Renormalisation and Precision Physics

Georg Weiglein

IPPP Durham

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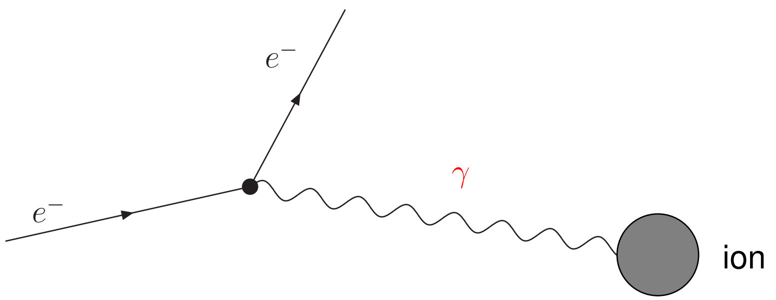
- Introduction
- Evaluation of quantum corrections
- Renormalisation procedure: absorption of infinities and parameter definitions
- Renormalisation of supersymmetric theories
- Precision physics in the SM and beyond
- Conclusions and outlook

The Standard Model of the electroweak and strong interactions

Standard Model (SM): quantum field theory

Classical field theory (e.g. classical electrodynamics):

Quantum field theory (e.g. QED): field is quantised,
field quantum: photon



Interaction: exchange of field quanta

Electroweak and strong interactions

Electroweak interaction:

Fermion fields: quarks: $\begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R,$ leptons: $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, e_R$

3 generations: $u, d, \quad s, c \quad t, b$

$\nu_e, e \quad \nu_\mu, \mu \quad \nu_\tau, \tau$

gauge bosons: γ, Z, W^+, W^-

Gauge group: $\mathrm{SU(2)_I \times U(1)_Y \supset U(1)_{em}}$

Strong interaction: QCD

quarks: q_r, q_g, q_b, gauge bosons: $g_1, \ldots g_8$: gluons, $\mathrm{SU(3)_C}$

Construction principle of the SM: gauge invariance

Example:

Quantum electrodynamics (QED)

free electron field: $\mathcal{L}_{\mathrm{Dirac}} = i\overline{\Psi}\gamma_\mu\partial^\mu\Psi - m\overline{\Psi}\Psi$

invariant under global gauge transformation: $\Psi \to e^{i\theta}\Psi$

Requirement of local gauge invariance:

gauge field A_μ introduced, $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$

gauge transformation: $\Psi \to e^{ie\lambda(x)}\Psi$, $A_\mu \to A_\mu + \partial_\mu\lambda(x)$

Construction of the QED Lagrangian

⇒ Lagrangian with interaction term:

$$\mathcal{L}_{\text{QED}} = \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{free photon field}} + \underbrace{\overline{\Psi}(i\gamma_\mu \partial^\mu - m)\Psi}_{\text{free electron field}} + \underbrace{\mathrm{e}\overline{\Psi}\gamma_\mu \Psi A^\mu}_{\text{interaction}}$$

free photon field free electron field interaction

invariant under local gauge transformations

mass term, $m^2 A^\mu A_\mu$: not gauge-invariant

⇒ A_μ: massless gauge field

Construction of a gauge-invariant Lagrangian for the SM

Problem: gauge fields Z, W^+, W^- are massive

explicit mass terms $\Leftrightarrow$ breaking of gauge invariance

$\Rightarrow$ Need additional concept:

Higgs mechanism, spontaneous electroweak symmetry breaking:

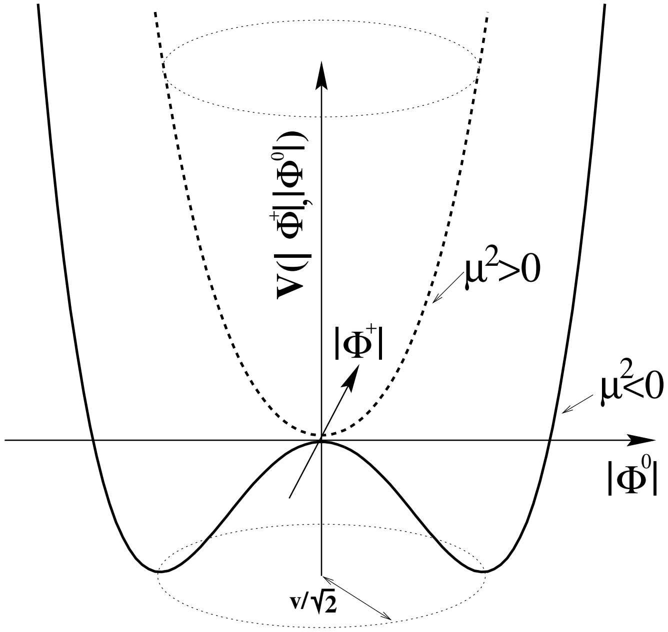
Scalar field postulated, gauge-invariant mass terms from coupling to Higgs field

Higgs sector of the Standard Model

scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential: $V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$

$\mu^2 < 0$: spontaneous symmetry breaking



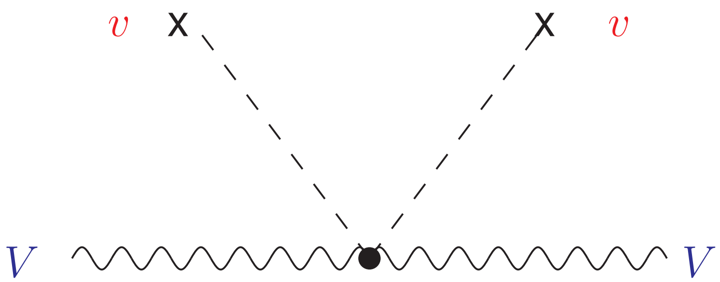
minimum of the potential at

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Gauge-invariant interaction with gauge fields

$$\mathcal{L}_{\rm Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi);$$ unitary gauge: $$\Phi = \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$VV\Phi\Phi$ coupling:



