Durham University

Recent Advances in QCD Event Generators

Peter Richardson

IPPP, Durham University

Introduction

- Monte Carlo event generators are essential for experimental particle physics.
- They are used for:
  - Comparison of experimental results with theoretical predictions;
  - Studies for future experiments.
- Often these programs are ignored by theorists and treated as black boxes by experimentalists.
- It is important to understand the assumptions and approximations involved in these simulations.

Introduction

- Experimental physicists need to be able to answer the following questions
  - Is the effect I'm seeing due to different models, or approximations, or is it a bug?
  - Am I measuring a fundamental quantity or merely a parameter of the simulation code?
- Theorists need to understand enough to be able ask
  - Have the experimentalists misused the Monte Carlo giving incorrect results?

Introduction

- For both the Tevatron and LHC we are interested in final states with large numbers of jets and leptons. For example
  - Top production
  - SUSY
- The backgrounds to these processes generally come from multiple QCD radiation giving jets.
- These QCD process are of course interesting in their own right.

Introduction

- In this talk I will start by describing the ideas behind Monte Carlo simulations.
- Recently there has been a lot of progress in two related areas:
  - Next-to-leading order simulation;
  - Matching leading order matrix elements;

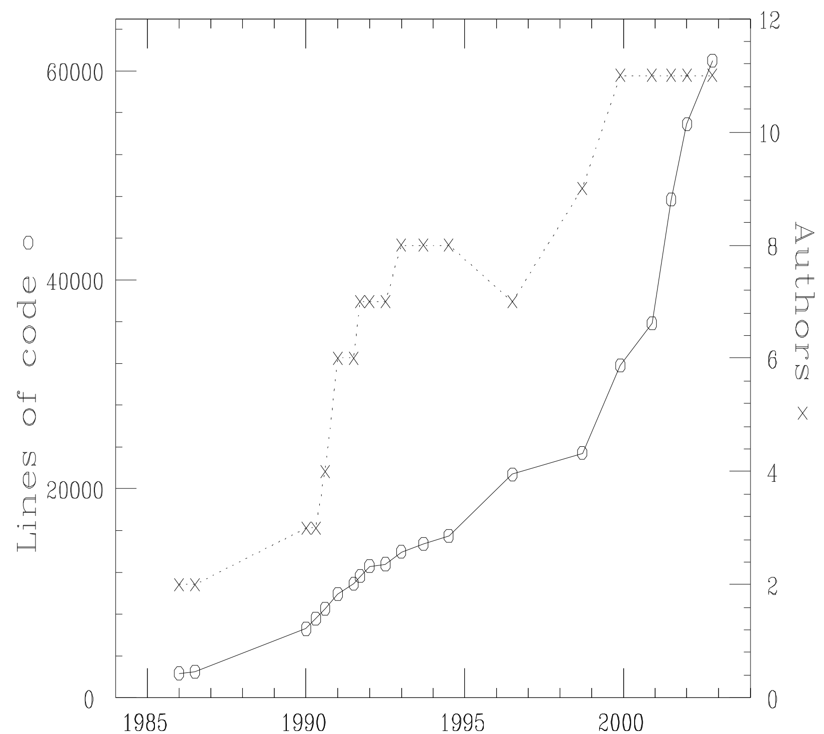
  which are aimed at improving the treatment of hard radiation.
- I will go on to discuss these and where they are of use.

Monte Carlo Event Generators

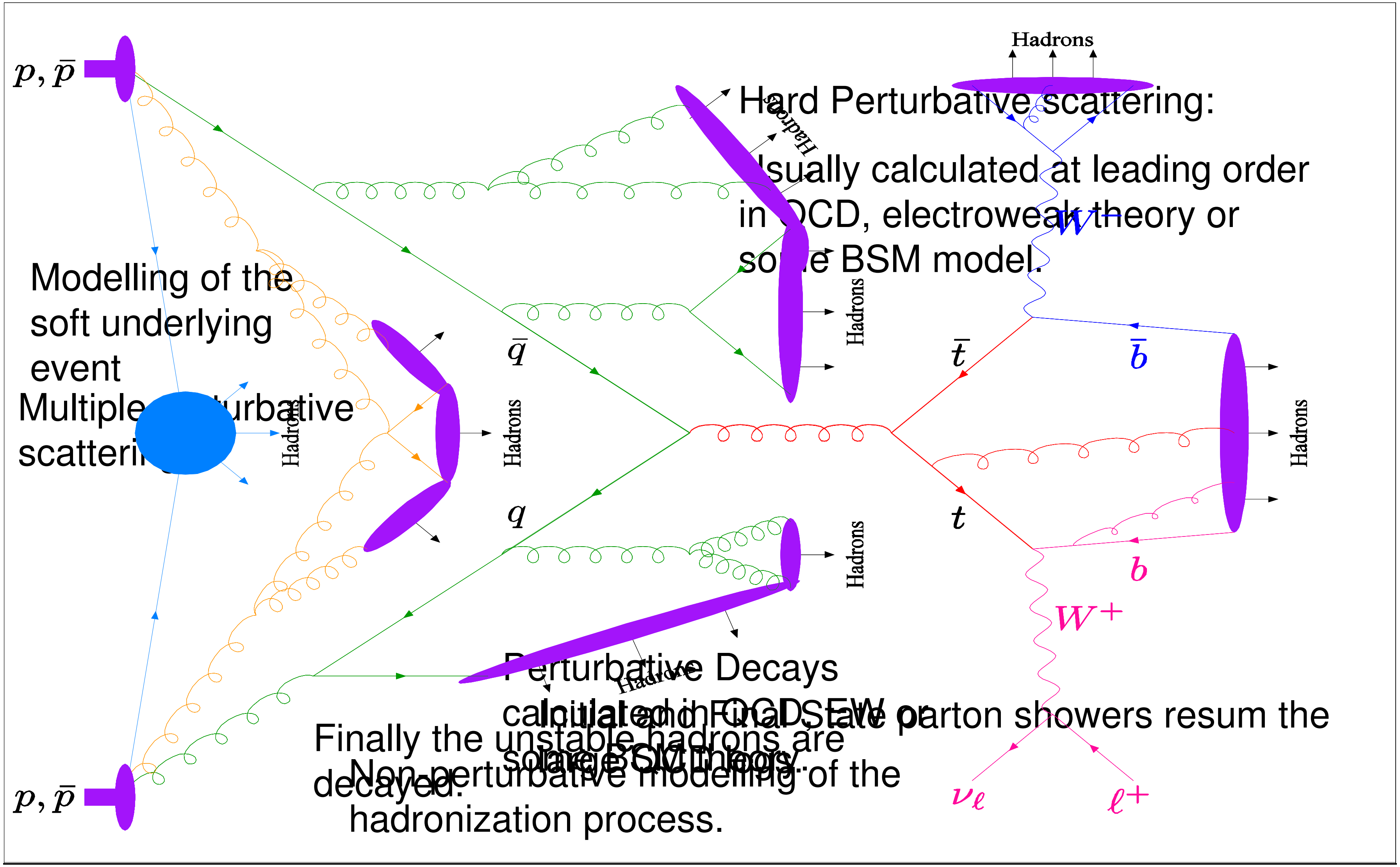
- There are a number of different Monte Carlo event generators in common use
  - ISAJET
  - PYTHIA
  - HERWIG
  - SHERPA
- They all split the event generation up into the same pieces.
- The models and approximations they use for the different pieces are of course different.

C++ Generators

- Most of these programs are written in Fortran 77, (some are even older.)
- There are ongoing projects to rewrite HERWIG and PYTHIA in C++.
- Some of the newer projects, SHERPA, are also in C++.



A Monte Carlo Event



Monte Carlo Event Generators

- All the event generators split the simulation up into the same phases:
  - Hard Process;
  - Parton Shower;
  - Secondary Decays;
  - Multiple Scattering/Soft Underlying Event;
  - Hadron Decays.
- I will breifly discuss the different models and approximations in the different programs.
- I will try and give a fair and objective comparision, but ear in mind that I'm one of the authors of HERWIG.

QCD Radiation

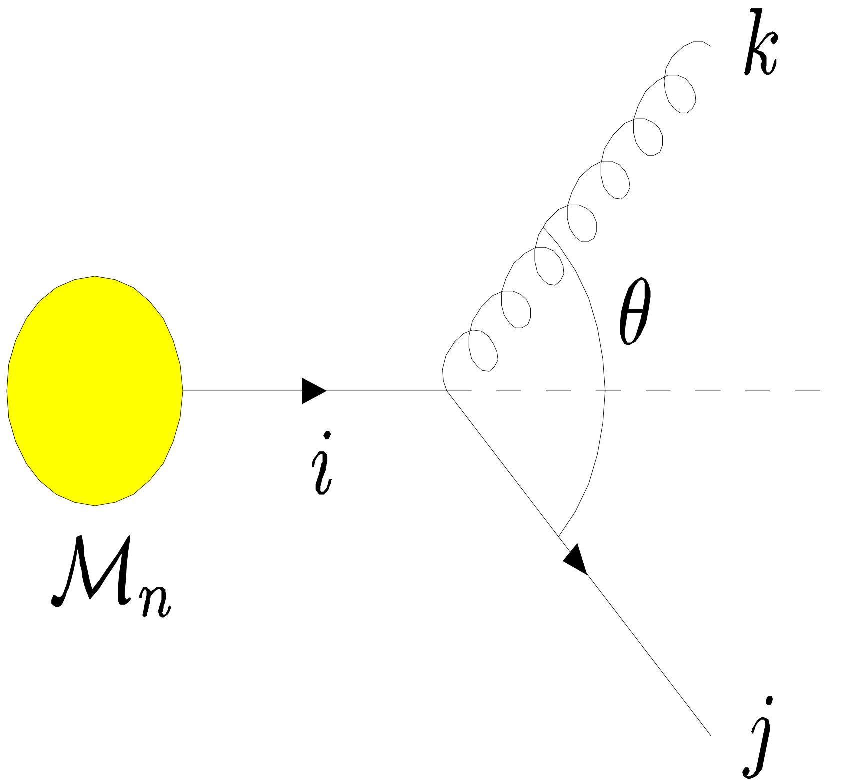
- It is impossible to calculate and integrate the matrix elements for large numbers of partons.
- Instead we treat the regions where the emission of QCD radiation is enhanced.
- This is soft and collinear radiation.
- The different generators differ in the sophistication of their simulation of this.

Collinear Singularities

- In the collinear limit the cross section for a process factorizes

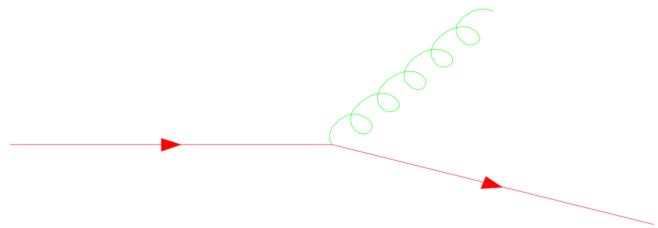
$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z)$$

  - $P_{ji}(z)$ is the DGLAP splitting function
- This expression is singular as $\theta \to 0$.
- What is a parton? (or what is the difference between a collinear pair and a parton)



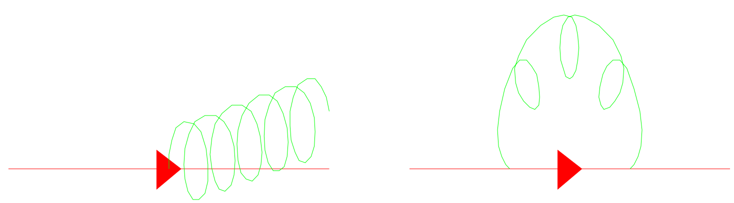
Collinear Singularities

- Introduce a resolution criterion, e.g. $k_T > Q_0$
- Combine the virtual corrections and unresolvable emission



Resolvable Emission

Finite



Unresolvable Emission

Finite

•Unitarity: Unresolved + Resolved =1

Monte Carlo Procedure

- Using this approach we can exponentiate the real emission piece.

$$\text{Unresolved} = 1 - \text{Resolved} \;¿\; 1 - \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z)$$

$$= \exp\left(- \int_{q^2}^{Q^2} \frac{dk^2}{k^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z) \right)$$

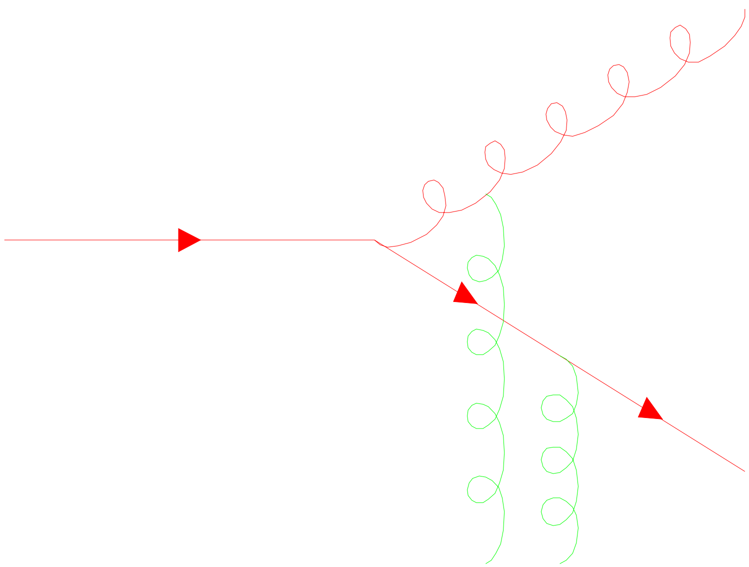
- This gives the Sudakov form factor which is the probability of evolving between two scales and emitting no resolvable radiation.
- More strictly it is the probability of evolving from a high scale to the cut-off with no resolvable emission.

