Lectures on Supersymmetry

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OC 204

Part 3:

- Properties of SUSY theories

- Prospects for SUSY searches at the next generation of colliders

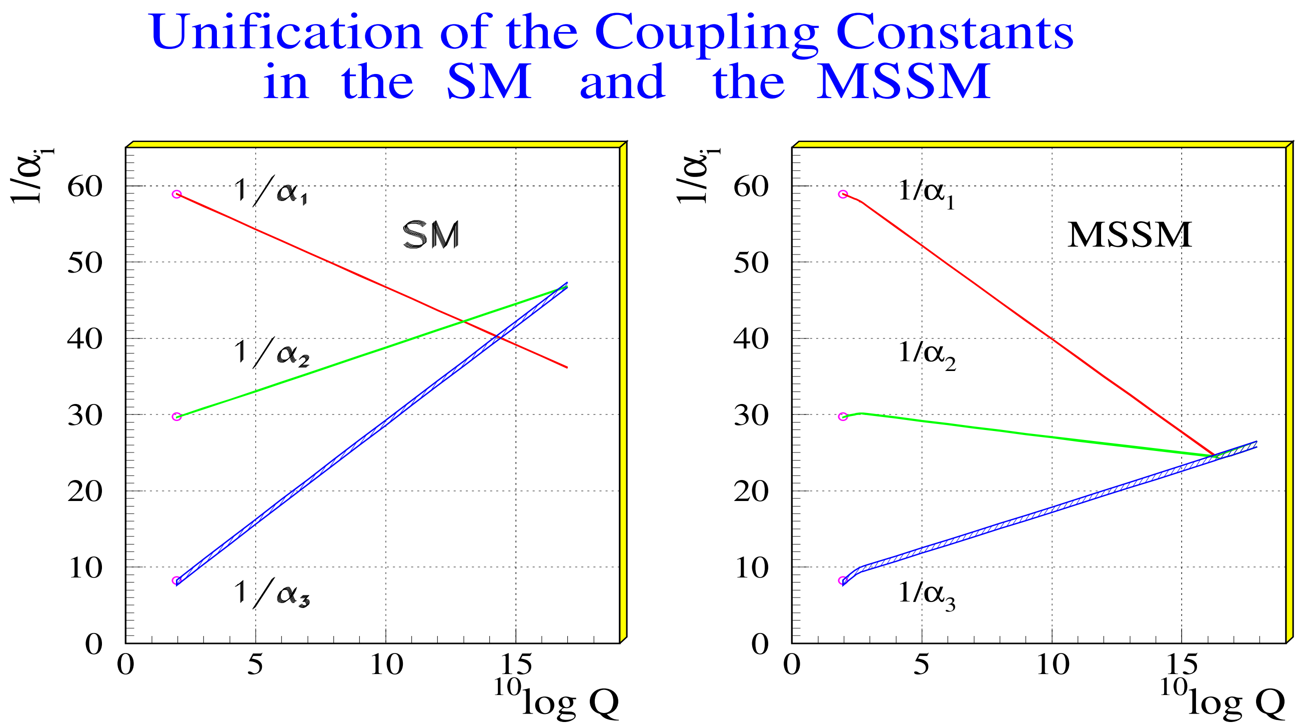
Properties of SUSY theories:

Gauge coupling unification:

Running of gauge couplings: $$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

Unification of couplings at high scale $\Leftrightarrow$ "Grand unified theories" (GUTs)

E.g.: SO(10) GUTs, can naturally accomodate right-handed neutrinos

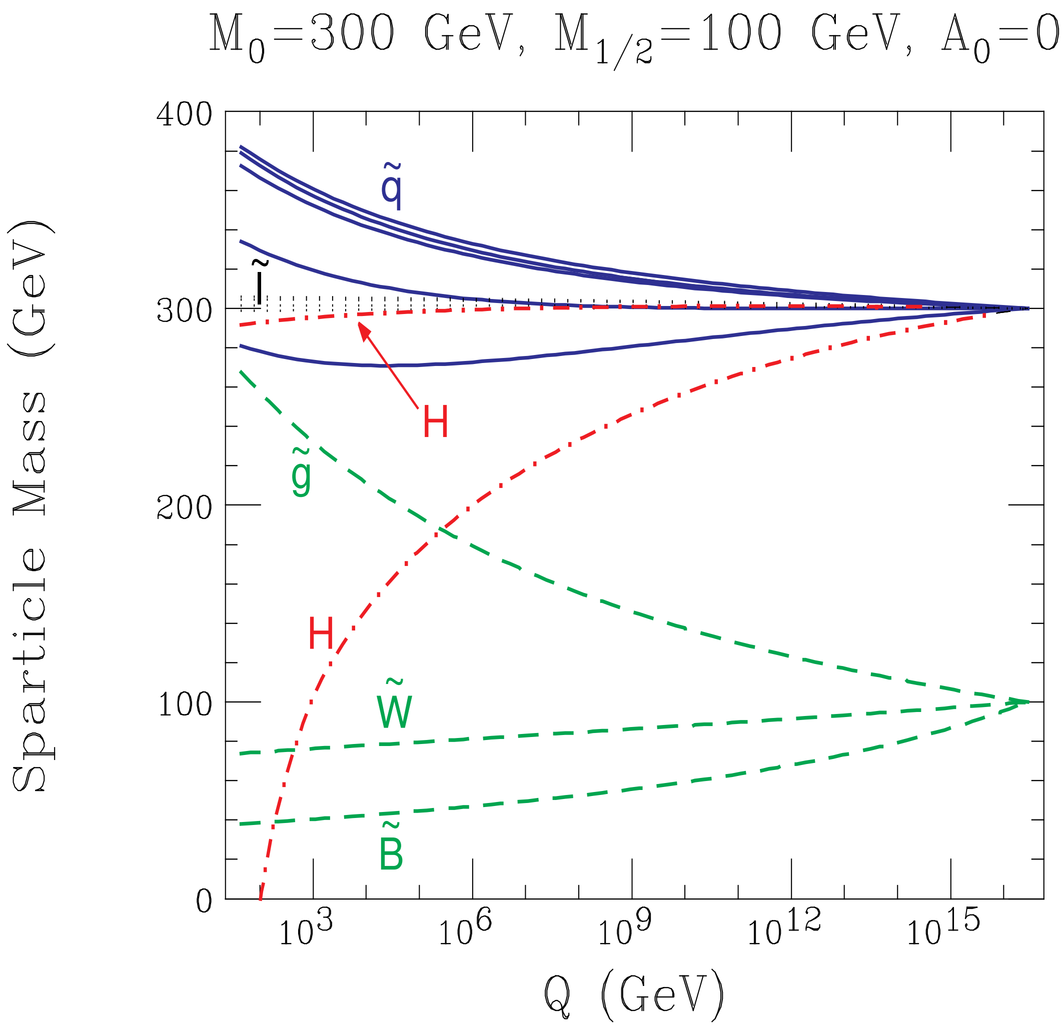


[*U. Amaldi, W. de Boer, H. Fürstenau '91*]

$\Rightarrow$ coupling constant unification in MSSM for $M_{\text{SUSY}} \lesssim 1$ TeV

Radiative electroweak symmetry breaking:

Universal boundary conditions at GUT scale,
renormalization group running down to weak scale



large corrections from
top-quark Yukawa coupling

$\Rightarrow$ $m^2_{H_u}$ driven to negative values

$\Rightarrow$ ew symmetry breaking

emerges naturally at scale $\sim 10^2$ GeV
for 100 GeV $\lesssim m_t \lesssim$ 200 GeV

R parity

Most general gauge-invariant and renormalizable superpotential with chiral superfields of the MSSM:

$$\mathcal{V} = \mathcal{V}_{\text{MSSM}} + \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D_k + \mu'^{i} L_i H_u}_{\text{violate lepton number}} + \underbrace{\frac{1}{2}\lambda''^{ijk} U_i D_j D_k}_{\text{violates baryon number}}$$

If both lepton and baryon number are violated

⇒ rapid proton decay

Minimal choice (MSSM) contains only terms in the Lagrangian with even number of SUSY particles

⇒ additional symmetry: "R parity"

⇒ all SM particles have even R parity, all SUSY particles have odd R parity

Phenomenological consequences of R parity conservation:

- SUSY particles can only be produced in pairs, e.g. $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$
- Decay of SUSY particles into SM particles
  + odd number of SUSY particles

  $\Rightarrow$ Lightest SUSY particle (LSP) is stable
- All SUSY particles will eventually decay into LSP

  $\Rightarrow$ Decay cascades of heavy SUSY particles
- LSP stable

  $\Rightarrow$ some LSP must have survived from Big Bang

  $\Rightarrow$ Weakly interacting massive particle ("WIMP")
  (otherwise ruled out from bounds on exotic isotopes etc.)