$\Rightarrow$ VV mass terms: $\frac{1}{2} g_2^2 v^2 \equiv M_{\rm W}^2, \quad \frac{1}{2}(g_1^2 + g_2^2) v^2 \equiv M_{\rm Z}^2$

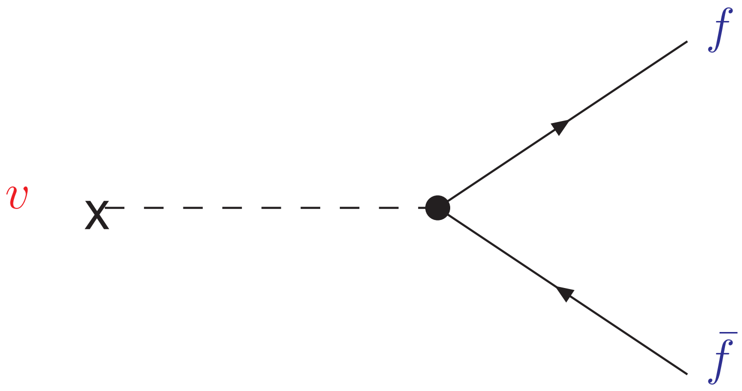
3 components of Higgs doublet

$\longrightarrow$ longitudinal components of $W^\pm$, Z

H: elementary scalar field, Higgs boson

Fermion masses, Higgs mass

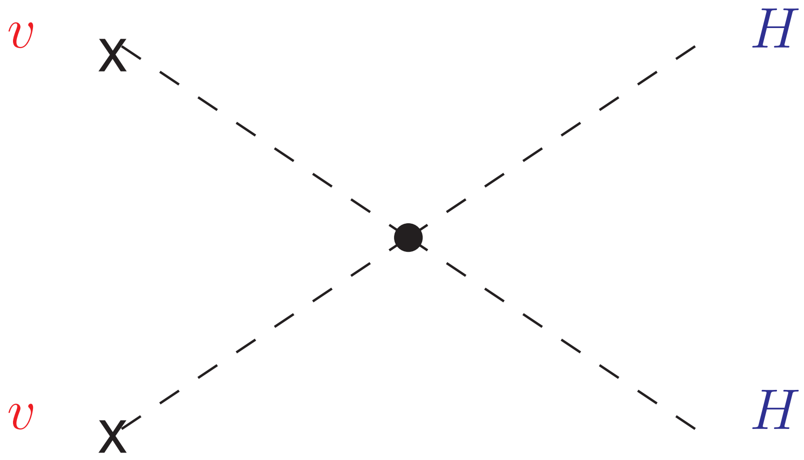
Fermion mass terms: Yukawa couplings



$$m_f = v\, g_f$$

free parameters

Mass of the Higgs boson: self-interaction



$$M_{\mathrm{H}} = v\sqrt{\lambda}$$

free parameter

$\Rightarrow$ Higgs couplings proportional to masses of the particles

Gauge-invariant Lagrangian of the SM

$$\mathcal{L}_{\mathrm{EW}}(\underbrace{g_2, g_1, v}_{M_{\mathrm{W}},\, M_{\mathrm{Z}},\, \alpha},\ \underbrace{\lambda}_{M_{\mathrm{H}}},\ \underbrace{g_f}_{m_f}) + \mathcal{L}_{\mathrm{QCD}}(\alpha_{\mathrm{s}})$$

Gauge invariance $\Rightarrow$ theory is renormalisable

[*G. 't Hooft '71*] [*G. 't Hooft, M. Veltman '72*] *Nobel prize '99*

$\Rightarrow$ theory can consistently been treated as a quantised field theory:

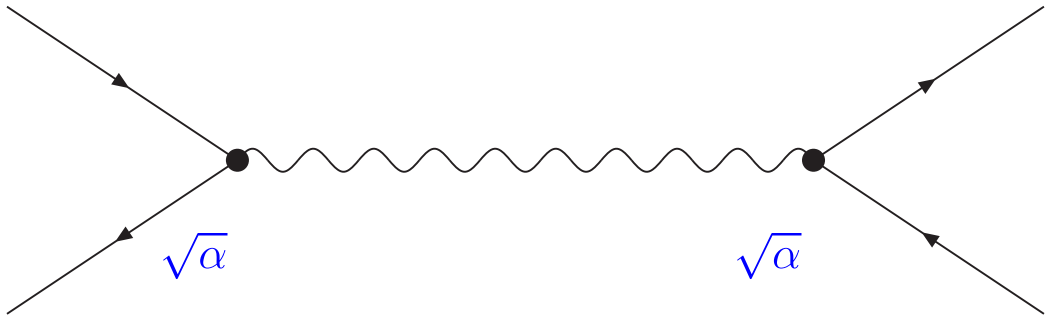
$\Rightarrow$ quantum effects can be evaluated

Perturbative evaluation of quantum field theories

Expansion in coupling constant: $\alpha \approx \frac{1}{137} \ll 1$

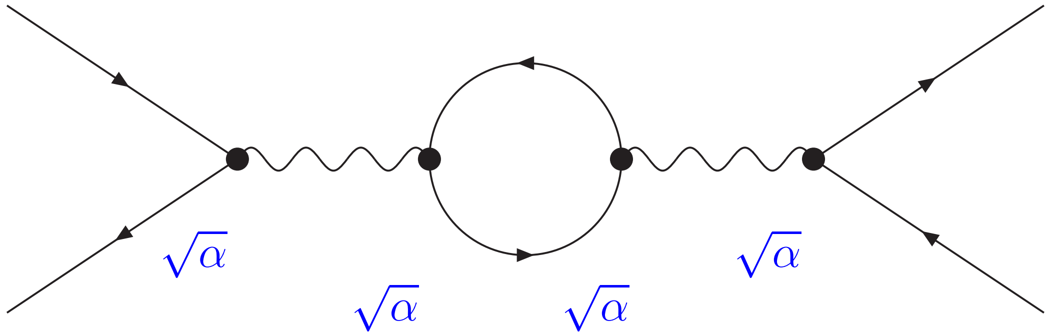
$\Leftrightarrow$ expansion about theory without interaction

lowest order,
classical limit



$\alpha \approx \frac{1}{137}$

quantum
corrections:
loop diagrams



$\mathcal{O}(\alpha)$ relative to lowest order

What can one learn from quantum corrections?

