Supersymmetry Parameter Analysis with Fittino

Philip Bechtle (SLAC)

Klaus Desch (Univ. Freiburg)

Peter Wienemann (DESY)

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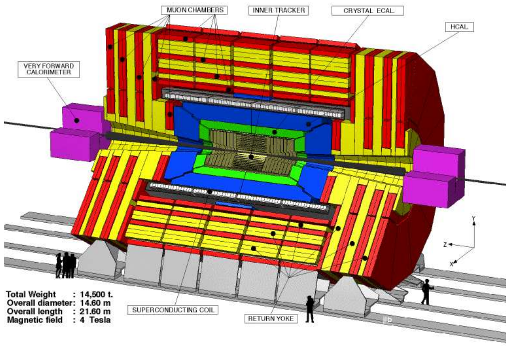
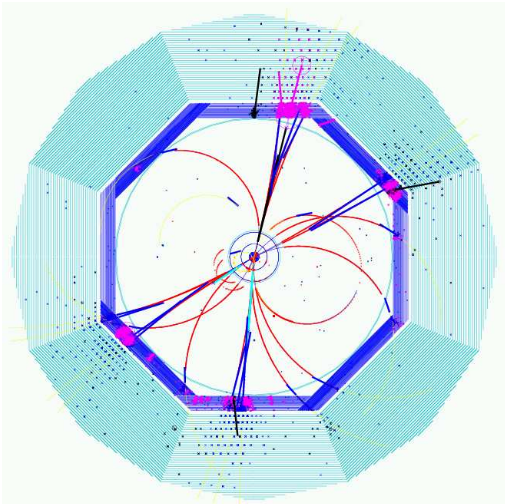
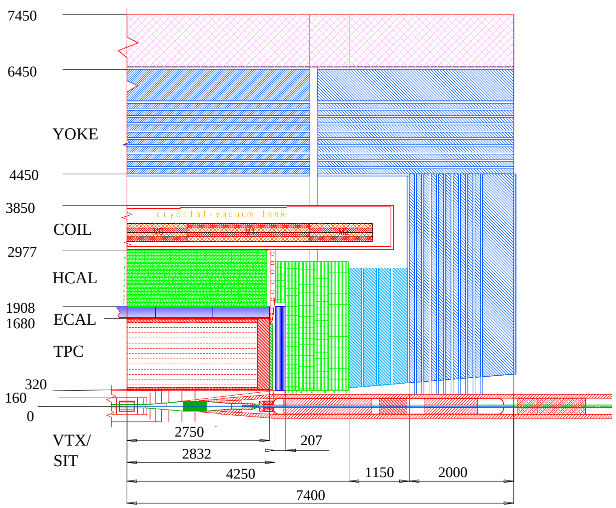
Outline

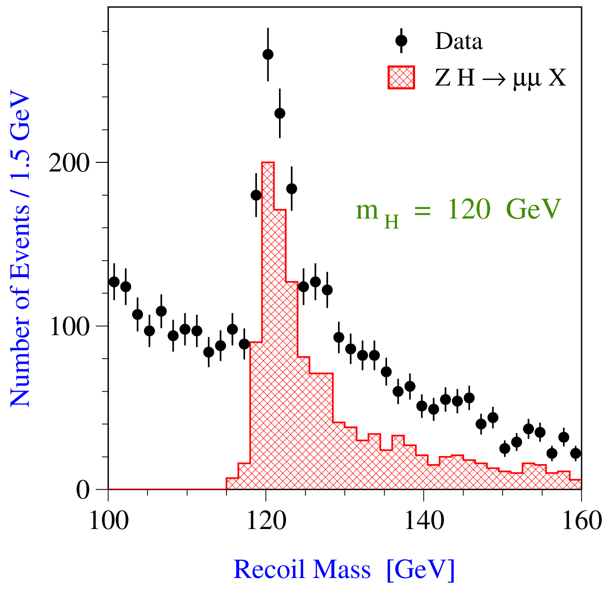
- Motivation and introduction to Fittino
- Parameter determination in Fittino
- Fittino results using ILC and LHC observables
- One more very convincing reason why life without the ILC is very boring
- Conclusions



