha_S(d_1)\alpha_S(d_2)...\alpha_S(d_3)}{\alpha_S(d_{\text{ini}})^n} \leq 1$$

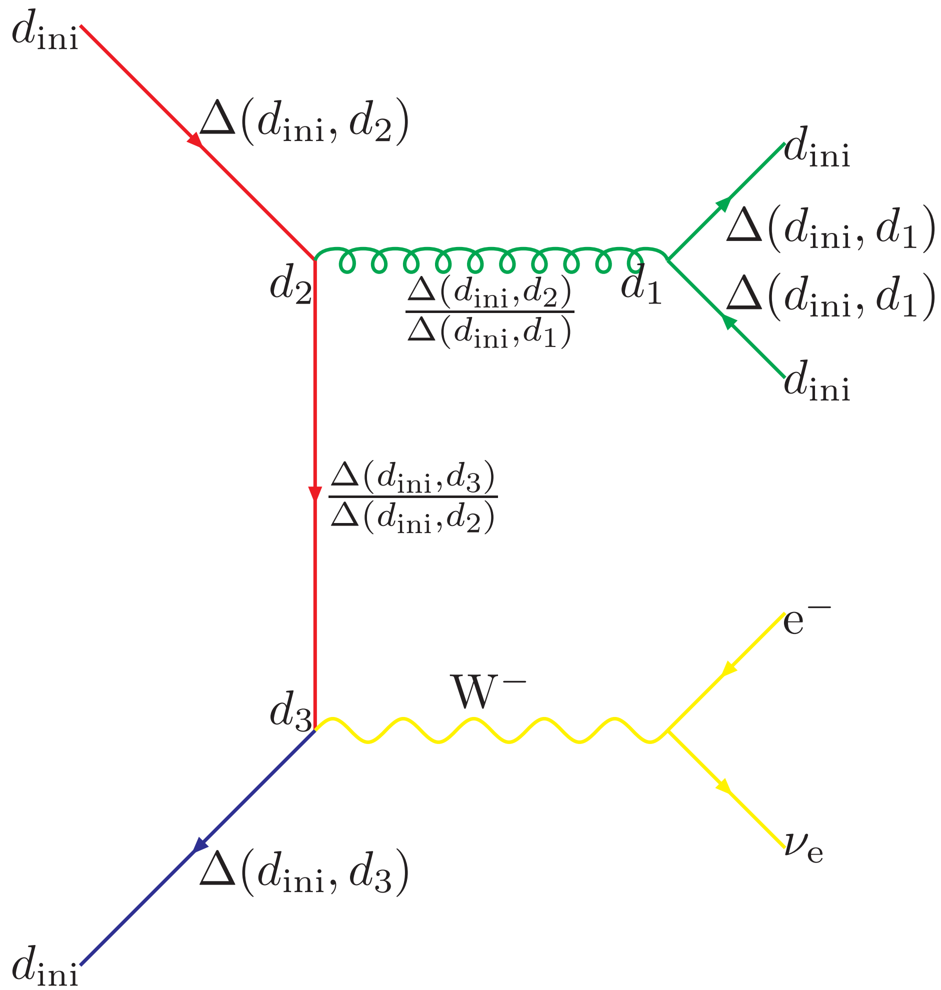


CKKW Procedure

5) Reweight the lines by a Sudakov factor

$$\frac{\Delta(d_{\text{ini}}, d_j)}{\Delta(d_{\text{ini}}, d_k)}$$

6) Accept the configuration if the product of the α_S and Sudakov weight is less than $R \in [0,1]$ otherwise return to step 1.



