Ambiguities in super-particle mass determinations

based on: B. K. Gjelsten, D. J. Miller, P. Osland

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LHC-ILC

Per Osland, CERN / University of Bergen

Assume Supersymmetry realized at the LHC/ILC

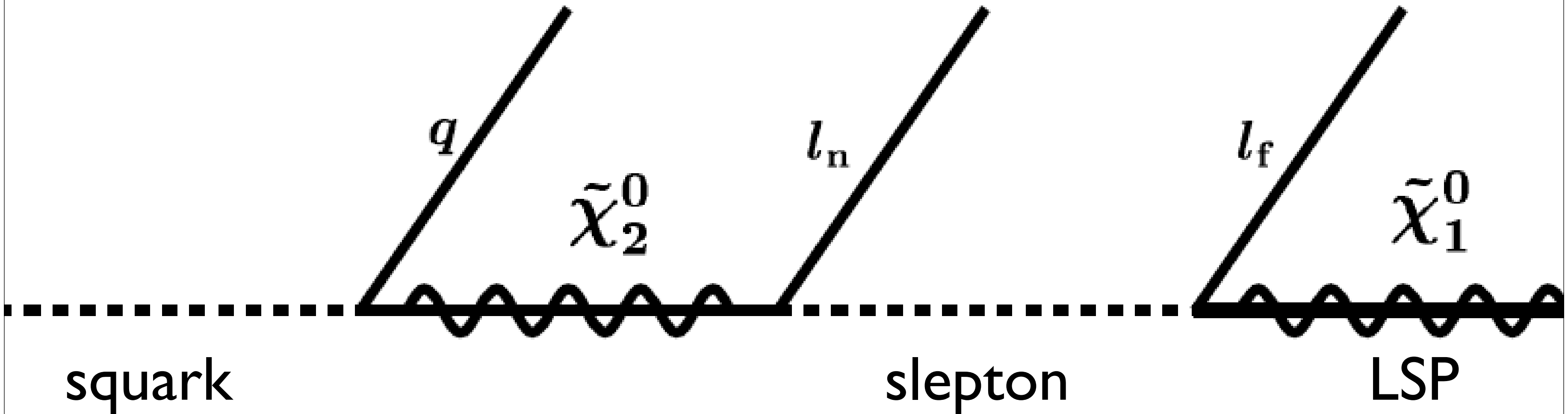
LHC: determine mass *differences* up to high values

ILC: can determine LSP mass with high precision

Synergy:

ILC provides more precision of masses determined at the LHC, and can resolve ambiguities in such mass measurements

“Easy” SPS 1a squark cascade



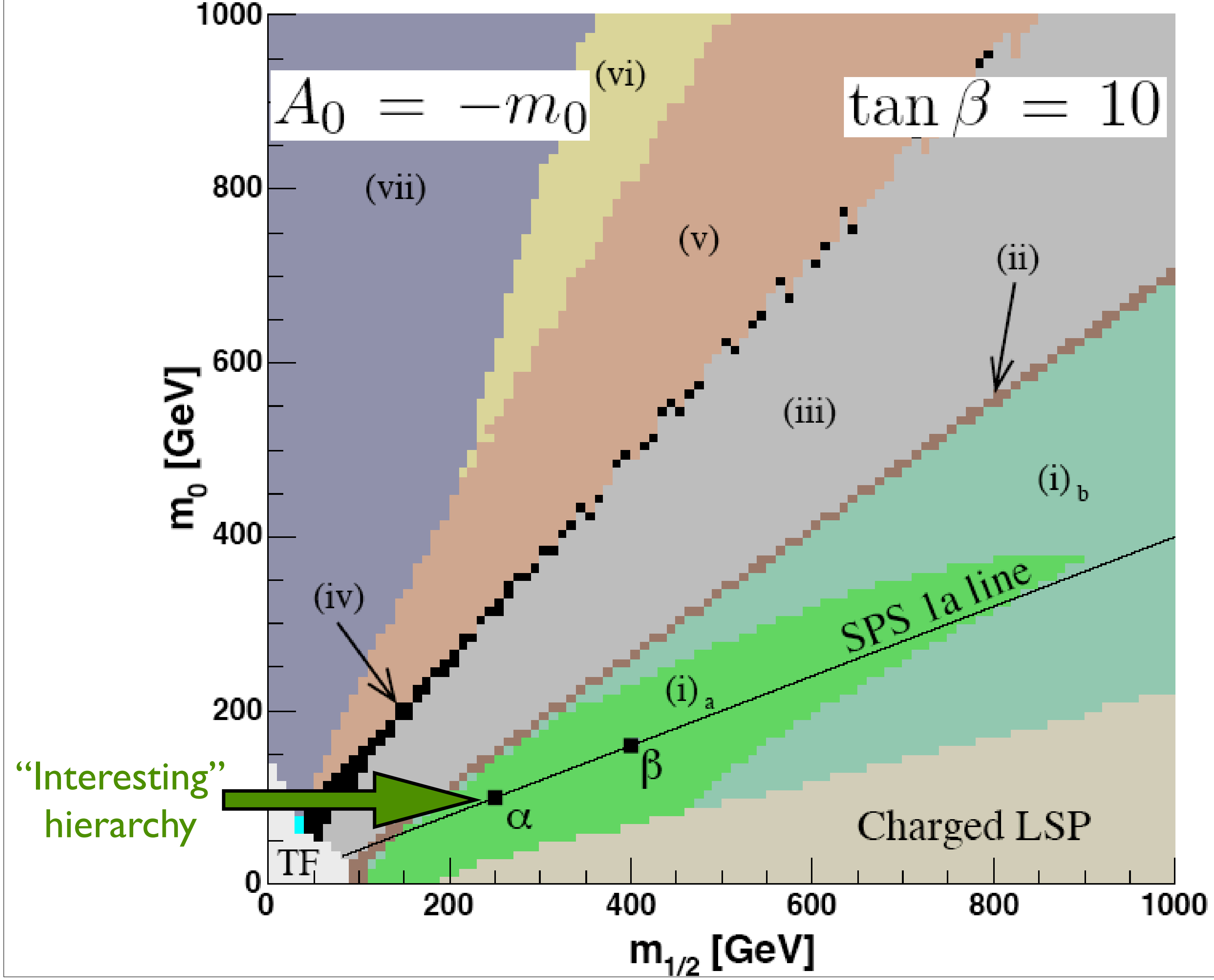
Detect: quark jet and two leptons

Aim: determine squark, slepton and neutralino masses

Question: Is this mass hierarchy “typical”?