Monte Carlo Procedure

- The key difference between the different Monte Carlo simulations is in the choice of the evolution variable.
- Evolution Scale
  - Virtuality, q^2
  - Transverse Momentum, k_T.
  - Angle, θ.
  - ….
- Energy fraction, z
  - Energy fraction
  - Light-cone momentum fraction
  - ….
- All are the same in the collinear limit.

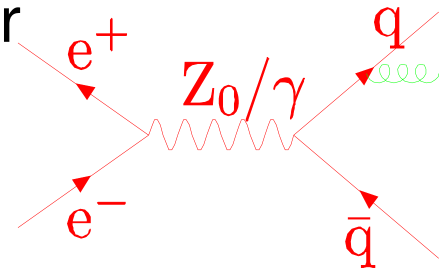
Soft Emission

- However we have only considered collinear emission. What about soft emission?
- In the soft limit the matrix element factorizes but at the amplitude level.
- Soft gluons come from all over the event.
- There is quantum interference between them.
- Does this spoil the parton shower picture?

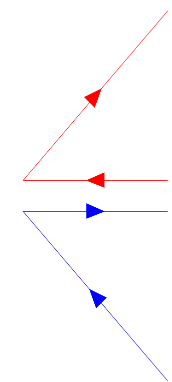


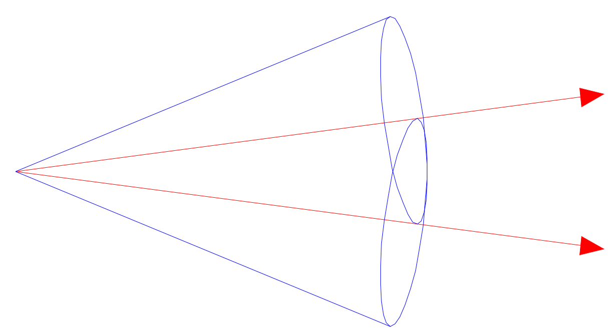
Angular Ordering

- There is a remarkable result that if we take the large number of colours limit much of the interference is destructive.
- In particular if we consider the colour flow in an event.
- QCD radiation only occurs in a cone up to the direction of the colour partner.
- The best choice of evolution variable is therefore an angular one.



Colour Flow





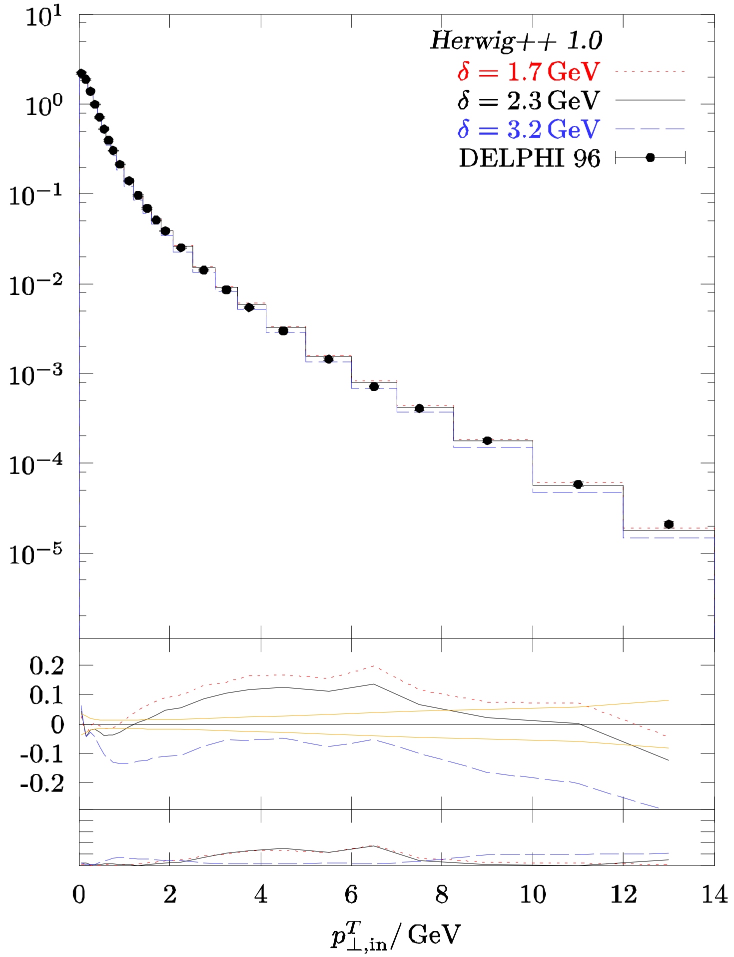
Emitter

Colour Partner

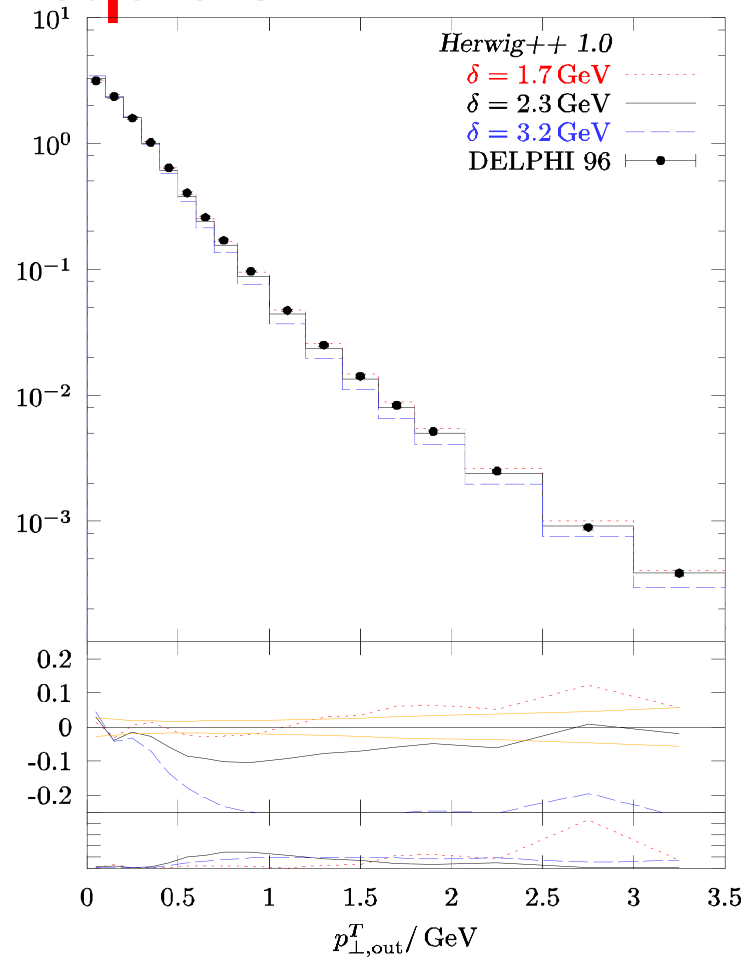
Parton Shower

- ISAJET uses the original parton shower algorithm which only resums collinear logarithms.
- HERWIG uses the angular ordered parton shower algorithm which resums both soft and collinear singularities.
- PYTHIA uses the collinear algorithm with an angular veto to try to reproduce the effect of the angular ordered shower.
- SHERPA uses the PYTHIA algorithm.

Event Shapes



Momentum transverse to the thrust axis in the event plane.



Momentum transverse to the thrust axis out of the event plane.

Parton Shower

- The collinear algorithm implemented in ISAJET does not give good agreement with data.
- In general event generators which include angular ordering, colour coherence, give the best agreement with data.

Dipole Showers

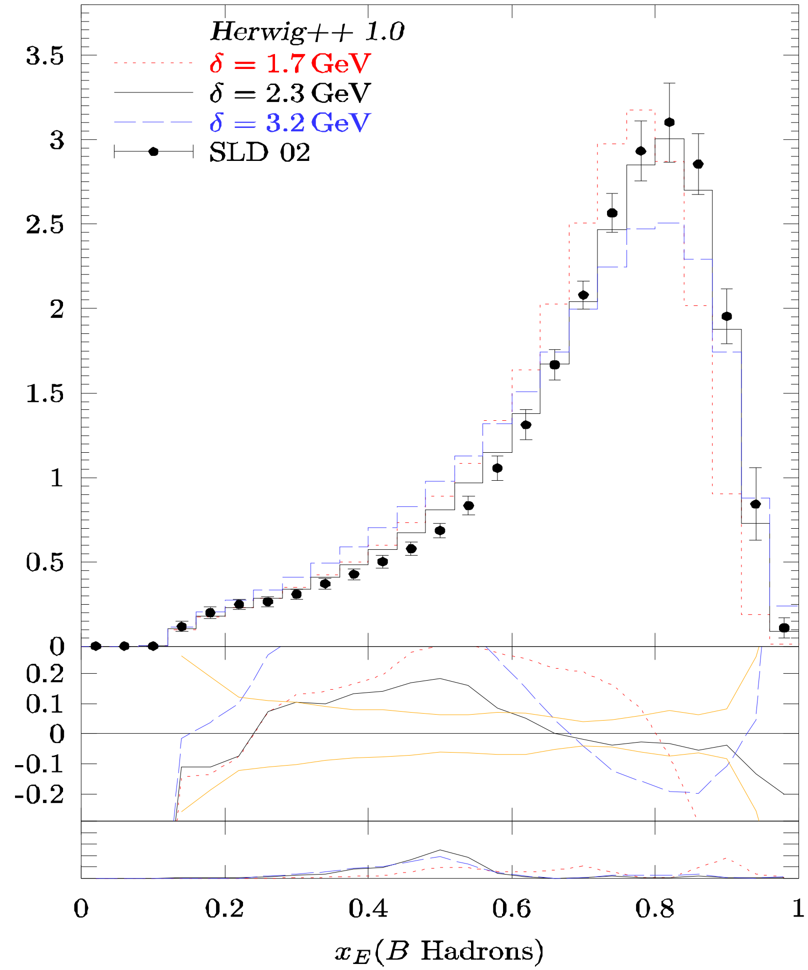
- The best agreement with the LEP data was obtained using ARIADNE which is based on the dipole approach.
- This is based on $2 \rightarrow 3$ splittings rather than $1 \rightarrow 2$ which makes it easier to conserve momentum.
- The soft and collinear are included in a consistent way.
- The initial state shower is more difficult in this approach though.

Parton Showers

- Much of the recent work on parton showers has been on simulating hard radiation which I will talk about later.
- There are however some other improvements.
- The major new ideas are
  - An improved coherent parton shower using massive splitting functions.
  - A transverse momentum ordered shower.

Herwig++ Shower

- Gieseke et. al.,
  JHEP 0402:005,2004
  JHEP 0312:045,2003.
- Gives an improved treatment of radiation from heavy particles, for example the b quark fragmentation function.
- This allows some radiation inside the 'dead-cone.'



P_T ordered shower

- T. Sjostrand hep-ph/0401061.
- Order the shower in transverse momentum rather than angle or virtuality.
- Still remains to shown that the coherence properties are correct.
- Can be used in new ideas in multiple scattering and the underlying event.
- T. Sjostrand, P.Z. Skands, hep-ph/0408302.

Hadronization

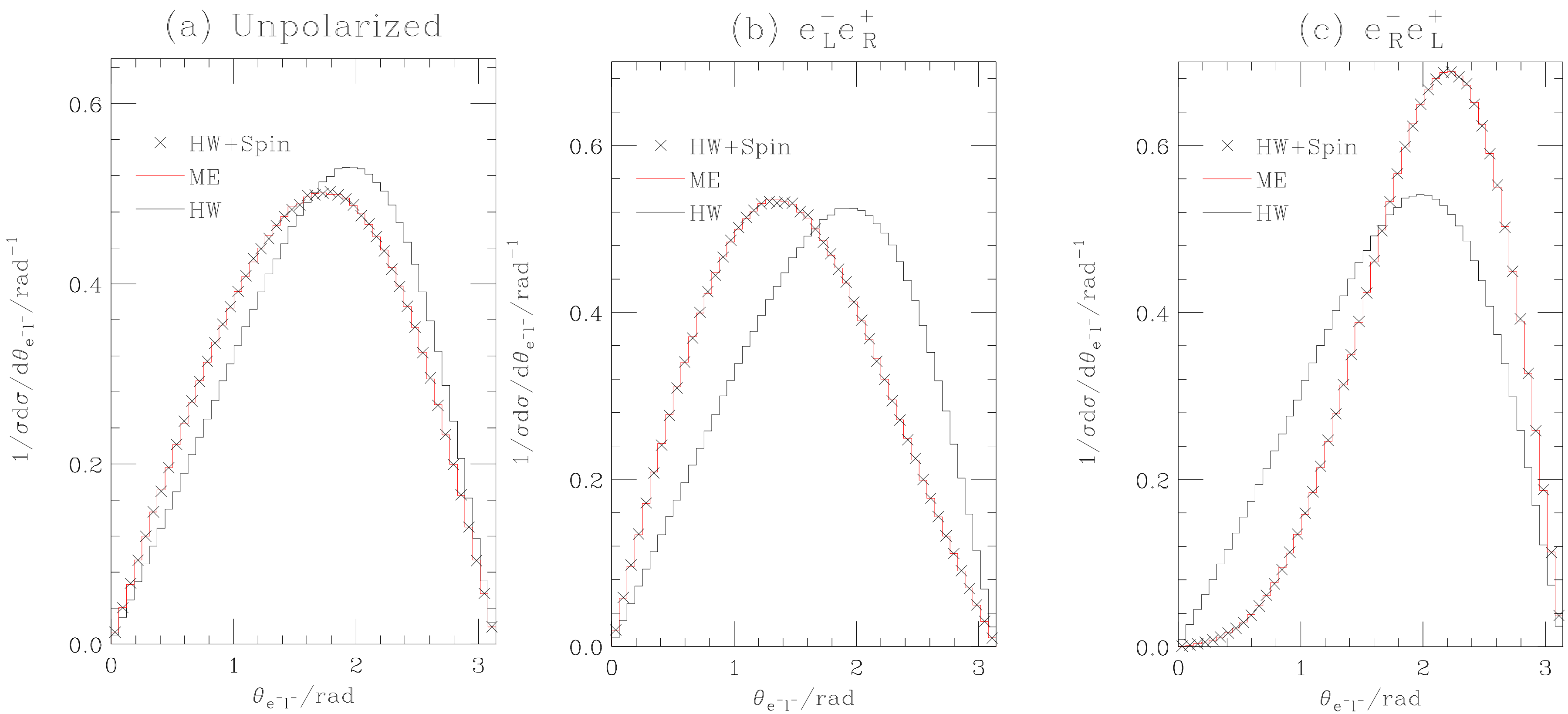
- As the hadronization is less important for what I will say later and there's been less progress I will only briefly mention the different models.
- ISAJET uses the original independent fragmentation model
- PYTHIA uses the Lund string model.
- HERWIG uses the cluster hadronization model.
- ARIADNE and SHERPA use the Lund model from PYTHIA.
- The independent fragmentation model cannot fit the LEP data.
- The cluster model gives good agreement with LEP data on event shapes but doesn't fit the identified particle spectrum as well.
- The Lund string model gives the best agreement with data.