- Inclusion of quantum effects $\Leftrightarrow$ more accurate theoretical predictions

  Large loop corrections:

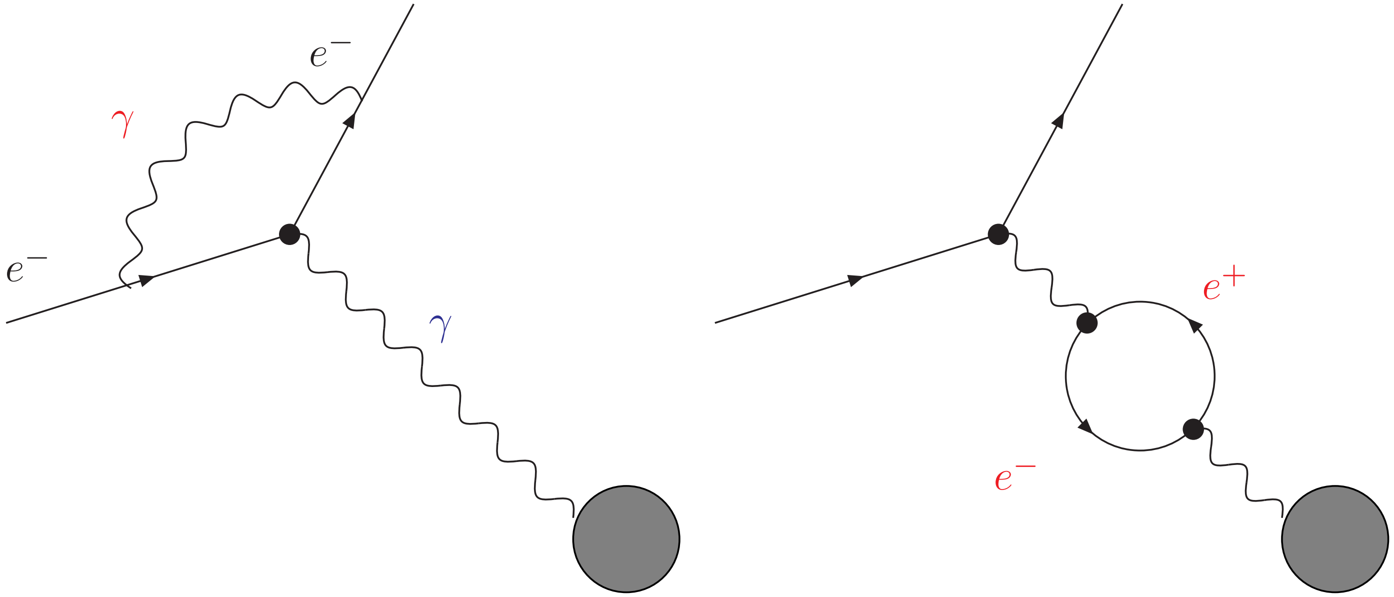
  - QCD corrections are often of $\mathcal{O}(100\%)$
  - EW enhancement factors: m_t^2, m_t^4, $\ldots$,
    large logarithms (involving two very different scales)
  - Per mille level corrections needed to match EW precision measurements

- Quantum effects provide sensitivity to the underlying structure of the theory

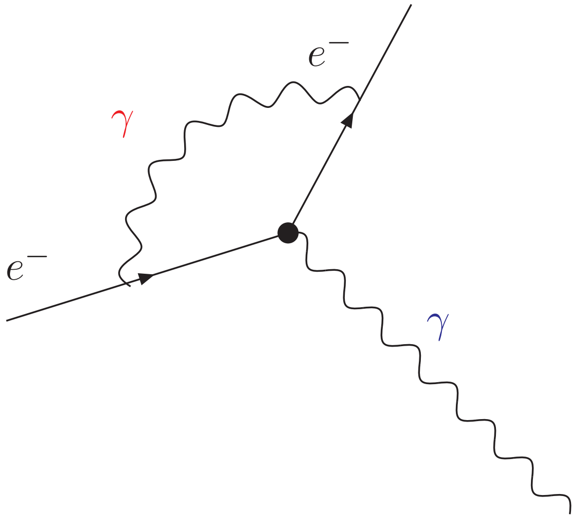
Example: electroweak precision physics

Historical: QED

Lamb shift:



$g-2$:



$$a = \frac{1}{2}(g-2)$$

$$a_{\text{exp}} = 1\,159\,652\,188(\pm 4) \times 10^{-12}$$

$$a_{\text{theo}} = 1\,159\,652\,157(\pm 28) \times 10^{-12}$$

Electroweak precision measurements

- LEP1/SLC: e^+e^- annihilation at the Z-boson resonance

  LEP1: $\sim 4 \times 10^6$ events/experiment

- LEP2: $e^+e^- \to W^+W^-$

- Tevatron: $q\bar{q}' \to W \to l\bar{\nu},\ q\bar{q}'$

  $$q\bar{q} \to t\bar{t},\ t \to W^+b \to \ldots$$

- low-energy experiments: muon decay, $g_\mu - 2$, b physics, . . .