Want “heavy” gluino and “heavy” neutralino $\tilde{\chi}_2^0$

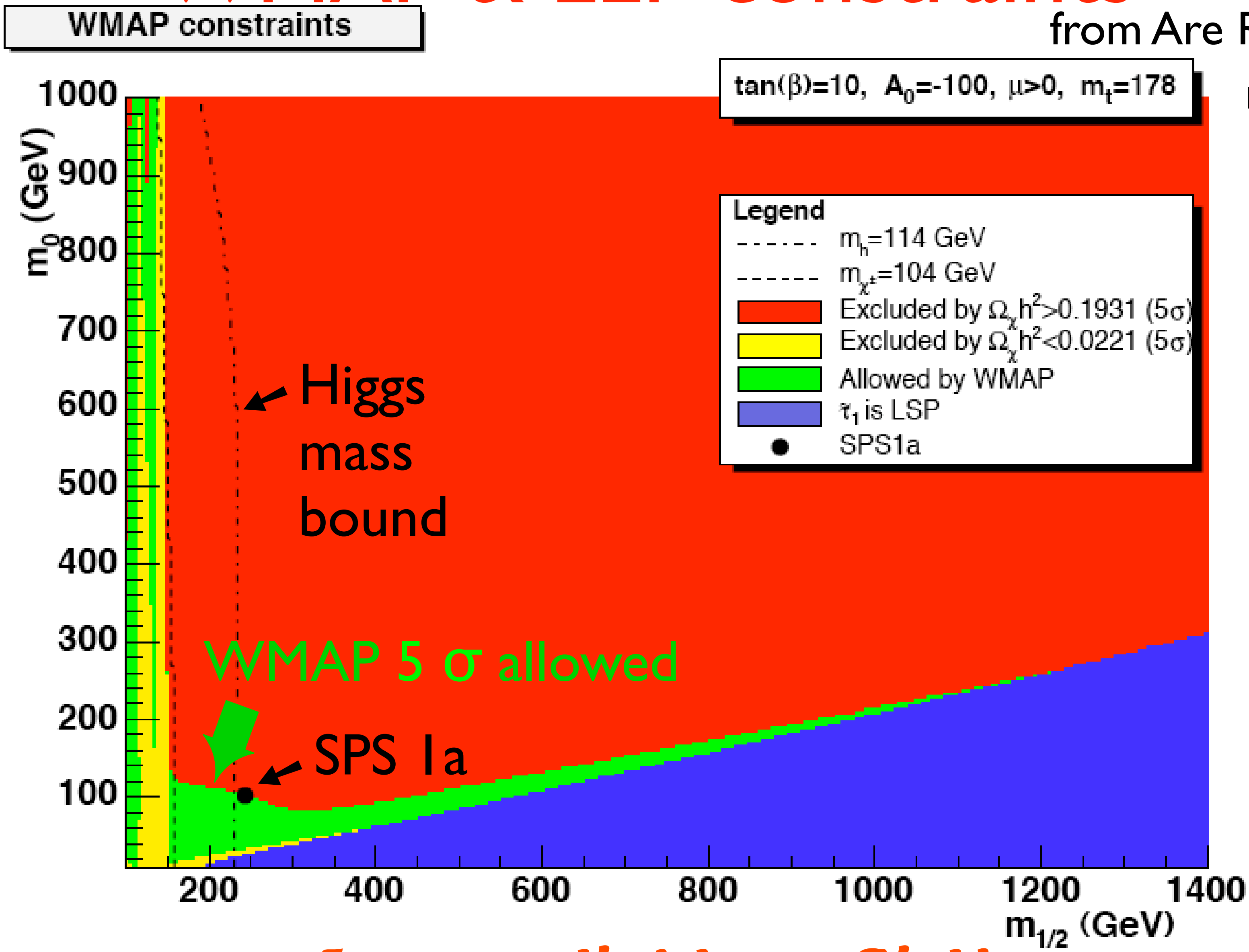
Hierarchies:



WMAP & LEP constraints

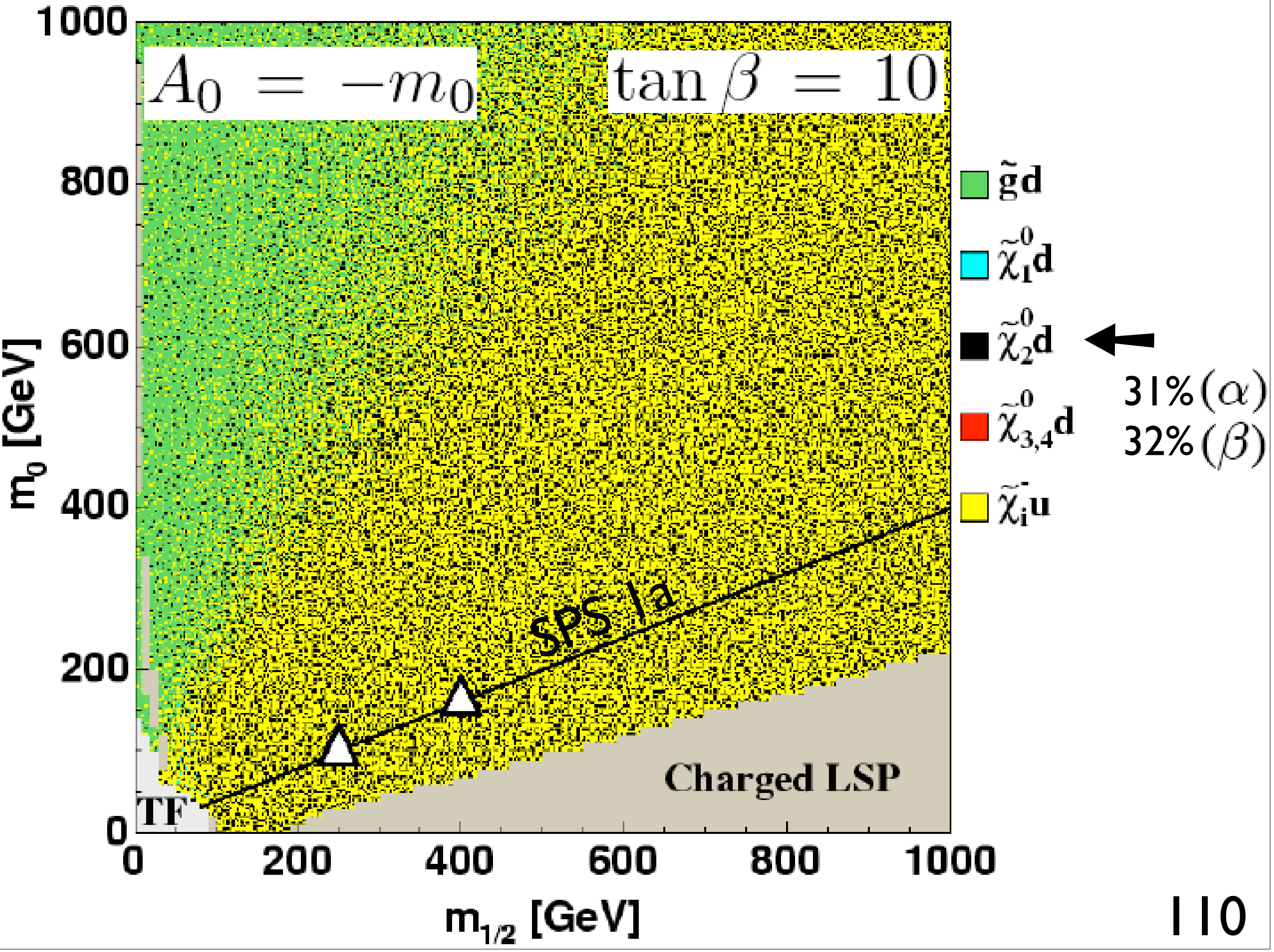
from Are Raklev

DarkSusy
ISASUSY



Ignore slight conflict!

Squark Branching Ratios ($\tilde{u}_L$)



SPS 1a (line)

$$m_0 = -A_0 = 0.4\, m_{1/2}$$

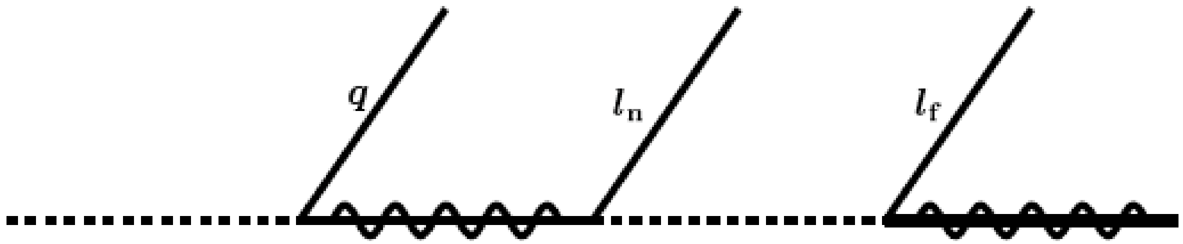
$$\tan\beta = 10\,, \qquad \mu > 0$$

Two particular points on the line:

$$(\alpha): \quad m_0 = 100 \text{ GeV}\,, \qquad m_{1/2} = 250 \text{ GeV}$$

$$(\beta): \quad m_0 = 160 \text{ GeV}\,, \qquad m_{1/2} = 400 \text{ GeV}$$

Quantifying the cascade:

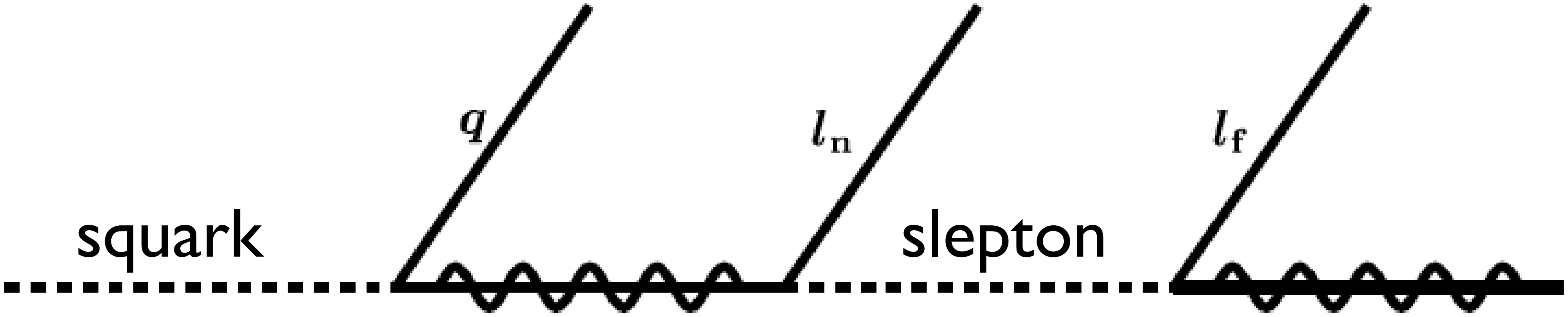


	$\sigma = 32.8$ pb	$\tilde{q}_L$	31.4%						1245 fb
(α)	$\sigma = 7.7$ pb	$\tilde{b}_1$	35.5%	$\tilde{\chi}_2^0$	12.1%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	329 fb
	$\sigma = 4.3$ pb	$\tilde{b}_2$	18.0%						94 fb
									1669 fb
	$\sigma = 3.21$ pb	$\tilde{q}_L$	31.9%						54.3 fb
(β)	$\sigma = 0.50$ pb	$\tilde{b}_1$	23.6%	$\tilde{\chi}_2^0$	5.3%	$\tilde{l}_R$	100%	$\tilde{\chi}_1^0$	6.3 fb
	$\sigma = 0.31$ pb	$\tilde{b}_2$	8.8%						1.5 fb
									62.0 fb