  $\Rightarrow$ Candidate for cold dark matter in the Universe (if $m_{\mathrm{LSP}} \lesssim 1$ TeV)
- LSP neutral, uncolored $\Rightarrow$ leaves no traces in collider detectors

  $\Rightarrow$ Typical SUSY signatures: "missing energy"

Relations between SUSY parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.:

gauge boson–fermion coupling

=

gaugino–fermion–sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters
(except M_W–M_Z interdependence)

MSSM:

- Upper bound on mass of lightest $\mathcal{CP}$-even Higgs boson

- Relations between neutralino and chargino masses

- Sfermion mass relations, e.g.

$$m^2_{\tilde{e}_L} = m^2_{\tilde{\nu}_L} - M^2_W \cos(2\beta)$$

All relations receive corrections from loop effects
$\Leftrightarrow$ effects of soft SUSY breaking, electroweak symmetry breaking

$\Rightarrow$ Experimental verification of parameter relations is a crucial test of SUSY!

The Higgs sector of the MSSM

Two Higgs doublets:

$$
\begin{aligned}
H_d &= \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix} = \begin{pmatrix} v_d + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_u &= \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_u + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}
\end{aligned}
$$

Higgs potential:

$$
\begin{aligned}
V = & \left(|\mu|^2 + m_{H_u}^2\right)\left(|h_u^0|^2 + |h_u^+|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(|h_d^0|^2 + |h_d^-|^2\right) \\
& + \left[b\,(h_u^+ h_d^- - h_u^0 h_d^0) + \text{h.c.}\right] \\
& + \underbrace{\frac{g^2 + g'^2}{8}}\left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2\right)^2 + \underbrace{\frac{g'^2}{2}}\left|h_u^+ h_d^{0*} + h_u^0 h_d^{-*}\right|^2
\end{aligned}
$$

gauge couplings, in contrast to SM

Mixing of $\mathcal{CP}$-even, $\mathcal{CP}$-odd, charged fields:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

$$\tan(2\alpha) = \tan(2\beta) \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

3 Goldstone bosons (as in SM): G^0, $G^\pm \longrightarrow$ longitudinal comp. of $W^\pm$, Z

$\Rightarrow$ Five physical states: $h^0, H^0, A^0, H^\pm$

h, H: neutral, $\mathcal{CP}$-even, A^0: neutral, $\mathcal{CP}$-odd, $H^\pm$: charged

Gauge-boson masses:

$$M_W^2 = \frac{1}{2} g'^2 (v_d^2 + v_u^2), \quad M_Z^2 = \frac{1}{2} (g^2 + g'^2)(v_d^2 + v_u^2), \quad M_\gamma = 0$$

Problem:

MSSM contains term $\mu H_d H_u$ in superpotential

μ: dimensionful parameter

For ew symmetry breaking required: $\mu \sim$ electroweak scale

But: no a priori reason for $\mu \neq 0$, $\mu \ll M_\mathrm{P}$

(problem mainly in GMSB svenario, easier to overcome in mSUGRA)

Possible solution:

μ related to v.e.v. of additional field

$\Rightarrow$ Introduction of extra singlet field S, v.e.v. $s \Rightarrow$ "NMSSM"

Superpotential: $\mathcal{V} = \lambda H_d H_u S + \frac{1}{3}\kappa S^3 + \ldots$

Physical states in NMSSM Higgs-sector:

S_1, S_2, S_3 (CP-even), P_1, P_2 (CP-odd), $H^\pm$

Parameters in MSSM Higgs potential V (besides g, g'):

$$v_d,\ v_u,\ \left(|\mu|^2 + m_{H_u}^2\right),\ \left(|\mu|^2 + m_{H_d}^2\right),\ b$$

relation for M_W^2, M_Z^2 $\Rightarrow$ 1 condition

minimization of V w.r.t. neutral Higgs fields h_d^0, h_u^0 $\Rightarrow$ 2 conditions

$\Rightarrow$ only two free parameters remain in V, conventionally chosen as

$$\tan\beta = \frac{v_u}{v_d}, \qquad M_\mathrm{A}^2 = -b(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{\mathrm{H}^\pm}$: no free parameters, can be predicted

In lowest order:

$$M_{\mathrm{H}^\pm}^2 = M_\mathrm{A}^2 + M_\mathrm{W}^2$$

Predictions for m_{h}, m_{H} from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M_{\mathrm{Higgs}}^{2,\mathrm{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1\phi_2}^2 \\ m_{\phi_1\phi_2}^2 & m_{\phi_2}^2 \end{pmatrix} =$$

$$\begin{pmatrix} M_{\mathrm{A}}^2 \sin^2\beta + M_{\mathrm{Z}}^2 \cos^2\beta & -(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2) \sin\beta \cos\beta \\ -(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2) \sin\beta \cos\beta & M_{\mathrm{A}}^2 \cos^2\beta + M_{\mathrm{Z}}^2 \sin^2\beta \end{pmatrix}$$

$$\Downarrow \quad \leftarrow \quad \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_{\mathrm{H}}^{2,\mathrm{tree}} & 0 \\ 0 & M_{\mathrm{h}}^{2,\mathrm{tree}} \end{pmatrix}$$

Tree-level result for m_{h}, m_{H}:

$$m^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2 \pm \sqrt{(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2)^2 - 4M_{\mathrm{Z}}^2 M_{\mathrm{A}}^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_{\mathrm{h}} \leq M_{\mathrm{Z}}$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_{h}, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$
\begin{aligned}
g_{\mathrm{hVV}} &= \sin(\beta-\alpha)\, g_{\mathrm{HVV}}^{\mathrm{SM}}, \qquad V = W^{\pm}, Z \\
g_{\mathrm{HVV}} &= \cos(\beta-\alpha)\, g_{\mathrm{HVV}}^{\mathrm{SM}} \\
g_{\mathrm{hAZ}} &= \cos(\beta-\alpha)\, \frac{g'}{2\cos\theta_{\mathrm{W}}} \\
g_{\mathrm{hb\bar{b}}}, g_{\mathrm{h}\tau^+\tau^-} &= -\frac{\sin\alpha}{\cos\beta}\, g_{\mathrm{Hb\bar{b}},\mathrm{H}\tau^+\tau^-}^{\mathrm{SM}} \\
g_{\mathrm{ht\bar{t}}} &= \frac{\cos\alpha}{\sin\beta}\, g_{\mathrm{Ht\bar{t}}}^{\mathrm{SM}} \\
g_{\mathrm{Ab\bar{b}}}, g_{\mathrm{A}\tau^+\tau^-} &= \gamma_5 \tan\beta\, g_{\mathrm{Ab\bar{b}}}^{\mathrm{SM}}
\end{aligned}
$$

$\Rightarrow$ $g_{\mathrm{hVV}} \leq g_{\mathrm{HVV}}^{\mathrm{SM}}$, $\quad g_{\mathrm{hVV}}$, g_{HVV}, g_{hAZ} cannot all be small

$g_{\mathrm{hb\bar{b}}}, g_{\mathrm{h}\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling possible

Higgs mass bounds in SUSY theories

MSSM predicts upper bound on m_{h}:

tree-level bound: $m_{\mathrm{h}} < M_{\mathrm{Z}}$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{\mathrm{e}\, m_{\mathrm{t}}}{2 M_{\mathrm{W}} s_{\mathrm{W}}}, \; \frac{\mathrm{e}\, m_{\mathrm{t}}^2}{M_{\mathrm{W}} s_{\mathrm{W}}}, \; \ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_{\mathrm{t}}^4 \ln \left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2} \right)$, $\mathcal{O}(100\%)$!

Present status of m_{h} prediction in the MSSM:

Complete one-loop and ‘almost complete’ two-loop result available

Upper bound on m_h in the MSSM:

"Unconstrained MSSM":

M_A, $\tan\beta$, 5 parameters in $\tilde{\mathrm{t}}$–$\tilde{\mathrm{b}}$ sector, μ, $m_{\tilde{\mathrm{g}}}$, M_2

Diagrammatic result: *FeynHiggs*

[*S. Heinemeyer, W. Hollik, G. W. '98, '00, '02, '06*]