Electroweak precision physics

Electroweak precision measurements:

$$
\begin{array}{lcll}
M_{\mathrm{W}}[\,\mathrm{GeV}] & = & 80.410 \pm 0.032 & 0.04\% \\
\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} & = & 0.23153 \pm 0.00016 & 0.07\% \\
\Gamma_{\mathrm{Z}}[\,\mathrm{GeV}] & = & 2.4952 \pm 0.0023 & 0.09\% \\
 & & & \\
M_{\mathrm{Z}}[\,\mathrm{GeV}] & = & 91.1875 \pm 0.0021 & 0.002\% \\
G_\mu[\,\mathrm{GeV}^{-2}] & = & 1.16637(1)\,10^{-5} & 0.0009\% \\
m_{\mathrm{t}}[\,\mathrm{GeV}] & = & 172.7 \pm 2.9 & 1.7\% \\
\ldots & & &
\end{array}
$$

Quantum effects of the theory:

Loop corr. to “pseudo-observables” $M_{\mathrm{W}}, \sin^2\theta_{\mathrm{eff}}, \ldots$: $\sim \mathcal{O}(1\%)$

$\Rightarrow$ EW prec. measurements test quantum effects of the theory

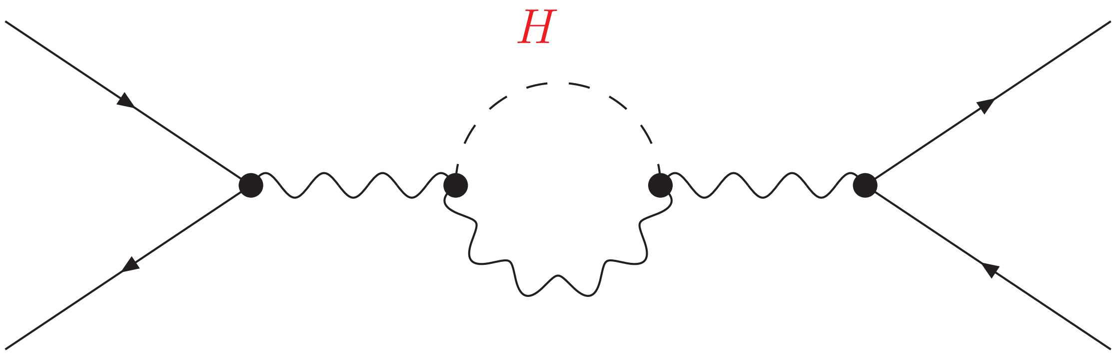
Comparison of electroweak precision data with theory predictions:

EW precision data:
$M_Z, M_W, \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}, \ldots$

Theory:
SM, MSSM, . . .

$\Downarrow$

Test of theory at quantum level: sensitivity to loop corrections



$\Downarrow$

Indirect constraints on unknown parameters: $M_H, \ldots$

Effects of “new physics”?

Indirect determination of m_{t} from precision data vs. direct measurement

Indirect det. of m_{t} from precision data: $m_{\mathrm{t}} = 179.4^{+12.1}_{-9.2}$ GeV

Direct measurement: $m_{\mathrm{t}} = 172.7 \pm 2.9$ GeV

Leading corrections to precision observables:

$$\sim m_{\mathrm{t}}^2$$

$$\sim \ln M_{\mathrm{H}}$$

$\Rightarrow$ For constraints on M_{H}, SUSY, ...: very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

– experimental error of input parameters: m_{t}, $\Delta\alpha_{\mathrm{had}}$, ...

Evaluation of quantum corrections

Problem in quantisation of gauge theories:

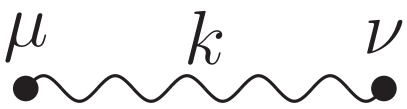
gauge fields contain unphysical degrees of freedom that must not be quantised

$\Rightarrow$ need to fix the gauge

$\Rightarrow$ "gauge-fixing term" added to the Lagrangian

$\Rightarrow$ gauge-boson propagator receives contribution from gauge-fixing term, e.g. photon propagator:

$$\Delta_{\mu\nu}(k) = \frac{-ig_{\mu\nu} + ik_\mu k_\nu/k^2(1-\xi^\gamma)}{k^2 + i\varepsilon} =$$



$\xi^\gamma = 1$: "'t Hooft–Feynman gauge"

Gauge fixing and Faddeev-Popov ghosts

Contributions of unphysical degrees of freedom of gauge fields are compensated by “Faddeev-Popov ghost fields”

⇒ Lagrangian of quantised gauge theory

$$\mathcal{L}_{\mathrm{EW}} + \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{FP}}$$

is invariant under Becchi-Rouet-Stora (BRS) symmetry

⇒ Symmetry relations for Green functions: “Slavnov–Taylor identities”, have a more complicated structure for QCD (and SM) than Ward identities of QED

Quantisation of the SM with “background-field method”: gauge fields in Lagrangian decomposed into quantum field and background field ⇒ QED-like Ward identities

[*A. Denner, S. Dittmaier, G. W. '95*]

Gauge fixing and Faddeev-Popov ghosts in the electroweak SM

$$
\begin{aligned}
\mathcal{L}_{\mathrm{gf}} &= -\frac{1}{2}\Big((F^{\gamma})^2 + (F^{\mathrm{Z}})^2 + F^+F^- + F^-F^+\Big), \\
F^{\gamma} &= (\xi_1^{\gamma})^{-\frac{1}{2}}\,\partial_\mu A^\mu + \frac{\xi^{\gamma\mathrm{Z}}}{2}\partial_\mu Z^\mu, \\
F^{\mathrm{Z}} &= \left(\xi_1^{\mathrm{Z}}\right)^{-\frac{1}{2}}\partial_\mu Z^\mu + \frac{\xi^{\mathrm{Z}\gamma}}{2}\partial_\mu A^\mu - \left(\xi_2^{\mathrm{Z}}\right)^{\frac{1}{2}} M_{\mathrm{Z}}\,\chi, \\
F^{\pm} &= \left(\xi_1^{\mathrm{W}}\right)^{-\frac{1}{2}}\partial_\mu W^{\pm\mu} \mp i\left(\xi_2^{\mathrm{W}}\right)^{\frac{1}{2}} M_{\mathrm{W}}\,\phi^{\pm}
\end{aligned}
$$

$$
\mathcal{L}_{\mathrm{FP}} = \sum_{a,b=\gamma,Z,\pm} \bar{u}^a \frac{\delta F^a}{\delta\theta^b} u^b = \int d^4y \sum_{a,b} \bar{u}^a(x)\frac{\delta F^a(x)}{\delta\theta^b(y)} u^b(y)
$$