Reduction (rate and BR) from (α) to (β)

squark cascade decay:

$$\tilde{q}_L \to \tilde{\chi}_2^0 \, q \to \tilde{l}_R \, q \, l_{\rm n} \to \tilde{\chi}_1^0 \, q \, l_{\rm n} \, l_{\rm f}$$



More invariants and endpoints:

$$m_{qll} \quad m_{ql_{\rm n}} \quad m_{ql_{\rm f}} \quad m_{ll}$$

Four endpoints and *four* masses:

$$m_{\tilde{q}_L} \quad m_{\tilde{\chi}_2^0} \quad m_{\tilde{l}_R} \quad m_{\tilde{\chi}_1^0}$$

B.C. Allanach et al, hep-ph/0007009 (conditions rephrased):

$$(m_{ll}^{\max})^2 = \frac{\left(m_{\tilde\chi_2^0}^2 - m_{\tilde l_R}^2\right)\left(m_{\tilde l_R}^2 - m_{\tilde\chi_1^0}^2\right)}{m_{\tilde l_R}^2}$$

one case

mass ratios of adjacent sparticles in chain

$$(m_{qll}^{\max})^2 = \begin{cases} \dfrac{\left(m_{\tilde q_L}^2 - m_{\tilde\chi_2^0}^2\right)\left(m_{\tilde\chi_2^0}^2 - m_{\tilde\chi_1^0}^2\right)}{m_{\tilde\chi_2^0}^2} & \text{for} \quad \dfrac{m_{\tilde q_L}}{m_{\tilde\chi_2^0}} > \dfrac{m_{\tilde\chi_2^0}}{m_{\tilde l_R}}\dfrac{m_{\tilde l_R}}{m_{\tilde\chi_1^0}} \quad (1) \\ \dfrac{\left(m_{\tilde q_L}^2 m_{\tilde l_R}^2 - m_{\tilde\chi_2^0}^2 m_{\tilde\chi_1^0}^2\right)\left(m_{\tilde\chi_2^0}^2 - m_{\tilde l_R}^2\right)}{m_{\tilde\chi_2^0}^2 m_{\tilde l_R}^2} & \text{for} \quad \dfrac{m_{\tilde\chi_2^0}}{m_{\tilde l_R}} > \dfrac{m_{\tilde l_R}}{m_{\tilde\chi_1^0}}\dfrac{m_{\tilde q_L}}{m_{\tilde\chi_2^0}} \quad (2) \\ \dfrac{\left(m_{\tilde q_L}^2 - m_{\tilde l_R}^2\right)\left(m_{\tilde l_R}^2 - m_{\tilde\chi_1^0}^2\right)}{m_{\tilde l_R}^2} & \text{for} \quad \dfrac{m_{\tilde l_R}}{m_{\tilde\chi_1^0}} > \dfrac{m_{\tilde q_L}}{m_{\tilde\chi_2^0}}\dfrac{m_{\tilde\chi_2^0}}{m_{\tilde l_R}} \quad (3) \\ \left(m_{\tilde q_L} - m_{\tilde\chi_1^0}\right)^2 & \text{otherwise} \quad (4) \end{cases}$$

four cases

$$\left(m_{ql(\text{low})}^{\max}, m_{ql(\text{high})}^{\max}\right) = \begin{cases} \left(m_{ql_\text{n}}^{\max}, m_{ql_\text{f}}^{\max}\right) & \text{for} \quad 2m_{\tilde l_R}^2 > m_{\tilde\chi_1^0}^2 + m_{\tilde\chi_2^0}^2 > 2m_{\tilde\chi_1^0}m_{\tilde\chi_2^0} \quad (1) \\ \left(m_{ql(\text{eq})}^{\max}, m_{ql_\text{f}}^{\max}\right) & \text{for} \quad m_{\tilde\chi_1^0}^2 + m_{\tilde\chi_2^0}^2 > 2m_{\tilde l_R}^2 > 2m_{\tilde\chi_1^0}m_{\tilde\chi_2^0} \quad (2) \\ \left(m_{ql(\text{eq})}^{\max}, m_{ql_\text{n}}^{\max}\right) & \text{for} \quad m_{\tilde\chi_1^0}^2 + m_{\tilde\chi_2^0}^2 > 2m_{\tilde\chi_1^0}m_{\tilde\chi_2^0} > 2m_{\tilde l_R}^2 \quad (3) \end{cases}$$

three cases

where
$$(m_{ql_n}^{\max})^2 = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)}{m_{\tilde{\chi}_2^0}^2}$$
$$(m_{ql_f}^{\max})^2 = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{\tilde{l}_R}^2}$$
$$(m_{ql(\mathrm{eq})}^{\max})^2 = \frac{(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}{(2m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2)}$$

Finally:

$$\begin{aligned}(m_{qll(\theta>\frac{\pi}{2})}^{\min})^2 = \Big[&(m_{\tilde{q}_L}^2 + m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}_R}^2)(m_{\tilde{l}_R}^2 - m_{\tilde{\chi}_1^0}^2) \\ &- (m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)\sqrt{(m_{\tilde{\chi}_2^0}^2 + m_{\tilde{l}_R}^2)^2(m_{\tilde{l}_R}^2 + m_{\tilde{\chi}_1^0}^2)^2 - 16m_{\tilde{\chi}_2^0}^2 m_{\tilde{l}_R}^4 m_{\tilde{\chi}_1^0}^2} + \\ &+ 2m_{\tilde{l}_R}^2(m_{\tilde{q}_L}^2 - m_{\tilde{\chi}_2^0}^2)(m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\chi}_1^0}^2)\Big](4m_{\tilde{l}_R}^2 m_{\tilde{\chi}_2^0}^2)^{-1}\end{aligned}$$

one case

θ is opening angle between leptons in $\tilde{l}_R$ rest frame

Over-all: 4×3 cases, denoted (1,1), (1,2), etc.

