Lectures on Supersymmetry

Georg Weiglein
OC 204

Part 1:

- Limitations of the Standard Model
- The SUSY algebra
- Superspace and superfields
- Construction of SUSY field theories
- The Minimal Supersymmetric Standard Model (MSSM)

The Standard Model (SM) cannot be the ultimate theory

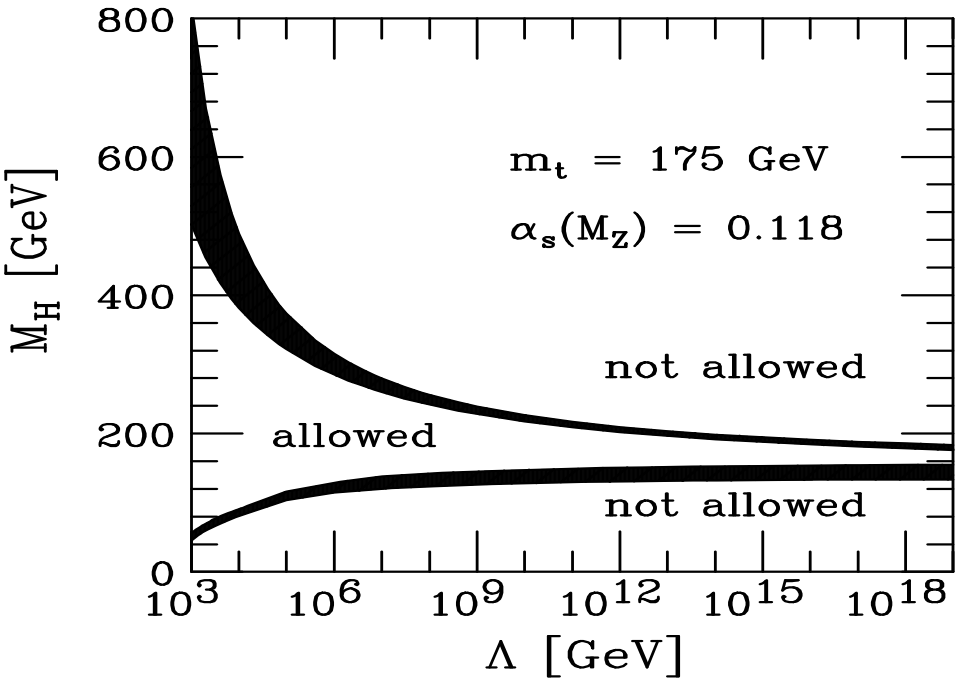
– doesn't contain gravity

– further problems: "hierarchy problem", ...

Up to which energy scale Λ can it be valid?

– $\Lambda < M_{\mathrm{Pl}} \approx 10^{19}$ GeV (inclusion of gravity effects necessary)

– stability of Higgs potential:
[*T. Hambye, K. Riesselmann '97*]



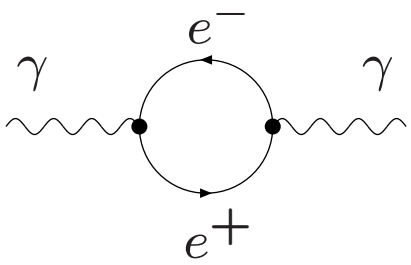
– hierarchy problem: Higgs mass unstable w.r.t. large quantum corrections
$\Rightarrow \delta M_{\mathrm{H}}^2 \sim \Lambda^2$

1. The hierarchy problem

Consider loop corrections to propagators $\Leftrightarrow$ corrections to particle masses

$$\Delta(p^2) \sim \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

Photon self-energy in QED:

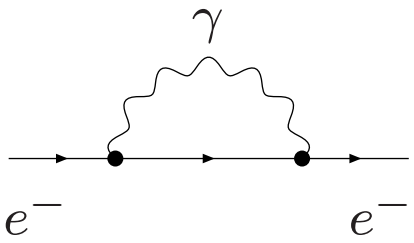


$$\Sigma^{\gamma\gamma}(0) = 0$$

consequence of U(1) gauge invariance of QED $\Rightarrow$ photon stays massless

$$\Delta^{-1}_{\gamma\gamma}(p^2) \to 0 \text{ for } p^2 \to 0$$

Electron self-energy in QED:



for $\Lambda \to \infty$: $\Sigma^{ee} \sim m_e \int^{\Lambda} \frac{dk}{k} \to \ln \Lambda$

$\Rightarrow$ logarithmically divergent correction to electron mass, δm_e

Within QED: divergence can be removed via renormalization
$\Rightarrow k \to \infty$ possible

QED as effective theory, underlying more fundamental theory at scale Λ
$\Rightarrow$ cutoff scale Λ

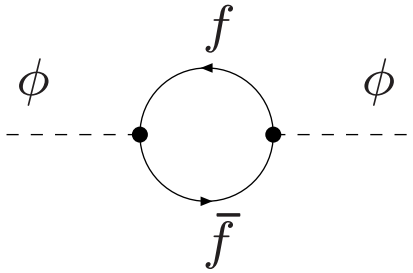
For $\Lambda = M_{Pl}$: $\delta m_e \approx 2 \frac{\alpha}{\pi} m_e \log(M_{Pl}/m_e) \approx 0.2\, m_e$

$\Rightarrow$ modest correction, proportional to m_e

reason: chiral symmetry in limit $m_e \to 0$, $\psi_e \to e^{i\gamma_5 \theta} \psi_e$

$\Rightarrow$ breaking proportional to m_e $\Rightarrow$ symmetry "protects" m_e

Contribution of heavy fermions to Higgs self-energy:



$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

for $\Lambda \to \infty$:

$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_P$: $\delta M_\phi^2 \sim M_P^2 \Rightarrow \delta M_\phi^2 \approx 10^{30}\, M_\phi^2$ ($M_\phi \lesssim 1$ TeV)

no additional symmetry for $M_\phi = 0$, no protection against large corrections

$\Rightarrow$ in general: scalar masses tend to be near highest mass scale of the theory

$\Rightarrow$ hierarchy problem, extreme fine-tuning necessary to get small M_ϕ

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_\phi^2 \approx \lambda \langle v_{\mathsf{GUT}} \rangle^2$

Even if $\lambda = 0$ at tree level: non-zero coupling regenerated by radiative corrections: $\delta M_\phi^2 \approx \mathcal{O}(\alpha/\pi) \langle v_{\mathsf{GUT}} \rangle^2$ (cf. 'Little Higgs' models)

Would need a symmetry to suppress *many* orders of perturbation theory

Hierarchy problem is not just a problem of the Higgs mass;

problem: why is $M_{\mathsf{W}} \ll M_{\mathsf{GUT}}, M_{\mathsf{Pl}}$, why is $V_{\mathsf{Coulomb}} \gg V_{\mathsf{Newton}}$?