Unphysical degrees of freedom:

χ, $\phi^{\pm}$: from Higgs pot., give longitudinal components to Z, $W^{\pm}$

u^{γ}, u^{Z}, $u^{\pm}$: Faddeev-Popov ghost fields, appear only in closed loops

Evaluation of higher-order corrections: algorithmic procedure

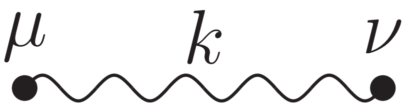
Lagrangian for specific model (SM, MSSM, . . .) yields Feynman rules for propagators and interaction vertices

$\Rightarrow$ mathematical expression

Example: Feynman rules for QED

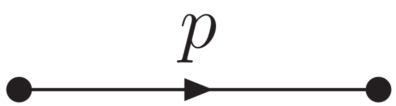
- photon propagator (Feynman gauge):

$$\Delta_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} =$$



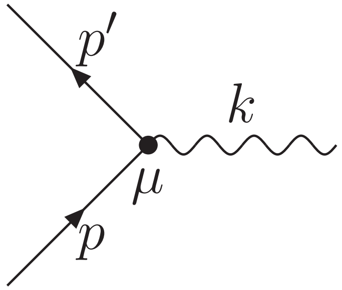
- electron/positron propagator:

$$S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon} =$$



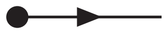
Feynman rules for QED

- electron–positron–photon vertex:



$$= (2\pi)^4 \delta^4(p - p' - k)(ie\gamma_\mu)$$

- ingoing and outgoing lines:

electron:  $= u(p, \sigma)$  $= \overline{u}(p, \sigma)$

positron:  $= \overline{v}(p, \sigma)$  $= v(p, \sigma)$

photon:  $= \epsilon_\mu(p, \lambda)$  $= \epsilon^*_\mu(p, \lambda)$

- minus sign for fermion loops, combinatorial factor
- integration over momenta of internal particles

δ-distr. $\Rightarrow$ momentum conservation

for loop diagrams $\Rightarrow \int \frac{d^4q}{(2\pi)^4}$ for each loop

Calculation of loop diagrams

Evaluation of Dirac algebra, Dirac traces:

$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu} \qquad \Rightarrow \mathrm{Tr}\,\{\gamma_\mu \gamma_\nu\} = 4 g_{\mu\nu}$$

Tensor-integral decomposition:

$$\int \frac{d^D q}{(2\pi)^D} \frac{q_\mu}{(q^2 - m^2)\,[(q+k)^2 - m^2]} = a\, k_\mu$$

$\Rightarrow$ Scalar integrals:

$$\begin{aligned} A_0(m^2) &= \frac{1}{i\pi^2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m^2} \\ B_0(p^2, m_1^2, m_2^2) &= \frac{1}{i\pi^2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - m_1^2)\,[(q+k)^2 - m_2^2]} \\ &\dots \end{aligned}$$

Evaluation of standard scalar integrals:

One-loop: reducible to A_0, B_0, C_0, D_0:

$$A_0, B_0 : \log(\sqrt{\ldots}); \quad C_0, D_0 : \log, \quad \mathrm{Li}_2(x) = \int_0^1 dt \frac{\ln t}{t - 1/x}$$

Two-loop: in general not expressible in terms of polylogarithmic functions [*R. Scharf '91*]

$\Rightarrow$ numerical evaluation, asymptotic expansions, ...

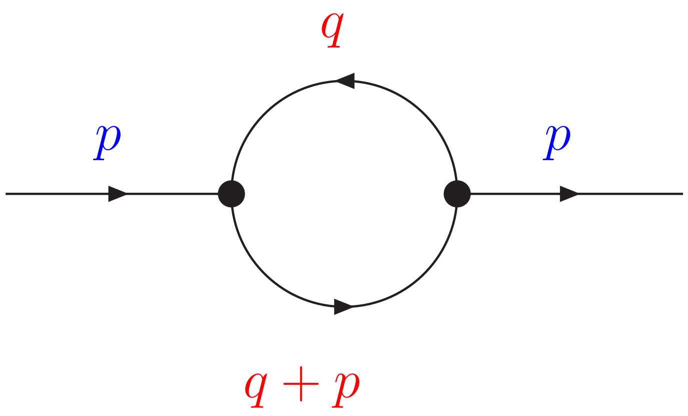
On-shell conditions for external legs

$\Rightarrow$ S-matrix elements

$|\mathcal{M}^2|$, phase-space integration

$\Rightarrow$ cross sections, life times

Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2 + i\varepsilon)\left[(q+k)^2 - m_2^2 + i\varepsilon\right]}$$

$$q \to \infty: \quad \sim \int_0^\infty \frac{q^3 dq}{q^4} = \int_0^\infty \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q!

$\Rightarrow$ theory in this form not physically meaningful

$\Rightarrow$ further concept needed: renormalisation

Renormalisable theories: infinities can consistently be absorbed into parameters of theory

Two step procedure: regularisation

Regularisation:

theory modified such that expressions become mathematically meaningful

$\Rightarrow$ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \;\to\; \int_0^\Lambda d^4q; \qquad \Lambda \to \infty \text{ at the end}$$

technically more convenient: dimensional regularisation

$$\int d^4q \;\to\; \int d^Dq, \qquad D = 4 - \varepsilon; \qquad D \to 4 \text{ at the end}$$

Two step procedure: renormalisation

Renormalisation:

original “bare” parameters replaced by renormalised parameters $+$ counterterms

reparametrization: $$\underbrace{g_0}_{\text{bare parameter}} = \underbrace{g}_{\text{renormalised parameter}} + \underbrace{\delta g}_{\text{counterterm}}$$

Renormalisable theory:

divergences compensated by counterterms

Two aspects of renormalisation:

- Absorption of divergences

- Determination of physical meaning of parameters order by order in perturbation theory

Why do the parameters (and fields) have to be renormalised?