Signal Simulation

- In general we have become very good at simulating signals, be that top quark production, SUSY or other BSM physics.
- In many cases the simulations, particularly in HERWIG, the simulation is very detailed including correlation effects.
- This should be good enough for top and is certainly good enough for things that haven't been seen yet.

Signal Simulation

Angle between the lepton in top decay and the beam for top pair production at a 500 GeV linear collider.



Hard Jet Radiation

- I’ve tried to show you that the parton shower is designed to simulate soft and collinear radiation.
- While this is the bulk of the emission we are often interested in the radiation of a hard jet.
- This is not something the parton shower should be able to do, although it often does better than we except.
- If you are looking at hard radiation HERWIG and PYTHIA will often get it wrong.

Hard Jet Radiation

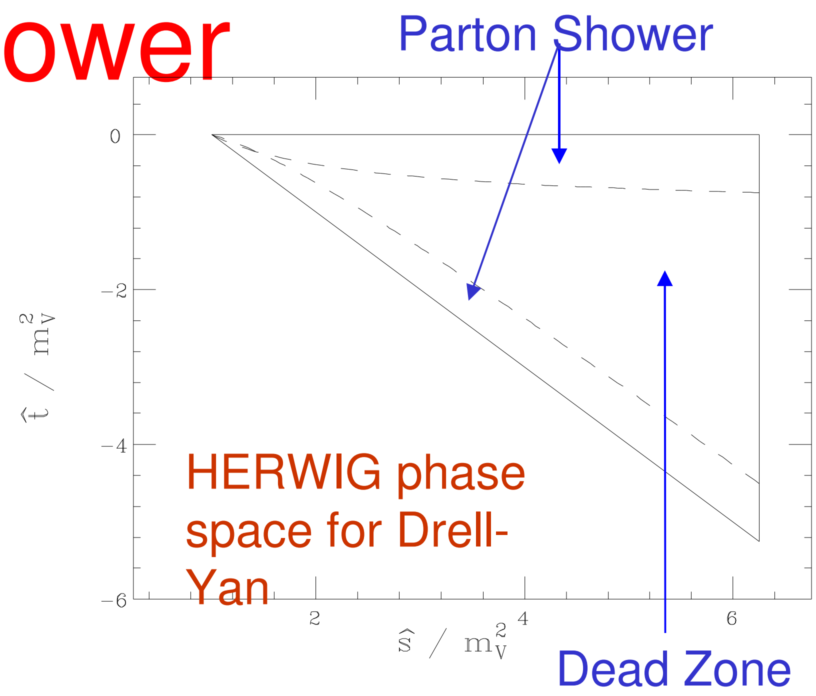
- Given this obvious failing of the approximations this is an obvious area to make improvements in the shower and has a long history.
- You will often here this called
  - Matrix Element matching.
  - Matrix Element corrections.
  - Merging matrix elements and parton shower
  - MC@NLO
- I will discuss all of these and where the different ideas are useful.

Hard Jet Radiation: General Idea

- Parton Shower (PS) simulations use the soft/collinear approximation:
  - Good for simulating the internal structure of a jet;
  - Can't produce high p_T jets.
- Matrix Elements (ME) compute the exact result at fixed order:
  - Good for simulating a few high p_T jets;
  - Can't give the structure of a jet.
- We want to use both in a consistent way, i.e.
  - ME gives hard emission
  - PS gives soft/collinear emission
  - Smooth matching between the two.
  - No double counting of radiation.

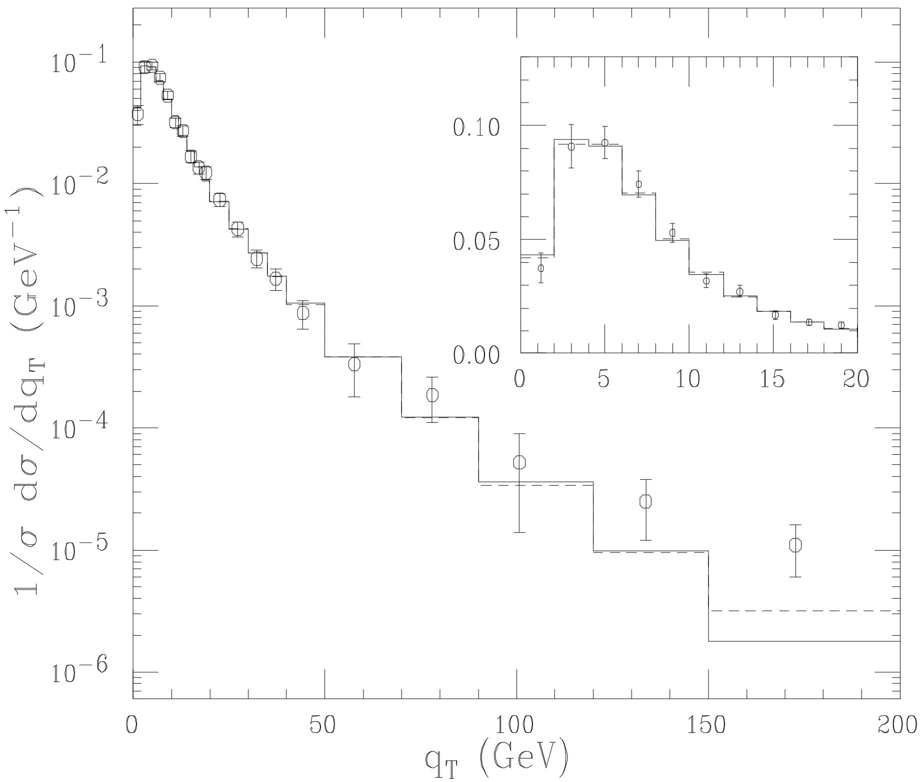
Matching Matrix Elements and Parton Shower

- The oldest approaches are usually called matching matrix elements and parton showers or the matrix element correction.
- Slightly different for HERWIG and PYTHIA.
- In HERWIG
  - Use the leading order matrix element to fill the dead zone.
  - Correct the parton shower to get the leading order matrix element in the already filled region.

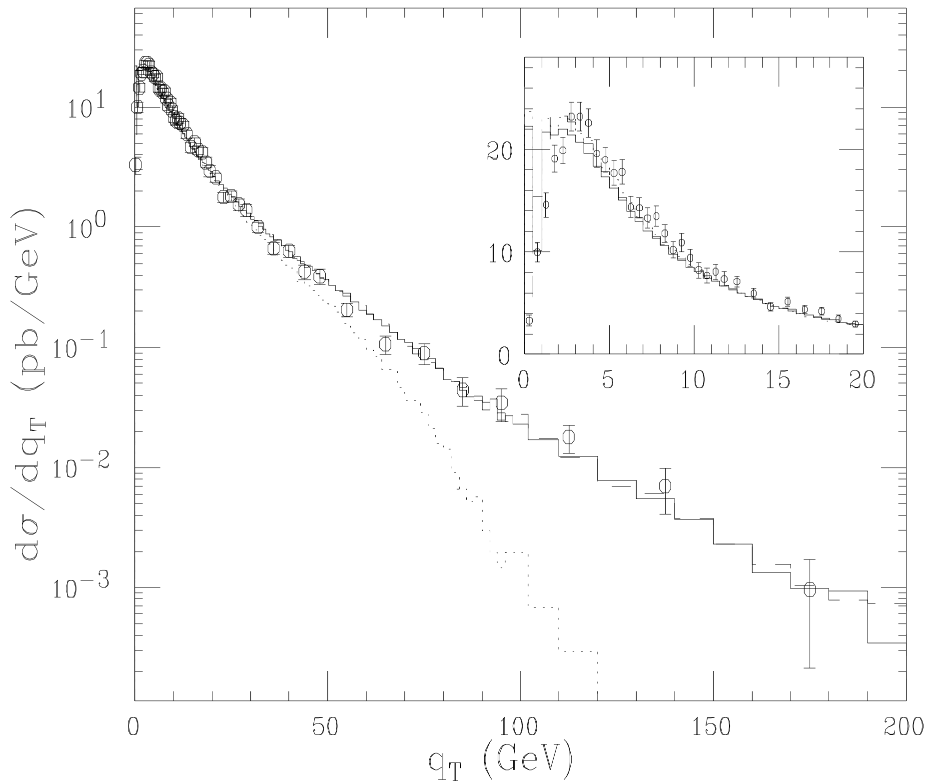


- PYTHIA fills the full phase space so only the second step is needed.

Matrix Element Corrections



W q_T distribution from D0



Z q_T distribution from CDF

G. Corcella and M. Seymour, Nucl.Phys.B565:227-244,2000.

Matrix Element Corrections

- There was a lot of work for both HERWIG and PYTHIA and the corrections for
  - e^+e^- to hadrons
  - DIS
  - Drell-Yan
  - Top Decay
  - Higgs Production
- There are problems with this
  - Only the hardest emission was correctly described
  - The leading order normalization was retained.

Recent Progress

- In the last few years there has been a lot of work addressing both of these problems.
- Two types of approach have emerged

3) NLO Simulation
   - NLO normalization of the cross section
   - Gets the hardest emission correct
4) Multi-Jet Leading Order
   - Still leading order.
   - Gets many hard emission correct.

NLO Simulation

- There has been a lot of work on NLO Monte Carlo simulations.
- However apart from some early work by Dobbs the only Frixione, Nason and Webber have produced code which can be used to generate results.
- I will therefore only talk about the work of Frixione, Nason and Webber.
- Most of this is taken from Bryan Webber's talk at the YETI meeting in Durham.

MC@NLO

- S. Frixione and B.R. Webber JHEP 0206 (2002) 029, hep-ph/0204244, hep-ph/0309186
- S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.
- http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/

MC@NLO

- MC@NLO was designed to have the following features.
  - The output is a set of fully exclusive events.
  - The total rate is accurate to NLO
  - NLO results for observables are recovered when expanded in α_s.
  - Hard emissions are treated as in NLO calculations.
  - Soft/Collinear emission are treated as in the parton shower.
  - The matching between hard emission and the parton shower is smooth.
  - MC hadronization models are used.

Toy Model

- I will start with Bryan Webber's toy model to explain MC@NLO to discuss the key features of NLO, MC and the matching.
- Consider a system which can radiate photons with energy with energy x with

$$0 \le x \le x_s \le 1$$

where x_s is the energy of the system before radiation.

- After radiation the energy of the system $x_s' = x_s - x$
- Further radiation is possible but photons don't radiate.

Toy Model

- Calculating an observable at NLO gives

$$\langle O \rangle = \lim_{\varepsilon \to 0} \int_0^1 dx x^{-2\varepsilon} O(x) \left[\left(\frac{d\sigma}{dx} \right)_B + \left(\frac{d\sigma}{dx} \right)_V + \left(\frac{d\sigma}{dx} \right)_R \right]$$

where the Born, Virtual and Real contributions are

$$\left(\frac{d\sigma}{dx} \right)_B = B\,\delta(x) \qquad \left(\frac{d\sigma}{dx} \right)_V = \alpha \left(\frac{B}{2\varepsilon} + V \right) \delta(x) \qquad \left(\frac{d\sigma}{dx} \right)_R = \alpha \frac{R(x)}{x}$$

α is the coupling constant and $\lim_{x \to 0} R(x) = B$

Toy Model

- In a subtraction method the real contribution is written as

$$\langle O \rangle_R = \alpha\, B O(0) \int_0^1 dx \frac{1}{x^{1+2\varepsilon}} + \alpha \int_0^1 dx \frac{O(x)R(x) - BO(0)}{x^{1+2\varepsilon}}$$

- The second integral is finite so we can set $\varepsilon = 0$

$$\langle O \rangle_R = -\alpha \frac{B}{2\varepsilon} O(0) + \alpha \int_0^1 dx \frac{O(x)R(x) - BO(0)}{x}$$

- The NLO prediction is therefore

$$\langle O \rangle_{sub} = \int_0^1 dx \left[\alpha\, O(x) \frac{R(x)}{x} + O(0)\left(B + \alpha V - \alpha \frac{B}{x} \right) \right]$$

Toy Monte Carlo

- In a MC treatment the system can emit many photons with the probability controlled by the Sudakov form factor, defined here as

$$\Delta(x_1, x_2) = \exp\left[-\alpha \int_{x_1}^{x_2} dx \frac{Q(x)}{x}\right]$$

where $Q(x)$ is a monotonic function which has

$$0 \le Q(x) \le 1 \qquad \lim_{x \to 0} Q(x) = 1 \qquad \lim_{x \to 1} Q(x) = 0$$

- $\Delta(x_1, x_2)$ is the probability that no photon can be emitted with energy x such that $x_1 \le x \le x_2$.