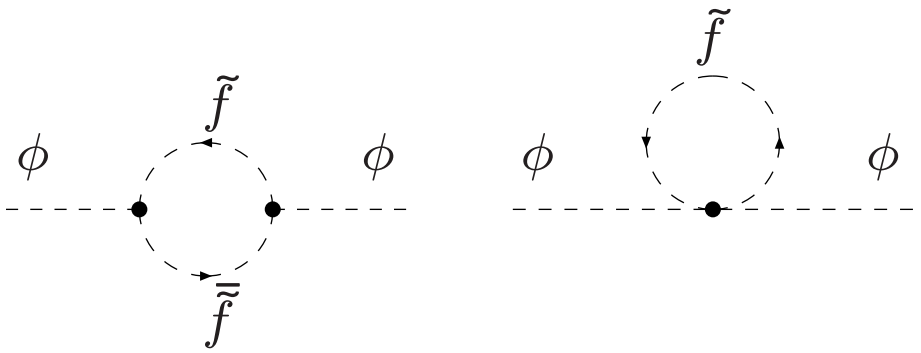
Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

SUSY: additional contributions from scalar fields:



$$\Sigma_{\tilde{f}}^{\phi\phi} \sim N(\tilde{f})\,\tilde{\lambda}_f \int d^4k \left(\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right) + \text{terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_{\tilde{f}}^{\phi\phi} \sim 2\, N(\tilde{f})\,\tilde{\lambda}_f\,\Lambda^2$

$\Rightarrow$ quadratic divergencies cancel for

$$N(\tilde{f}_L) = N(\tilde{f}_R) = N(f)$$
$$\tilde{\lambda}_f = \lambda_f^2$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

For $m_{\tilde{f}}^2 = m_f^2 + \Delta^2$, $\tilde{\lambda}_f = \lambda_f^2$, "soft SUSY breaking"

$$\Rightarrow \Sigma_{f+\tilde{f}}^{\phi\phi} \sim N(f)\, \lambda_f^2\, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Other recent proposals for solving hierarchy problem in models with extra spatial dimensions:

E.g.: Gravity in large extra dimensions

[*N. Arkani-Hamed, S. Dimopoulos, G. Dvali '98*]

Gravity propagates in δ large extra spatial dimensions

fundamental Planck scale in $4+\delta$-dimensional space, M_*, is of order 1 TeV

$$M_{\mathrm{Pl}}^2 = V_\delta \, M_*^{2+\delta}$$

$\Rightarrow$ large value of M_{Pl} related to relatively large volume V_δ (10^{-3} m–10^{-15} m for $\delta = 2$–6) of extra dimensions

2. The SUSY algebra

Symmetry: a group of transformations that leave Lagrangian invariant

generators of the group fulfill certain algebra

Noether's theorem: symmetries $\Leftrightarrow$ conservation laws

Unification of fundamental interactions?

electroweak and strong interactions:

described by gauge theories: internal symmetries

γ, Z, $W^{\pm}$: spin 1

gravity:

described by general relativity: invariance under space–time transformations

graviton G: spin 2

Space–time symmetries: Poincaré group

Lorentz transformations:

$M^{\rho\nu} = -M^{\nu\rho}$ (6 generators), translations P^{ρ} (4 generators)

continous group: Lie-group

particle states characterized by eigenvalues of mass and spin

For later use it is convenient to describe spin $\frac{1}{2}$ particles not in terms of 4-component 'Dirac spinors' but in terms two 2-component 'Weyl spinors'

Spinor representation of the Lorentz group

Form (complex) linear combinations of the generators of rotations and Lorentz boosts such that commutation relations become

$$\left[J_i^{\pm}, J_j^{\pm}\right] = i\epsilon_{ijk}J^{\pm,k}; \quad \left[J_i^{+}, J_j^{-}\right] = 0$$

$\Rightarrow$ two SU(2) algebras, commute with each other

$\Rightarrow$ Lorentz group is equivalent to a (complexified) version of SU(2) $\times$ SU(2) ($\rightarrow$ SL(2, C))

$\Rightarrow$ Representations of Lorentz group are labelled by two 'spins', j_1, j_2, where $j_1, j_2 = 0, \frac{1}{2}, 1, \ldots$ as usual for SU(2)

Basic representations:

$(\frac{1}{2}, 0)$: LEFT-handed 2-component Weyl spinor, ψ_α

$(0, \frac{1}{2})$: RIGHT-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

The two component Weyl spinors ψ_α (left-handed) and $\bar{\psi}^{\dot{\alpha}}$ (right-handed) transform under Lorentz transformations as follows:

$$\begin{aligned} \psi'_\alpha &= M_\alpha{}^\beta \psi_\beta; & \bar{\psi}'_{\dot{\alpha}} &= (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} \\ \psi'^\alpha &= (M^{-1})_\beta{}^\alpha \psi^\beta; & \bar{\psi}'^{\dot{\alpha}} &= (M^{*-1})_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} \end{aligned}$$

where $M = \exp(i\frac{\vec{\sigma}}{2}(\vec{\vartheta} - i\vec{\varphi}))$ and $\vec{\vartheta}$ and $\vec{\varphi}$ are the three rotation angles and boost parameters, respectively