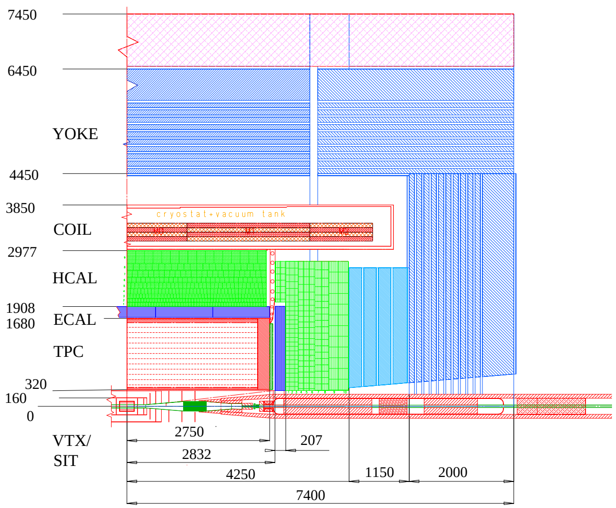


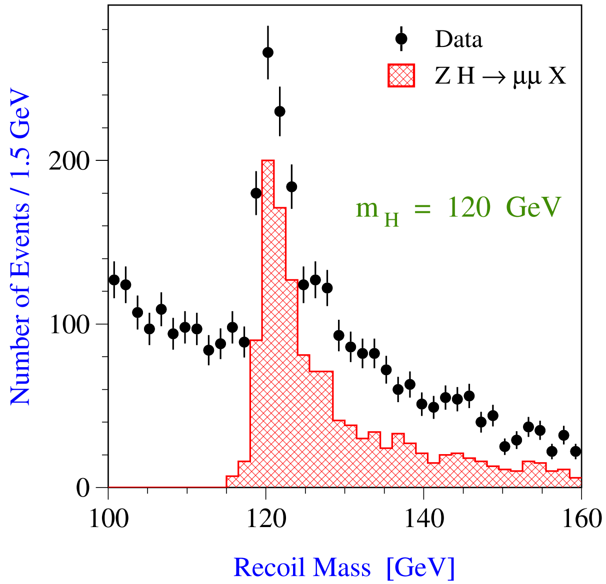
Linking Measurements and Theory



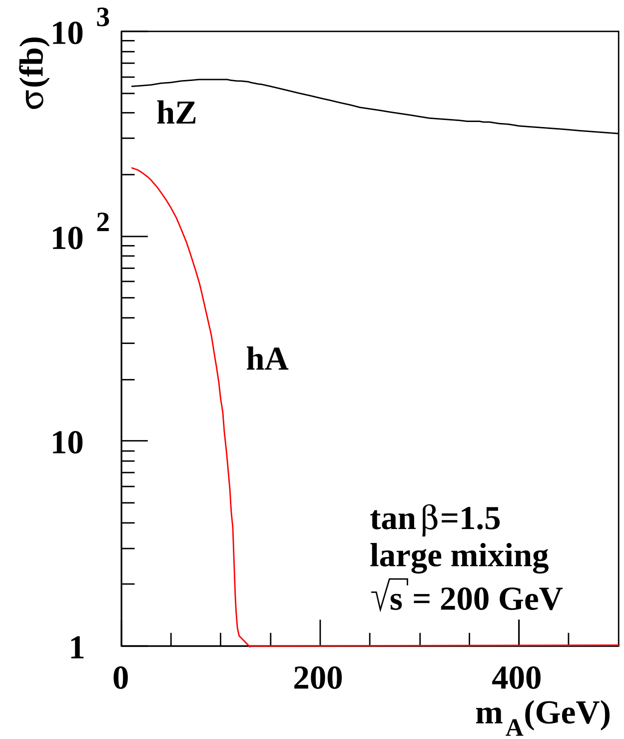


Linking Measurements and Theory

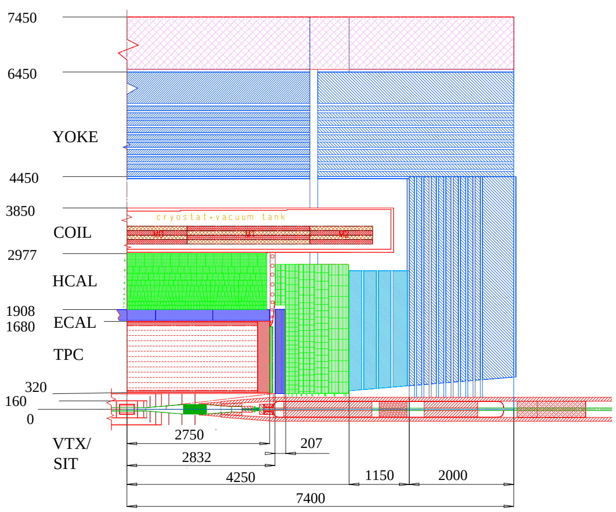


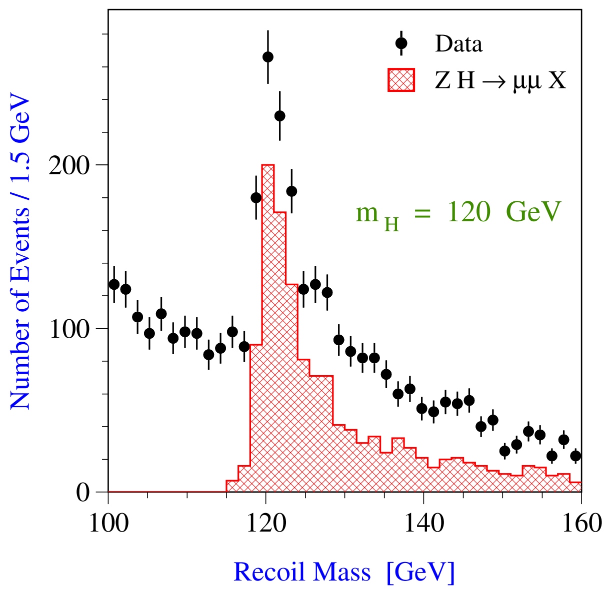


$$
\begin{aligned}
V_{\text{Higgs}} \quad = \quad & m_{1H}^2|H_1|^2 + m_{2H}^2|H_2|^2 \\
& -m_{12}^2(\epsilon_{ij}H_1^iH_2^j + h.c.) \\
& +\frac{1}{8}(g^2+g'^2)(|H_1|^2 - \\
& |H_2|^2)^2 + \frac{1}{2}g^2|H_1^*H_2|^2
\end{aligned}
$$



Linking Measurements and Theory

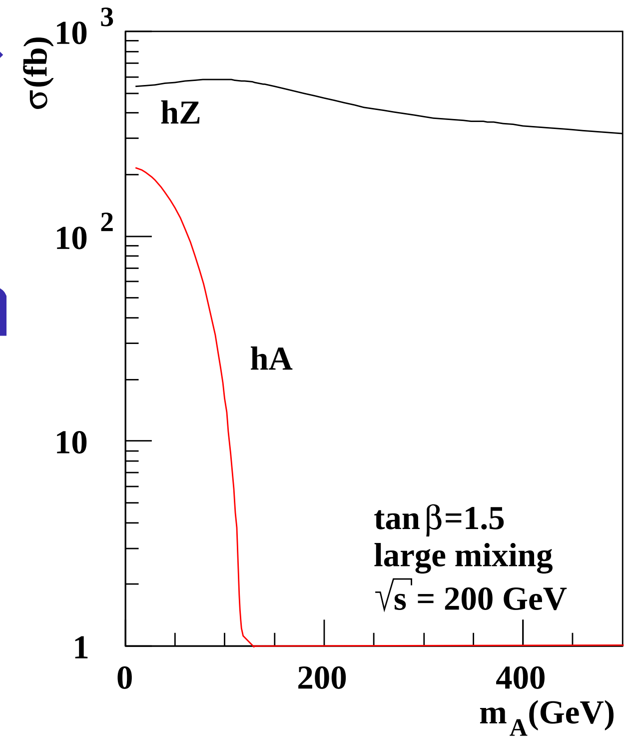




$$V_{\text{Higgs}} = m_{1H}^2|H_1|^2 + m_{2H}^2|H_2|^2 - m_{12}^2(\epsilon_{ij}H_1^iH_2^j + h.c.) + \frac{1}{8}(g^2+g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g^2|H_1^*H_2|^2$$

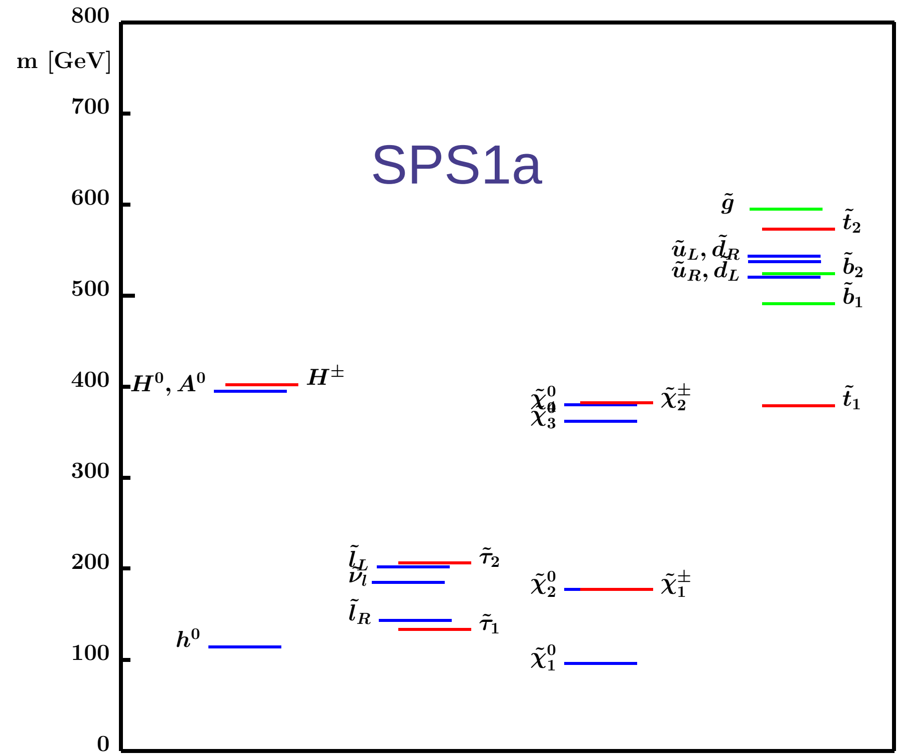
Link:

Parameter Determination



MSSM Physics at the ILC

- Higgs physics:
  - light Higgs mass precision $\approx 50\,\mathrm{MeV}$
  - light Higgs BR $\approx 2\,\%$
  - Discovery of heavy Higgs bosons $> 400\,\mathrm{GeV}$
- Sleptons: $\Delta m_{\tilde{\mu}_R} = 0.2\,\mathrm{GeV}$
- Gauginos: $\Delta m_{\chi_1^\pm} = 0.5\,\mathrm{GeV}$
- Squarks: $\Delta m_{\tilde{t}_1} = 2\,\mathrm{GeV}$
- + precise analysis of quantum numbers (Higgs parity...)
- + precise analysis of couplings from BR and σ
- indirect prediction of particle masses (LHC/ILC interplay)
- Precision of interpretation has to match precision of measurement