Toy MC@NLO

- We want to interface NLO to MC. Naïve first try
  - $O(0) \Rightarrow$ start MC with 0 real emissions: F^0_{MC}
  - $O(x) \Rightarrow$ start MC with 1 real emission at *x*: $F^1_{MC}(x)$
- So that the overall generating functional is

$$\int_0^1 dx \left[F^0_{MC} \left(B + a V - \frac{a B}{x} \right) + F^1_{MC}(x) \frac{a R(x)}{x} \right]$$

- This is wrong because MC with no emissions will generate emission with NLO distribution

$$\left(\frac{d\sigma}{dx} \right)_{MC} = a B \frac{Q(x)}{x}$$

Toy MC@NLO

- We must subtract this from the second term

$$F_{MC@NLO} = \int_0^1 dx \left[F^0_{MC} \left(B + \alpha V + \frac{\alpha B (Q(x) - 1)}{x} \right) + F^1_{MC}(x) \frac{\alpha (R(x) - BQ(x))}{x} \right]$$

- This prescription has many good features:
  - The added and subtracted terms are equal to $O(\alpha)$
  - The coefficients of F^0_{MC} and F^1_{MC} are separately finite.
  - The resummation of large logs is the same as for the Monte Carlo renormalized to the correct NLO cross section.

  However some events may have negative weight.

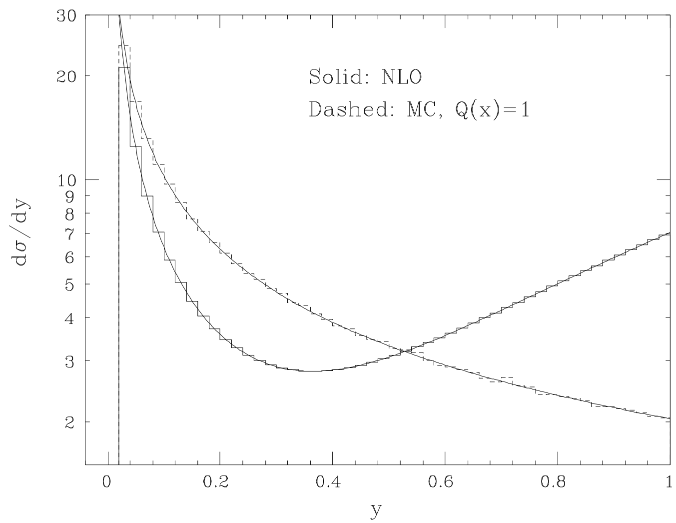
Toy MC@NLO Observables

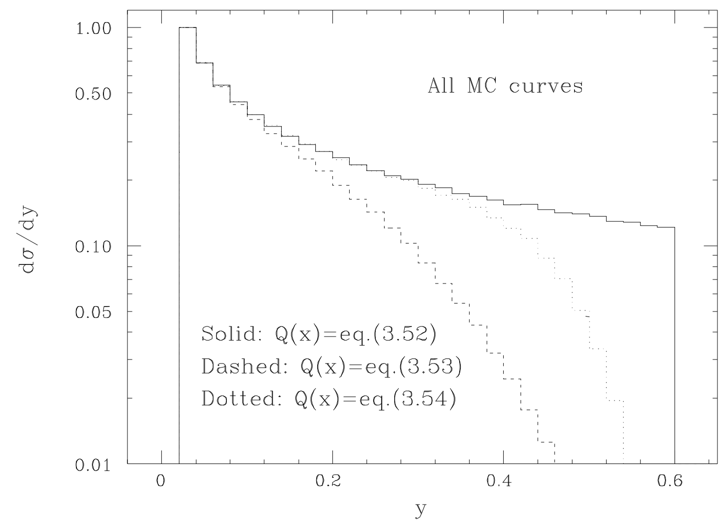
- As an example of an "exclusive" observable consider the energy *y* of the hardest photon in each event.
- As an "inclusive" observable consider the fully inclusive distributions of photon energies, *z*
- Toy model results shown are for

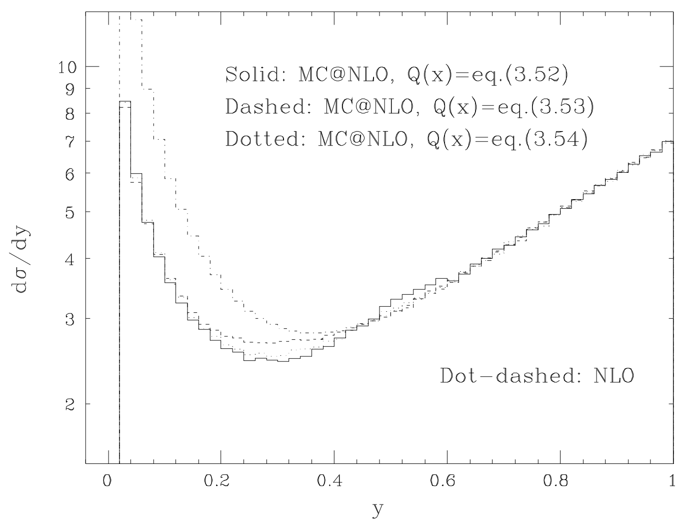
$$a = 0.3, \qquad B = 2, \qquad V = 1,$$

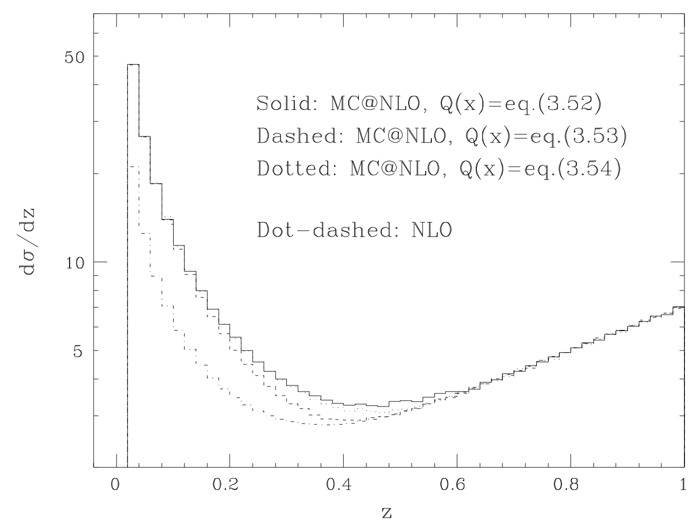
$$R(x) = B + x\left(1 + \frac{x}{2} + 20x^2\right)$$

Toy MC@NLO Observables









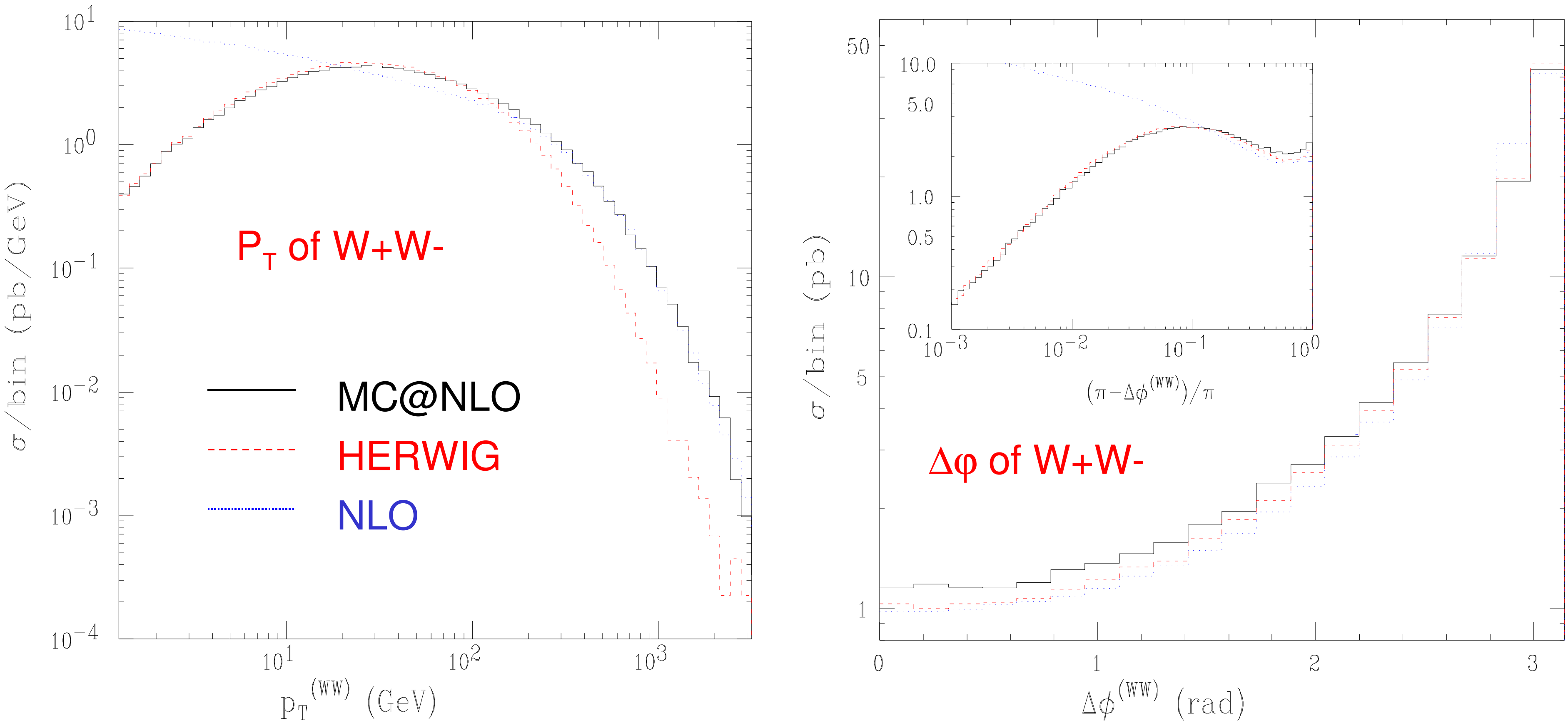
Real QCD

- For normal QCD the principle is the same we subtract the shower approximation to the real emission and add it to the virtual piece.
- This cancels the singularities and avoids double counting.
- It's a lot more complicated.

Real QCD

- For each new process the shower approximation must be worked out, which is often complicated.
- While the general approach works for any shower it has to be worked out for a specific case.
- So for MC@NLO only works with the HERWIG shower algorithm.
- It could be worked out for PYTHIA or Herwig++ but this remains to be done.

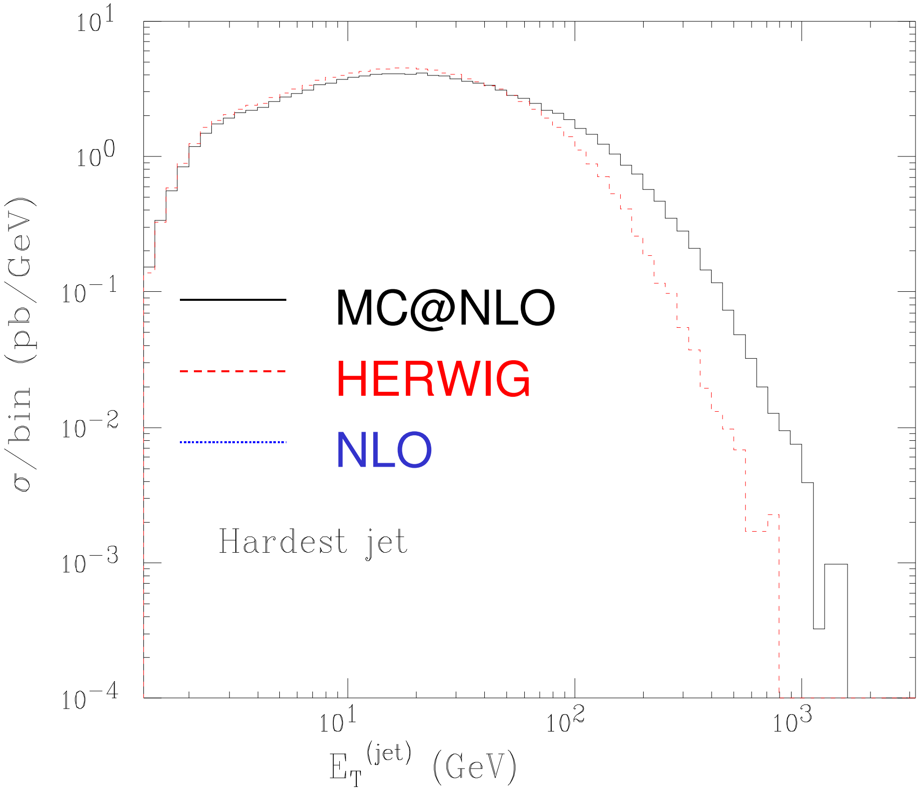
W^+W^- Observables

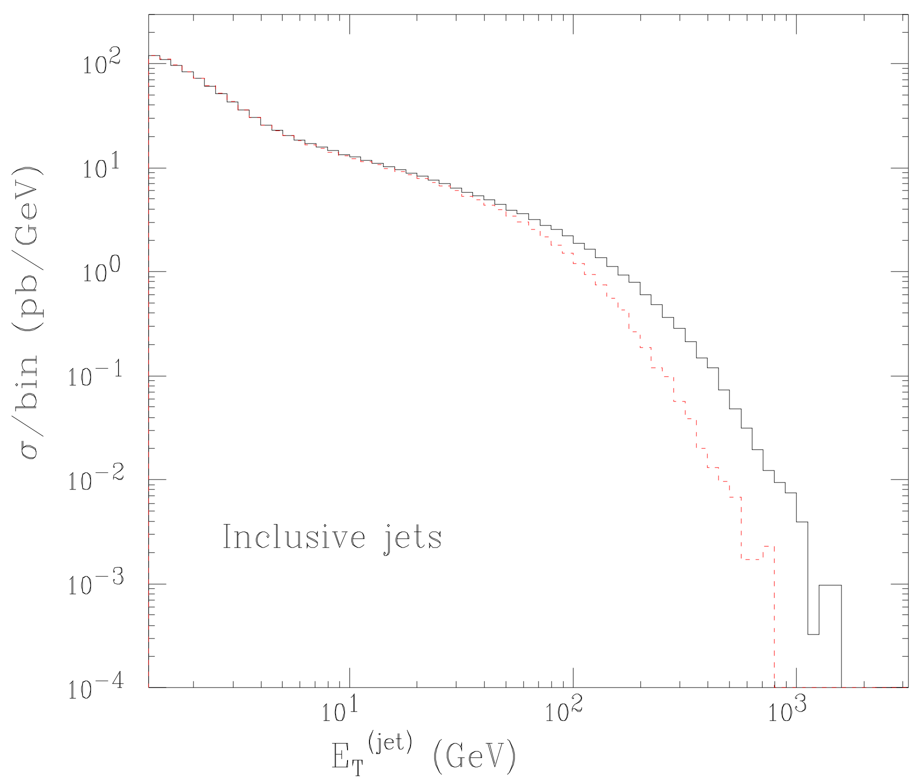


MC@NLO gives the correct high P_T result and soft resummation.