$\Rightarrow$ spinors with undotted indices (first two components of Dirac spinor) transform according to $(\frac{1}{2}, 0)$-representation of Lorentz group,

spinors with dotted indices (last two components of Dirac spinor) transform according to $(0, \frac{1}{2})$-representation

Internal symmetries, e.g. gauge symmetries

QED: $\psi \to e^{iQ\lambda}\psi$, $A_\mu \to A_\mu + \partial_\mu \lambda$, generator: Q, group U(1)

QCD: SU(3), SM: SU(2) $\times$ U(1)

generators T^a of internal symmetries satisfy Lie algebra, $\left[T^a, T^b\right] = i f^{abc} T_c$

'Trivial' extension of Poincaré group, T^a commute with Poincaré generators

$[P^\mu, T^a] = 0$, $[M^{\mu\nu}, T^a] = 0$

$\Rightarrow$ direct product

(Poincaré group) $\otimes$ (internal symmetry group)

Particle states characterized by maximal set of commuting observables:

$$|\underbrace{m, s; \vec{p}, s_3}; \underbrace{Q, I, I_3, Y, \ldots}\rangle$$

space–time internal

quantum numbers

Direct product $\Rightarrow$ no irreducible multiplets containing particles with different mass or different spin

$\Rightarrow$ Look for extension of space–time symmetry with new generators Q_α such that

$[Q_\alpha, M^{\mu\nu}] \neq 0$ or $[Q_\alpha, P^\mu] \neq 0$

But: general "no go theorems" of QFT

Coleman–Mandula theorem '67:

Any Lie-group containing Poincaré group P and internal symmetry group G must be direct product $P \otimes G$

No go theorem can be evaded if instead of Lie-group (generators fulfill commutator relations):

[*Gol'fand, Likhtman '71*] [*Volkov, Akulov '72*] [*Wess, Zumino '73*]

$$[\ldots, \ldots] \rightarrow \{\ldots, \ldots\}$$

Anticommutator: $\{A, B\} = AB + BA$

$\Rightarrow$ 'graded Lie algebra', 'superalgebra'

Haag, Lopuszanski, Sohnius theorem '75:

no direct symmetry transformations between fields with different integer spins

$\Rightarrow$ particles with different spin in the same multiplet only possible for SUSY theories, $Q|\text{boson}\rangle = |\text{fermion}\rangle$, $Q|\text{fermion}\rangle = |\text{boson}\rangle$

symmetry generator Q: fermionic operator, needs to have spin 1/2

E.g.:

$$\begin{array}{ccccc} \text{spin } 2 & \rightarrow & \text{spin } \frac{3}{2} & \rightarrow & \text{spin } 1 \\ \text{graviton} & & \text{gravitino} & & \text{photon} \end{array}$$

Q changes spin (behavior under spatial rotations) by $\frac{1}{2}$

$\Rightarrow$ SUSY transformation influences in general both space–time and internal quantum numbers

Simplest case:
only one fermionic generator Q_α (and conjugate $\bar{Q}_{\dot{\beta}}$, $\alpha = 1, 2$, $\dot{\beta} = 1, 2$)

$\Rightarrow$ $N = 1$ SUSY algebra:

$$
\begin{aligned}
[Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot{\beta}}, P_\mu\right] &= 0 \\
[Q_\alpha, M^{\mu\nu}] &= i\,(\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta \\
\left\{Q_\alpha, Q_\beta\right\} = \left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} &= 0 \\
\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu
\end{aligned}
$$

Haag, Lopuszanski, Sohnius theorem $\Rightarrow$ only possibility

$P_0 = H$, $[Q_\alpha, H] = 0$ $\Rightarrow$ conserved charge

$\Rightarrow$ SUSY: symmetry that relates bosons to fermions

unique extension of Poincaré group of symmetries of $D = 4$ relativistic QFT

'generalizes classical notion of a dimension to quantum domain'

'so beautiful it must be true . . . '

Consequences of the SUSY algebra

Global SUSY transformation:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = \underbrace{2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu}_{\text{constant translation in space–time}}$$

If SUSY transformations are made local

$\Rightarrow$ space–time transformation differing from point to point

Invariance under local SUSY transformations:

$\Rightarrow$ invariance under local coordinate change

$\Rightarrow$ general relativity

$\Rightarrow$ local SUSY includes gravity, called "supergravity"

In the following: mostly global SUSY transformations considered (flat space–time)

Q_α changes spin of particle by $\frac{1}{2}$

$Q_\alpha|\text{boson}\rangle = |\text{fermion}\rangle$, $Q_\alpha|\text{fermion}\rangle = |\text{boson}\rangle$

Consider fermionic state $|f\rangle$ with mass m: bosonic state $|b\rangle = Q_\alpha|f\rangle$

$P^2|f\rangle = m^2|f\rangle$

$\Rightarrow P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle$

$\Rightarrow$ for each fermionic state there is a bosonic state with the same mass

$\Rightarrow$ states are paired bosonic $\leftrightarrow$ fermionic

More on spinors:

The components of the spinors are Grassmann variables, i.e. anticommuting c-numbers

Raising and lowering of indices through the totally antysymmetric ϵ-tensor:

$$\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \; \epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta; \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta; \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}; \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}};$$

In particular

$$\epsilon^\alpha{}_\beta = \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^\alpha{}_\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of two Grassmann spinors is defined through

$$\theta\zeta \equiv \theta^\alpha\zeta_\alpha = \theta^\alpha\epsilon_{\alpha\beta}\zeta^\beta = -\epsilon_{\alpha\beta}\zeta^\beta\theta^\alpha = \zeta^\beta\epsilon_{\beta\alpha}\theta^\alpha = \zeta^\beta\theta_\beta = \zeta\theta$$

$$\bar{\theta}\bar{\zeta} \equiv \bar{\theta}_{\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}} = -\bar{\zeta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}} = \bar{\zeta}\bar{\theta}$$

The γ-matrices are defined by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot\alpha} \\ (\bar\sigma^\mu)^{\dot\alpha\alpha} & 0 \end{pmatrix}; \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

where $(\sigma^\mu)_{\alpha\dot\alpha} \equiv \{1, \sigma^1, \sigma^2, \sigma^3\}_{\alpha\dot\alpha}$; $(\bar\sigma^\mu)_{\alpha\dot\alpha} \equiv \{1, -\sigma^1, -\sigma^2, -\sigma^3\}_{\alpha\dot\alpha}$;

$(\sigma_\mu)_{\alpha\dot\alpha} = g_{\mu\nu}(\sigma^\nu)_{\alpha\dot\alpha}$; $(\bar\sigma_\mu)_{\alpha\dot\alpha} = g_{\mu\nu}(\bar\sigma^\nu)_{\alpha\dot\alpha}$.