(9 of 12 are realized)

Complication 1:

The two leptons can not be distinguished

For each event, form

$$m_{ql(\mathrm{low})} < m_{ql(\mathrm{high})}$$

well defined

Complication 2:

For some invariants, there are multiple cases: endpoint formula depends on mass ratios

Complication 3:

Multiple squark masses; widths

Complication 4:

Endpoints not always linearly independent

Inverting endpoint formulas

Endpoint formulas can be inverted

Complications:

- nonlinear (rational, sqrt)
- several cases (mass regions)

$m_{qll}^{\max}$ four cases

$m_{ql(\mathrm{low})}^{\max}, m_{ql(\mathrm{high})}^{\max}$ three cases

Example:

Region (1,1):

$m_{qll}^{\max}$

$m_{ql(\mathrm{low})}^{\max}, m_{ql(\mathrm{high})}^{\max}$

$$m^2_{\tilde\chi^0_1} = \frac{(b^2-d^2)(b^2-c^2)}{(c^2+d^2-b^2)^2}a^2$$

$$m^2_{\tilde l_R} = \frac{c^2(b^2-c^2)}{(c^2+d^2-b^2)^2}a^2$$

$$m^2_{\tilde\chi^0_2} = \frac{c^2d^2}{(c^2+d^2-b^2)^2}a^2$$

$$m^2_{\tilde q_L} = \frac{c^2d^2}{(c^2+d^2-b^2)^2}(c^2+d^2-b^2+a^2)$$

$$a = m_{ll}^{\max}\,, \qquad b = m_{qll}^{\max}\,, \qquad c = m_{ql(\mathrm{low})}^{\max}\,, \qquad d = m_{ql(\mathrm{high})}^{\max}\,.$$

Inverting endpt formulas, cont

- If 4 endpts & 4 masses, (if linear) unique solution
- May have more endpts, system overconstrained
- Endpts have (different) uncertainties
- Use inversion formulas for start point of fit
- Composite formulas: multiple solutions!

LHC simulation

- ISAJET 7.58 defines low-energy model
- PYTHIA 6.2 with CTEQ 5L: Monte Carlo sample
- ATLFAST 2.60 simulates ATLAS detector
- precuts:
  - At least three jets, satisfying: $p_T^{\text{jet}} > 150, 100, 50$ GeV
  - $E_{T,\text{miss}} > \max(100 \text{ GeV}, 0.2 M_{\text{eff}})$ with $M_{\text{eff}} \equiv E_{T,\text{miss}} + \sum_{i=1}^{3} p_{T,i}^{\text{jet}}$
  - Two isolated opposite-sign same-flavour leptons (e or μ), satisfying $p_T^{\text{lep}} > 20, 10$ GeV

SM background: 95% $t\bar{t}$

Aim: determine/study expected accuracy

Extraction of masses

- simulate 10,000 ATLAS ‘experiments’
- focus on statistical uncertainty
- each endpoint: gaussian distribution
- invert endpoint formulas, fit masses
- what is chance of finding correct minimum?

Following Allanach et al, each endpt $E_i^{\exp}$ taken as:

$$E_i^{\exp} = E_i^{\mathrm{nom}} + A_i \sigma_i^{\mathrm{stat}} + B \sigma_i^{\mathrm{scale}}$$

A, B picked from gaussian distribution, mean 0, width 1

One A for each endpoint,
one B for m_{ll}, other B for endpoints involving jets

determine masses

Minimize:

$$\Sigma = [\mathbf{E}^{\exp} - \mathbf{E}^{\mathrm{th}}(\mathbf{m})]^T \mathbf{W} [\mathbf{E}^{\exp} - \mathbf{E}^{\mathrm{th}}(\mathbf{m})]$$

