**Problem sheet 2 (please hand in on 01.02.07)**

1) Use the representation of the SUSY generators $Q_\alpha$, $\bar{Q}_{\dot{\alpha}}$ as differential operators derived in the lecture (see page 28 of the lecture notes) to prove

$$\left\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\right\} = 2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu \; .$$

Note: the anti-commutator is understood to act on a superfield $\phi$.

2) A Grassmann variable, $\theta$, is an anti-commuting object: $\{\theta, \theta\} = 0$.

a) What does this imply for the Taylor expansion of a function $\phi(\theta)$?

b) What follows from the usual translational invariance,

$$\int_{-\infty}^{\infty} \mathrm{d}x \; \phi(x) = \int_{-\infty}^{\infty} \mathrm{d}x \; \phi(x+c),$$

when applied to Grassmann variables, for $\int_{-\infty}^{\infty} \mathrm{d}\theta$?
The normalization is chosen such that $\int_{-\infty}^{\infty} \mathrm{d}\theta \, \theta = 1$ holds. What does this yield for

$$\int \mathrm{d}\theta \, \phi(\theta) \quad \text{and} \quad \frac{\partial}{\partial\theta}\phi(\theta) \quad ?$$

c) For a Grassmann spinor $\theta^\alpha$ we define

$$\mathrm{d}^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta}\mathrm{d}\theta^\alpha \mathrm{d}\theta^\beta.$$

Compute $\int \mathrm{d}^2\theta \, \phi(\theta)$, where $\phi(\theta) = a + \theta\psi + \theta\theta \, f$.

3) Derive the transformation of the component fields $\varphi(x)$, $\psi(x)$, $F(x)$ of the chiral supermultiplet

$$\begin{aligned} \phi(x,\theta,\bar{\theta}) &= \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar{\theta}) \\ & \quad - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\partial_\mu\varphi(x) - (\theta\theta)F(x) \end{aligned} \tag{1}$$

under an infinitesimal SUSY transformation.