- The SM Lagrangian contains free parameters that are not predicted by the theory:

  α, M_{W}, M_{Z}, M_{H} $+$ fermion masses $+$ four parameters of the quark-mixing matrix ($+$ four parameters of the lepton-mixing matrix if right-handed neutrinos are included)

- The bare parameters appearing in the Lagrangian have no physical meaning

- One needs n physical observables (masses, cross sections, ...) to fix n free parameters of the Lagrangian

  (further observables $\Leftrightarrow$ test of the theory)

Why do the parameters (and fields) have to be renormalised?

- Only relations between observables are physically meaningful, not relations between observables and bare parameters

- Relations between physical observables include loop effects to all orders; one cannot “switch off” the interactions in nature

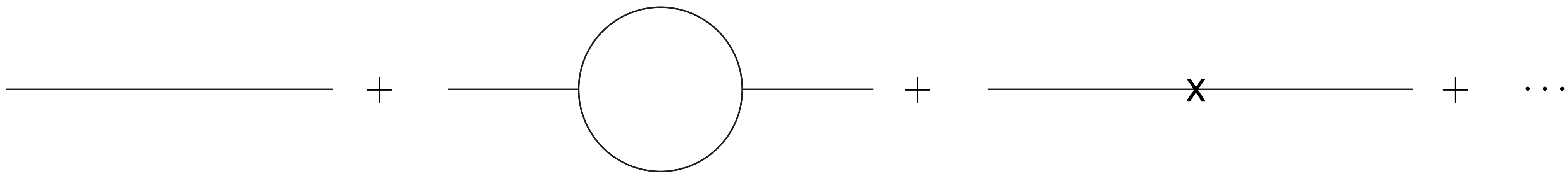
$\Rightarrow$ Need to define the meaning of the parameters in the Lagrangian in every loop order

Example: mass renormalisation

Renormalisation of the mass parameter: $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:



$p^2 - m^2$ $\qquad\qquad$ $\Sigma(p^2)$ $\qquad\qquad$ $-\,\delta m^2$

Pole of the propagator: $p^2 - m^2 + \Sigma(p^2) - \delta m^2 = 0$

On-shell and MS renormalisation

"On-shell renormalisation": $\delta m^2 = \Sigma(p^2)\big|_{p^2=m^2}$

$\Rightarrow$ pole of propagator for $p^2 = m^2 \Rightarrow m$: physical mass

Counterterm contains divergence: expansion in $\varepsilon \equiv 4 - D$, 1-loop:

$$\delta m^2 = \underbrace{a\,\frac{1}{\varepsilon}}_{\substack{\text{divergent} \\ \text{for } D \to 4}} + \underbrace{b\,\varepsilon^0}_{\text{finite}} + \underbrace{c\,\varepsilon}_{\substack{\to 0 \\ \text{for } D \to 4}} + \ldots$$

Other renormalisation prescription:

$\delta m^2 = a\frac{1}{\varepsilon}$ "minimal subtraction" (MS)

Renormalisation group equations

Different renormalisation prescriptions $\Leftrightarrow$ different parametrizations of result

Physical result must be independent of renormalisation prescription, e.g.

$$\Sigma^{\text{ren}}(p^2 = \underbrace{\mu^2}_{\text{renormalisation scale}}) \equiv \Sigma(p^2 = \mu^2) - \delta m^2 = 0$$

$\Rightarrow$ for physical observable σ:

$$\mu \frac{d}{d\mu} \sigma\left(p_i, g(\mu), m(\mu), \mu\right) = 0$$

"renormalisation group equation"

Renormalisation group equations

$\Rightarrow$ Explicit dependence of σ on μ compensated by μ-dependence of $g(\mu)$, $m(\mu)$

$$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right), \quad \beta = \text{const.}$$

$g(\mu)$: “running coupling”, $m(\mu)$: “running mass”

$\Rightarrow$ effective coupling $\alpha(Q)$ varies with scale Q

In exact theory (incl. all orders) all schemes are equivalent

Truncation of perturbative expansion at certain order

$\Rightarrow$ results depend on ren. scheme, ren. scale μ

ren. scheme dependence is of the order of uncalculated higher orders

Running coupling of QED

$$\alpha(Q) = \frac{\alpha(Q_0)}{1 + 4\pi\beta\,\alpha(Q_0)\,\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$\beta < 0$ in QED: $\beta \equiv b/(16\pi^2)$, $b = -4/3$ (1-loop)

$\Rightarrow$ $\alpha(Q)$ increases for increasing Q ($Q > Q_0$),

$\alpha(m_\mathrm{e}) \approx 1/137$, $\alpha(M_\mathrm{Z}) \approx 1/128$

Scale Λ_QED where QED coupling becomes strong:

$$\alpha(Q) = \frac{1}{4\pi\beta\,\ln\left(\frac{Q^2}{\Lambda^2_\mathrm{QED}}\right)}$$

$\Lambda_\mathrm{QED} \approx 10^{274}\ \mathrm{GeV} \gg M_\mathrm{Pl}(\approx 10^{19}\ \mathrm{GeV})$

$\Rightarrow$ QED is weakly coupled up to the Planck-scale

Running coupling of QCD

$$b^{\mathrm{QCD}} = \frac{1}{3}(11 N_C - 2 n_f) > 0 \text{ for } n_f = 6$$

$\Rightarrow$ $\alpha_{\mathrm{s}}(Q) \to 0$ for $Q \to \infty$: "asymptotic freedom"

For large Q: perturbation theory applicable, description in terms of quarks and gluons useful