The Fit Program Fittino

- Determine the low-energy MSSM Lagrangian parameters from the observables from the ILC and LHC in a global fit
- Use full theoretical precision, all available loop effects
- Bottom-up approach, no assumption on SUSY breaking mechanism
- To be unbiased: Use no prior knowledge of the parameters at any step
- Provide easy user interface for measurements, parameter definitions and output
- Goals:
  - Unambiguous parameter determination without human bias?
  - Determine precision of parameter measurements
  - Test the necessary experimental and theoretical precision
- More information in `http://www-flc.desy.de/fittino/`
- Similar Program: SFitter by R. Lafaye, T. Plehn and D. Zerwas

Observables known to Fittino

- Fittino can fit to any combination of the following observables:
  - Masses of SM and MSSM particles
  - Edges in mass spectra
  - Particle widths
  - Branching fractions, sums of branching fractions
  - Cross-sections (here: only ILC)
  - Any product of cross-sections and branching fractions
  - Ratios of branching fractions
- Correlations among observables can be specified
- Limits on masses of unobserved particles can be specified

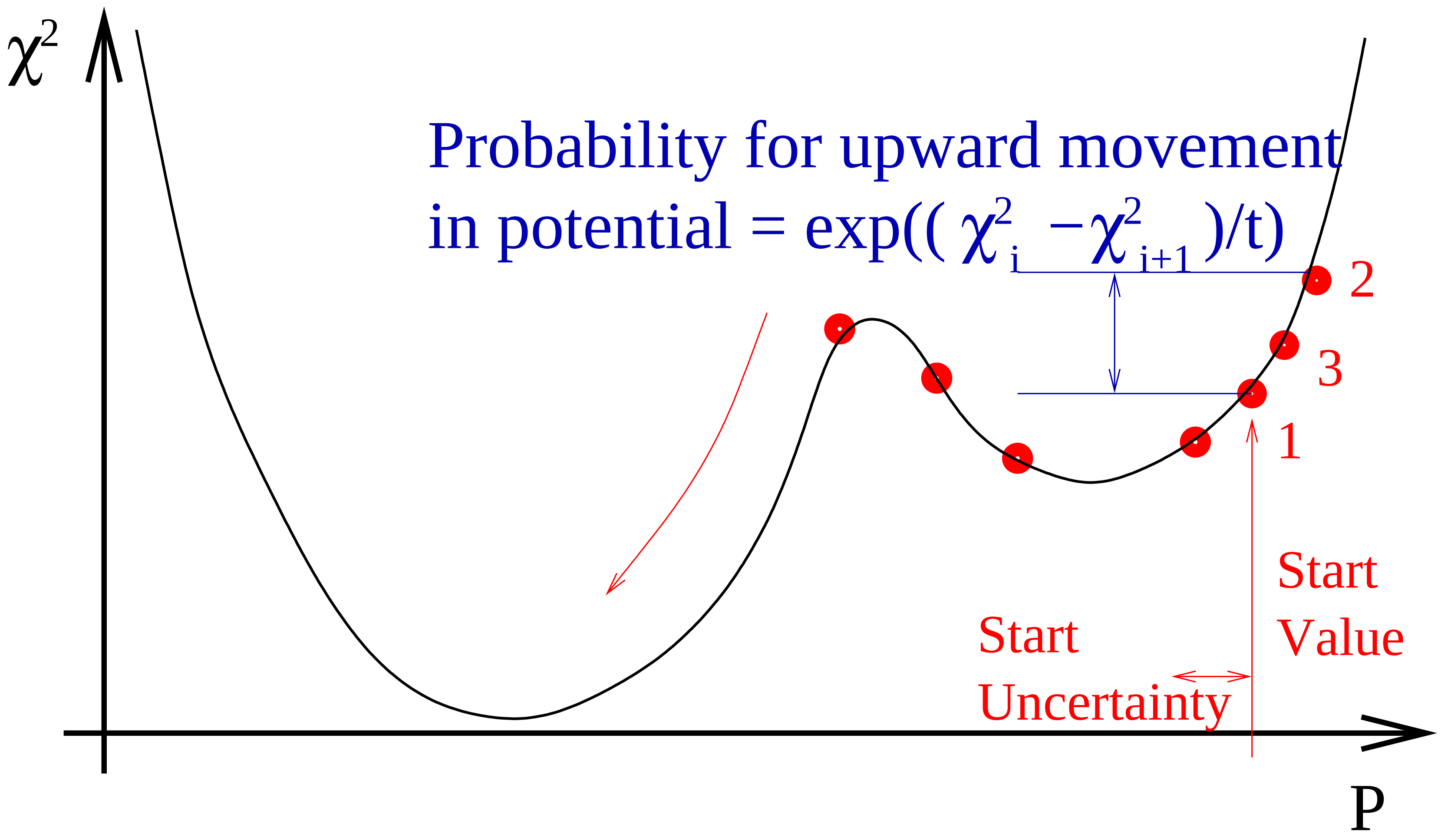
The Iterative Fit Procedure

- Challenge: A χ^2 fit with ≈ 20 parameters probably does not converge without good start values
- Solution: Iterative fit procedure
  - Tree-level estimates of parameters P_i from observables O_j
  - Subsector fits or simulated annealing to approach/find minimal χ^2
  - Global fit to refine minimum and find global parameter uncertainties and correlations
- Additional Features of Fittino:
  - Pulls: Determine χ^2 and pull distributions from toy experiments independent check of uncertainties, biases and correlations
  - Determination of most important observables for each parameter determination
  - 2d scans and uncertainty contours

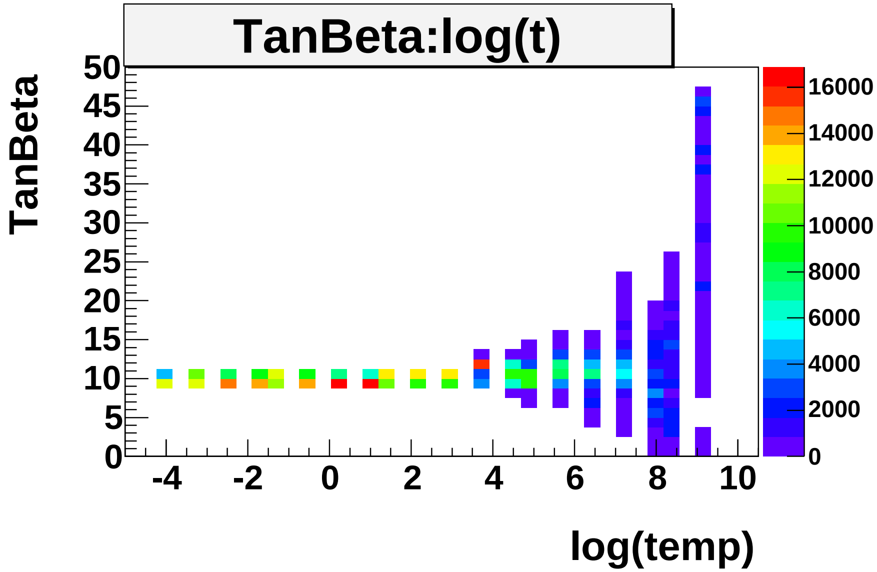
Don’t Ever Believe Minuit!