S. Frixione and B.R. Webber JHEP 0206(2002) 029, hep-ph/0204244, hep-ph/0309186

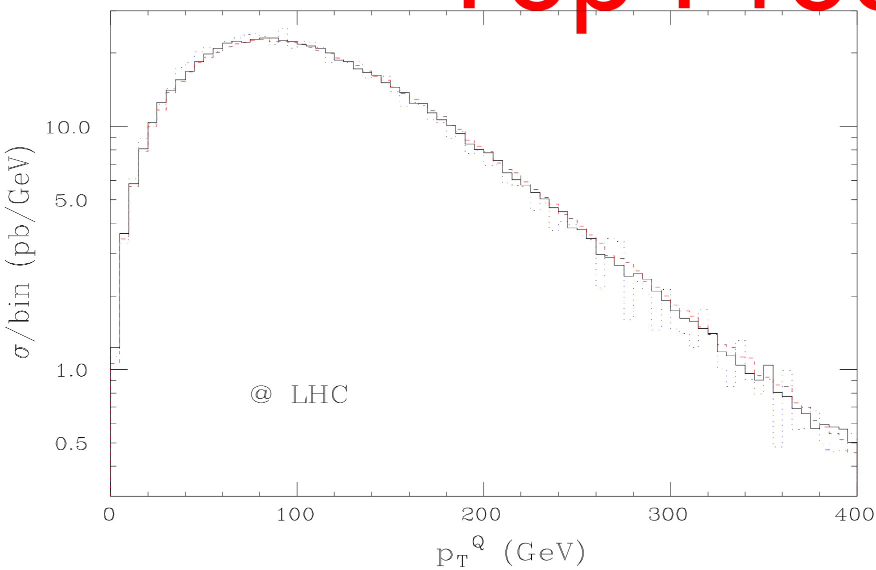
W^+W^- Jet Observables

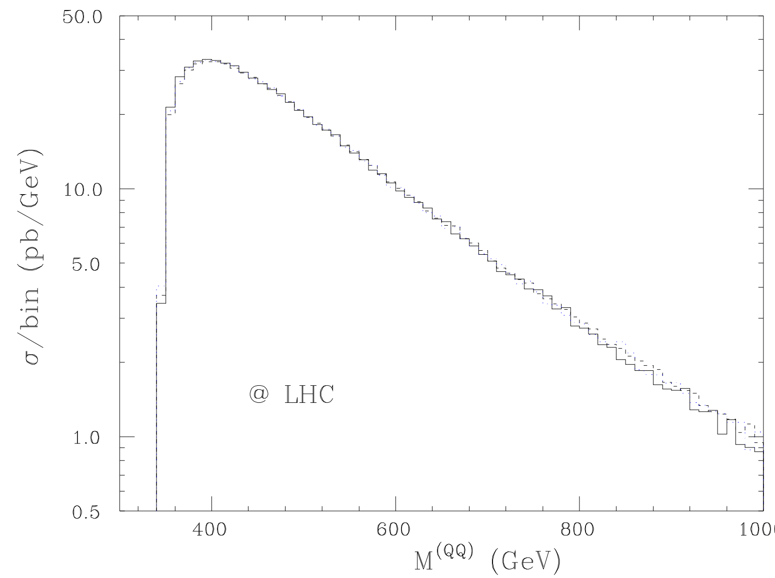


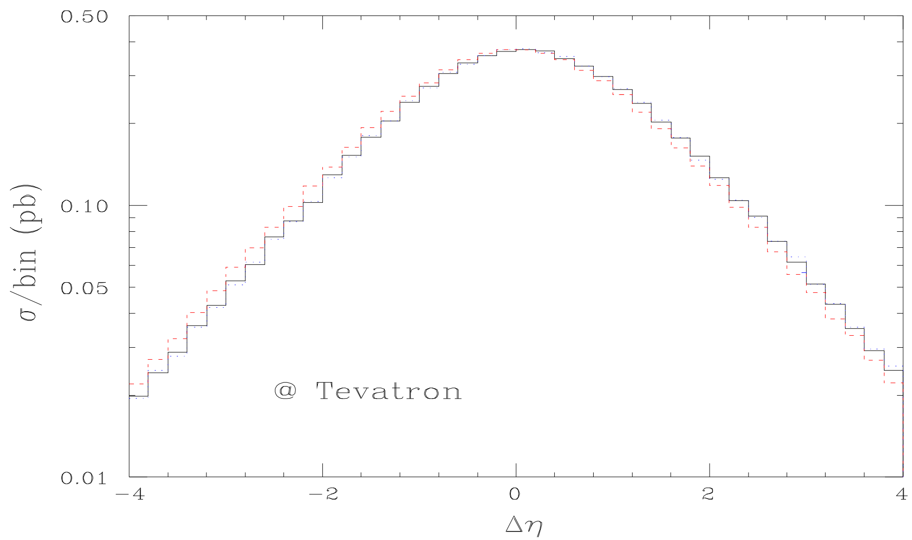


S. Frixione and B.R. Webber JHEP 0206(2002) 029, hep-ph/0204244, hep-ph/0309186

Top Production







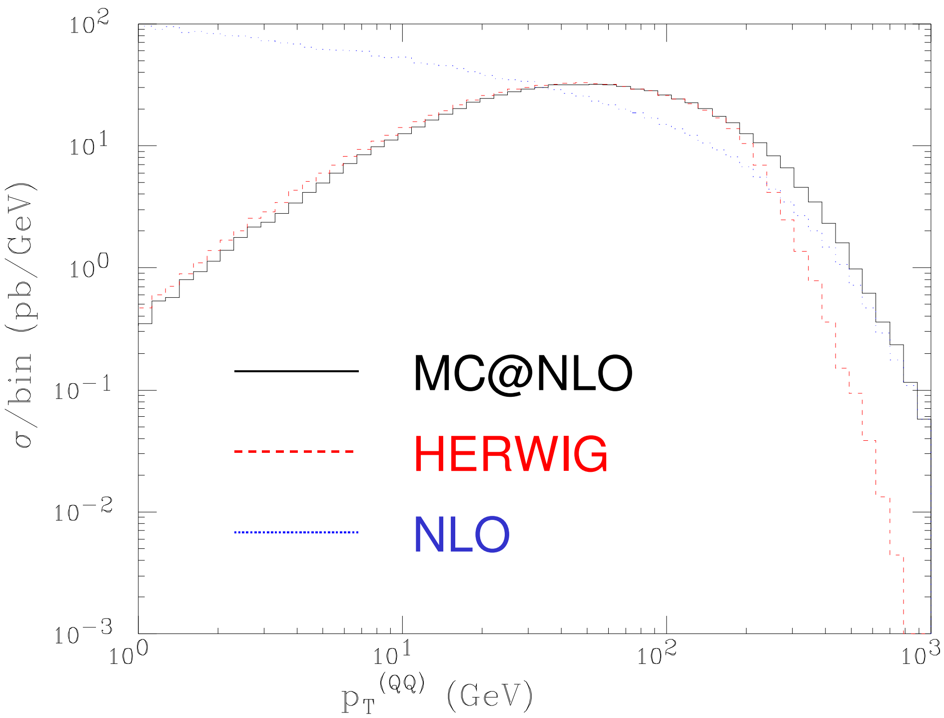
MC@NLO

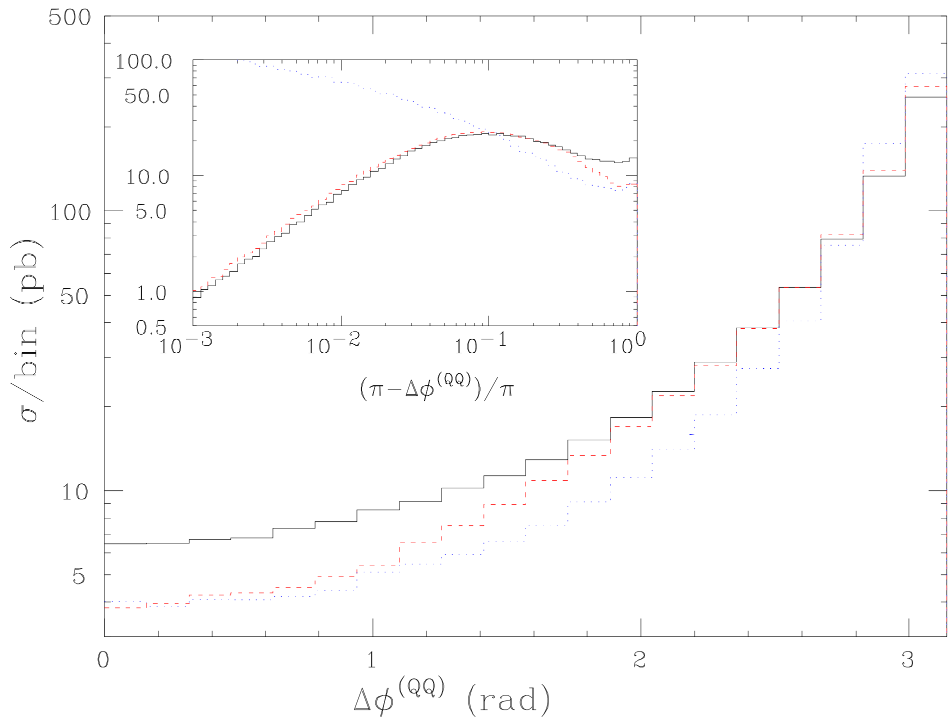
HERWIG

NLO

S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.