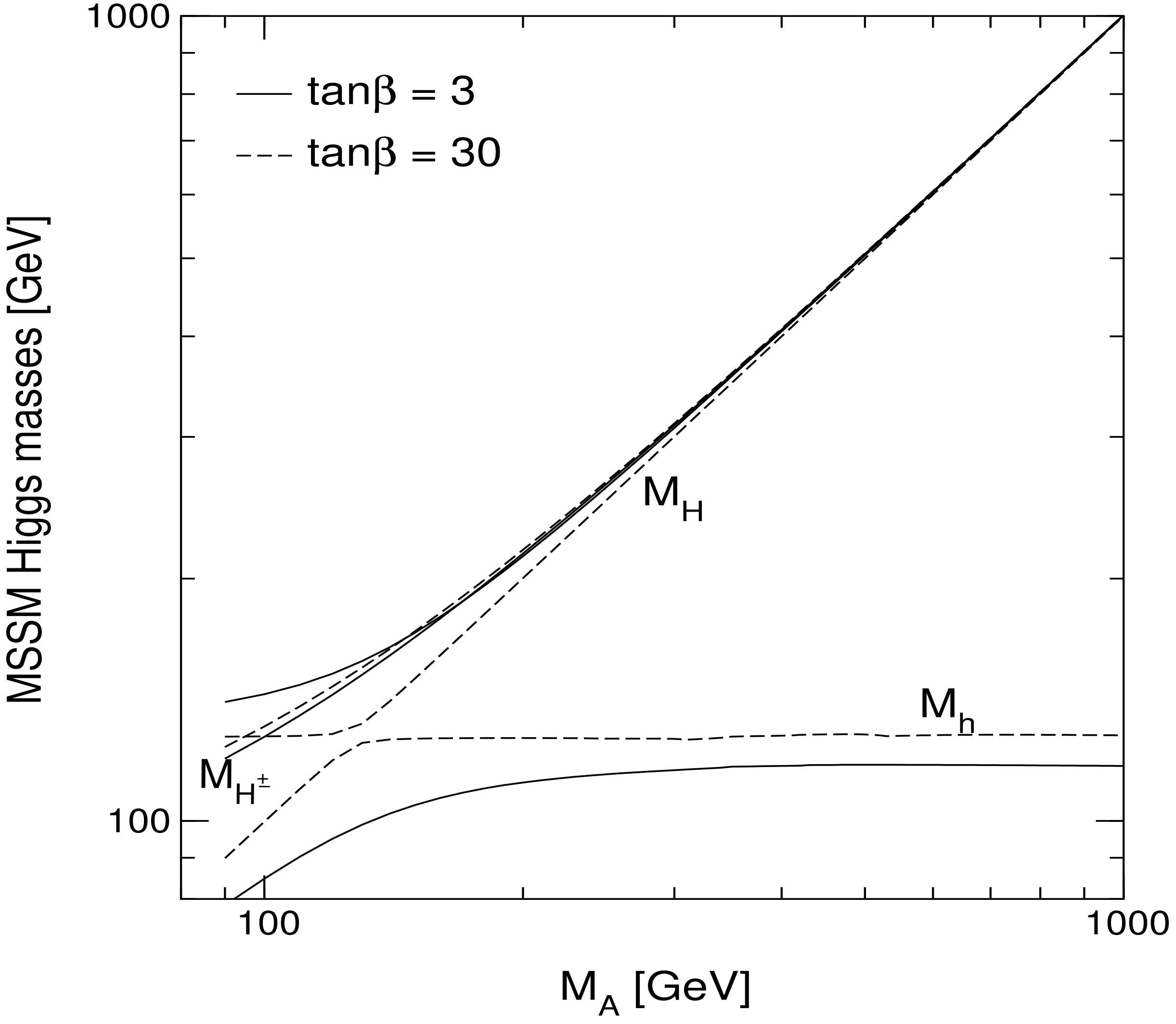
`http://www.feynhiggs.de`

$$m_\mathrm{h} \lesssim 129\,\mathrm{GeV}$$

for $m_\mathrm{t} = 171.4\,\mathrm{GeV}$

no theoretical uncertainties from unknown higher orders included

Upper bound on m_{h} saturated for large M_{A}, large $\tan\beta$, significant mixing in $\tilde{\mathrm{t}}$ sector



Upper bound $m_{\mathsf{h}} \lesssim 131$ GeV reduced by

$\approx 6, 11, 8$ GeV in mSUGRA, GMSB, AMSB scenarios

[*S. Ambrosanio, A. Dedes, S. Heinemeyer, S. Su, G. W. '01*]

Upper bound on m_{h} in extensions of MSSM: $m_{\mathsf{h}} \lesssim 200$ GeV

[*G. Kane, C. Kolda, J. Wells '93*] [*J. Espinosa, M. Quirós '93, '98*]

$\Rightarrow$ **SUSY requires light Higgs boson:**

definite and robust prediction of SUSY models

testable at next generation of colliders

The LSP as a dark matter candidate

Astrophysical data (cosmic microwave background, . . .) $\Rightarrow$ existence of non-baryonic cold dark matter in the Universe

SUSY with R parity conservation $\Rightarrow$ LSP relic density falls naturally in favoured range if $m_{\mathsf{LSP}} \lesssim 1$ TeV

Recent result (WMAP)

[*C. Bennet et al. '03*], [*D. Spergel et al. '03*]

$\Rightarrow$ cold dark matter density: $\Omega_{\mathsf{CDM}} h^2 = 0.1126^{+0.0161}_{-0.0181}$ at 95% C.L.
(h: expansion rate)

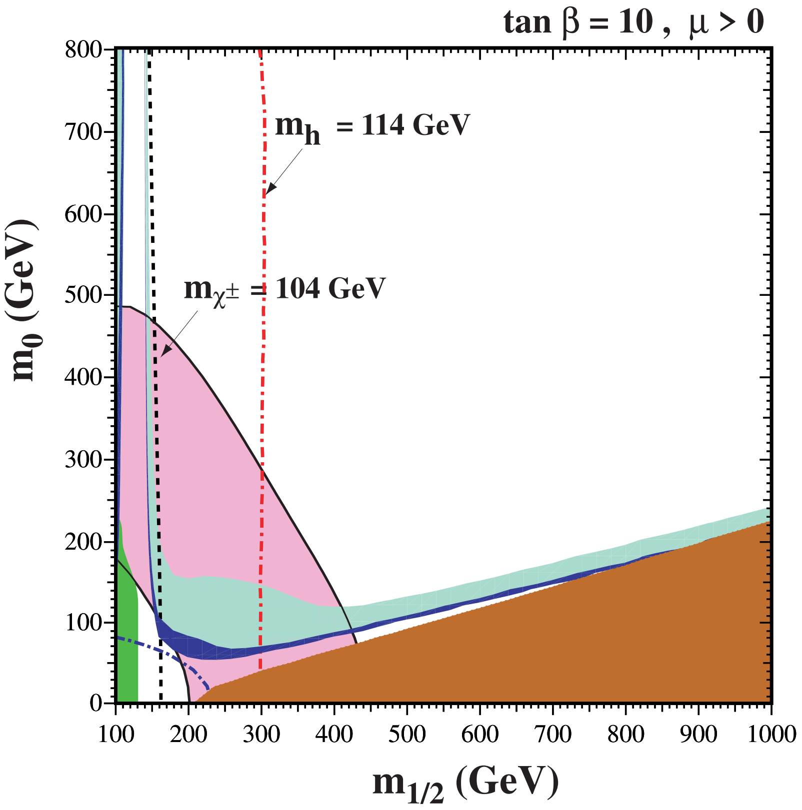
$\Rightarrow$ Constraints on the SUSY parameter space

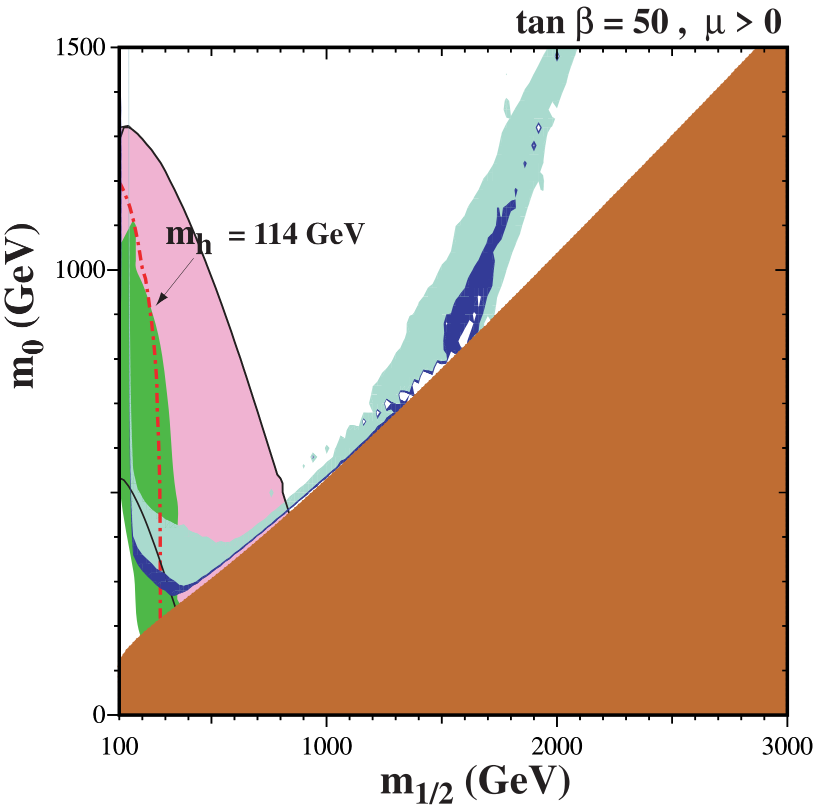
Example: mSUGRA scenario with $\mu > 0$, $A_0 = 0$, $\tan\beta = 10, 50$

Parameter region allowed by the CDM constraint in the $m_{1/2}$–m_0 plane with and without the new WMAP data

(also shown: favoured region from $(g_\mu - 2)$, region excluded by $b \to s\gamma$)

[*J. Ellis, K. Olive, Y. Santoso, V. Spanos '03*]