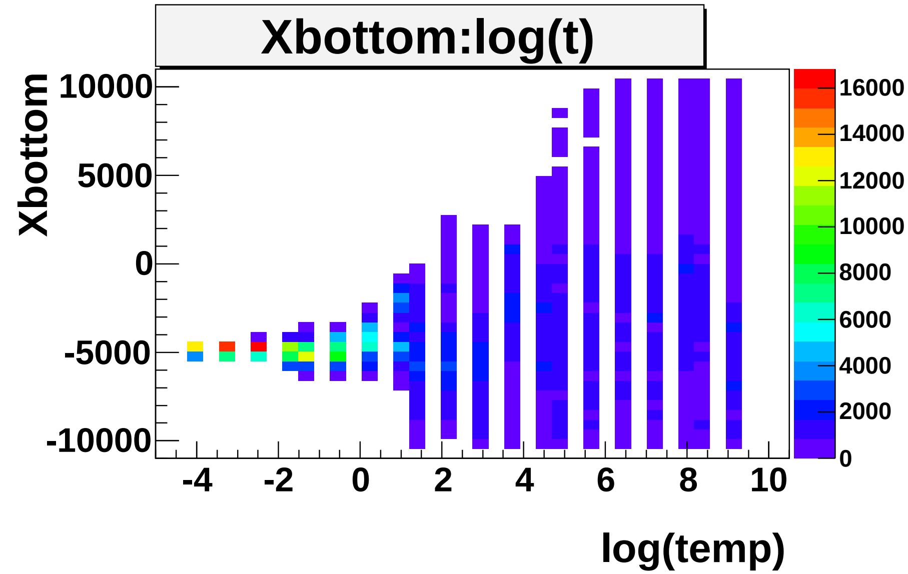
- Typically in complex fits it does not converge to the true minimum
- Even if you can check that it does: What about the uncertainties and parameter correlations?
- Clearly don’t believe Minuit, if Error matrix accurate is not claimed
- If Error matrix accurate: Check error matrix!
  - Clearly something is suspicious, if one line and row is 0 apart from diagonal element
  - Even if this is not the case it might be wrong
- Always create pull distributions

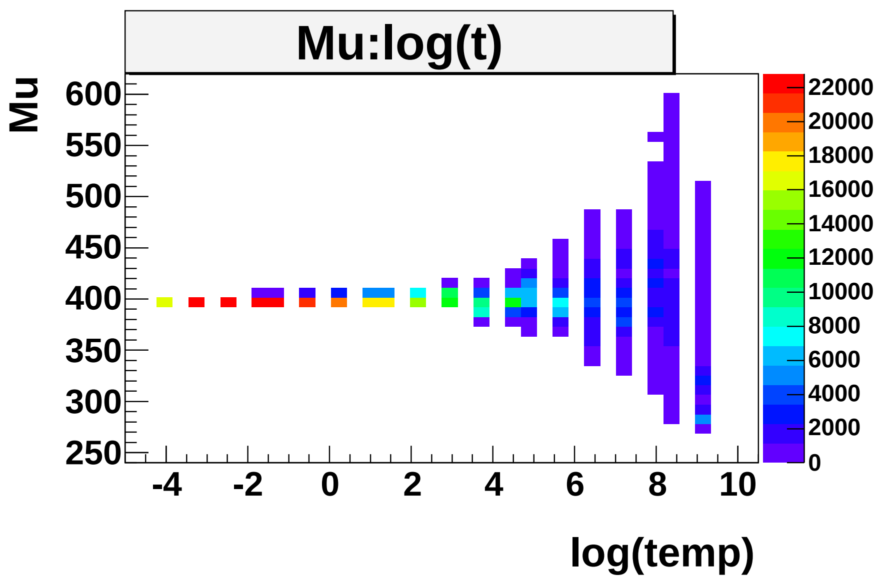
Simulated Annealing

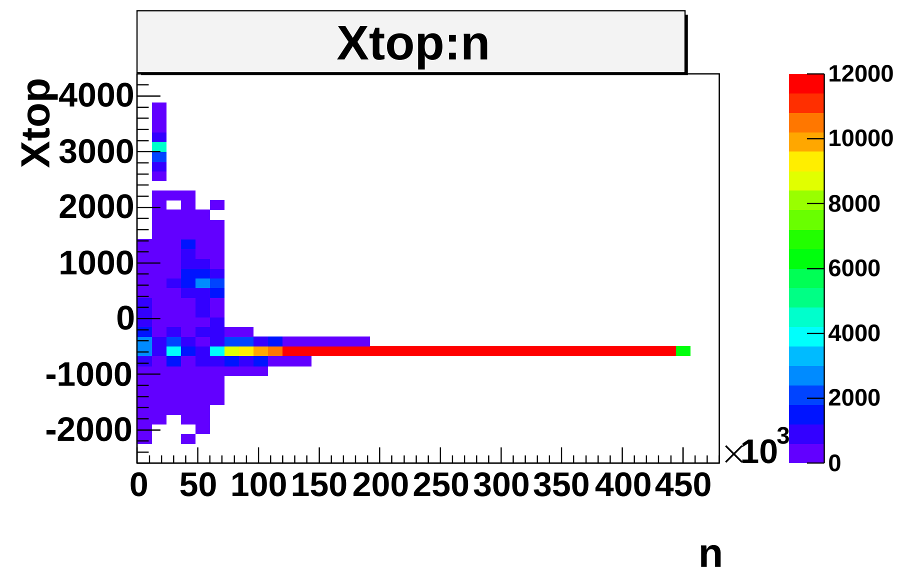


Variable parameter scan









The SPS1a' Fit: Inputs

- Observables: for the SPS1a' scenario:
  - SM observables $m_Z, m_W, G_F, m_t, \ldots$
  - Higgs sector masses from 500 GeV and 1 TeV ILC
  - All accessible sparticle and gaugino masses from LHC and ILC with realistic uncertainties (hep-ph/0410364)
  - ILC $\sigma \times \mathrm{BR}$ at 400,500,1000 GeV, polarisation LR, RL, LL and RR
  - absolute h BR's and σ (hep-ph/0106315)
- Assumptions for this test:
  - Unification in the first two generations, no complex phases, no squark mixing across flavours
- Two fits:
  - No theory uncertainty
  - Theory uncertainty on all masses and $2\times$ larger σ uncertainties
- Use SPheno (by W. Porod) as SUSY calculator

ILC+LHC Parameter Measurement

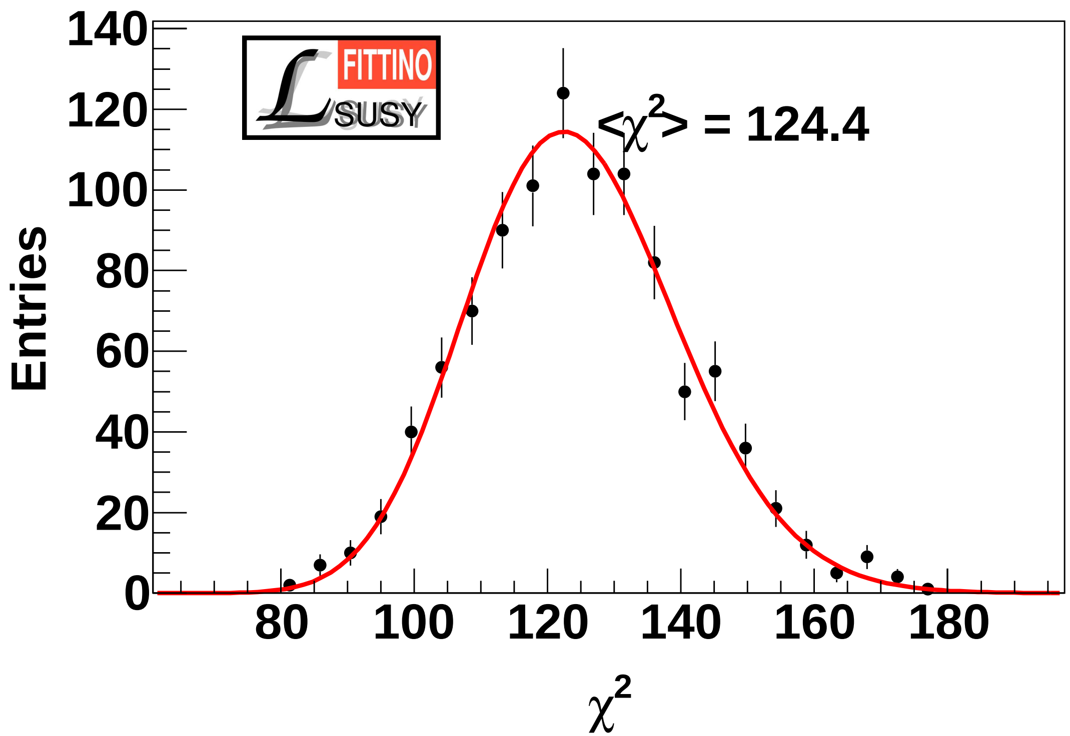
Don't read all numbers!