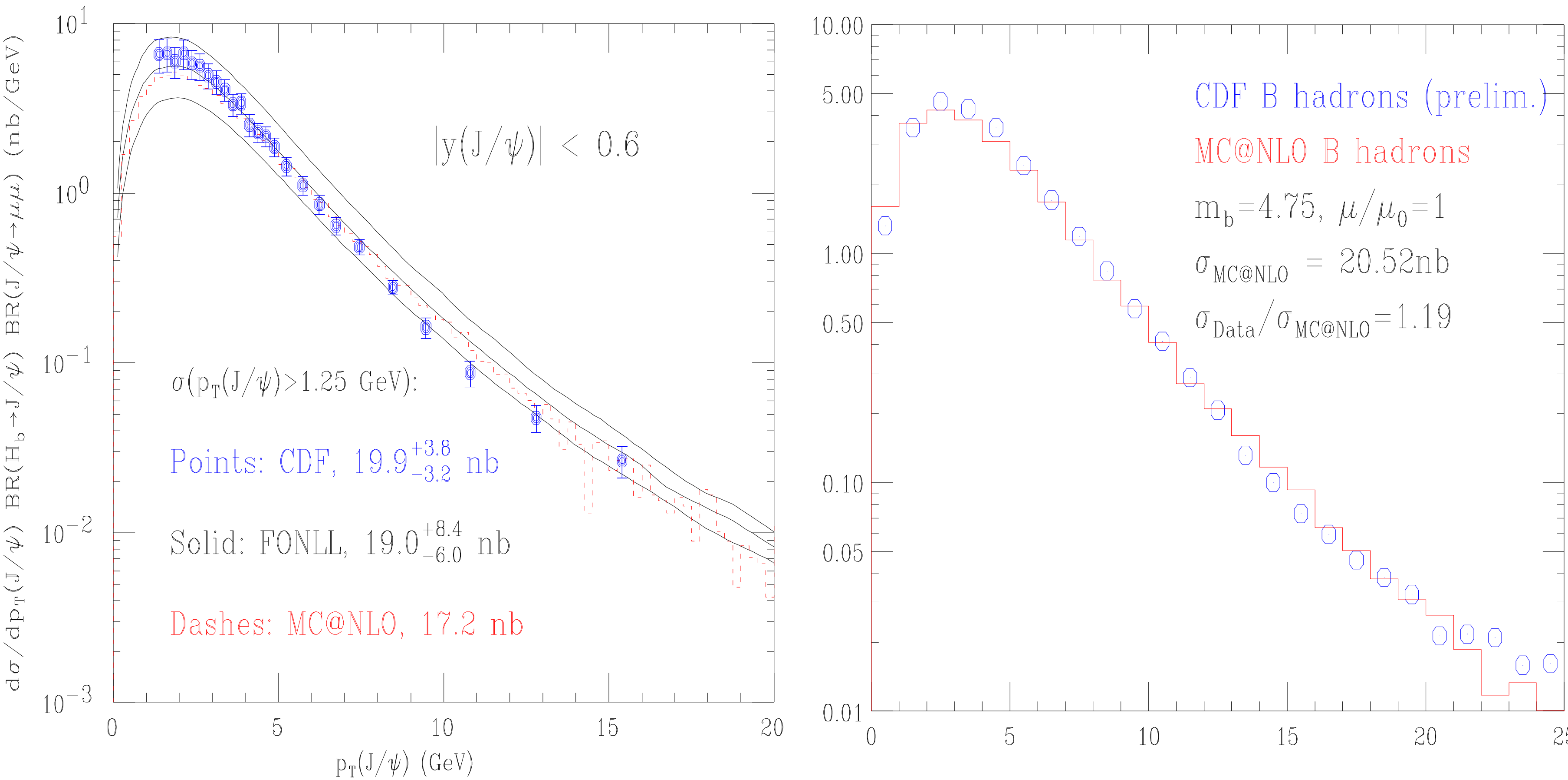
Top Production at the LHC





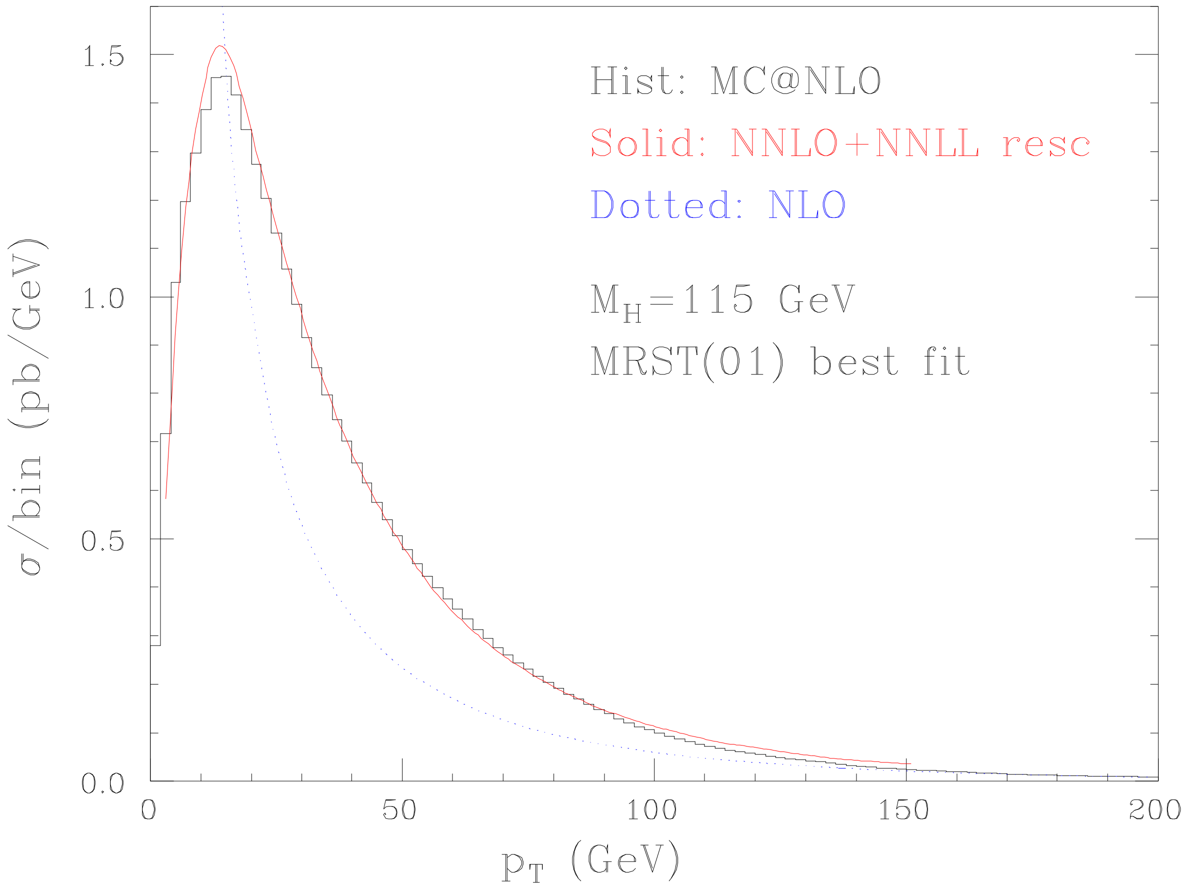
S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.

B Production at the Tevatron



S. Frixione, P. Nason and B.R. Webber, JHEP 0308(2003) 007, hep-ph/0305252.

Higgs Production at LHC



S. Frixione and B.R. Webber JHEP 0206(2002) 029, hep-ph/0204244, hep-ph/0309186

NLO Simulation

- So far MC@NLO is the only implementation of a NLO Monte Carlo simulation.
- Recently there have been some ideas by Paulo Nason JHEP 0411:040,2004.
- Here there would be no negative weights but more terms would be exponentiated beyond leading log.
- This could be an improvement but we will need to see physical results.

Multi-Jet Leading Order

- While the NLO approach is good for one hard additional jet and the overall normalization it cannot be used to give many jets.
- Therefore to simulate these processes use matching at leading order to get many hard emissions correct.
- I will briefly review the general idea behind this approach and then show some results.

CKKW Procedure

- Catani, Krauss, Kuhn and Webber JHEP 0111:063,2001.
- In order to match the ME and PS we need to separate the phase space:
- One region contains the soft/collinear region and is filled by the PS;
- The other is filled by the matrix element.
- In these approaches the phase space is separated using in k_T-type jet algorithm.

Durham Jet Algorithm

- For all final-state particles compute the resolution variables

$$d_{kB} \approx E_k^2\,\theta_{kB}^2 \approx k_{\text{¿}kB}^2 \qquad \theta_{kB}^2 \to 0$$

$$d_{kl} \approx \min\left(E_k^2, E_l^2\right)\,\theta_{kl}^2 \approx k_{\text{¿}kl}^2 \qquad \theta_{kl}^2 \to 0$$

- The smallest of these is selected. If d_{kl} is the smallest the two particles are merged. If d_{kB} is the smallest the particle is merged with the beam.
- This procedure is repeated until the minimum value is above some stopping parameter d_{cut} .
- The remaining particles and pseudo-particles are then the hard jets.

CKKW Procedure

- Radiation above a cut-off value of the jet measure is simulated by the matrix element and radiation below the cut-off by the parton shower.

2) Select the jet multiplicity with probability

$$P_n = \frac{\sigma_n}{\sum_{k=0}^{N} \sigma_k}$$

where σ_n is the n-jet matrix element evaluated at resolution d_{ini} using d_{ini} as the scale for the PDFs and α_S, n is the jet of jets

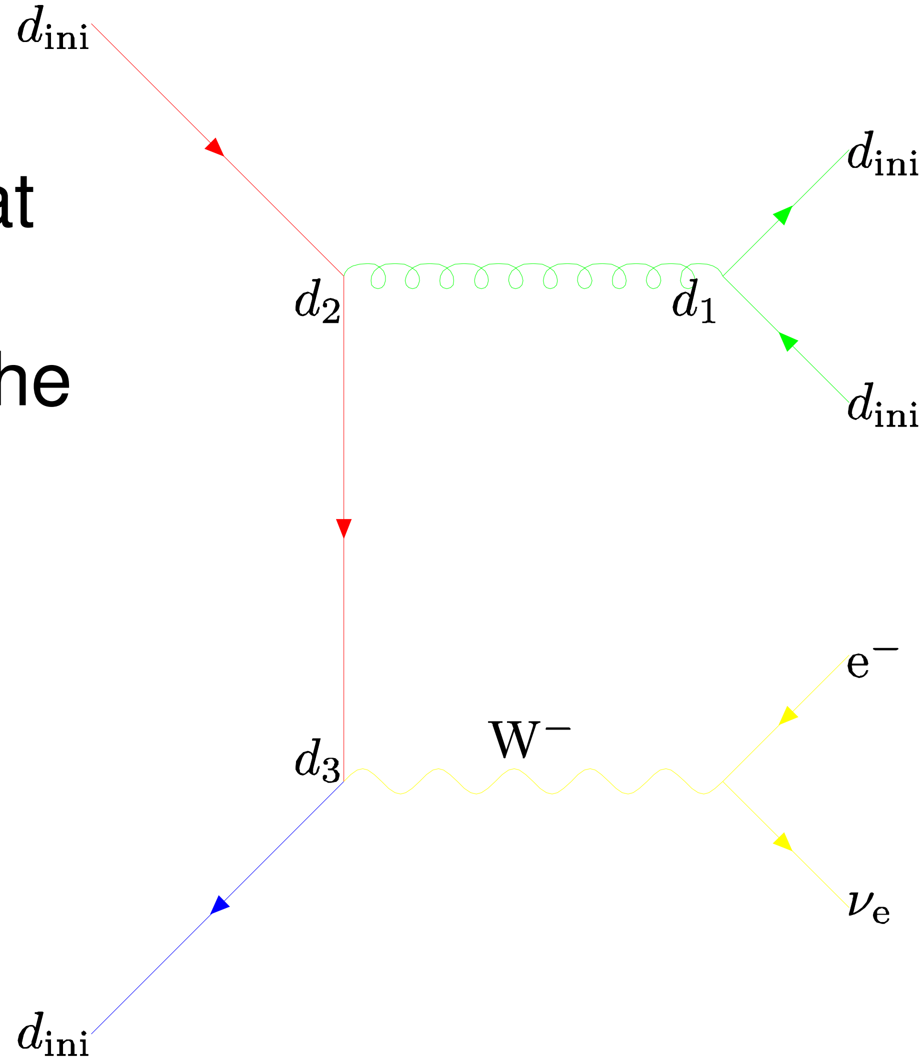
6) Distribute the jet momenta according the ME.

CKKW Procedure

1) Cluster the partons to determine the values at which 1,2,..*n*-jets are resolved. These give the nodal scales for a tree diagram.

2) Apply a coupling constant reweighting.

$$\frac{\alpha_s(d_1)\,\alpha_s(d_2)\ldots\alpha_s(d_3)}{\alpha_s(d_{\mathrm{ini}})^n} \leq 1$$

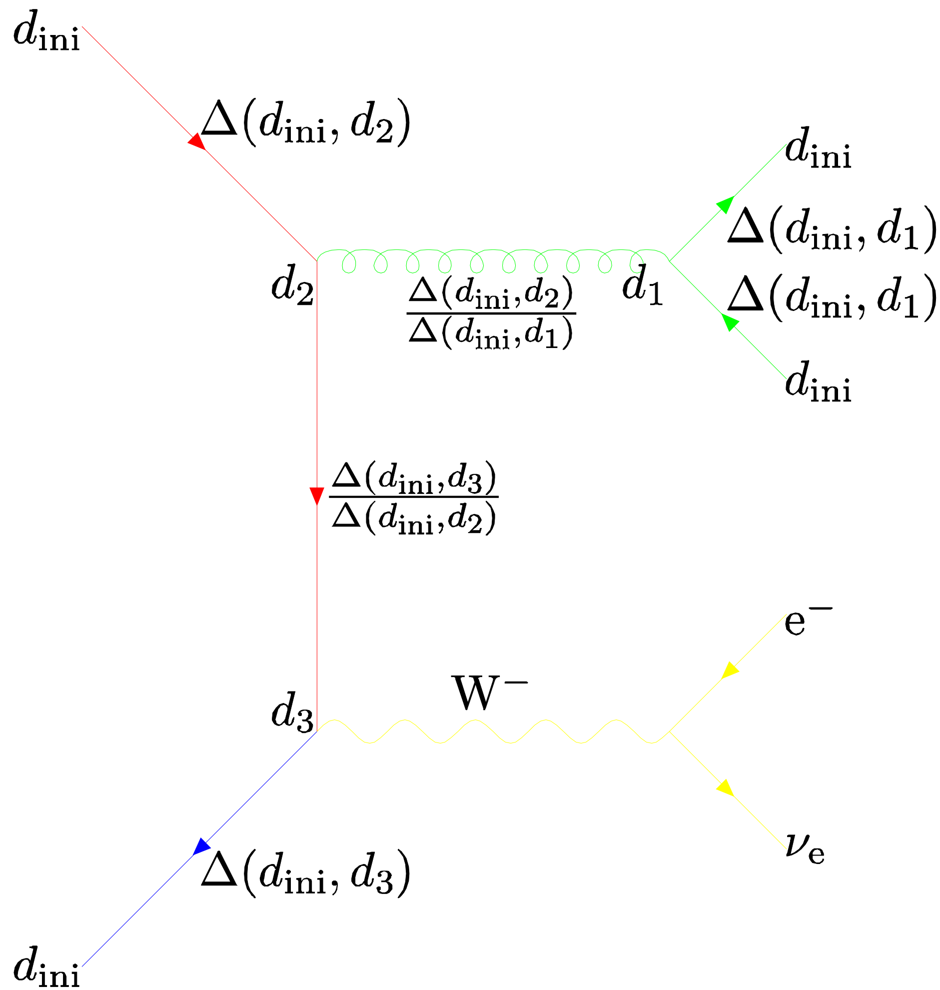


CKKW Procedure

1) Reweight the lines by a Sudakov factor

$$\frac{\Delta(d_{\text{ini}}, d_j)}{\Delta(d_{\text{ini}}, d_k)}$$

4) Accept the configuration if the product of the α_S and Sudakov weight is less than $R\in[0,1]$ otherwise return to step 1.