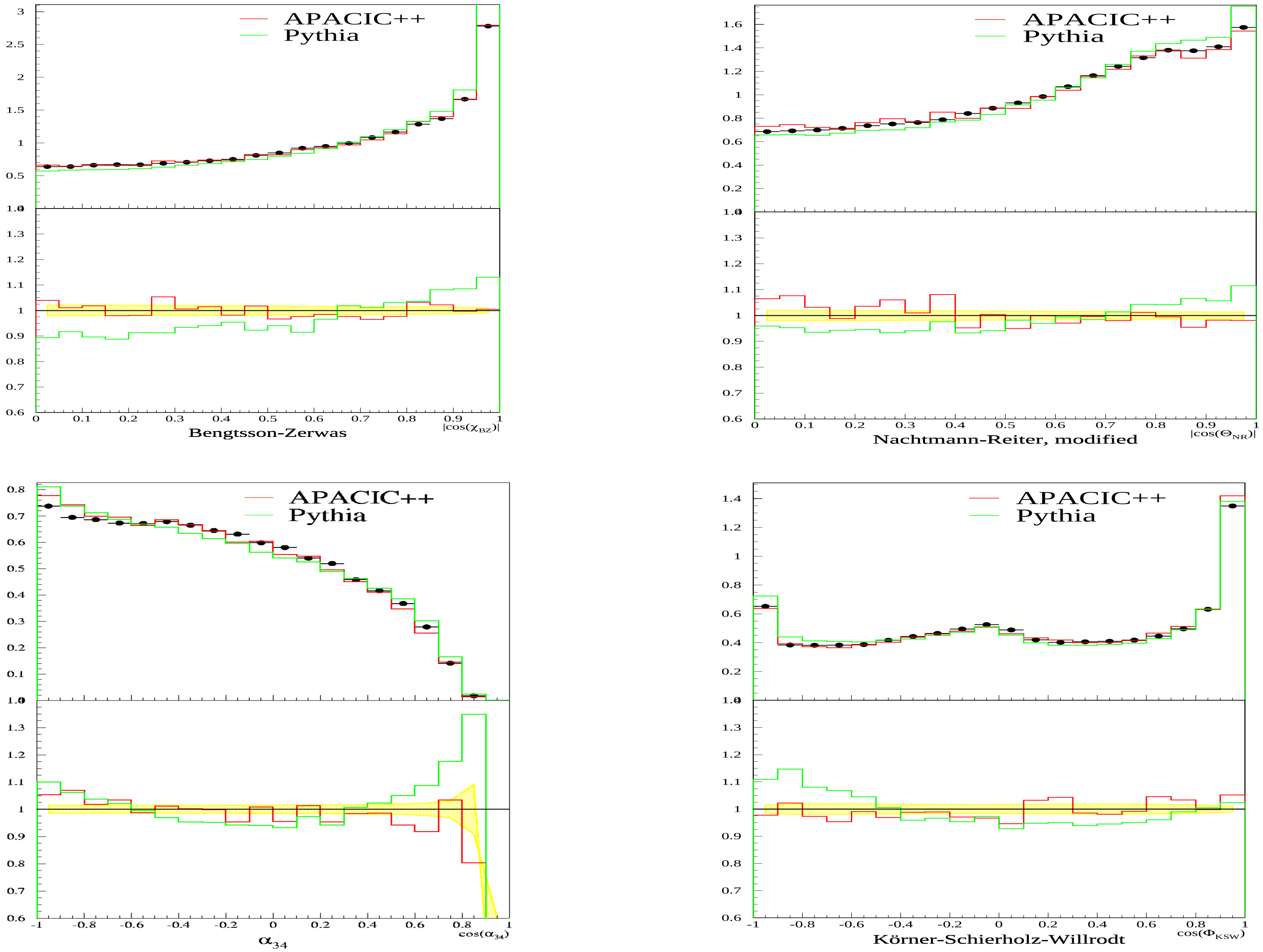
CKKW Procedure

7) Generate the parton shower from the event starting the evolution of each parton at the scale at which it was created and vetoing emission above the scale d_{ini} .