The convention for the metric is: $g^{\mu\nu} = g_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$.

The γ-matrices have the usual commutation relation, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

This follows from

$$(\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma^\nu)^{\dot\alpha\alpha} = \text{Tr}(\sigma^\mu\bar\sigma^\nu) = 2g^{\mu\nu}.$$

Furthermore,

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu), \\ \bar\sigma^{\mu\nu} &= \frac{1}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu). \end{aligned}$$

The Lorentz covariant expressions involving 4-component Dirac spinors can be written in two-component notation using

$$\Phi = \begin{pmatrix} \lambda_\alpha \\ \bar{\phi}^{\dot{\alpha}} \end{pmatrix}; \quad \overline{\Psi} = \begin{pmatrix} \chi^\alpha & \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$$

$$\begin{aligned}
\overline{\Psi}\Phi &= \chi\lambda + \bar{\psi}\bar{\phi} = \chi^\alpha \lambda_\alpha + \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^5\Phi &= \bar{\psi}\bar{\phi} - \chi\lambda = \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \chi^\alpha\lambda_\alpha \\
\overline{\Psi}\gamma^\mu\Phi &= \chi\sigma^\mu\bar{\phi} - \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^5\Phi &= \chi\sigma^\mu\bar{\phi} + \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} + \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^\nu\Phi &= \chi\sigma^\mu\bar{\sigma}^\nu\lambda + \bar{\psi}\bar{\sigma}^\mu\sigma^\nu\bar{\phi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma})^{\dot{\alpha}\beta}\lambda_\beta + \bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}(\sigma^\nu)_{\beta\dot{\beta}}\bar{\phi}^{\dot{\beta}}
\end{aligned}$$

Further consequences of the SUSY algebra

$$\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$$

$$\Rightarrow \left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} \bar{\sigma}_\nu^{\dot{\beta}\alpha} = 2 \underbrace{\sigma^\mu_{\alpha\dot{\beta}} \bar{\sigma}_\nu^{\dot{\beta}\alpha}}_{2g^\mu{}_\nu} P_\mu = 4P_\nu$$

$$\nu = 0 \Rightarrow H = P_0 = \tfrac{1}{4}\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} \bar{\sigma}_0^{\dot{\beta}\alpha} = \tfrac{1}{4}\left(\left\{Q_1, Q_1^\dagger\right\} + \left\{Q_2, Q_2^\dagger\right\}\right)$$

where $\bar{Q}_{\dot{\alpha}} = (Q_\alpha)^\dagger$

$\left\{Q_i, Q_i^\dagger\right\} = Q_i Q_i^\dagger + Q_i^\dagger Q_i$: hermitean operator, eigenvalues ≥ 0

$\Rightarrow$ for any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$

spectrum of H is bounded from below, ≥ 0

$\Rightarrow$ no negative eigenvalues

State with lowest energy: vacuum state $|0\rangle$

if vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q

$\Rightarrow$ vacuum has zero energy, $\langle 0|H|0\rangle = E_{\text{vac}} = 0$

For spontaneous symmetry breaking: vacuum state is not invariant

$\Rightarrow$ If (global) SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$,
then $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ non-vanishing vacuum energy

SUSY multiplets:

Particularly important: massless representations

go to reference frame where $P^\mu = (E, 0, 0, E)$

$\Rightarrow Q_1, Q_{\dot{1}} = 0$ as operators; thus left with $Q_2, Q_{\dot{2}}$ only

define $a = \frac{1}{\sqrt{4E}} Q_2$, $a^\dagger = \frac{1}{\sqrt{4E}} \bar{Q}_{\dot{2}}$

$\Rightarrow$ annihilation and creation operators: $\{a, a^\dagger\} = 1, \{a, a\} = 0, \{a^\dagger, a^\dagger\} = 0$

start with state of lowest helicity λ_0

application of $a^\dagger \Rightarrow$ one additional state with helicity $\lambda_0 + \frac{1}{2}$

$\Rightarrow$ one fermionic + one bosonic state

(N SUSY generators $\Rightarrow 2^{N-1}$ bosonic and 2^{N-1} fermionic states)

$\Rightarrow$ equal number of bosonic and fermionic states in supermultiplet

Most relevant multiplets (possess also CPT conjugate 'mirrors'):

- chiral supermultiplet: $-\frac{1}{2}, 0$

  Weyl fermion (quark, lepton, . . .) + complex scalar (squark, slepton)

- vector supermultiplet: $-1, -\frac{1}{2}$

  Gauge boson (massless vector) + Weyl fermion (gaugino)

- graviton supermultiplet: $-2, -\frac{3}{2}$

  graviton + gravitino

3. Superspace and superfields

For compact representation of SUSY transformations

$\Rightarrow$ 'fermionic variables' θ^α, $\bar{\theta}^{\dot{\alpha}}$ introduced: anticommuting c-numbers (Grassmann variables)

E.g.: translations, generators P_μ, parameters of transformation x^μ

$N = 1$ SUSY, generators Q_α, $\bar{Q}_{\dot{\alpha}}$, parameters of transformation θ^α, $\bar{\theta}^{\dot{\alpha}}$