$\mathbf{W}$ inverse error/correlation matrix

SPS 1a (α) $\Delta\Sigma \leq 1$

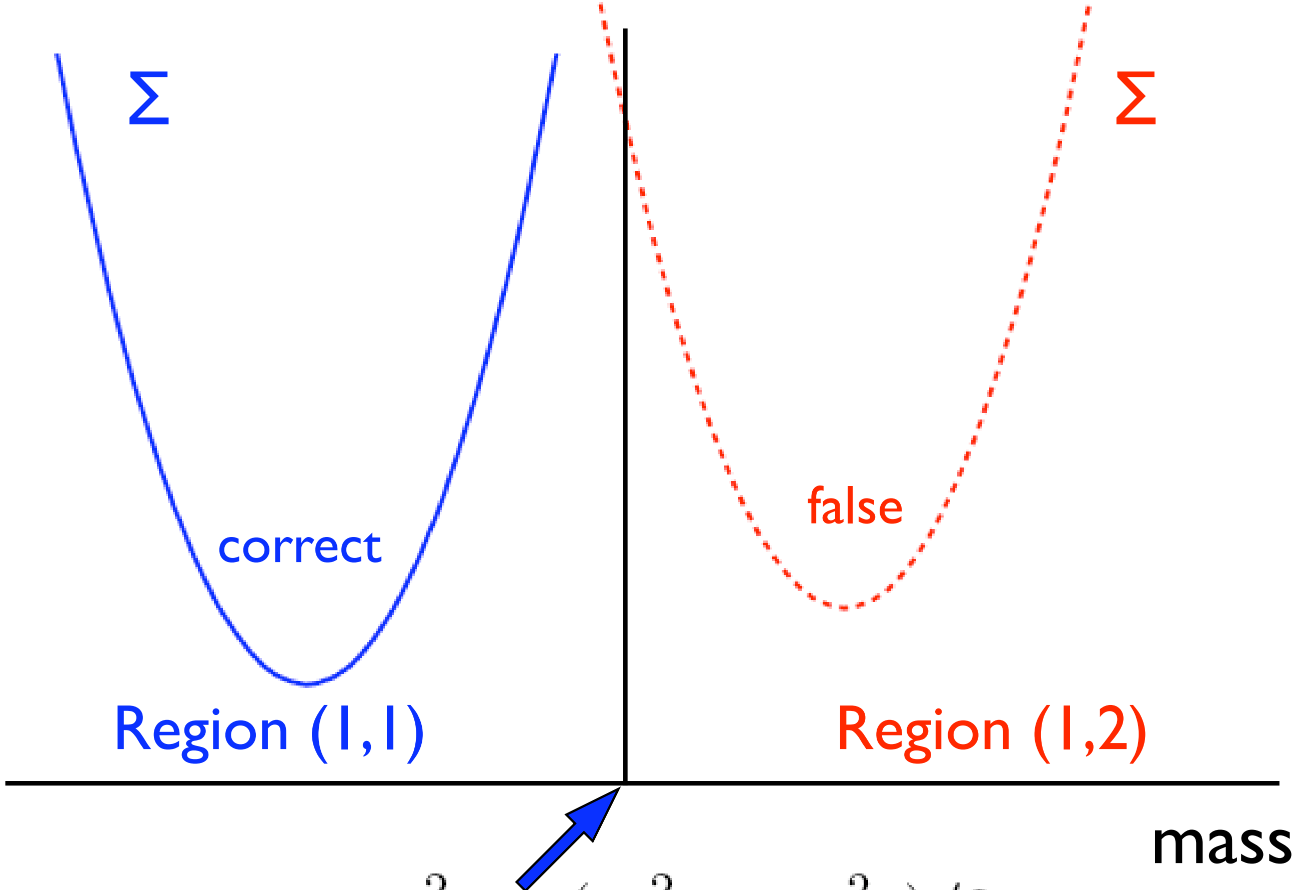
nominal correct fit false fit

	Nom	(1,1) $\langle m \rangle$	σ	γ_1	(1,2) $\langle m \rangle$	σ	γ_1
$m_{\tilde{\chi}_1^0}$	96.1	96.3	3.8	0.2	85.3	3.4	0.1
$m_{\tilde{l}_R}$	143.0	143.2	3.8	0.2	130.4	3.7	0.1
$m_{\tilde{\chi}_2^0}$	176.8	177.0	3.7	0.2	165.5	3.4	0.1
$m_{\tilde{q}_L}$	537.2	537.5	6.1	0.1	523.2	5.1	0.1
$m_{\tilde{b}_1}$	491.9	492.4	13.4	0.0	469.6	13.3	0.1
$m_{\tilde{l}_R} - m_{\tilde{\chi}_1^0}$	46.9	46.9	0.3	0.0	45.1	0.7	-0.2
$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$	80.8	80.8	0.2	0.0	80.2	0.3	-0.1
$m_{\tilde{q}_L} - m_{\tilde{\chi}_1^0}$	441.2	441.3	3.1	0.0	438.0	2.7	0.0
$m_{\tilde{b}_1} - m_{\tilde{\chi}_1^0}$	395.9	396.2	12.0	0.0	384.4	12.0	0.1

Note: Three lightest masses are very correlated

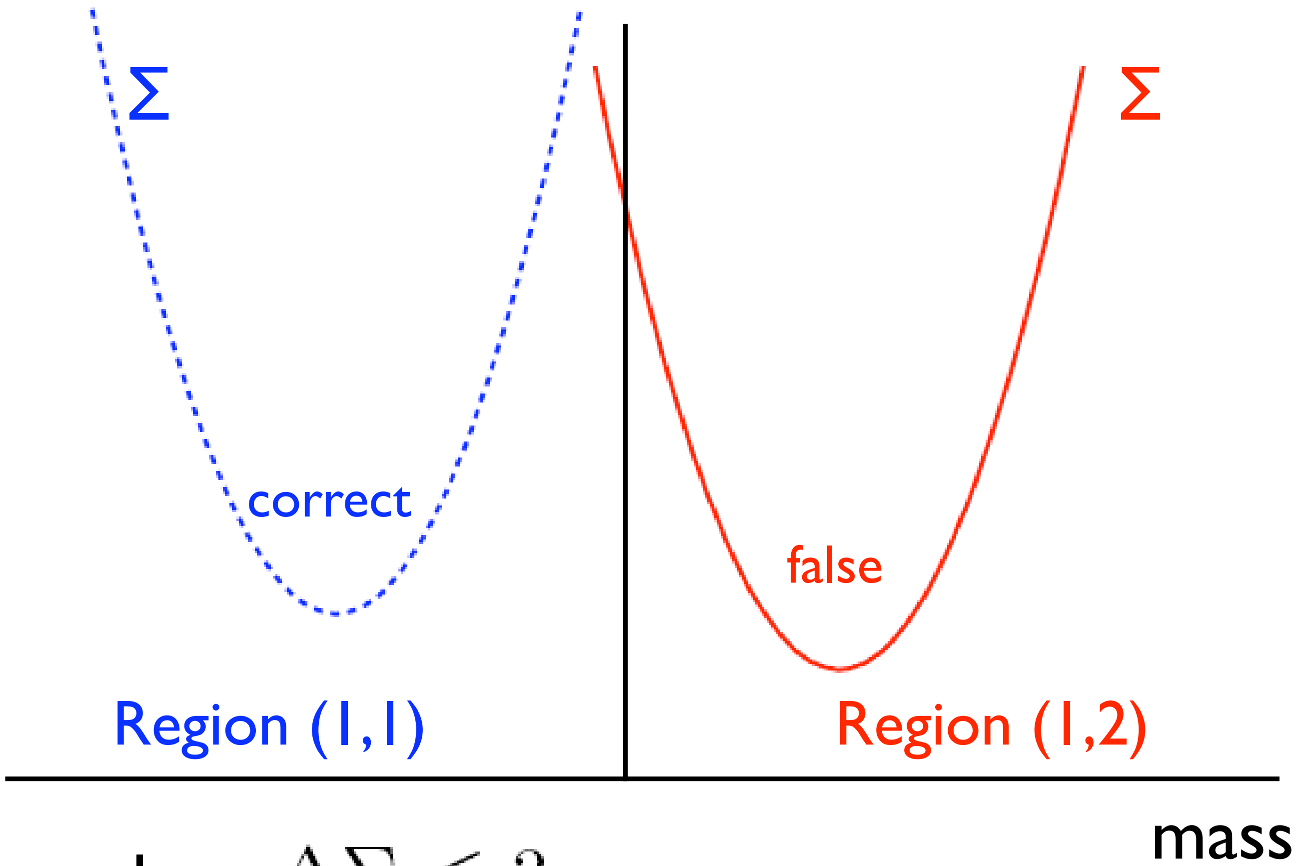
Problem due to compositeness of formulas:

If masses are close to border of ‘region’, may find a similar-quality or better minimum in ‘other’ region



$$m^2_{\tilde{l}_R} = (m^2_{\tilde{\chi}^0_1} + m^2_{\tilde{\chi}^0_2})/2$$

Measurement 'error' may interchange minima



Example: $\Delta\Sigma \leq 3$

30% chance of finding *two* minima

SPS 1a $(\beta)\ \Delta\Sigma \leq 1$

more difficult than (α)

nominal

correct fit

					1 solution			2 solutions					
		(1,1)			(1,2)/(1,3)/B			(1,2)			(1,3)		
	Nom	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1	$\langle m \rangle$	σ	γ_1
$\tilde{\chi}_1^0$	161	438	88	0.9	175	35	1.0	161	22	0.3	166	27	0.6
$\tilde{l}_R$	222	518	85	0.7	236	37	0.8	221	24	0.3	223	28	0.5
$\tilde{\chi}_2^0$	299	579	85	0.7	313	35	1.0	299	22	0.3	304	27	0.6
$\tilde{q}_L$	826	1146	104	0.8	843	44	0.9	826	30	0.3	835	36	0.5
$\tilde{l}_R - \tilde{\chi}_1^0$	61	81	1.8	-0.3	61	4.4	0.4	61	1.9	-0.2	57	1.3	-0.2
$\tilde{\chi}_2^0 - \tilde{\chi}_1^0$	138	141	0.9	0.1	138	0.6	0.2	138	0.5	0.0	138	0.5	0.0
$\tilde{q}_L - \tilde{\chi}_1^0$	665	708	17	0.1	668	10	0.5	665	9	0.1	669	10	0.2

false fit

"correct" fit

(three solns not resolved)

false fit

How likely is a false minimum?

Depends on cut $\Delta\Sigma$ (level of confidence)

SPS 1a (α)

	# Minima	*(1,1)*	*(1,2)*
$\Delta\Sigma \leq 0$	1.00	90%	10%
$\Delta\Sigma \leq 1$	1.12	94%	17%
$\Delta\Sigma \leq 3$	1.30	97%	33%
$\Delta\Sigma \leq \infty$	1.88	99%	88%

Example: $\Delta\Sigma \leq 3$

30% chance of finding *two* minima

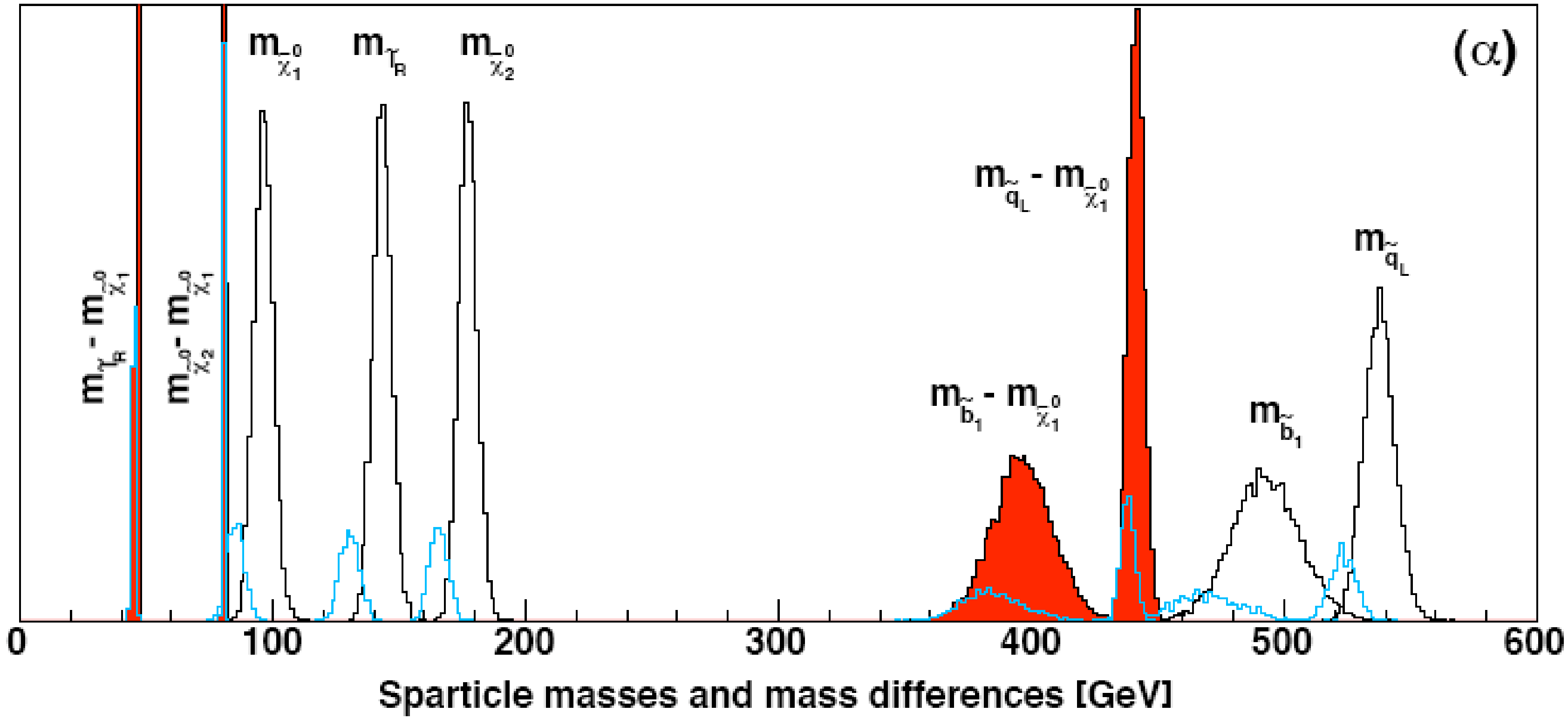
SPS 1a (β)

			1 sol			2 sol
	# Min	*(1,1)*	*(1,2)*	*(1,3)*	B	*(1,2)&(1,3)*
$\Delta\Sigma \leq 0$	1.0	3%	60%	25%	12%	0%
$\Delta\Sigma \leq 1$	1.2	5%	52%	18%	12%	16%
$\Delta\Sigma \leq 3$	1.4	13%	46%	14%	12%	28%
$\Delta\Sigma \leq 99$	2.3	99%	41%	13%	12%	34%

non-negligible probability of finding wrong minimum

non-negligible probability of finding more than one minimum

Masses and mass differences



black: correct solution

red: mass differences

blue-green: false solution (area prop to probability)