$\alpha_{\mathrm{s}}(Q)$ becomes strong for small Q:

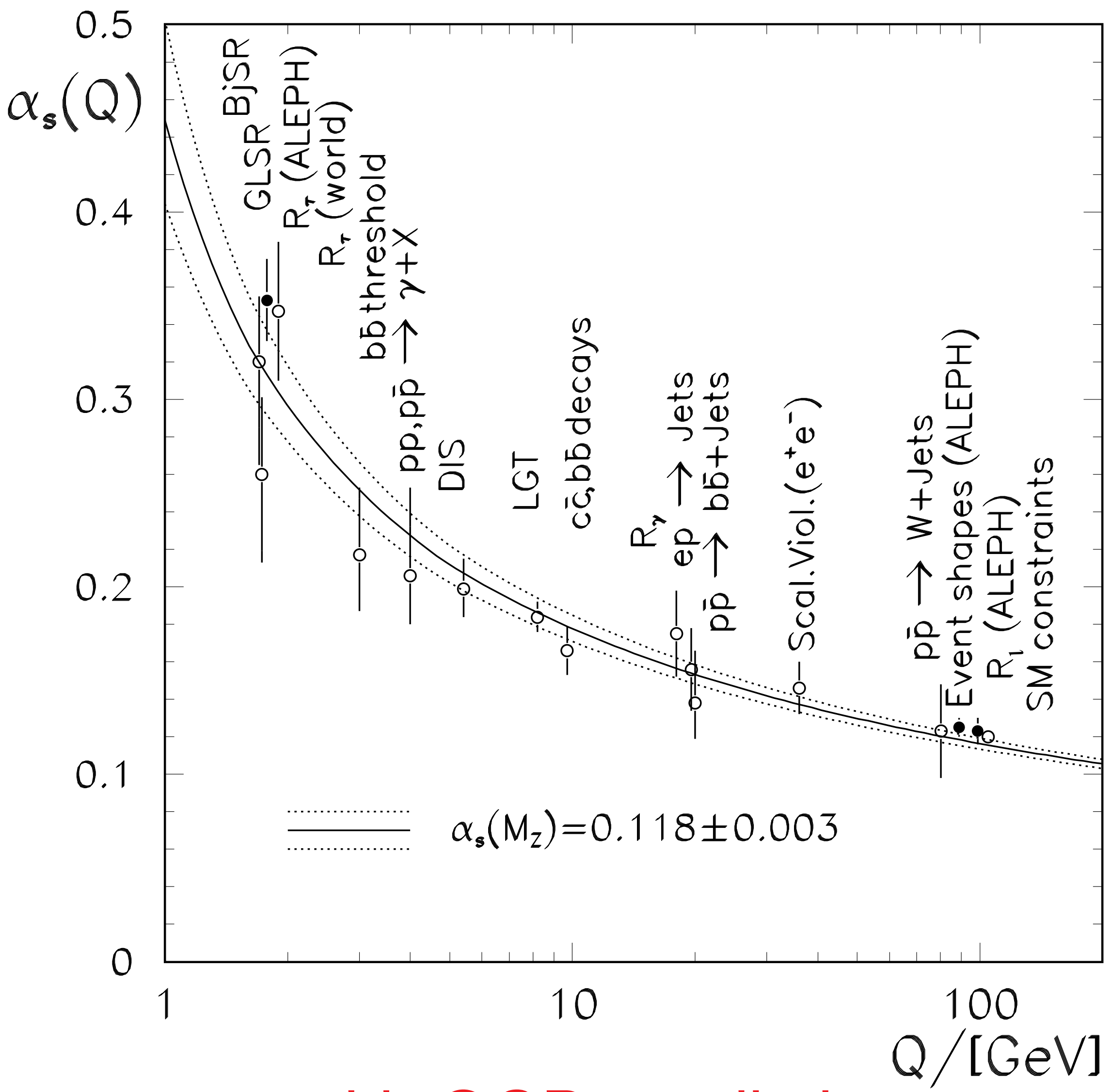
quarks and gluons bound together to form colourless hadrons

$\Rightarrow$ "confinement"

$$\alpha_{\mathrm{s}}(Q) = \frac{12\pi}{(11 N_C - 2 n_f) \ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}$$

For scales of $\mathcal{O}(\Lambda_{\mathrm{QCD}}) \approx 100$ MeV: $\alpha_{\mathrm{s}}(Q)$ so large that perturbation theory not applicable

Running of $\alpha_s(Q)$*:*



⇒ Very good agreement with QCD prediction

Which theories are renormalisable?

General structure of divergences:

$$\sim \int d^4q \frac{N(q, \ldots, p, \ldots)}{(q^2 - m_1^2 + i\varepsilon)\left[(q+k)^2 - m_2^2 + i\varepsilon\right] \cdots}$$

High-energy behaviour of integrands of Feynman integrals determined by loop integration, internal propagators, momentum-dependent vertices

For Feynman graph G (one-particle irreducible) with E_ϕ external scalars, E_ψ external fermions and E_V external vector particles one obtains "superficial degree of divergence" from power counting and combinatorics
(all integration momenta $\to \infty$ simultaneously)

Renormalisable theories

$$\omega(G) = 4 - E_\phi - \frac{3}{2}E_\psi - E_V - \sum_v [g_v]$$

$\sum_v$: sum over all vertices of G

$[g_v]$: mass dimension of coupling constant at vertex v

assumes that scalar and vector-boson propagators behave as $1/q^2$ and fermion propagators as $1/q$ for large q

All UV divergences are controlled by superficial degree of divergence; sub-divergences treated iteratively order by order

Necessary cond. for renormalisability: $[g_v] \geq 0$ for all couplings

$\Rightarrow$ only finite number of classes of diagrams that are divergent

$\Rightarrow$ all UV divergences can be absorbed by ren. of finite number of parameters