Parameter	"True" value	Fit value	Uncertainty (exp.)	Uncertainty (exp.+theor.)
$\tan\beta$	10.00	10.00	0.11	0.15
μ	400.4 GeV	400.4 GeV	1.2 GeV	1.3 GeV
X_τ	−4449. GeV	−4449. GeV	20. GeV	30. GeV
$M_{\tilde{e}_R}$	115.60 GeV	115.60 GeV	0.27 GeV	0.50 GeV
$M_{\tilde{\tau}_R}$	109.89 GeV	109.89 GeV	0.41 GeV	0.60 GeV
$M_{\tilde{e}_L}$	181.30 GeV	181.30 GeV	0.10 GeV	0.12 GeV
$M_{\tilde{\tau}_L}$	179.54 GeV	179.54 GeV	0.14 GeV	0.19 GeV
X_t	−565.7 GeV	−565.7 GeV	3.1 GeV	15.4 GeV
X_b	−4935. GeV	−4935. GeV	1284. GeV	1825. GeV
$M_{\tilde{u}_R}$	503. GeV	503. GeV	24. GeV	27. GeV
$M_{\tilde{b}_R}$	497. GeV	497. GeV	8. GeV	15. GeV
$M_{\tilde{t}_R}$	380.9 GeV	380.9 GeV	2.5 GeV	3.9 GeV
$M_{\tilde{u}_L}$	523. GeV	523. GeV	10. GeV	15. GeV
$M_{\tilde{t}_L}$	467.7 GeV	467.7 GeV	3.1 GeV	5.1 GeV
M_1	103.27 GeV	103.27 GeV	0.06 GeV	0.14 GeV
M_2	193.45 GeV	193.45 GeV	0.10 GeV	0.15 GeV
M_3	569. GeV	569. GeV	7. GeV	7. GeV
$m_{A_{run}}$	312.0 GeV	311.9 GeV	4.6 GeV	6.9 GeV
m_t	178.00 GeV	178.00 GeV	0.050 GeV	0.108 GeV
χ^2 for unsmeared observables: 5.3×10^{-5}				

FITTINO SUSY

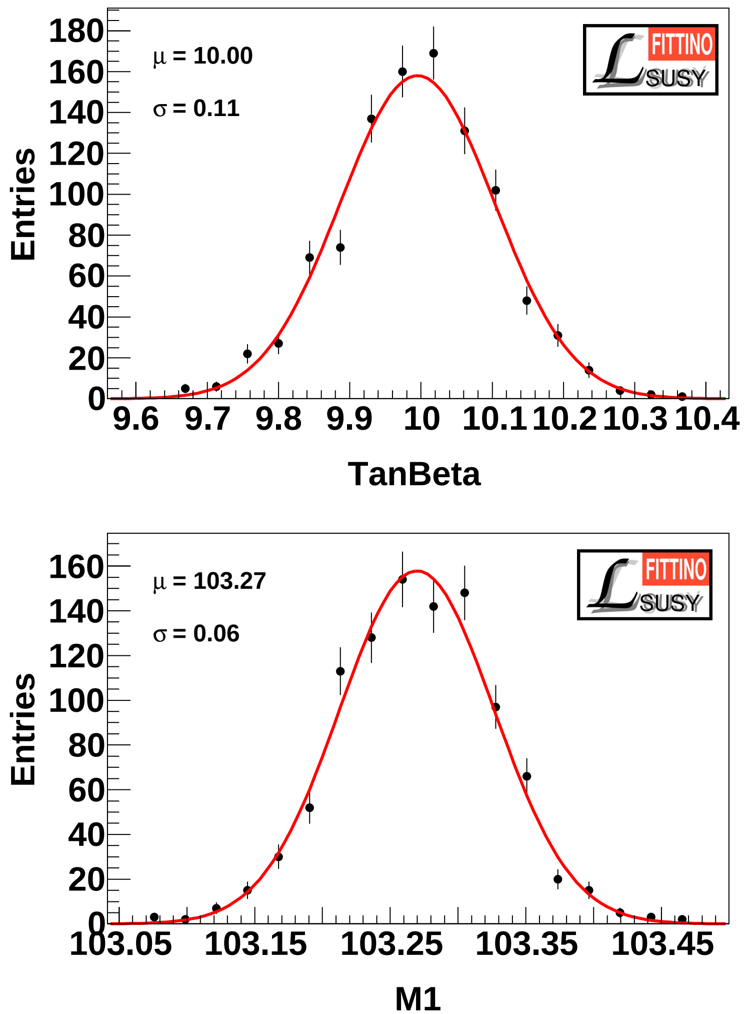
χ^2 and Toy Fit Distributions

χ^2 distribution:



for n.d.f. = 126

Toy Fits:



Importance of Observables

Parameter Value	Total $\Delta\chi^2$	Observable	Contribution to the $\Delta\chi^2$ in %
$\tan\beta$	5.0	$\sigma(e_L^- e_R^+ \to H^\pm H^\mp \to t\bar{b}\bar{t}b)$ 1 TeV	31.1
10.00 ± 0.11		$\sigma(e_L^- e_R^+ \to HA \to b\bar{b}b\bar{b})$ 1 TeV	9.61
		m_h	8.12
$M_{\tilde{e}_R}$	27.7	$m_{\tilde{e}_R}$	89.3
115.60 ± 0.27 GeV		$m_{\tilde{\mu}_R}$	5.58
		$\sigma(e_L^- e_R^+ \to \tilde{\mu}_R^- \tilde{\mu}_L^+ \to \chi_1^0 \mu^- \chi_1^0 \mu^+)$ 400 GeV	0.74
$M_{\tilde{q}_R}$	36.6	$m_{\tilde{e}_R}$	53.6
501.6 ± 23.6 GeV		$m_{\tilde{\mu}_R}$	3.34
		$\sigma(e_L^- e_R^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^- \to \bar{\nu}_\tau \chi_1^0 \tau^- \nu_\tau \chi_1^0 \tau^+)$ 1 TeV	3.25
		$m_{\tilde{q}_R}$	2.42
m_t	1.2	m_t	80.7
178.00 ± 0.05 GeV		m_h	18.6
		$\sigma(e_L^- e_R^+ \to \tilde{t}_1^- \tilde{t}_1^+ \to \chi_1^0 \tau^- \bar{\nu}_\tau \bar{b} \chi_1^0 \tau^+ \nu_\tau b)$ 1 TeV	0.13

Warning: Partial fits are not reliable

- ILC 400 and 500 data only, n.d.f = 75:

Parameter	"True" value	Fit with correctly fixed parameters	Fit with incorrectly fixed parameters	Uncertainty
		Fixed parameters		
X_t	-565.7 GeV	-565.7 GeV	-30.0 GeV	fixed
X_b	-4934.8 GeV	-4934.8 GeV	-4000.0 GeV	fixed
$M_{\tilde{q}_R}$	501.6 GeV	501.6 GeV	600.0 GeV	fixed
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
M_3	568.9 GeV	568.9 GeV	700.0 GeV	fixed
$m_{\mathrm{A}_\mathrm{run}}$	312.0 GeV	312.0 GeV	400.0 GeV	fixed
m_t	178.0 GeV	178.0 GeV	178.0 GeV	fixed
		Additional parameters of fit excluding Higgs sector observables		
$\tan\beta$	10.00	10.00	11.1	0.47
μ	400.39 GeV	400.388 GeV	388.3 GeV	3.1 GeV
X_τ	-4449.2 GeV	-4449.2 GeV	-4447.8 GeV	37.2 GeV
$M_{\tilde{e}_R}$	115.60 GeV	115.602 GeV	113.74 GeV	0.06 GeV
$M_{\tilde{\tau}_R}$	109.89 GeV	109.89 GeV	107.77 GeV	0.48 GeV
$M_{\tilde{e}_L}$	181.30 GeV	181.304 GeV	181.76 GeV	0.04 GeV
$M_{\tilde{\tau}_L}$	179.54 GeV	179.54 GeV	179.99 GeV	0.14 GeV
M_1	103.271 GeV	103.271 GeV	103.11 GeV	0.05 GeV
M_2	193.446 GeV	193.445 GeV	193.49 GeV	0.12 GeV
χ^2		1.8×10^{-5}	5.89	