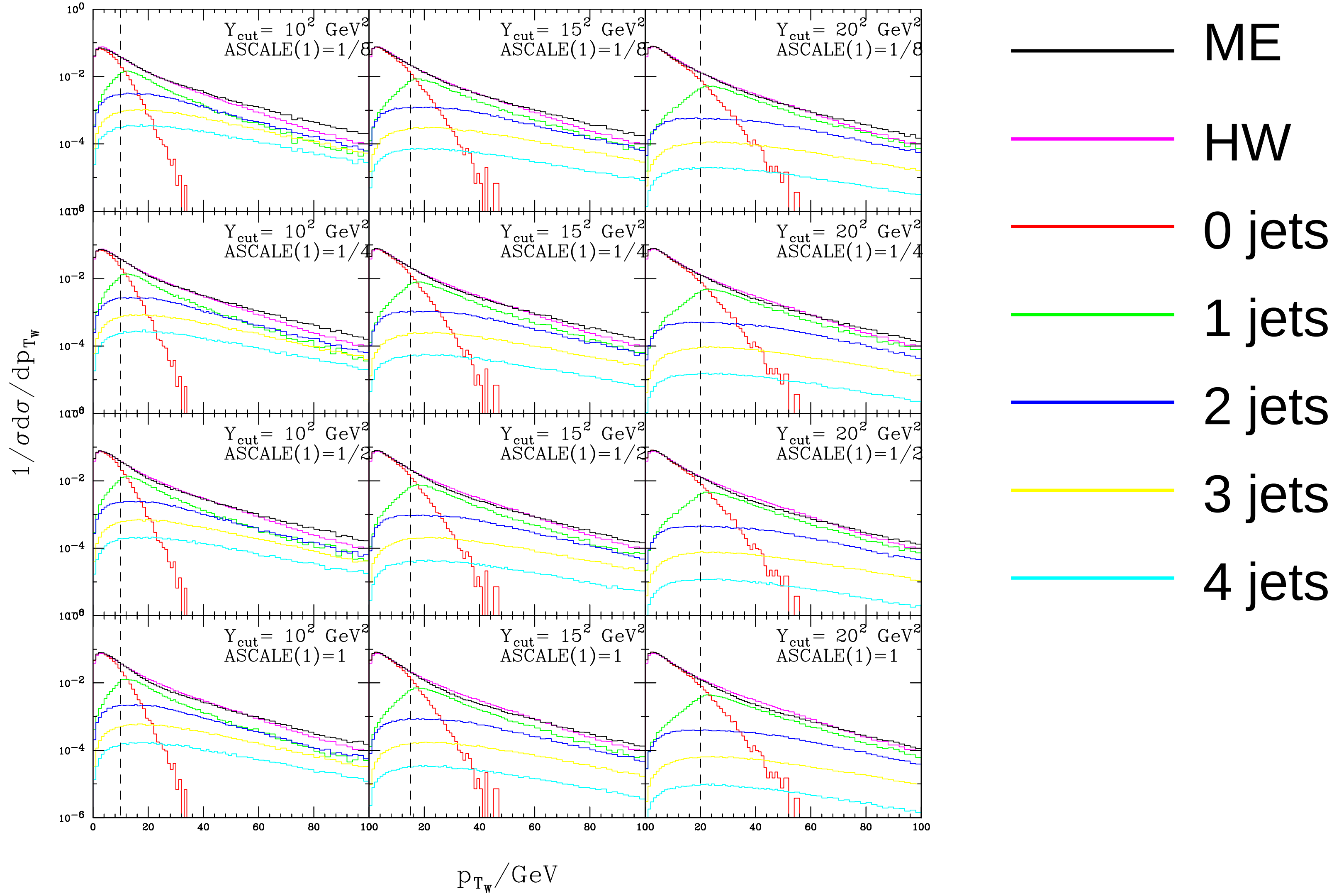
CKKW Procedure

- Although this procedure ensures smooth matching at the NLL log level are still choices to be made:
  - Exact definition of the Sudakov form factors.
  - Scales in the strong coupling and α_S.
  - Treatment of the highest Multiplicity matrix element.
  - Choice of the k_T algorithm.
- In practice the problem is understanding what the shower is doing and treating the matrix element in the same way.

