Problem sheet 3 (please hand in on 08.02.07)

1) Consider the MSSM with the superpotential

$$\mathcal{V}_{\rm MSSM} = U\mathbf{y_u}QH_u + D\mathbf{y_d}QH_d + E\mathbf{y_e}LH_d + \mu H_u H_d$$

and the soft breaking terms

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - (bH_uH_d + \text{h.c.}) \\ &\quad - \left(\tilde{u}_R \mathbf{a_u} \widetilde{Q} H_u - \tilde{d}_R \mathbf{a_d} \widetilde{Q} H_d - \tilde{e}_R \mathbf{a_e} \widetilde{L} H_u\right) + \text{h.c.} \\ &\quad + \text{terms without Higgs.}\end{aligned}$$

Restricting to the scalar field content of the superfields H_u, H_d, they can be decomposed as

$$H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}.$$

Show that the potential in the neutral Higgs sector (i.e. restricting to terms containing only neutral Higgs bosons) is given by

$$\begin{aligned}V &= \left(|\mu|^2 + m_{H_u}^2\right)\left|h_u^0\right|^2 + \left(|\mu|^2 + m_{H_d}^2\right)\left|h_d^0\right|^2 \\ &\quad - \left(b\, h_u^0 h_d^0 + \text{h.c.}\right) + \frac{g^2 + g'^2}{8}\left(\left|h_u^0\right|^2 - \left|h_d^0\right|^2\right)^2,\end{aligned}$$

where g and g' are the couplings of SU(2) and U(1)_Y, respectively.

2) The superpotential $\mathcal{V}_{\rm MSSM}$ given in problem 1 is not the most general gauge-invariant and renormalizable superpotential that can be constructed with the chiral superfields U, D, Q, L, E, H_u and H_d. Find all gauge-invariant and renormalizable terms that are not included in $\mathcal{V}_{\rm MSSM}$.