Comparison of LHC and ILC+LHC

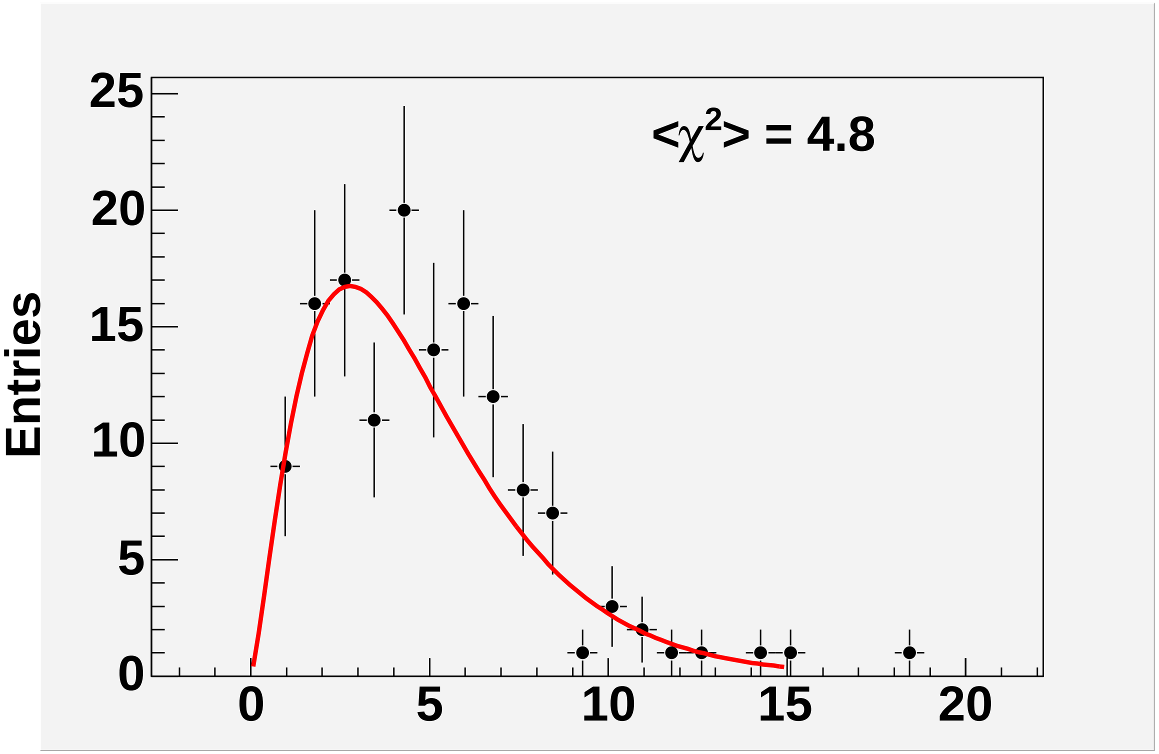
For SPS1a':

Compare the ability of LHC only and ILC+LHC to understand what is discovered:

- Use ILC+LHC inputs and results as before
- For the LHC (considered as LHC friendly assumptions):
  - SM observables as before
  - Mass measurements and precision as in LHC/ILC report `hep-ph/0410364`
  - + χ_1^+ mass measurement and precision from G. Polesello *et al* `hep-ph/0312318`
  - Ratios of Higgs branching fractions from M. Dührssen ATLAS Note
- Then plot relative size of uncertainties (RMS of toy fits) and bias (LHC only uncertainties normalized to 1, biases and ILC+LHC uncertainties normalized accordingly)

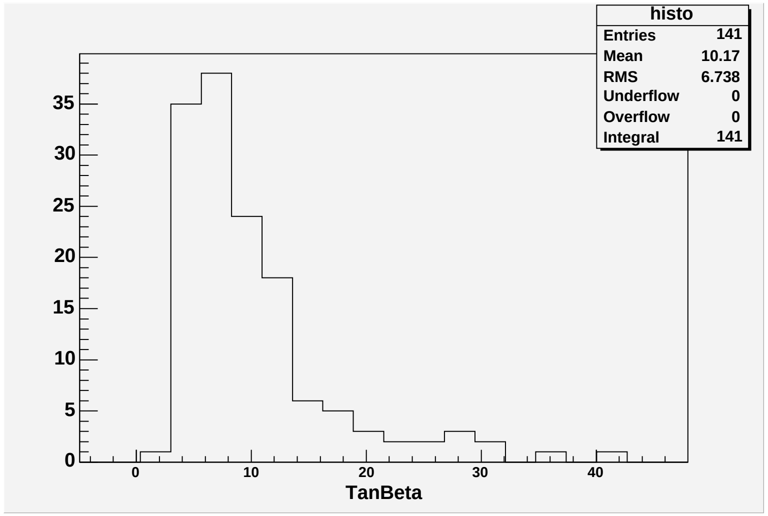
LHC only Pull Distributions

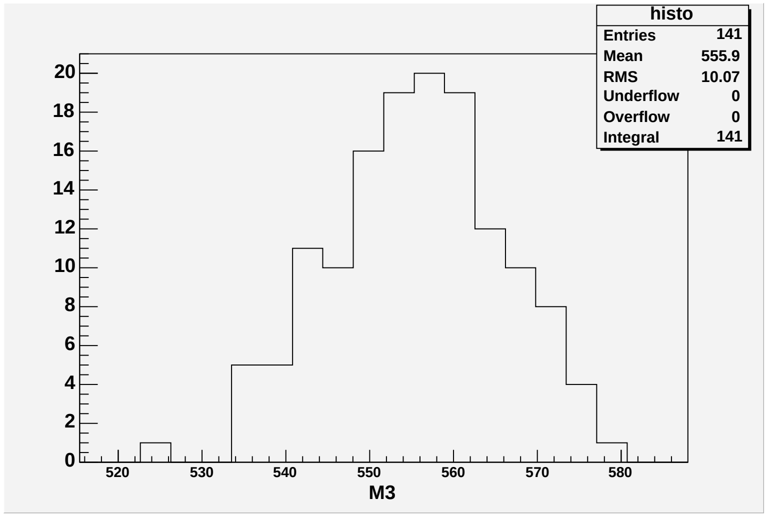
χ^2 distribution:



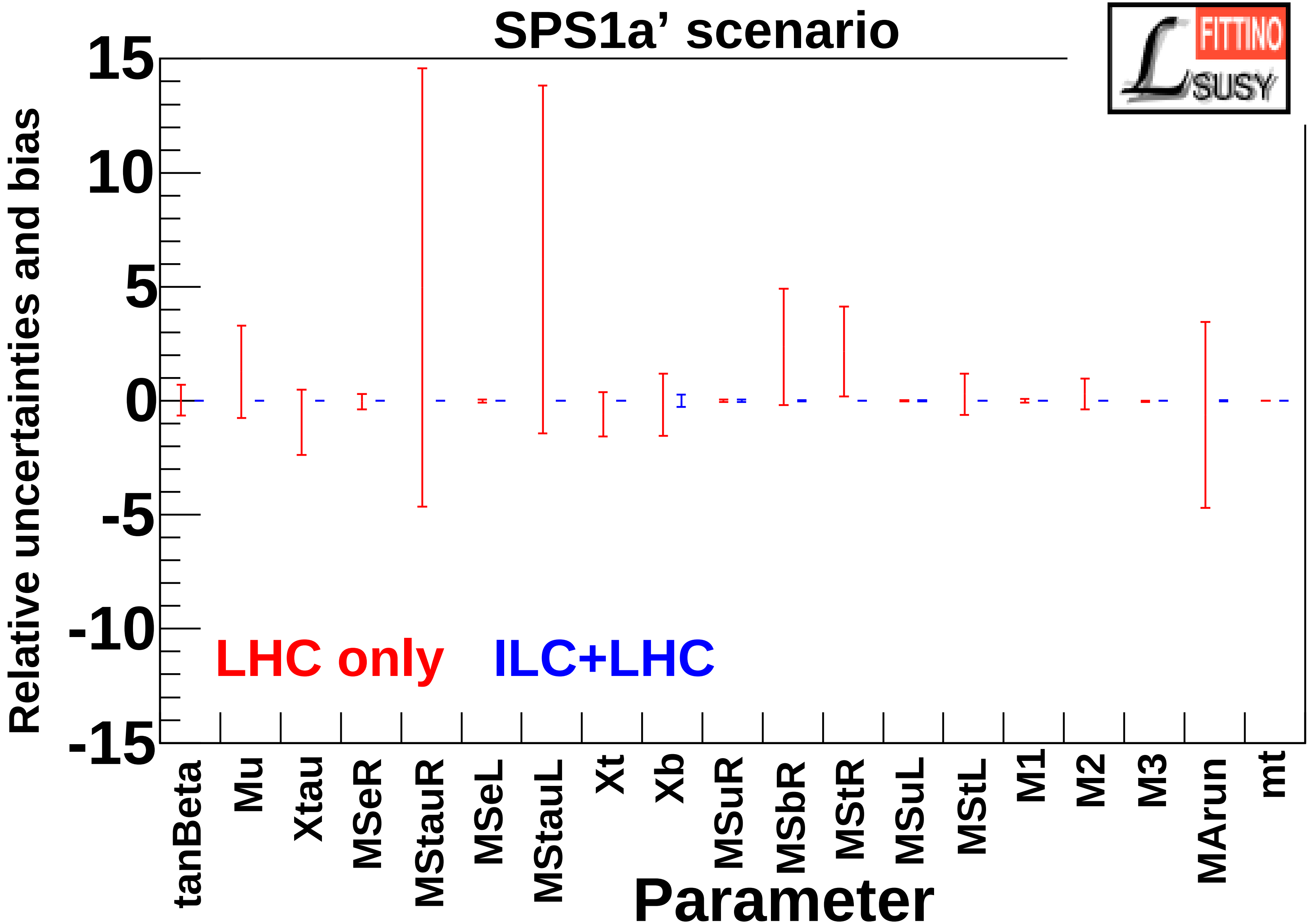
for n.d.f. = 2

Pulls:

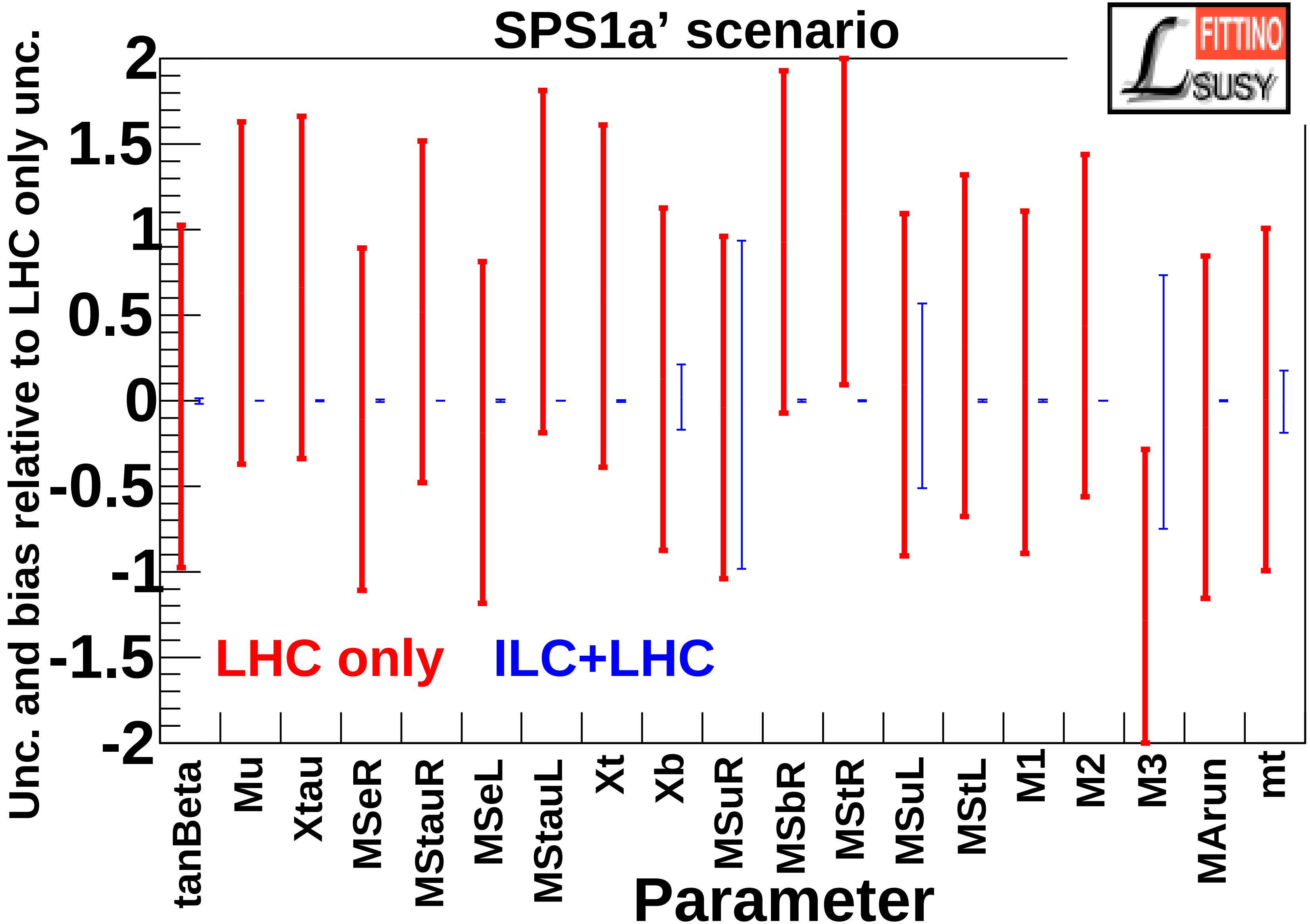




Comparison of LHC and ILC+LHC



Comparison of LHC and ILC+LHC



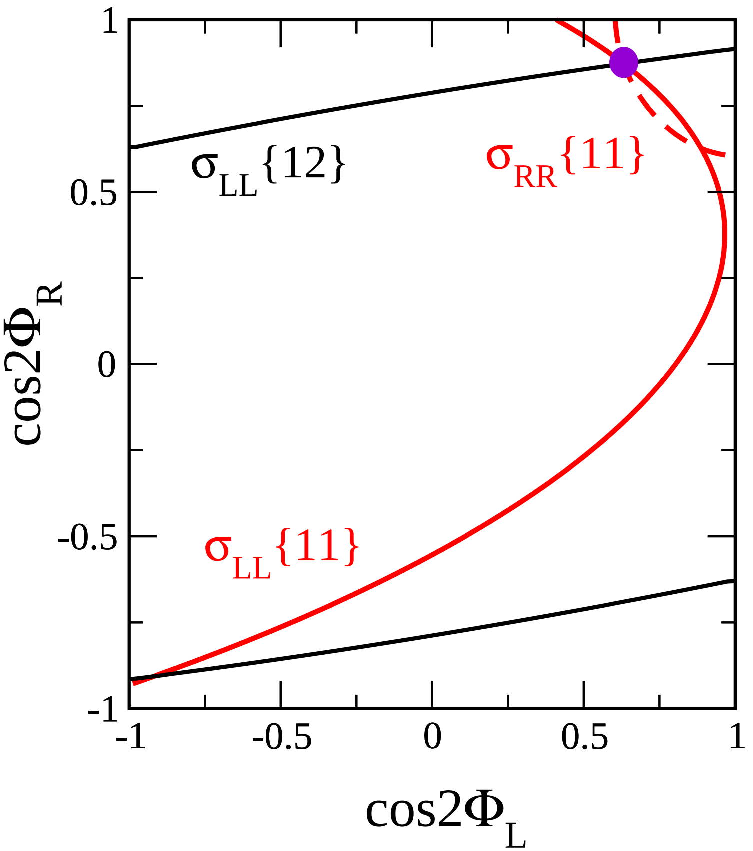
Conclusions

- Fittino measures SUSY Lagrangian parameters using many LHC and ILC observables, taking loop-corrections into account
- Fittino uses tree-level estimates, subspace fits and Simulated Annealing, allows to find the correct parameters in highly complex parameter space
- Lessons from Fittino:
  - Parameter determination is possible (non-trivial!)
  - Precise analysis of SUSY parameters requires high ILC precision
  - Theoretical Uncertainties can strongly affect parameter uncertainties
  - Fittino can be used to identify regions with need of theoretical or experimental improvements
  - Fittino could be used to optimize $\mathcal{L}(\sqrt{s}, \mathrm{pol.})$
- Fittino is available from `http://www-flc.desy.de/fittino`, see also `hep-ph/0412012`
- ILC + LHC can be the era of deeper understanding of new physics

Related Information

- Fittino:
  http://www-flc.desy.de/fittino/
  hep-ph/0412012 (submitted to *Comp. Phys. Commun.*)
- SFitter:
  http://sfitter.web.cern.ch/SFITTER/
- SPA:
  http://spa.desy.de/spa/
- SPheno:
  http://www-theorie.physik.unizh.ch/˜porod/SPheno.html
- Results from Fittino:
  DESY-THESIS-2004-040
  Publication to follow soon

Tree-Level Estimates: Gaugino Sector



from Zerwas *et al*, hep-ph/0211076

- Measure chargino and Neutralino masses
- Use the chargino cross-sections at different beam polarisations to determine the *pseudo-observables* $\cos 2\phi_L$, $\cos 2\phi_R$ (Chargino mixing angles)

$$|\mu| = m_W \left(\Sigma + \Delta(\cos 2\phi_L + \cos 2\phi_R)\right)^{\frac{1}{2}}$$

$$\tan\beta = \left(\frac{1 + \Delta(\cos 2\phi_R - \cos 2\phi_L)}{1 - \Delta(\cos 2\phi_R - \cos 2\phi_L)}\right)^{\frac{1}{2}}$$

$$M_2 = m_W \left(\Sigma - \Delta(\cos 2\phi_L + \cos 2\phi_R)\right)^{\frac{1}{2}}$$

$$M_3 = m_{\tilde{g}}$$

$$\Sigma = (m^2_{\chi^\pm_2} + m^2_{\chi^\pm_1})/2m^2_W - 1$$

$$\Delta = (m^2_{\chi^\pm_2} - m^2_{\chi^\pm_1})/4m^2_W$$

- Get M_1 from neutralino system

The Slepton and Squark Sector

- Problem: hard to get 3rd gen. squark mixing on tree-level
- Estimate squark and slepton mass parameters with assumption $A = 0$:

$$
\begin{aligned}
M_{\tilde{\mathrm{t}}_L} &= -m_{\mathrm{Z}}{}^2 \cos 2\beta\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) - m_{\mathrm{t}}^2 + \frac{1}{2}\left(m_{\tilde{\mathrm{t}}_1}^2 + m_{\tilde{\mathrm{t}}_2}^2\right) \\
&= m_{\mathrm{Z}}{}^2 \cos 2\beta\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right) - m_{\mathrm{b}}^2 + \frac{1}{2}\left(m_{\tilde{\mathrm{b}}_1}^2 + m_{\tilde{\mathrm{b}}_2}^2\right) \\
M_{\tilde{\mathrm{t}}_R} &= -m_{\mathrm{Z}}{}^2 \cos 2\beta \frac{2}{3}\sin^2\theta_W - m_{\mathrm{t}}^2 + \frac{1}{2}\left(m_{\tilde{\mathrm{t}}_1}^2 + m_{\tilde{\mathrm{t}}_2}^2\right) \\
M_{\tilde{\mathrm{b}}_R} &= m_{\mathrm{Z}}{}^2 \cos 2\beta \frac{1}{3}\sin^2\theta_W - m_{\mathrm{b}}^2 + \frac{1}{2}\left(m_{\tilde{\mathrm{b}}_1}^2 + m_{\tilde{\mathrm{b}}_2}^2\right) \\
X_{\mathrm{t}} &= -\mu/\tan\beta \\
X_{\mathrm{b}} &= -\mu\tan\beta
\end{aligned}
$$

- Slepton sector treated analogously

LHC only fit: Results

Don't read all numbers!

Parameter	"True" value	ILC Fit value	Uncertainty (ILC+LHC)	Uncertainty (LHC only)
$\tan\beta$	10.00	10.00	0.11	6.7
μ	400.4 GeV	400.4 GeV	1.2 GeV	$811.$ GeV
X_τ	$-4449.$ GeV	$-4449.$ GeV	$20.$ GeV	$6368.$ GeV
$M_{\tilde{e}_R}$	115.60 GeV	115.60 GeV	0.27 GeV	$39.$ GeV
$M_{\tilde{\tau}_R}$	109.89 GeV	109.89 GeV	0.41 GeV	$1056.$ GeV
$M_{\tilde{e}_L}$	181.30 GeV	181.30 GeV	0.10 GeV	12.9 GeV
$M_{\tilde{\tau}_L}$	179.54 GeV	179.54 GeV	0.14 GeV	$1369.$ GeV
X_t	-565.7 GeV	-565.7 GeV	3.1 GeV	$548.$ GeV
X_b	$-4935.$ GeV	$-4935.$ GeV	$1284.$ GeV	$6703.$ GeV
$M_{\tilde{u}_R}$	$503.$ GeV	$503.$ GeV	$24.$ GeV	$25.$ GeV
$M_{\tilde{b}_R}$	$497.$ GeV	$497.$ GeV	$8.$ GeV	$1269.$ GeV
$M_{\tilde{t}_R}$	380.9 GeV	380.9 GeV	2.5 GeV	$753.$ GeV
$M_{\tilde{u}_L}$	$523.$ GeV	$523.$ GeV	$10.$ GeV	$19.$ GeV
$M_{\tilde{t}_L}$	467.7 GeV	467.7 GeV	3.1 GeV	$424.$ GeV
M_1	103.27 GeV	103.27 GeV	0.06 GeV	8.0 GeV
M_2	193.45 GeV	193.45 GeV	0.10 GeV	$132.$ GeV
M_3	$569.$ GeV	$569.$ GeV	$7.$ GeV	10.1 GeV
$m_{\mathrm{A_{run}}}$	312.0 GeV	311.9 GeV	4.6 GeV	$1272.$ GeV
m_t	178.00 GeV	178.00 GeV	0.050 GeV	0.27 GeV
χ^2 for unsmeared observables: 5.3×10^{-5}				

Simulated Annealing