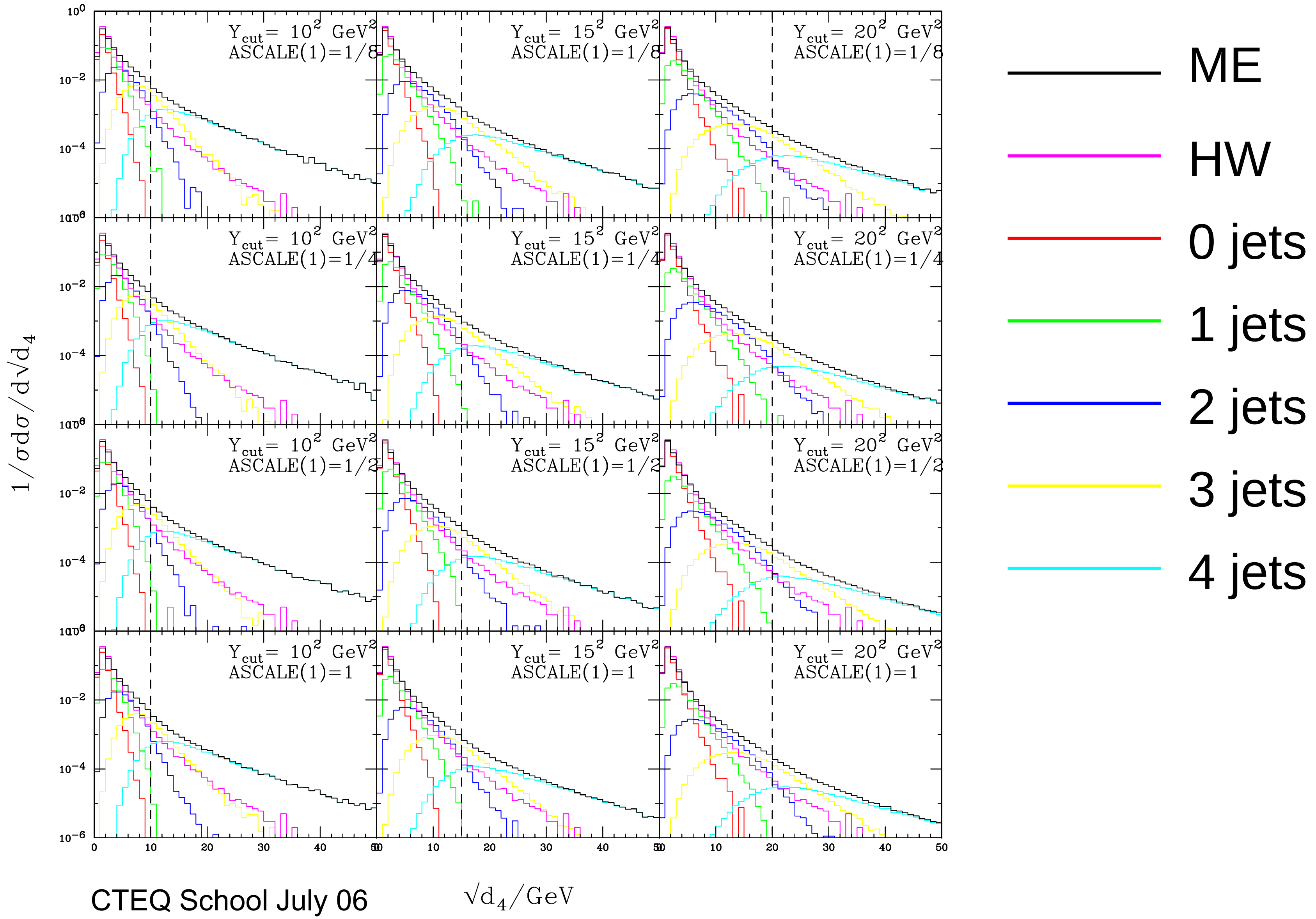
e^+e^- Results from SHERPA



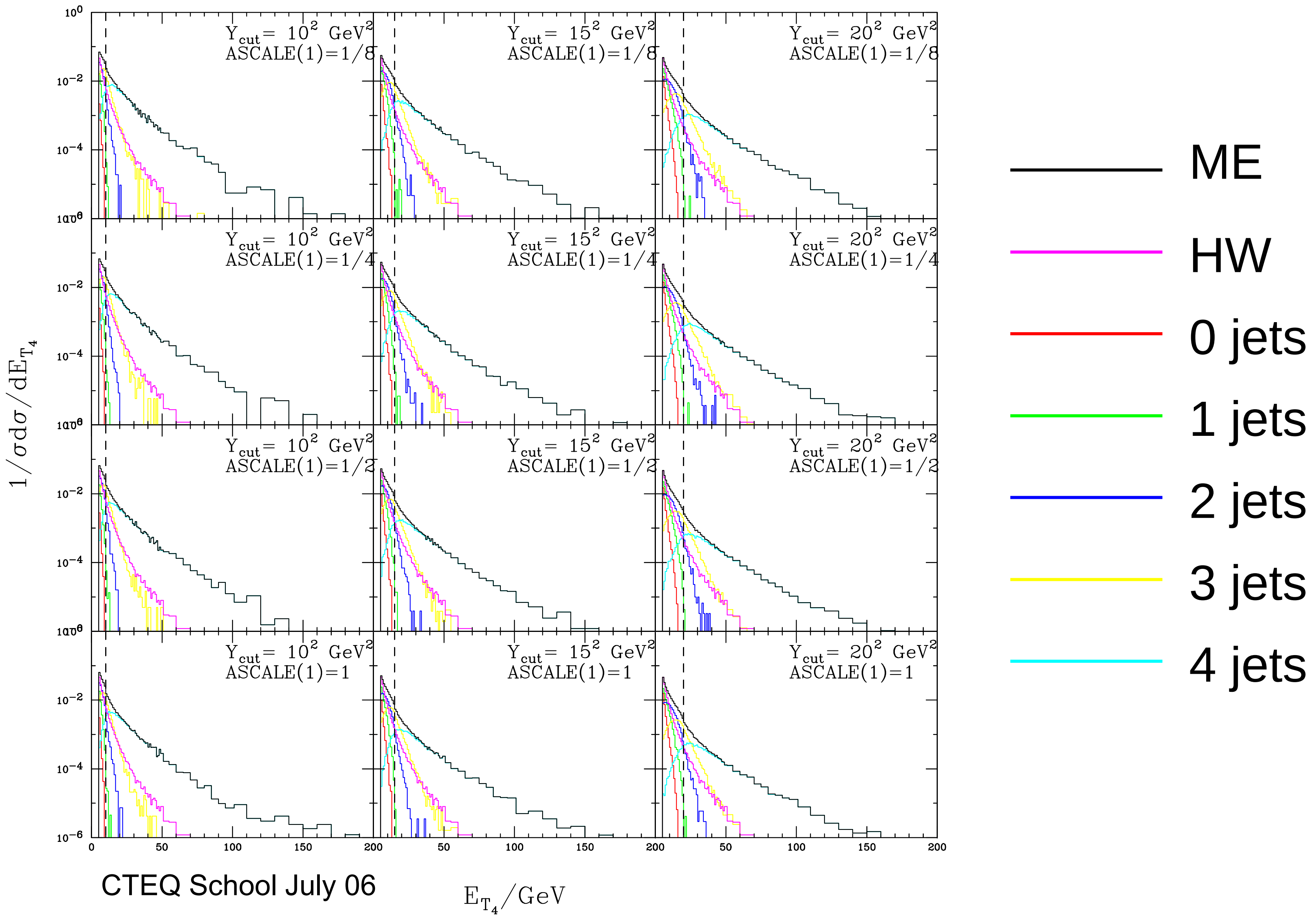
p_T of the W at the Tevatron from HERWIG



Tevatron p_T of the 4th jet from HERWIG



LHC E_T of the 4th jet from HERWIG



What Should I use?

- Hopefully this lecture will help you decide which of the many different tools is most suitable for a given analysis.
  - Only soft jets relative to hard scale MC
  - Only one hard jet MC@NLO or old style ME correction
  - Many hard jets CKKW.
- The most important thing is to think first before running the simulation.

Summary

- In this afternoon's lecture we have looked at
  - The basic parton shower algorithm
  - Colour Coherence
  - Backward Evolution
  - Next-to-leading Order simulations
  - Matrix Element matching
- On Thursday we will go on and look at the non-perturbative parts of the simulation.