$\Rightarrow$ Extension of 4-dim. space–time by coordinates θ^α, $\bar{\theta}^{\dot{\alpha}}$: superspace

Point in superspace: $X = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Superfield: $\phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Taylor expansion in Grassmann variable: $\theta^\alpha\theta^\beta\theta^\gamma = 0$

$\Rightarrow$ Taylor expansion terminates after second term, i.e. $\phi(\theta) = a + \theta\psi + \theta\theta\, f$
$(\theta\theta \equiv \theta^\alpha\theta_\alpha)$

Integration: $\int d\theta = 0,\ \int d\theta\, \theta = 1$

$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta$

$$\Rightarrow \quad \int d^2\theta\, \phi(\theta) = \int d^2\theta(a + \theta\psi + \theta\theta\, f) = f$$

With Grassmann variables:

SUSY algebra can be written in terms of commutators only

$$\begin{aligned} [\theta Q, \bar\theta\bar Q] &= 2\theta\sigma^\mu\bar\theta P_\mu \\ [\theta Q, \theta Q] = [\bar\theta\bar Q, \bar\theta\bar Q] &= 0 \\ [P^\mu, \theta Q] = [P^\mu, \bar\theta\bar Q] &= 0 \end{aligned}$$

$\Rightarrow$ can be treated like Lie-group with anticommuting parameters

SUSY transformations

Group element of finite SUSY transformation:

$$S(y,\xi,\bar{\xi}) = \exp i\left(\xi Q + \bar{\xi}\bar{Q} - y^\mu P_\mu\right)$$

in analogy to group elements for Lie-groups

ξ, $\bar{\xi}$ are independent of x^μ: global SUSY transformation

Transformation of superfield: $S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta})$

Use Hausdorff's formula ($e^A e^B = \ldots$) and SUSY algebra

$$\Rightarrow \quad S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta}) = \phi(x^\mu + y^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \xi+\theta, \bar{\xi}+\bar{\theta})$$

representations of generators obtained from infinitesimal transformation of superfield

$$\Rightarrow \quad P_\mu = i\partial_\mu, \quad Q_\alpha = -i\,\partial_\alpha + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = i\,\bar{\partial}_{\dot{\alpha}} - (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

Definition of covariant derivatives:

$$D_\alpha = -i\,\partial_\alpha - (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = i\,\bar{\partial}_{\dot{\alpha}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

Anticommutation relations of Q_α, D_α:

$$\{Q_\alpha, D_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = \{Q_\alpha, \bar{D}_{\dot{\beta}}\} = 0$$
$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$
$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0$$
$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu = 2(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu$$
$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu = -2(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu$$

$\Rightarrow$ D_α, $\bar{D}_{\dot{\alpha}}$ anticommute with SUSY generators

$\Rightarrow$ are invariant under SUSY transformations

$$[(\xi Q + \bar{\xi}\bar{Q}), D_\alpha] = 0, \quad [(\xi Q + \bar{\xi}\bar{Q}), \bar{D}_{\dot{\alpha}}] = 0$$

General superfield in component form

Most general form of field depending on x, θ, $\bar{\theta}$:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) \;=\; & \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta F(x) + \bar{\theta}\bar{\theta}H(x) + \theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) \\ & + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\xi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)\end{aligned}$$

Further terms vanish because of $\theta\theta\theta = 0$

Components (can be complex):

φ, F, H, D: scalar fields

A_{μ}: vector field

ψ, $\bar{\chi}$, $\bar{\lambda}$, ξ: Weyl-spinorfields

$\Rightarrow$ Too many components in 4-dim. for irreducible representation of SUSY with spin ≤ 1 (chiral or vector multiplet)

$\Rightarrow$ representation is reducible
(not all component fields mix with each other under SUSY transf.)

$\Rightarrow$ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield
conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LHχSF)

$D_\alpha\Phi = 0$: right-handed chiral superfield (RHχSF)

$\Phi = \Phi^\dagger$: vector superfield

Usefulness of two-component spinors:

SM fermions: left-handed and right-handed components transform differently

$\Rightarrow$ need superfields with only two fermionic degrees of freedom

$\Rightarrow$ chiral superfields describe left- or right-handed component of SM fermion + scalar partner

LHχSF in components:

$$\phi(x,\theta,\bar{\theta}) = \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta})$$
$$- \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^{\mu}\partial_{\mu}\varphi(x) - (\theta\theta)F(x)$$

φ, F: scalar fields, ψ: Weyl-spinor field

F: auxiliary field, unphysical (has mass dimension 2)

RHχSF:

$$\bar{\phi}(x,\theta,\bar{\theta}) = \Big(\phi(x,\theta,\bar{\theta})\Big)^{\dagger}$$

Transformation of component fields under infinitesimal SUSY transf.:

$$\delta\phi(x,\theta,\bar{\theta}) = i(\xi Q + \bar{\xi}\bar{Q})\phi(x,\theta,\bar{\theta})$$

Comparison with

$$\delta\phi(x,\theta,\bar{\theta}) = \delta\varphi + \sqrt{2}\theta\delta\psi - \ldots - (\theta\theta)\delta F$$

$\Rightarrow$ determination of $\delta\varphi$, $\delta\psi$, δF:

$$\begin{aligned}
\delta\varphi &= \sqrt{2}\xi\psi \qquad \text{boson} \rightarrow \text{fermion} \\
\delta\psi_\alpha &= -\sqrt{2}F\xi_\alpha - i\sqrt{2}(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\varphi \qquad \text{fermion} \rightarrow \text{boson} \\
\delta F &= \partial_\mu(-i\sqrt{2}\psi\sigma^\mu\bar{\xi}) \qquad F \rightarrow \text{total derivative}
\end{aligned}$$

Vector superfield in components:

$$
\begin{aligned}
V(x,\theta,\bar\theta) &= c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta v_\mu(x) \\
&+ \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(\bar\theta\bar\theta)(M(x)-iN(x)) \\
&+ i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\
&+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right)
\end{aligned}
$$