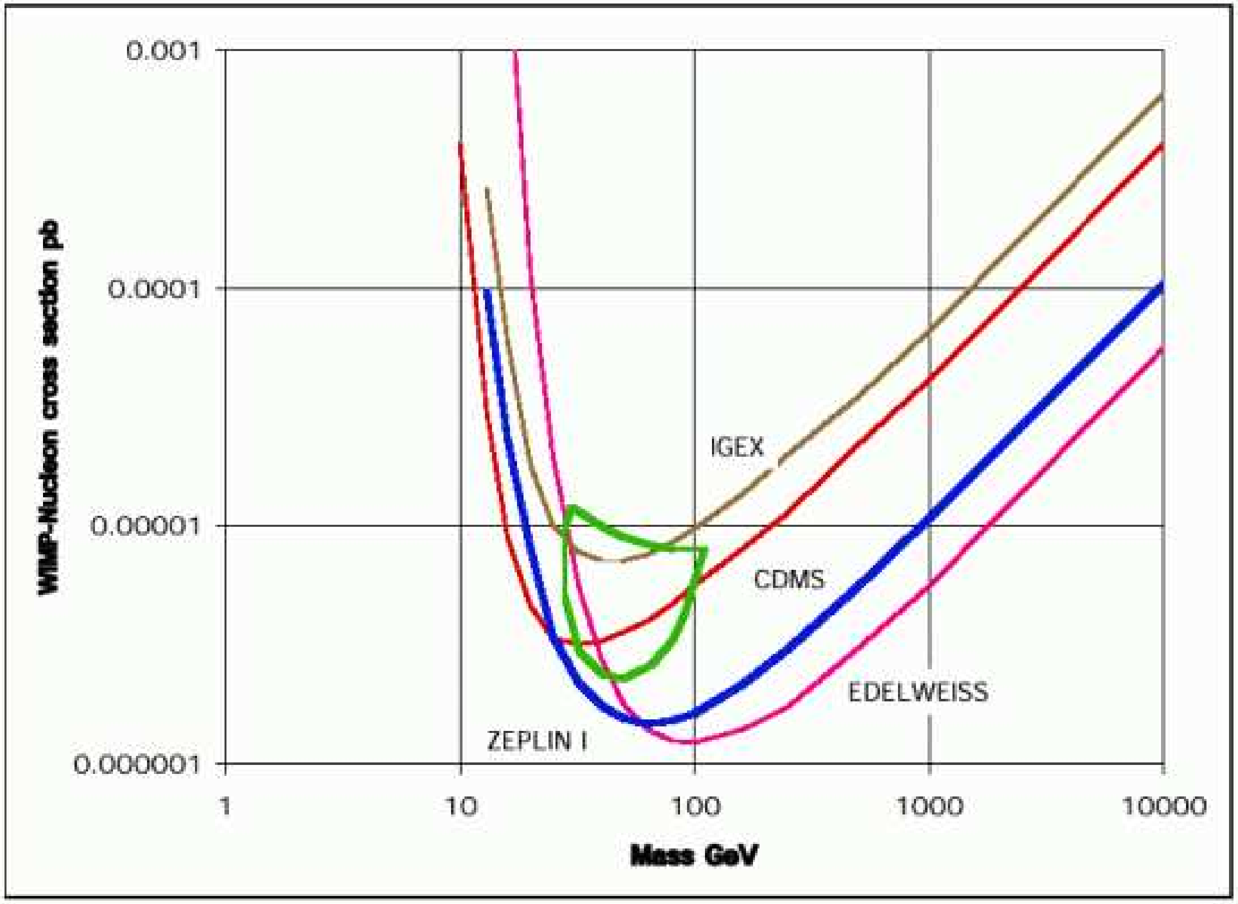


⇒ mSUGRA scenario consistent with all experimental constraints

Direct search for dark matter:

E.g.: scattering off nuclei $\Rightarrow$ measurement of nuclear recoil

Present limits in the plane of m_{WIMP} and the WIMP–nucleon cross section σ_{SI} from different experiments

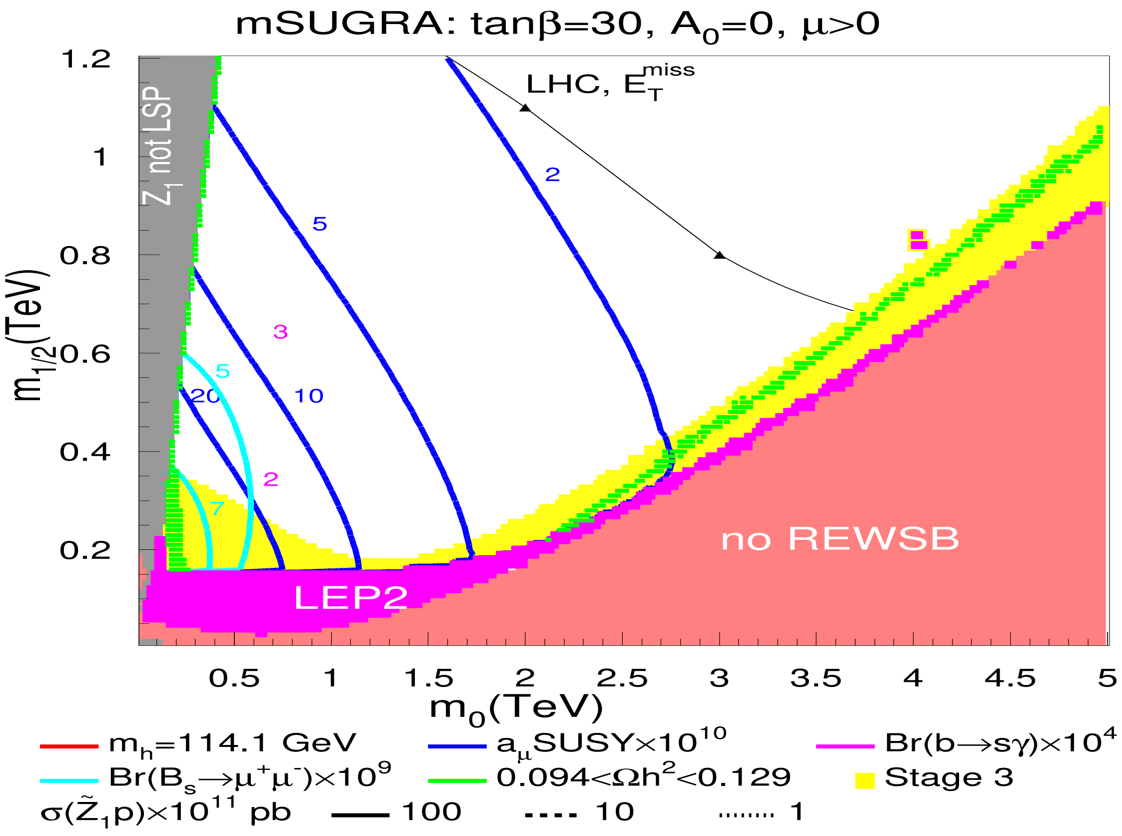


$\Rightarrow$ Present limits on σ_{SI} ("Stage 1" dark matter experiments): $\approx 10^{-5}$–10^{-6} pb

Anticipated sensitivity of possible future dark matter experiments ("Stage 3"): $\sigma_{SI} \approx 10^{-9}$–$10^{-10}$ pb

Accessible parameter region (yellow) in mSUGRA scenario ($\tan\beta = 30$, $\mu > 0$, $A_0 = 0$) with Stage 3 sensitivity vs. CDM allowed region and LHC sensitivity

[*H. Baer, C. Bal'azs, A. Belyaev, J. O'Farrill '03*]



Electroweak precision tests: SM vs. MSSM

Electroweak precision measurements:

$M_{\mathrm{Z}}[\,\mathrm{GeV}]$	$=$	91.1875 ± 0.0021	0.002%
$G_\mu[\,\mathrm{GeV}^{-2}]$	$=$	$1.16637(1)\,10^{-5}$	0.0009%
$m_{\mathrm{t}}[\,\mathrm{GeV}]$	$=$	171.4 ± 2.1	1.2%
$M_{\mathrm{W}}[\,\mathrm{GeV}]$	$=$	80.398 ± 0.025	0.03%
$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$	$=$	0.23150 ± 0.00016	0.07%
$\Gamma_{\mathrm{Z}}[\,\mathrm{GeV}]$	$=$	2.4952 ± 0.0023	0.09%
$\ldots$			

Quantum effects of the theory: loop corrections: $\sim \mathcal{O}(1\%)$

SM: M_{H} is free parameter

precise measurement of M_{W}, $\sin^2\theta_{\mathrm{eff}}$, $\ldots \Rightarrow$ constraints on M_{H}

MSSM: m_{h} is predicted

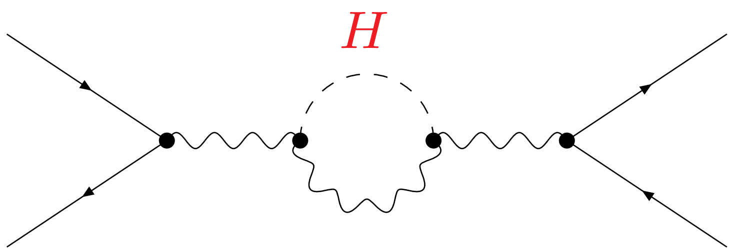
precise meas. of M_{W}, $\sin^2\theta_{\mathrm{eff}}$, m_{h}, $\ldots \Rightarrow$ constr. on $m_{\tilde{\mathrm{t}}}$, $\theta_{\tilde{\mathrm{t}}}$, $m_{\tilde{\mathrm{b}}}$, $\theta_{\tilde{\mathrm{b}}}$, $\ldots$

Comparison of ew precision data with theory:

EW precision data:	Theory:
$M_Z, M_W, \sin^2\theta_{\rm eff}^{\rm lept}, \ldots$	SM, MSSM, . . .