CKKW Procedure

1) Generate the parton shower from the event starting the evolution of each parton at the scale at which it was created and vetoing emission above the scale d_{ini}.

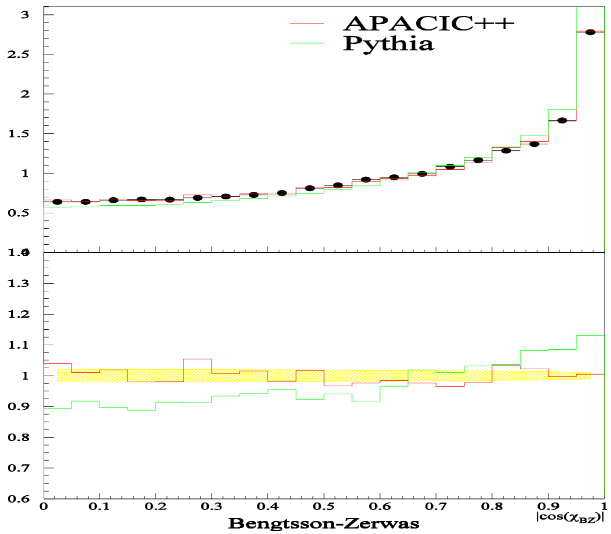
CKKW Procedure

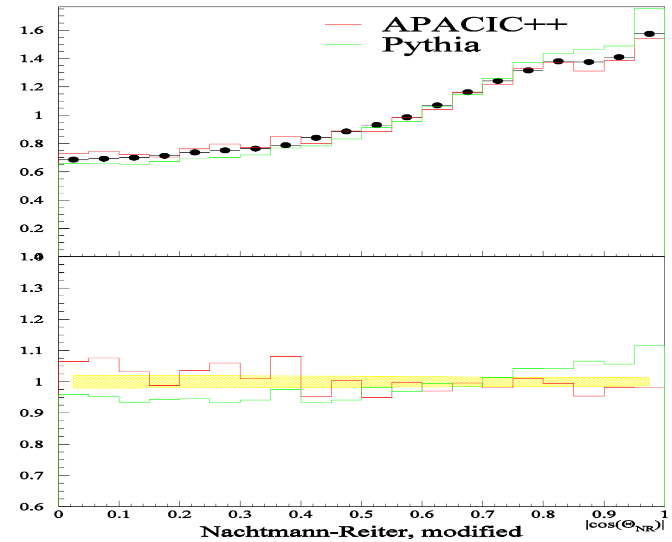
- Although this procedure ensures smooth matching at the NLL log level are still choices to be made:
  - Exact definition of the Sudakov form factors.
  - Scales in the strong coupling and α_S.
  - Treatment of the highest Multiplicity matrix element.
  - Choice of the k_T algorithm.
- In practice the problem is understanding what the shower is doing and treating the matrix element in the same way.

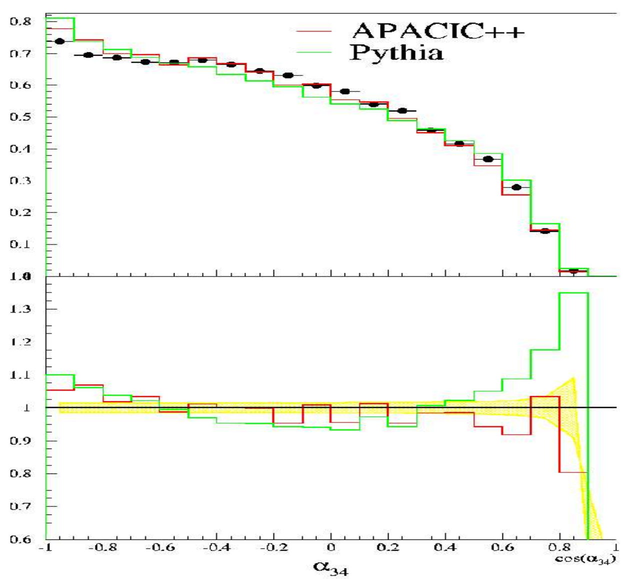
CKKW Procedure

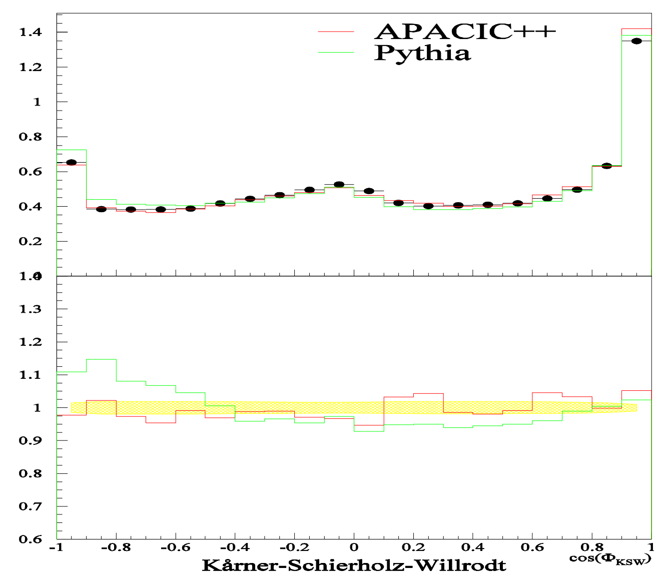
- A lot of work has been done mainly by
  - Frank Krauss et. al. (SHERPA)
  - Leif Lonnblad (ARIADNE)
  - Steve Mrenna (PYTHIA)
  - Peter Richardson (HERWIG)