$v_\mu(x)$: real vector field $\rightarrow$ describes gauge boson
(V must transform as adjoint representation of gauge group)

Number of components can be reduced by SUSY gauge transformation:

$$e^{gV} \longrightarrow e^{-ig\Lambda^\dagger} e^{gV} e^{ig\Lambda},$$

where $\Lambda(x,\theta,\bar\theta)$ is a chiral superfield and g is the gauge coupling

can perform a transformation such that $\chi(x) = c(x) = M(x) = N(x) \equiv 0$:
"Wess–Zumino gauge"

Wess–Zumino gauge removes many unphysical degrees of freedom

still leaves "ordinary" gauge freedom, e.g. $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \lambda(x)$ for abelian theory

Wess–Zumino gauge not preserved under SUSY transformations

Transformation of component fields under infinitesimal SUSY transf.:

$$\delta\lambda = \ldots, \quad \delta v_\mu = \ldots, \ldots, \quad \delta D = -\xi\sigma^\mu \partial_\mu \bar{\lambda}(x) - \partial_\mu \lambda(x) \sigma^\mu \bar{\xi}$$

$\Rightarrow$ D transforms into a total derivative

4. Supersymmetric Lagrangians

Aim: construct an action that is invariant under SUSY transformations:

$$\delta \int d^4x \mathcal{L}(x) = 0$$

Satisfied if $\mathcal{L} \longrightarrow \mathcal{L} +$ total derivative

F and D terms (the terms with the largest number of θ and $\bar{\theta}$ factors) of chiral and vector superfields transform into a total derivative under SUSY transformations

$\Rightarrow$ Use F-terms and D-terms to construct an invariant action:

$$S = \int d^4x \left(\int d^2\theta \mathcal{L}_f + \int d^2\theta d^2\bar{\theta} \mathcal{L}_d \right)$$

If Φ is a LHχSF $\Rightarrow$ Φ^n is also a LHχSF (since $\bar{D}_{\dot{\alpha}} \Phi^n = 0$ for $\bar{D}_{\dot{\alpha}} \Phi = 0$)

$\Rightarrow$ products of chiral superfields are chiral superfields, products of vector superfields are vector superfields

F-term Lagrangian:

$$\mathcal{L}_F = \int d^2\theta \sum_{ijk} \left(a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \right) + \text{ h.c.}$$

Terms of higher order in Φ_i lead to non-renormalizable Lagrangians

$\Rightarrow$ F-term Lagrangian contains mass terms, scalar–fermion interactions ($\rightarrow$ superpotential), but no kinetic terms

D-term Lagrangian:

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} V$$

$\Rightarrow$ D-term Lagrangian contains kinetic terms

Example: the Wess–Zumino Lagrangian

Construction of Lagrangian from chiral superfields Φ_i

$\Rightarrow$ Φ_i, $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$

$\Phi_i^\dagger\Phi_i$: vector superfield, $(\Phi_i^\dagger\Phi_i)^\dagger = \Phi_i^\dagger\Phi_i$

$$\left[\Phi_i^\dagger\Phi_i\right]_{\theta\theta\bar\theta\bar\theta} = F^\dagger F + (\partial_\mu\varphi^*)(\partial^\mu\varphi) + \frac{i}{2}(\psi\sigma^\mu\partial_\mu\bar\psi - \partial_\mu\psi\sigma^\mu\bar\psi) + \partial_\mu(\ldots)$$

Auxiliary field F can be eliminated via equations of motion

$$\begin{aligned}\Rightarrow \quad \mathcal{L} \;=\; & \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar\psi_i - (\partial_\mu\psi_i)\sigma^\mu\bar\psi_i) - \frac{1}{2}m_{ij}(\psi_i\psi_j + \bar\psi_i\bar\psi_j) \\ & + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) - \sum_i \left|a_i + m_{ij}\varphi_j + \lambda_{ijk}\varphi_j\varphi_k\right|^2 \\ & - \lambda_{ijk}\varphi_i\psi_j\psi_k - \lambda_{ijk}^\dagger\varphi_i^*\bar\psi_j\bar\psi_k\end{aligned}$$

Lagrangian for scalar fields φ_i and spinor fields ψ_i with the same mass m_{ii} contains couplings of type $hf\bar{f}$ and $\tilde{h}\bar{\tilde{h}}\tilde{f}$ with the same strength

$\Rightarrow$ SUSY implies relations between masses and couplings

$\mathcal{L}$ can be rewritten as kinetic part + contribution of superpotential $\mathcal{V}$:

$$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$$

$$\begin{aligned} \Rightarrow \quad \mathcal{L} \;=\; & \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar{\psi}_i - (\partial_\mu\psi_i)\sigma^\mu\bar{\psi}_i) + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) \\ & - \sum_i \left|\frac{\partial\mathcal{V}}{\partial\varphi_i}\right|^2 - \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi_i^*\partial\varphi_j^*}\bar{\psi}_i\bar{\psi}_j \end{aligned}$$

$\mathcal{V}$ determines all interactions and mass terms

Special case $a_i = 0$: Wess–Zumino model

In order to include vector bosons need to construct Lagrangian containing also vector superfields

Further requirement: gauge invariance

$$\int d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi \longrightarrow \int d^2\theta d^2\bar{\theta}\Phi^\dagger e^{2gV}\Phi$$

Kinetic terms for gauge fields from field strength

$\Rightarrow$ Supersymmetric Lagrangian in the Wess–Zumino gauge for chiral superfields ϕ_i (with component fields $\varphi_i,\ \psi_i$) and vector superfields V^a (with component fields $v_\mu^a,\ \lambda^a$):

$$\begin{aligned}
\mathcal{L} \;=\; & (D_\mu\varphi)_i^*(D^\mu\varphi)_i + \frac{i}{2}\psi_i\sigma^\mu(D_\mu\bar{\psi})_i - \frac{i}{2}(D_\mu\psi)_i\sigma^\mu\bar{\psi}_i \\
& - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\bar{\lambda})^a - \frac{i}{2}(D_\mu\lambda)^a\sigma^\mu\bar{\lambda}^a \\
& + \sqrt{2}ig\,\bar{\psi}_i\lambda^a T^a_{ij}\varphi_j - \sqrt{2}ig\,\varphi_i^* T^a_{ij}\psi_j\lambda^a \\
& - \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi_i^*\partial\varphi_j^*}\bar{\psi}_i\bar{\psi}_j - V(\varphi_i,\varphi_j^*)
\end{aligned}$$

$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$: holomorphic function of chiral sf.