$\Downarrow$

Test of theory at quantum level:



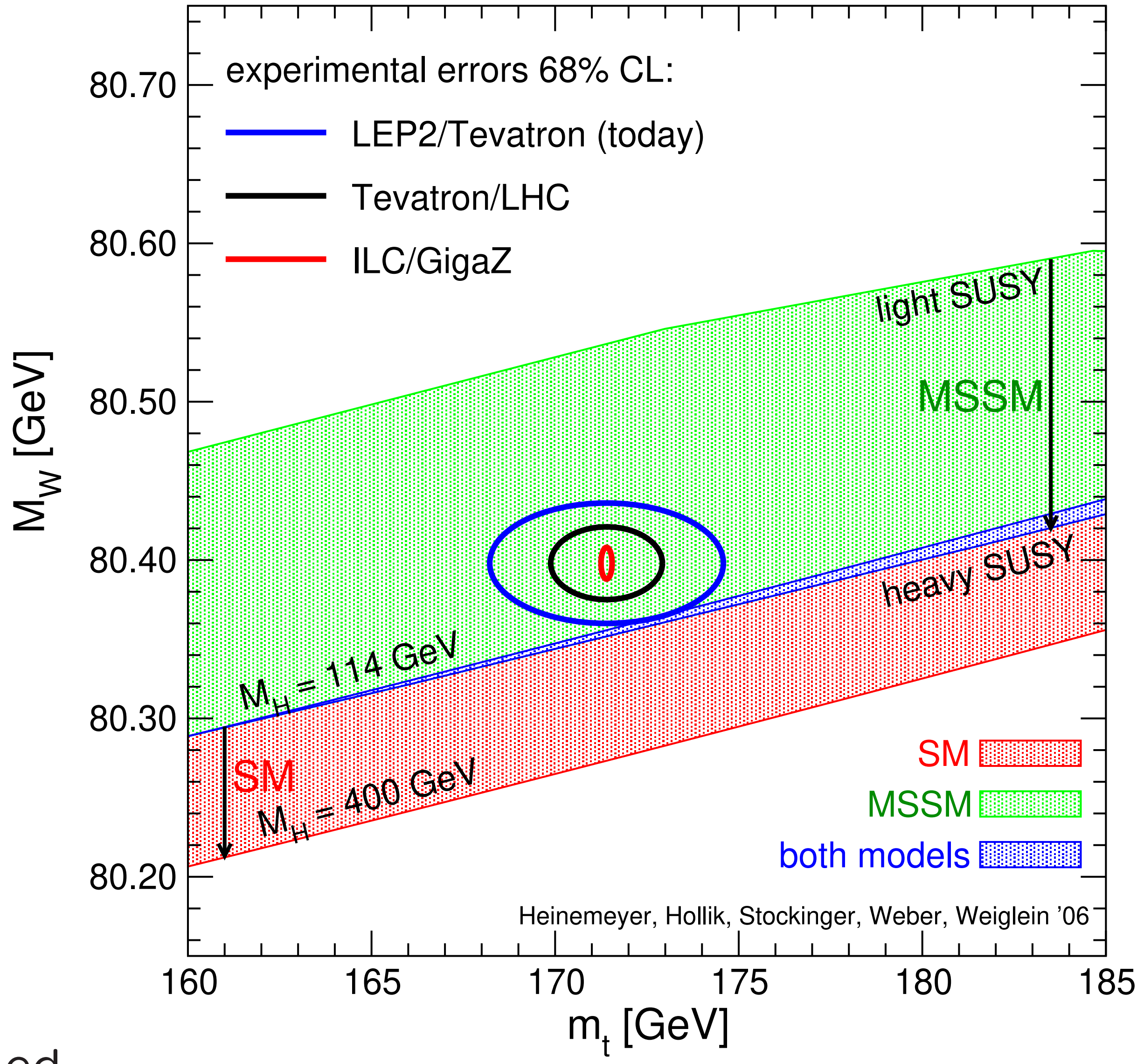
$\Downarrow$

Improve indirect constraints on unknown parameters: M_H, $m_{\tilde{t}}$, . . .

effects of “new physics”?

Electroweak precision tests: SM vs. MSSM

Prediction for M_W in the SM and the MSSM:

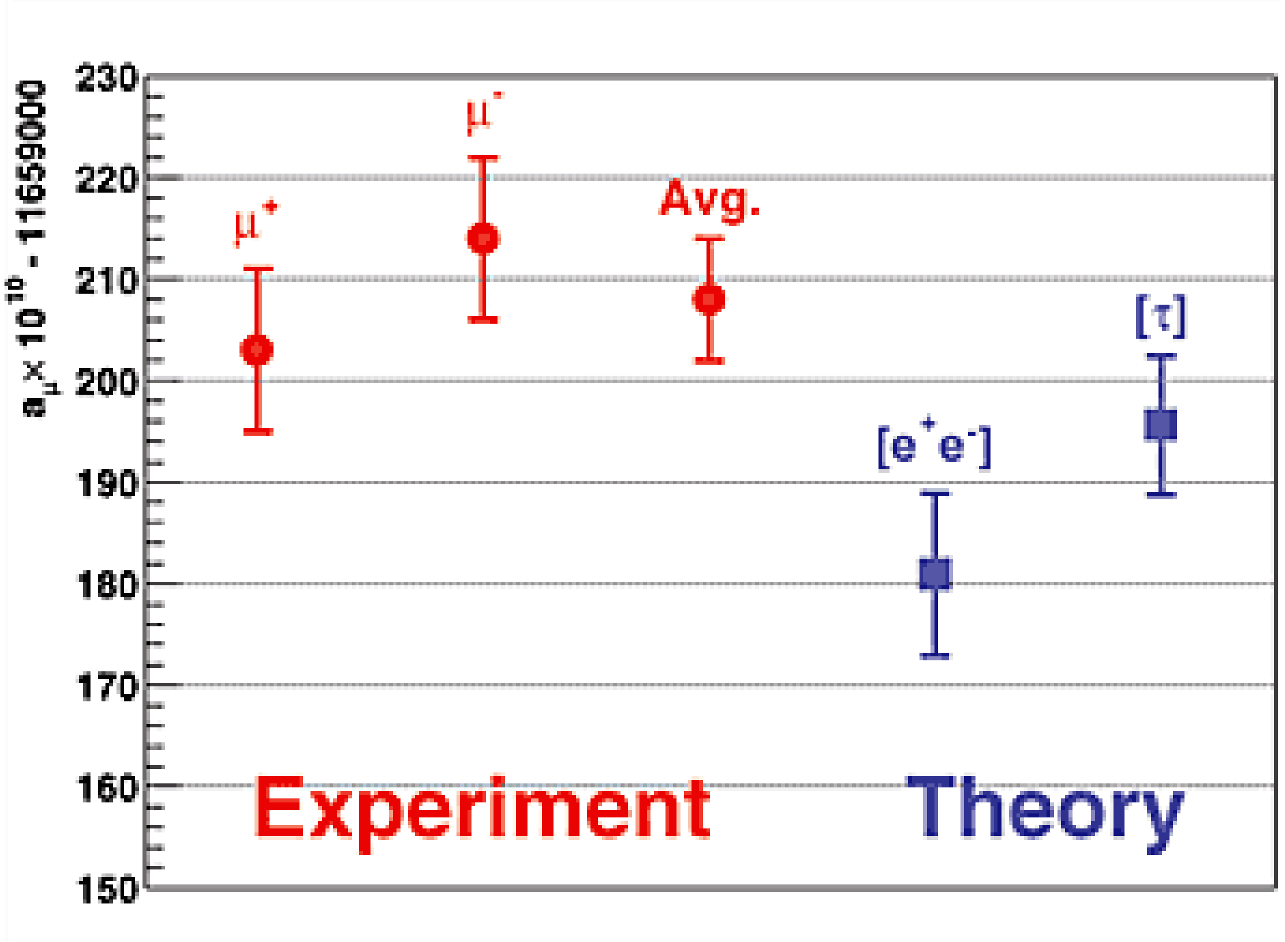


SM: M_H varied

MSSM: SUSY parameters varied

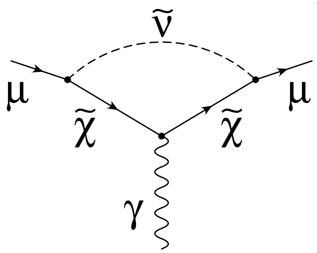
Experimental result for anomalous magnetic moment of the muon vs. SM prediction:

[*Muon $(g-2)$ Collaboration '04*]



$\Rightarrow a_\mu(\text{exp}) - a_\mu(\text{SM})$: 2.7 σ deviation (sizable hadronic uncertainties)

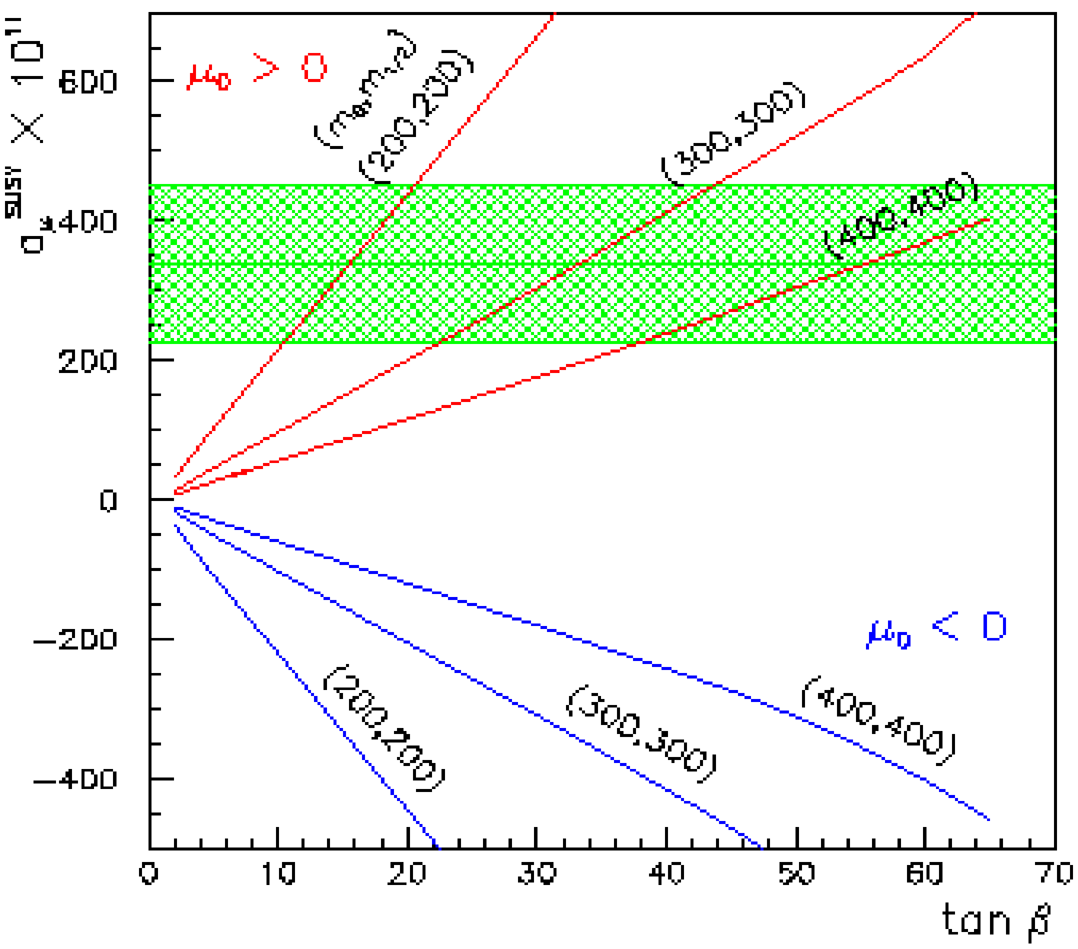
Compatible with effect of SUSY contributions to a_μ:



mSUGRA predictions for $(g_\mu - 2)$:

[*W. de Boer, C. Sander '02*]

$$a_\mu = \frac{1}{2}(g_\mu - 2)$$
$$a_\mu - a_\mu^{\rm SM} = (338 \pm 112) \times 10^{-11}$$



$\Rightarrow$ Preference for $\mu > 0$

note: influenced by hadronic uncertainties of SM result

Prospects for SUSY searches at the next generation of colliders

Limits on SUSY particles from LEP, Tevatron Run I: $\mathcal{O}(100\,\text{GeV})$

But: some SUSY particles can be much lighter if they have small coupling to Z boson

E.g.:

- no strict lower bound on lightest neutralino if "GUT relation" between M_1 and M_2 is relaxed
- A light sbottom with $m_{\tilde{b}_1} \approx 5$ GeV is in agreement with electroweak precision tests, Higgs searches, . . .
  [*M. Carena, S. Heinemeyer, C. Wagner, G. W. '01*]
- . . .

SUSY searches at the Tevatron, Run II:

compared to Run I: $\approx 100\times$ higher luminosity, slightly increased energy (1.8 $\to$ 2 TeV)

Limited mass window in which discovery of SUSY particles above Run I is possible

Best prospects for:

– 'Trilepton signal': $\tilde{\chi}_2^0 \tilde{\chi}_1^+ \to \ell^+ \ell^- \tilde{\chi}_1^0 \ell^+ \nu \chi_1^0$

– $\tilde{t}$, $\tilde{b}$ searches

– light SUSY Higgs h in region of large $\tan\beta$

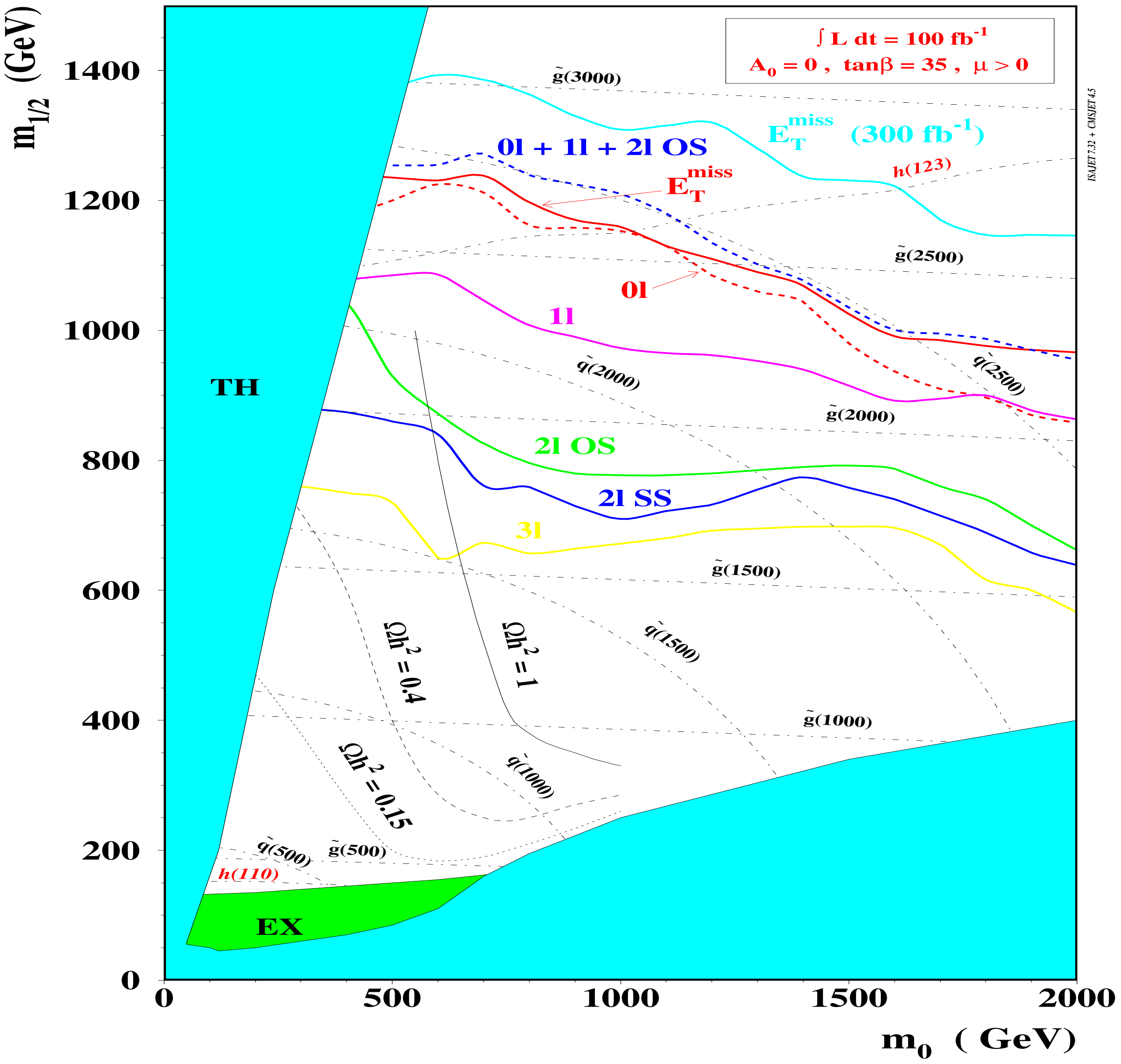
SUSY searches at the LHC:

Dominated by production of colored particles: gluino, squarks

Very large mass range in the searches for jets + missing energy

$\Rightarrow$ gluino, squarks accessible up to 2–3 TeV

Discovery reach contours in m_0–$m_{1/2}$ plane (mSUGRA scenario) for various final states with 100 fb^{-1}: [*CMS '99*]



⇒ discovery of SUSY particles expected if low-energy SUSY is realized

Production of SUSY particles at the LHC will in general result in complicated final states, e.g.

$$\tilde{g} \to \bar{q}\tilde{q} \to \bar{q}q\tilde{\chi}_2^0 \to \bar{q}q\tilde{\tau}\tau \to \bar{q}q\tau\tau\tilde{\chi}_1^0$$

Production of uncolored particles via cascade decays often dominates over direct production

Many states are produced at once

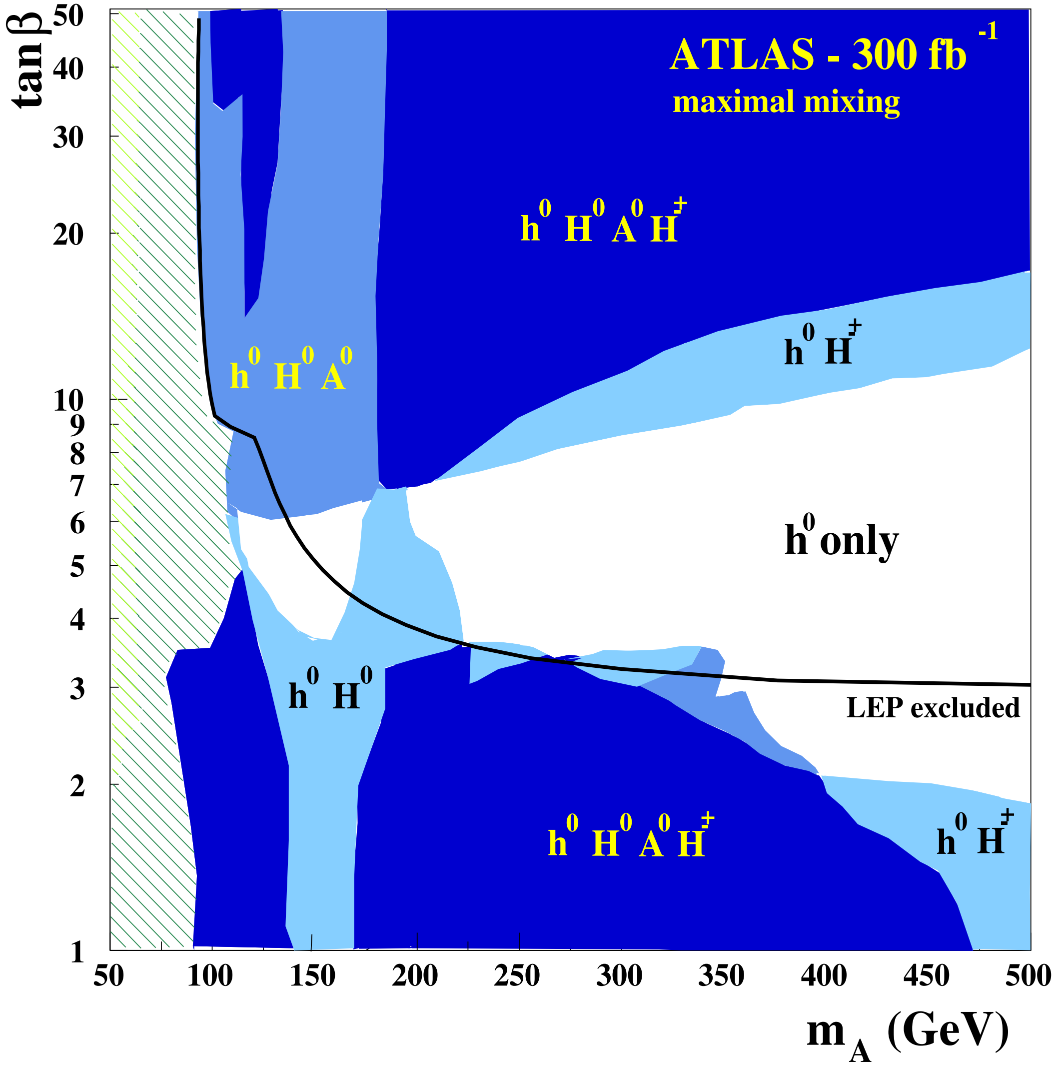
$\Rightarrow$ **Main background for SUSY is SUSY itself!**

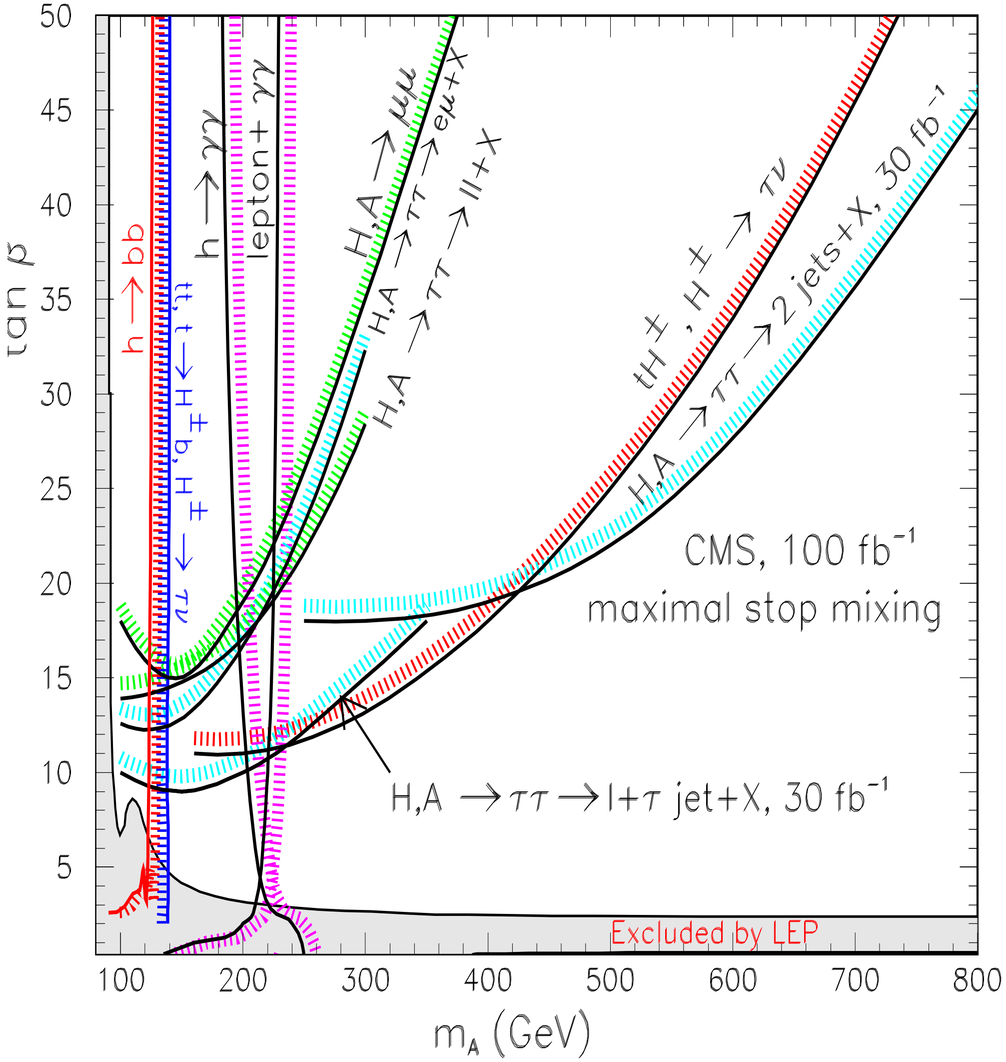
Searches for MSSM Higgs bosons:

good prospects for detecting light Higgs h

H/A discovery possible in significant part of parameter space

MSSM Higgs discovery contours in M_A–$\tan\beta$ plane (m_h^{max} benchmark scenario): [ATLAS '99] [CMS '03]





In order to establish SUSY experimentally:

Need to demonstrate that:

– every particle has superpartner

– their spins differ by 1/2

– their gauge quantum numbers are the same

– their couplings are identical

– mass relations hold

. . .

⇒ Precise measurements of masses, branching ratios, cross sections, angular distributions, . . . mandatory for

- establishing SUSY experimentally
- disentangling patterns of SUSY breaking

Requires clean experimental environment, high luminosity, beam polarization, . . .