LC input ("fixing" LSP mass)

SPS 1a (α)

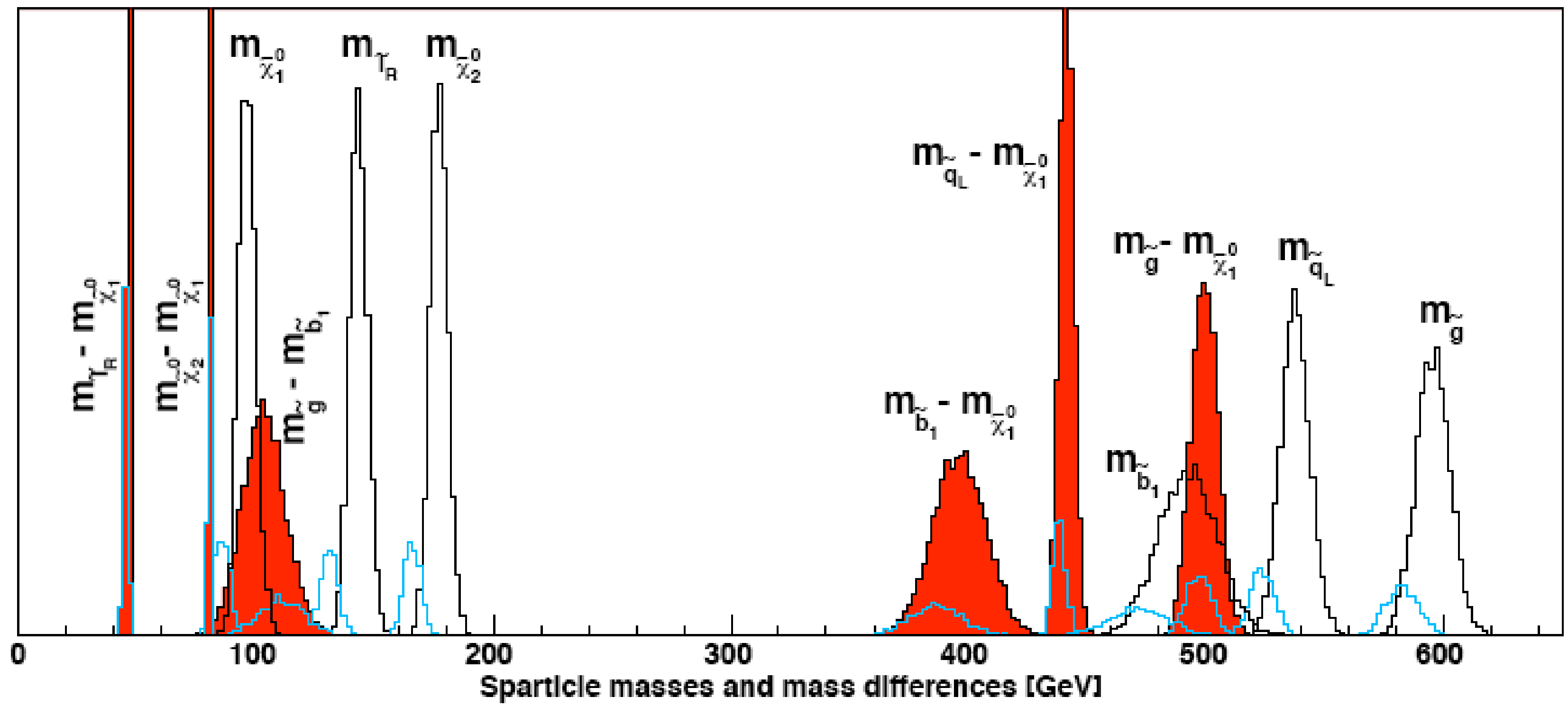
	Nom	(1,1) $\langle m \rangle$	(1,1) σ
$\tilde{\chi}_1^0$	96.05	96.05	0.05
$\tilde{l}_R$	142.97	142.97	0.29
$\tilde{\chi}_2^0$	176.82	176.82	0.17
$\tilde{q}_L$	537.25	537.2	2.5
$\tilde{b}_1$	491.92	492.1	11.7

Masses in GeV

SPS 1a (β)

		1 solution		2 solutions			
		(1,2)/(1,3)/B		(1,2)		(1,3)	
	Nom	$\langle m \rangle$	σ	$\langle m \rangle$	σ	$\langle m \rangle$	σ
$\tilde{\chi}_1^0$	161.02	161.02	0.05	161.02	0.05	161.02	0.05
$\tilde{l}_R$	221.86	221.15	3.26	222.22	1.32	217.48	1.01
$\tilde{\chi}_2^0$	299.05	299.15	0.57	299.11	0.53	299.05	0.52
$\tilde{q}_L$	826.29	826.1	6.3	825.9	5.8	828.6	5.5

Including gluino



Masses less precise than mass differences

'Other' masses (not $\tilde{g}$ and $\tilde{b}_1$) from squark analysis

SPS 1a (α)

	Nom	*(1,1)* Mean	RMS
$m_{\tilde{\chi}_1^0}$	96.05	96.05	0.05
$m_{\tilde{l}_R}$	142.97	142.97	0.29
$m_{\tilde{\chi}_2^0}$	176.82	176.82	0.17
$m_{\tilde{q}_L}$	537.2	537.2	2.5
$m_{\tilde{b}_1}$	491.9	491.9	10.9
$m_{\tilde{g}}$	595.2	595.2	5.5
$m_{\tilde{g}} - m_{\tilde{b}_1}$	103.3	103.3	9.0

Mass values (all in GeV) from LHC+LC. Occurrences of *(1,2)* solutions are reduced to $\sim 1\%$, and left out.

Summary

- SPS 1a SUSY masses can be determined with precision 4-10 GeV
- Non-zero probability of fitting wrong minimum
  (could be off by 10-20 GeV)
- Gluino mass can be obtained using two b jets
- LC input on LSP mass (σ = 50 MeV) removes ambiguity
- LC input increases precision from 6 GeV (~15 GeV if wrong minimum) to 2.5 GeV