e^+e^- Results from SHERPA

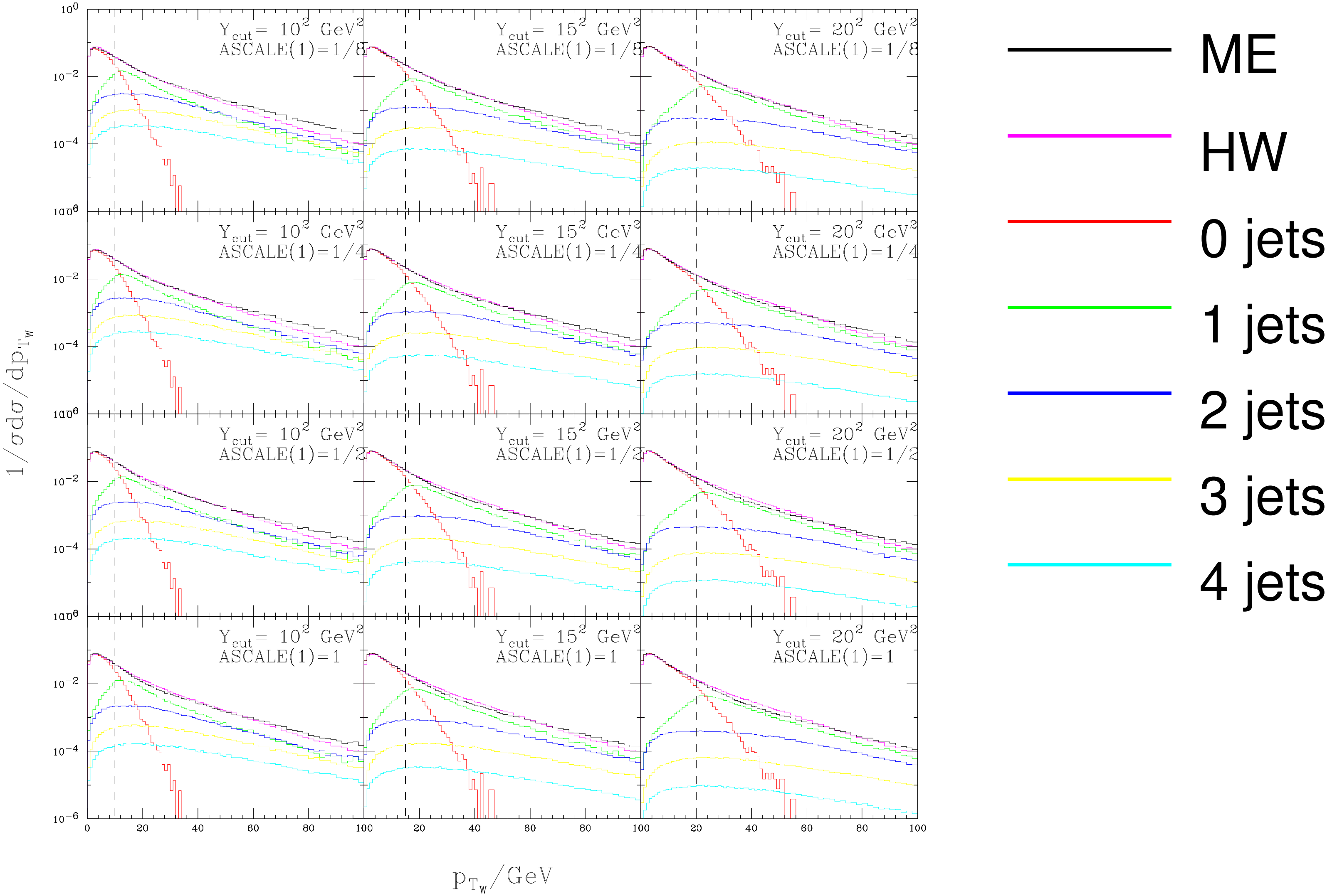




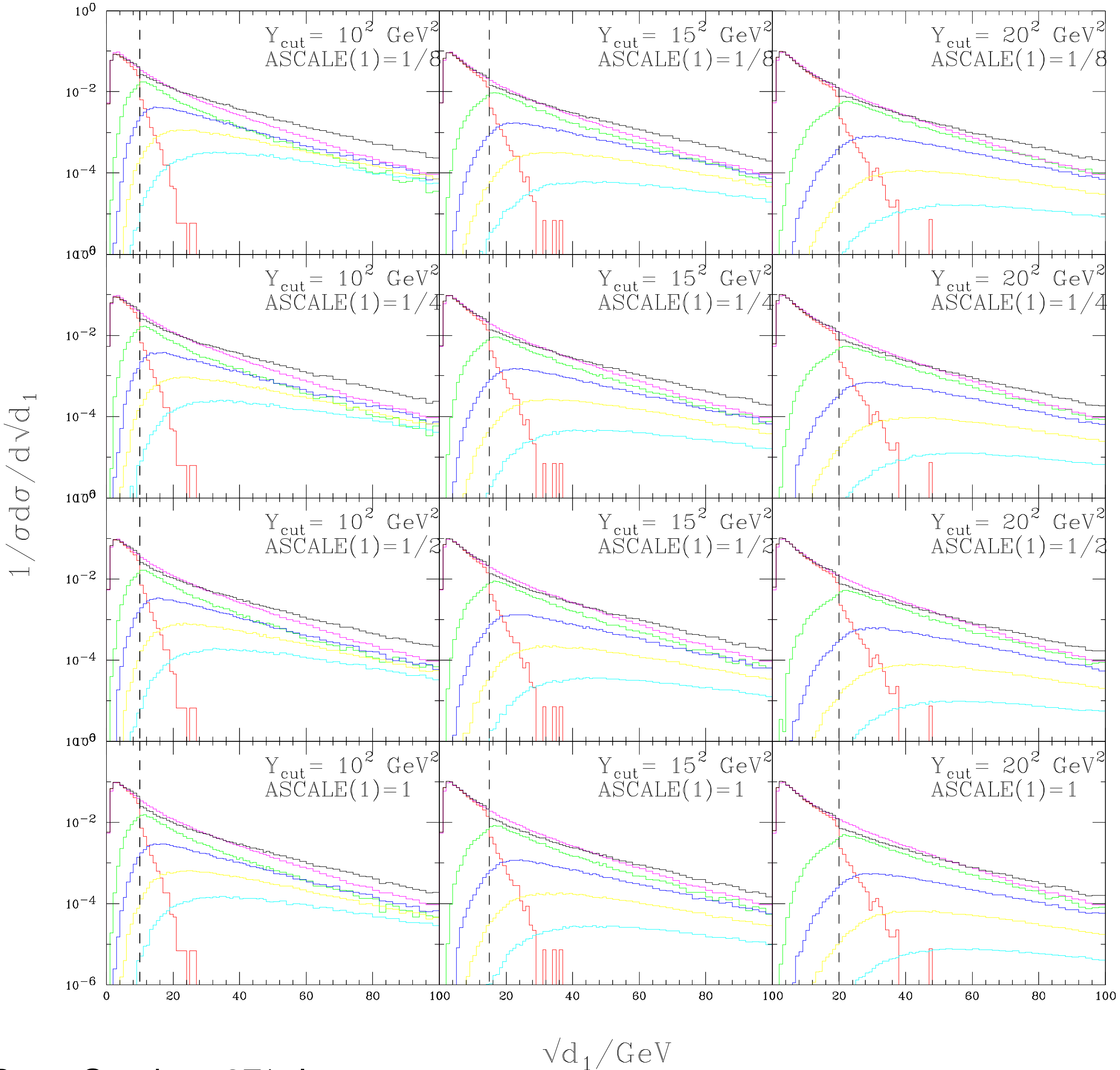




p_T of the W at the Tevatron



p_T of the hardest jet at the Tevatron



ME

HW

0 jets

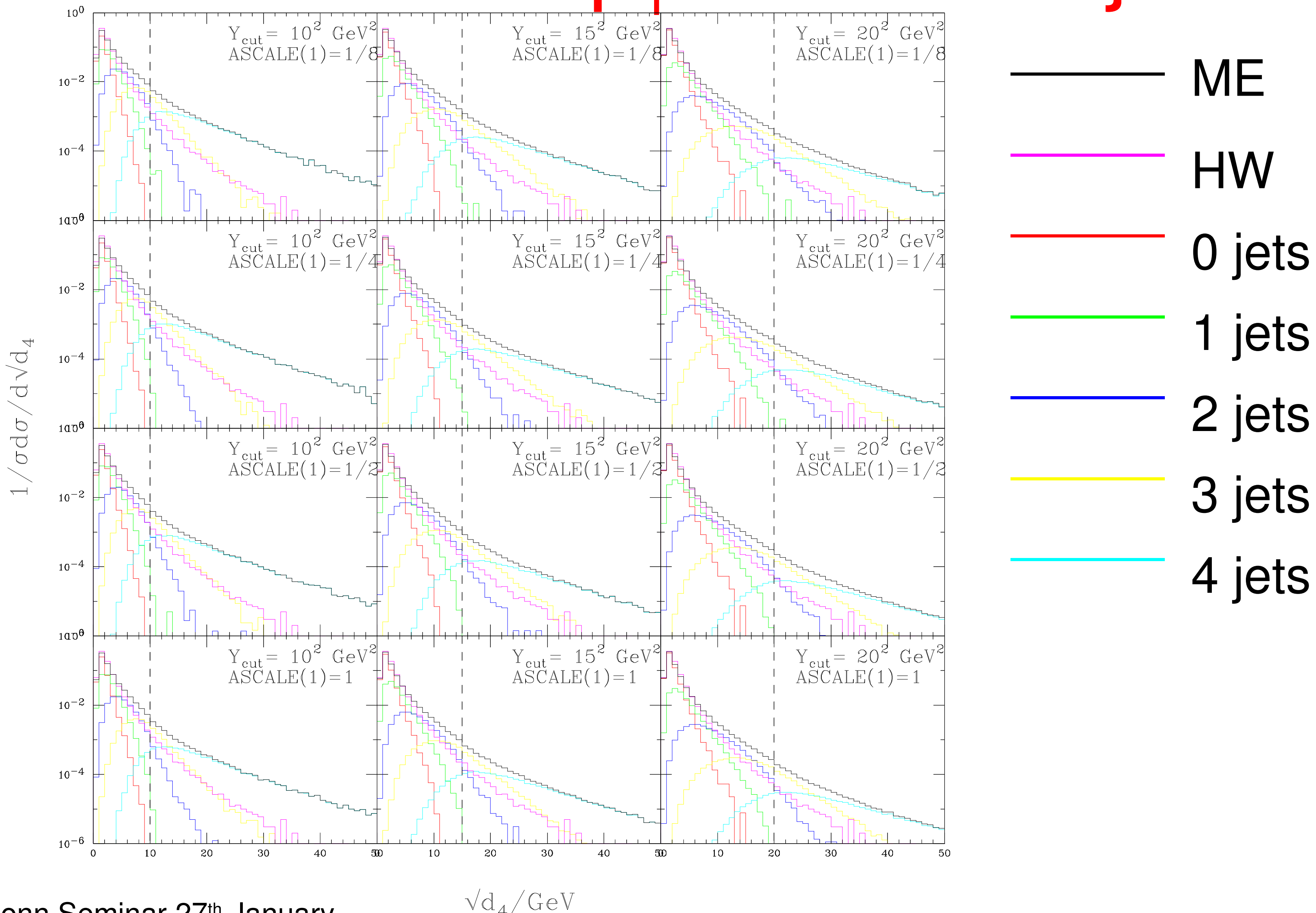
1 jets

2 jets

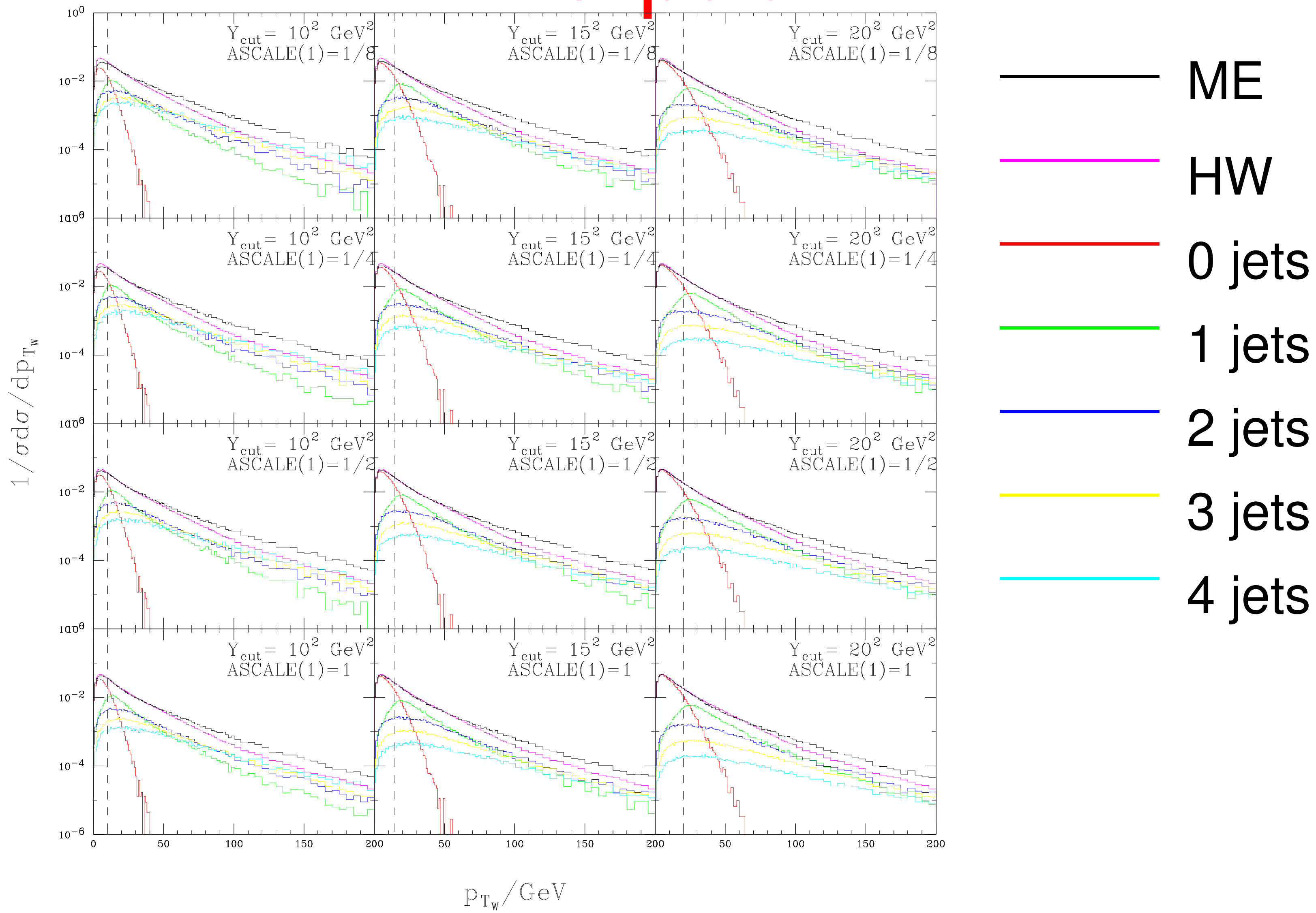
3 jets

4 jets

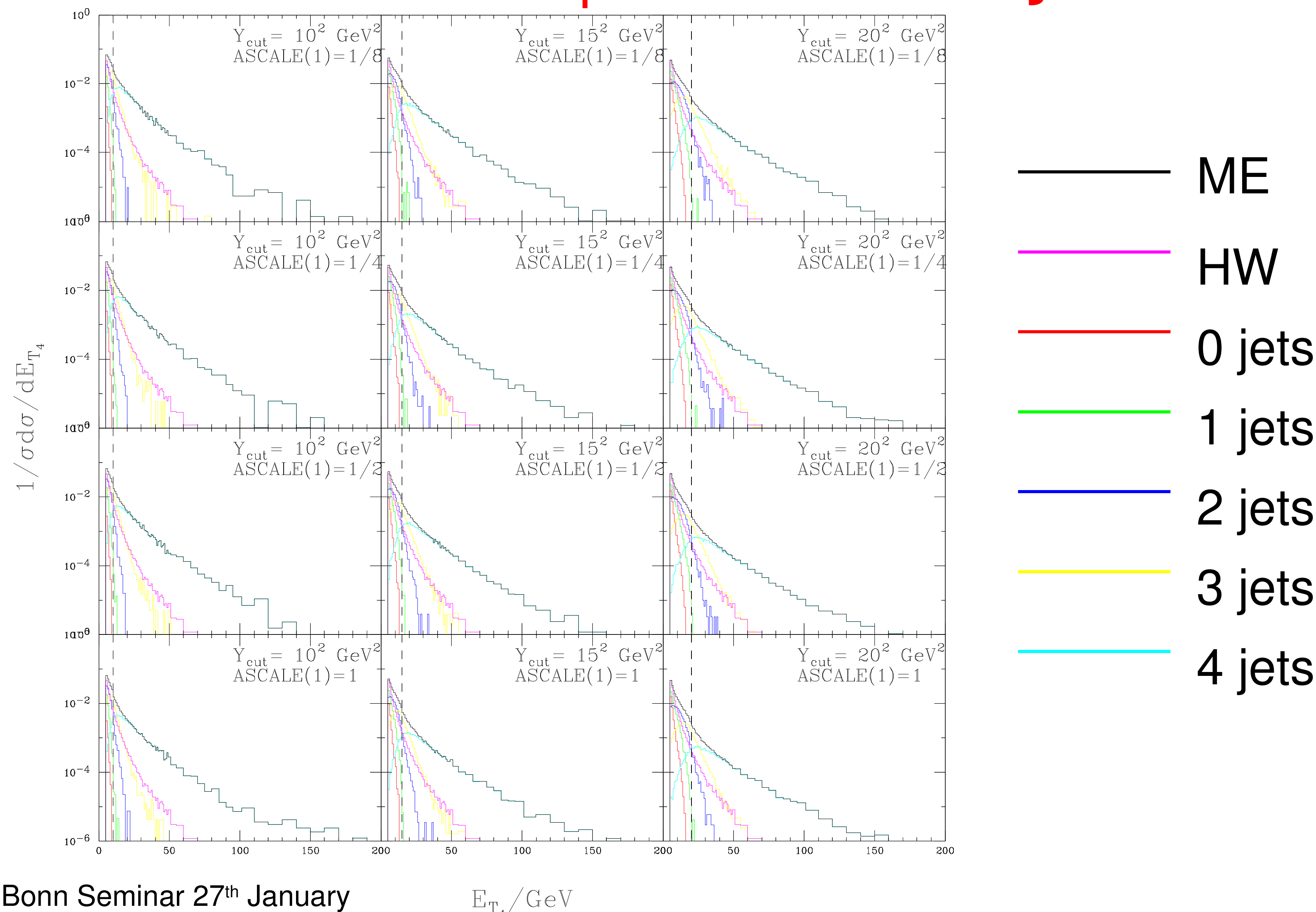
Tevatron p_T of the 4th jet



LHC pt of W


$Y_{cut} = 10^2$ GeV^2
ASCALE(1)=1/8
$Y_{cut} = 15^2$ GeV^2
ASCALE(1)=1/8
$Y_{cut} = 20^2$ GeV^2
ASCALE(1)=1/8
$Y_{cut} = 10^2$ GeV^2
ASCALE(1)=1/4
$Y_{cut} = 15^2$ GeV^2
ASCALE(1)=1/4
$Y_{cut} = 20^2$ GeV^2
ASCALE(1)=1/4
$Y_{cut} = 10^2$ GeV^2
ASCALE(1)=1/2
$Y_{cut} = 15^2$ GeV^2
ASCALE(1)=1/2
$Y_{cut} = 20^2$ GeV^2
ASCALE(1)=1/2
$Y_{cut} = 10^2$ GeV^2
ASCALE(1)=1
$Y_{cut} = 15^2$ GeV^2
ASCALE(1)=1
$Y_{cut} = 20^2$ GeV^2
ASCALE(1)=1
$1/\sigma d\sigma/dp_{T_W}$
p_{T_W}/GeV
ME
HW
0 jets
1 jets
2 jets
3 jets
4 jets

LHC E_T of the 4th jet



What Should I use?

- Hopefully this talk will help you decide which of the many different tools is most suitable for a given analysis.
  - Only soft jets relative to hard scale MC
  - Only one hard jet MC@NLO or old style ME correction
  - Many hard jets CKKW.
- The most important thing is to think first before running the simulation.

Future

- Clearly much progress has been made with MC@NLO.
- The matching of many jets needs improved understanding of the shower and matching but is promising for many processes.
- Progress has been made with SHERPA.
- Hopefully the new Herwig++ and pT ordered PYTHIA shower's will have better properties for the matching.

Future

- The Monte Carlo community is very small.
- There are three major projects
  - HERWIG (3 permanent staff, 3 postdocs, 1 student, ~3FTE)
  - PYTHIA (3 permanent staff, 1 postdoc,~2FTE)
  - SHERPA (1 permanent staff, 4 students,~4FTE)
- Given the large demand for both support and development this is not sustainable in the long term.
- We know how to construct the tools for the LHC.
- It may well be that everything we need will not be ready due to lack of manpower.