SUSY searches at the International Linear Collider (ILC):

Clean signatures, small backgrounds

Thresholds for pair production of SUSY particles

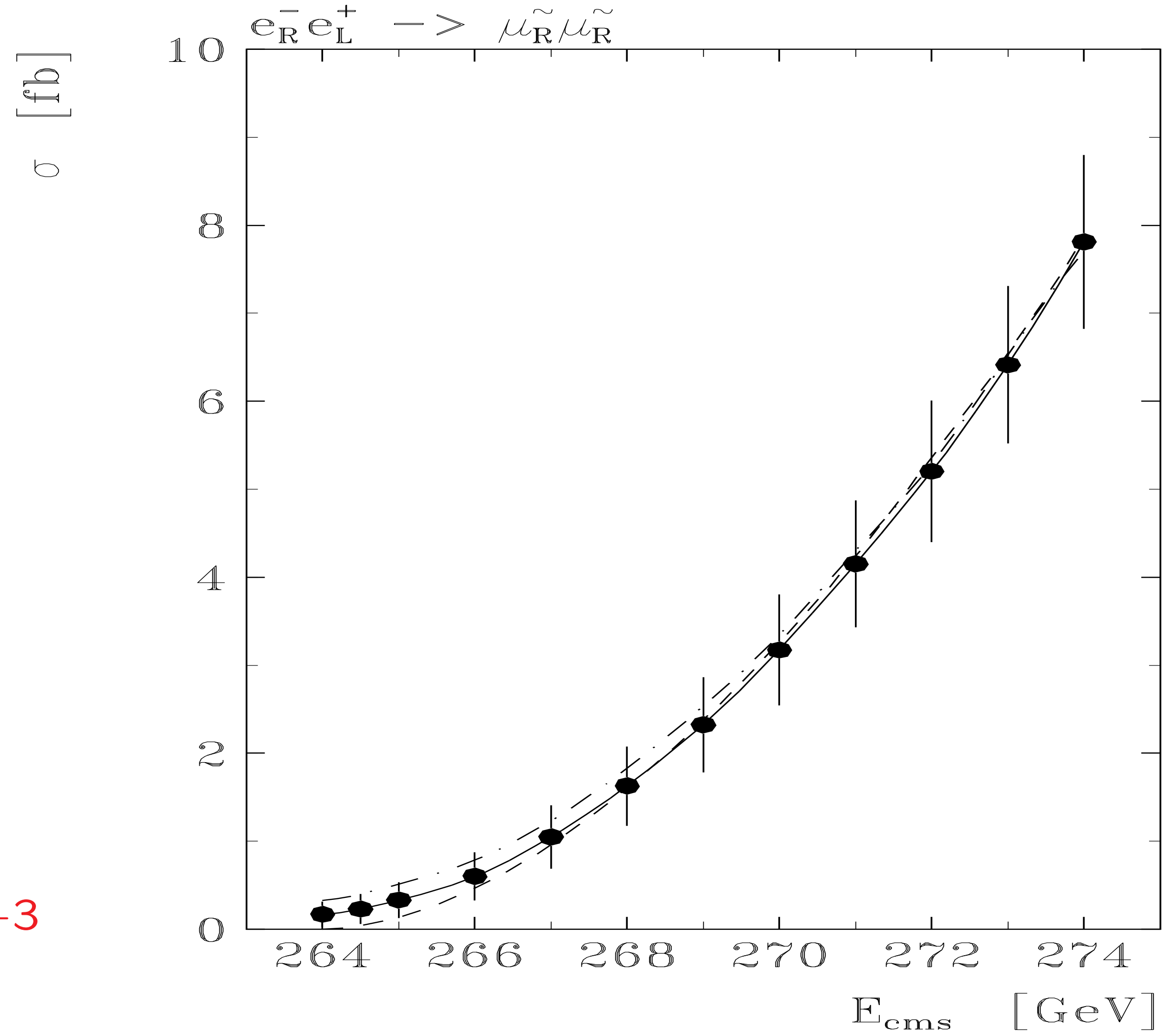
$\Rightarrow$ precise determination of mass and spin of SUSY particles, mixing angles, complex phases, . . .

Good prospects for production of uncolored particles

$\Rightarrow$ LHC / ILC complementarity

Examples for SUSY physics at the ILC:

Determination of mass and spin of $\tilde{\mu}_R$ from production at threshold:
[*TESLA TDR '01*]

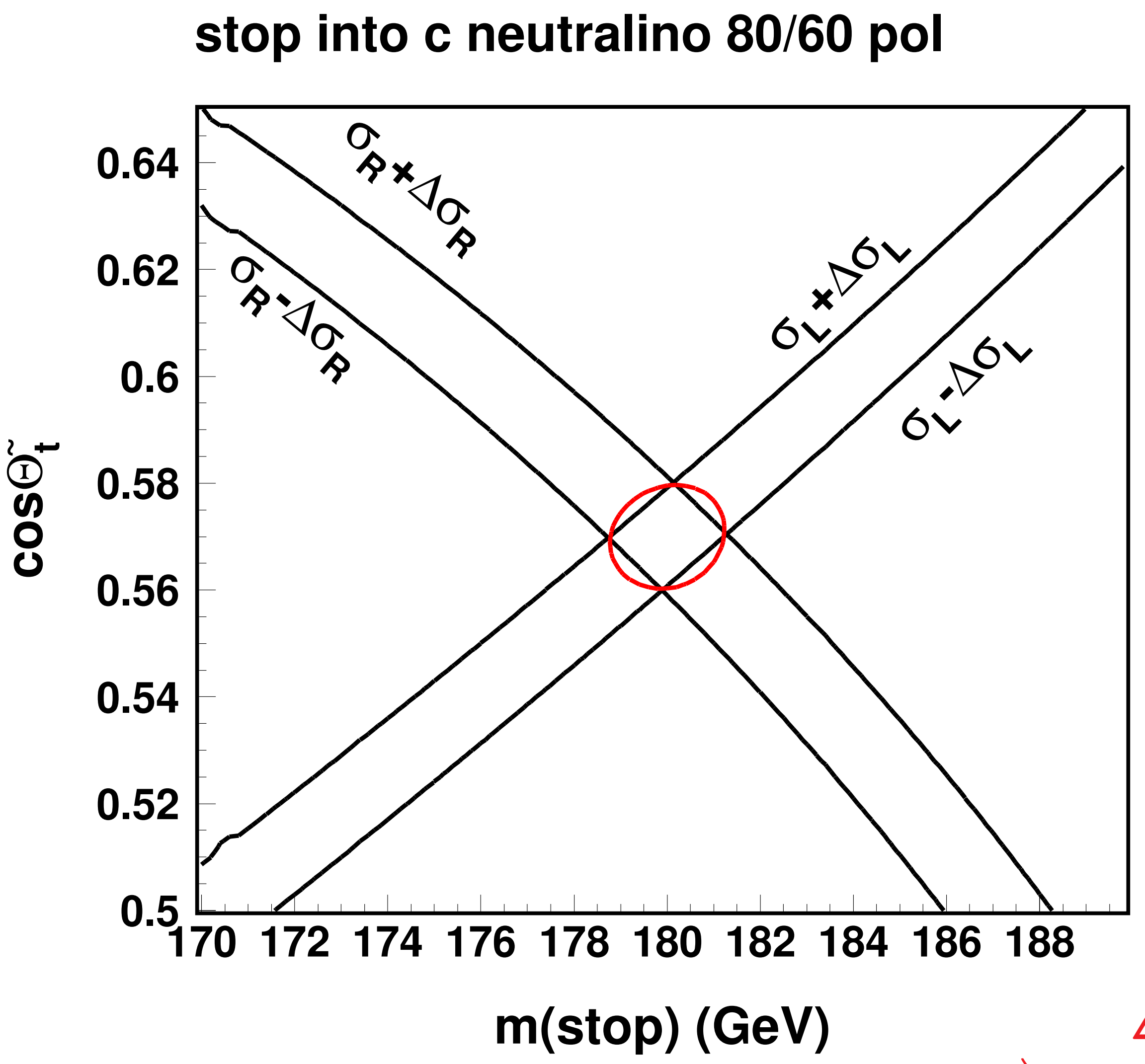


$$\Rightarrow \qquad \frac{\Delta m_{\tilde{\mu}_R}}{m_{\tilde{\mu}_R}} < 1 \times 10^{-3}$$

$\Rightarrow$ test of $J = 0$ hypothesis

Determination of $m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$ from $\sigma(e^+e^- \to \tilde{t}_1\tilde{t}_1)$ with polarized beams:

[*R. Keränen, H. Nowak, A. Sopczak '00*]



$$\Rightarrow \quad \frac{\Delta m_{\tilde{t}_1}}{m_{\tilde{t}_1}} \approx 0.5\%, \qquad \frac{\Delta \cos\theta_{\tilde{t}}}{\cos\theta_{\tilde{t}}} \approx 1.5\%$$

Outlook:

- Low-energy Supersymmetry continues to be our best bet for physics beyond the Standard Model

- Data rules:

  We need experimental information from Tevatron, LHC, ILC,
  ν experiments, dark matter searches, low-energy experiments, ...
  to verify / falsify models of new physics understanding

  The LHC is scheduled to start data taking a 14 TeV next year!

- The experiments in the next years will bring a decisive test of our ideas about low-energy SUSY

$\Rightarrow$ Very exciting prospects for the coming years