The potential V is the sum of the F-terms and D-terms and reads

$$V(\varphi_i, \varphi_j^*) = \sum_i \left| \frac{\partial \mathcal{V}}{\partial \varphi_i} \right|^2 + \frac{1}{2} g^2 \sum_a (\varphi_i^* T_{ij}^a \varphi_j + \xi^a)^2$$

All couplings are determined either by gauge couplings or the superpotential

E.g.: D-term in potential V: quartic scalar interaction is given by the gauge coupling (in contrast to the SM case)

The ξ^a-term (called the Fayet-Iliopoulos term) can be present only for $U(1)$ gauge fields

The requirement of gauge invariance imposes constraints on the coefficients a_i, m_{ij} and λ_{ijk} of the superpotential $\mathcal{V}$

The (gauge) covariant derivatives are (f^{abc} are the structure constants of the gauge group)

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig\, v_\mu^a T_{ij}^a \varphi_j,\ (D_\mu \psi)_i = \partial_\mu \psi_i + ig\, v_\mu^a T_{ij}^a \psi_j,\ (D_\mu \lambda)^a = \partial_\mu \lambda^a - g f^{abc} v_\mu^b \lambda^c$$

Summary on construction of SUSY Lagrangians:

Construct Lagrangians for $N = 1$ SUSY from chiral superfields and vector superfields:

multiplets containing pairs of fields with the same mass, spin differs by $\frac{1}{2}$

Fermion Yukawa interactions and scalar quartic self-interactions are determined by the superpotential

Gauge interactions determine couplings of the gauge fields

$\Rightarrow$ Many relations between couplings

$N > 1$ SUSY

So far: $N = 1$ SUSY, simplest case, only one fermionic generator and its hermitean adjoint: Q_α, $\bar{Q}^{\dot{\beta}}$

$\Rightarrow$ one superpartner for photon: photino

N-extended SUSY: N generators Q^A_α, $\bar{Q}^{\dot{\beta}}_B$, $A = 1, \ldots, N$

$\Rightarrow$ N superpartners for the photon, ...

Generalization of anticommutator relation:

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{\alpha\beta} X^{AB}$$

$X^{AB} = -X^{BA}$: "central charges"

Problem:

helicity $+\frac{1}{2}$ and helicity $-\frac{1}{2}$ fermions are in same supermultiplet
(e.g.: hypermultiplet for $N = 2$ SUSY)

$\Rightarrow$ helicity $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions need to transform in the same way under gauge transformations

not possible for chiral fermions of electroweak theory

$\Rightarrow$ $N > 1$ SUSY theories are 'non-chiral'

$\Rightarrow$ $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

However: $N > 1$ SUSY have interesting properties

e.g.: $N = 4$ SUSY field theory (flat space) is finite

Seiberg–Witten solution in $N = 2$ SUSY, ...

Soft SUSY breaking

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ in a realistic model: SUSY must be broken

Only satisfactory way for model of SUSY breaking:
spontaneous SUSY breaking

Specific SUSY-breaking schemes (see below) in general yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft SUSY-breaking terms

Soft SUSY-breaking terms: do not alter dimensionless couplings
(i.e. dimension of coupling constants of soft SUSY-breaking terms > 0)

$\Rightarrow$ no quadratic divergences (in all orders of perturbation theory)

scale of SUSY-breaking terms: $M_{\rm SUSY} \lesssim 1$ TeV

Classification of possible soft breaking terms:

[*L. Girardello, M. Grisaru '82*]

- scalar mass terms: $m^2_{\phi_i} |\phi_i|^2$
- trilinear scalar interactions: $A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$
- gaugino mass terms: $\frac{1}{2}m\bar{\lambda}\lambda$
- bilinear terms: $B_{ij}\phi_i\phi_j + \text{h.c.}$
- linear terms: $C_i\phi_i$

$\Rightarrow$ relations between dimensionless couplings unchanged

no additional mass terms for chiral fermions

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergencies

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenological description for universal breaking terms (FCNC, . . .)

Scenarios for SUSY breaking $\Rightarrow$ prediction for soft SUSY-breaking terms in terms of small set of parameters

Experimental determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking

5. The Minimal Supersymmetric Standard Model (MSSM)

MSSM: superpartners for SM fields

SM matter fermions are in different representation of gauge group than gauge bosons

$\Rightarrow$ need to be placed in different superfields

$\Rightarrow$ no SM fermion is a gaugino

Fermions, sfermions:

use definition of chiral superfields via left-handed fermions
($\Rightarrow$ the conjugates of right-handed ones appear)

LHχSF Q: quark, squark SU(2) doublets

LHχSF U: up-type quark, squark singlets

LHχSF D: down-type quark, squark singlets

LHχSF L: lepton, slepton SU(2) doublets

LHχSF E: lepton, slepton singlets

$\Rightarrow$ one generation of SM fermions and their superpartners described by five LHχSFs

Gauge bosons, gauginos:

Vector superfields:

gluons g and gluinos $\tilde{g}$

W bosons $W^{\pm}$, W^0 and winos $\tilde{W}^{\pm}$, $\tilde{W}^0$

B boson B^0 and bino $\tilde{B}^0$

Higgs bosons, higgsinos:

LHχSF

In MSSM: two Higgs doublets needed $\Rightarrow$ two LHχSFs

Comparison with SM case:

$$\mathcal{L}_{\text{SM}} = \underbrace{m_d \bar{Q}_L H d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \tilde{H} u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \tilde{H} = i\sigma_2 H^*, \quad H \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \tilde{H} \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L H^*$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ H_d and H_u needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Chiral supermultiplets of the MSSM:

		spin 0	spin $\frac{1}{2}$	$(\mathrm{SU}(3)_c, \mathrm{SU}(2), \mathrm{U}(1)_Y)$
squarks and quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, \frac{1}{6})$
	U	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3}, 1, -\frac{2}{3})$
	D	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3}, 1, \frac{1}{3})$
sleptons and leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -\frac{1}{2})$
	E	$\tilde{e}_R^*$	$e_R^\dagger$	$(1, 1, 1)$
higgs and higgsinos	H_u	(h_u^+, h_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$	$(1, 2, \frac{1}{2})$
	H_d	(h_d^0, h_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$	$(1, 2, -\frac{1}{2})$

Vector supermultiplets:

	spin $\frac{1}{2}$	spin 1	$(\mathrm{SU}(3)_c, \mathrm{SU}(2), \mathrm{U}(1)_Y)$
gluinos and gluons	$\tilde{g}$	g	$(8,1,0)$
winos and W-bosons	$\widetilde{W}^{\pm}, \widetilde{W}^{0}$	$W^{\pm}, W^{0}$	$(1,3,0)$
bino and B-boson	$\widetilde{B}$	B	$(1,1,0)$

Superpotential:

$$\mathcal{V}_{\mathrm{MSSM}} = U\mathbf{y_u}QH_u + D\mathbf{y_d}QH_d + E\mathbf{y_e}LH_d + \mu H_u H_d$$

$\mathbf{y_i}, \mathbf{a_i}$: Yukawa couplings, 3×3 matrices in family space

All terms have to be invariant under all gauge groups, $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)$, $\mathrm{U}(1)_Y$

$\Rightarrow$ need $Y = 0$ for all terms

$H_u H_d \equiv (H_u)_a (H_d)_b \epsilon^{ab}$ (a, b: weak isospin indices), . . .

$\mathcal{V}_{\text{MSSM}}$ is not the most general gauge-invariant superpotential

contains only terms that are necessary to build a realistic model

Gauge interactions introduce only terms with even number of superpartners

The same holds for minimal version of $\mathcal{V}_{\text{MSSM}}$

$\Rightarrow$ MSSM has further symmetry: "R-parity"

all SM-particles and Higgs bosons: even R-parity, $P_R = +1$

all superpartners: odd R-parity, $P_R = -1$

Soft breaking terms:

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} \;=\; & -\frac{1}{2}\Big(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}\widetilde{W} + M_3\tilde{g}\tilde{g}\Big) + \text{h.c.} \\
& - m^2_{H_u} H_u^\dagger H_u - m^2_{H_d} H_d^\dagger H_d - (bH_uH_d + \text{h.c.}) \\
& - \Big(\tilde{u}_R \mathbf{a_u} \widetilde{Q} H_u - \tilde{d}_R \mathbf{a_d} \widetilde{Q} H_d - \tilde{e}_R \mathbf{a_e} \widetilde{L} H_d\Big) + \text{h.c.} \\
& - \widetilde{Q}^\dagger \mathbf{m^2_Q} \widetilde{Q} - \widetilde{L}^\dagger \mathbf{m^2_L} \widetilde{L} - \tilde{u}_R \mathbf{m^2_u} \tilde{u}^*_R - \tilde{d}_R \mathbf{m^2_d} \tilde{d}^*_R - \tilde{e}_R \mathbf{m^2_e} \tilde{e}^*_R
\end{aligned}
$$

Most general parameterization of SUSY-breaking terms that keep relations between dimensionless couplings unchanged $\Rightarrow$ no quadratic divergences

Gaugino mass terms, scalar mass terms, and terms like in superpotential for all scalars

$\mathbf{m^2_i}$: 3×3 matrices in family space

$\Rightarrow$ many new parameters

Particle content of the MSSM:

Superpartners for Standard Model particles:

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin 0}$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \qquad \tilde{\chi}^{\pm}_{1,2} \qquad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:

Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Breaking of SU(2) $\times$ U(1)$_Y$ (electroweak symmetry breaking)

$\Rightarrow$ fields with different SU(2) $\times$ U(1)$_Y$ quantum numbers can mix if they have the same SU(3)$_c$, U(1)$_{\text{em}}$ quantum numbers

Squark mixing:

Stop, sbottom mass matrices ($X_\mathrm{t} = A_\mathrm{t} - \mu/\tan\beta$, $X_\mathrm{b} = A_\mathrm{b} - \mu\tan\beta$):

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m^2_\mathrm{t} + DT_1 & m_\mathrm{t} X_\mathrm{t} \\ m_\mathrm{t} X_\mathrm{t} & m^2_{\tilde{t}_R} + m^2_\mathrm{t} + DT_2 \end{pmatrix} \quad \Rightarrow \quad m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}$$

$$\mathcal{M}^2_{\tilde{\mathrm{b}}} = \begin{pmatrix} m^2_{\tilde{b}_L} + m^2_\mathrm{b} + DB_1 & m_\mathrm{b} X_\mathrm{b} \\ m_\mathrm{b} X_\mathrm{b} & m^2_{\tilde{b}_R} + m^2_\mathrm{b} + DB_2 \end{pmatrix} \quad \Rightarrow \quad m_{\tilde{\mathrm{b}}_1}, m_{\tilde{\mathrm{b}}_2}, \theta_{\tilde{\mathrm{b}}}$$

off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

$\Rightarrow$ mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

gauge invariance $\Rightarrow m_{\tilde{t}_L} = m_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}, m_{\tilde{\mathrm{b}}_1}, m_{\tilde{\mathrm{b}}_2}, \theta_{\tilde{\mathrm{b}}}$

Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

$\Rightarrow$ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

$\Rightarrow$ only one new parameter

$\Rightarrow$ MSSM predicts mass relations between neutralinos and charginos