Multiplicative renormalisation; example ϕ^4 theory

$$\mathcal{L}(\phi_0, \partial\phi_0; m_0, \lambda_0) = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{4!}\lambda_0\phi_0^4$$

multiplicative renormalisation transformation

$$\begin{aligned} m_0(m, \lambda, \delta) &= Z_m(m, \lambda, \delta)m, \quad \lambda_0(m, \lambda, \delta) = Z_\lambda(m, \lambda, \delta)\lambda \\ \phi_0 &= Z_\phi^{1/2}(m, \lambda, \delta)\phi \end{aligned}$$

renormalised Lagrangian

$$\mathcal{L} = \mathcal{L}(\phi_0, \partial\phi_0; m_0, \lambda_0) = \mathcal{L}(Z_\phi^{1/2}\phi, Z_\phi^{1/2}\partial\phi_0; Z_m m, Z_\lambda\lambda)$$

perturbation theory: $Z_i = 1 + \delta Z_i$

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi; m, \lambda) + \mathcal{L}_{\text{ct}}(\phi, \partial\phi; m, \lambda; \delta Z_\phi, \delta Z_m, \delta Z_\lambda)$$

same form as $\mathcal{L}$ but renormalised param.

counterterm Lagrangian $\to$ counterterm diagrams

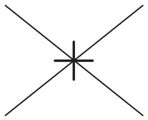
Feynman rules contain renormalised parameters and counterterms

Renormalised Lagrangian of ϕ^4 theory

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{4!}\lambda_0\phi_0^4 \\
&= \frac{1}{2}Z_\phi\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}Z_\phi Z_m m^2\phi^2 - \frac{1}{4!}Z_\phi^2 Z_\lambda\lambda\phi^4 \\
&= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 \\
&\quad - \frac{1}{2}\delta Z_m m^2\phi^2 + \frac{1}{2}\delta Z_\phi[\partial_\mu\phi\partial^\mu\phi - m^2\phi^2] - \frac{1}{4!}(2\delta Z_\phi + \delta Z_\lambda)\lambda\phi^4 \\
&\quad + \mathcal{O}((\delta Z)^2)
\end{aligned}
$$

$\Rightarrow$ Feynman rules for counterterms

$$i[\delta Z_\phi(p^2 - m^2) - \delta Z_m m^2] + \mathcal{O}((\delta Z)^2)$$



$$-\,i(\delta Z_\lambda + 2\delta Z_\phi)\lambda + \mathcal{O}((\delta Z)^2)$$

Renormalisability of gauge theories

- Gauge theories are renormalisable according to power counting (dimensionless couplings; gauge-boson propagators $\sim 1/k^2$)

$$\omega(G) = 4 - E_\phi - \frac{3}{2}E_\psi - E_V - E_u$$

- Unphysical degrees of freedom $\Rightarrow$ must proof that unphysical states do not contribute to the physical S-matrix

  needs definition of physical states (invariant under BRS transf.)

- gauge-fixed Lagrangian depends on gauge choice and gauge parameters $\Rightarrow$ proof that physical S-matrix is gauge-independent

- proofs rely on BRS invariance $\Rightarrow$ proof that theory can be renormalised such that it is invariant under (renormalised) BRS invariance

Multiplicative renormalisation of a gauge theory

$$
\begin{aligned}
A^a_{0,\mu} &= Z_A^{1/2} A^a_\mu, \qquad \xi_0 = Z_A \xi, \qquad g_0 = Z_g g \\
u^a_0 &= Z_u^{1/2} u^a, \qquad \bar{u}^a_0 = Z_u^{1/2} \bar{u}^a \\
\psi_0 &= Z_\psi^{1/2} \psi, \qquad m_0 = Z_m m
\end{aligned}
$$

- gauge-fixing parameter is renormalised by the same constant as the gauge field $\Rightarrow$ gauge-fixing term $-(\partial^\mu A^a_\mu)^2/(2\xi)$ is not renormalised
- field renormalisation for Faddeev–Popov ghost fields needed to render all Green functions finite (not needed for S-matrix elements)
- gauge coupling in different vertices is renormalised in the same way (consequence of gauge/BRS invariance)
- additional non-gauge coupling constants, like 4-scalar coupling λ need independent renormalisation

Renormalised Lagrangian is invariant under (renormalised) BRS transf.

Anomalies

Anomaly = breakdown of a classical symmetry upon quantisation

anomalies destroy BRS symmetry and thus renormalisability

$\Rightarrow$ renormalisable gauge theories must be free of anomalies

condition for cancellation of (axial) anomaly (γ_5 anomaly)

$$A^{abc} = \mathrm{Tr}\left(T^a_{\mathrm{L}}\left\{T^b_{\mathrm{L}}, T^c_{\mathrm{L}}\right\}\right) - \mathrm{Tr}\left(T^a_{\mathrm{R}}\left\{T^b_{\mathrm{R}}, T^c_{\mathrm{R}}\right\}\right) = 0$$

$T^a_{\mathrm{L}}, T^a_{\mathrm{R}}$: repr. matrices for left-handed, right-handed fermion fields

vector theory (QED, QCD): $T^a_{\mathrm{L}} = T^a_{\mathrm{R}} \Rightarrow A^{abc} = 0$

Standard Model: anomaly cancellation requires $\sum_f Q_f = 0$

single family: $(0-1) + N_{\mathrm{c}}\left(\frac{2}{3} - \frac{1}{3}\right) = -1 + \frac{N_{\mathrm{c}}}{3} = 0$

$\Rightarrow$ three quark colours ($N_{\mathrm{c}} = 3$) are needed

Non-renormalisable theories

If $[g_v] < 0$ for at least one coupling

$\Rightarrow$ every Green function gets divergent once sufficiently many couplings with $[g_v] < 0$ have been inserted

$\Rightarrow$ class of divergent diagrams grows order by order

$\Rightarrow$ need more and more free parameters to absorb divergences

$\Rightarrow$ limited predictive power

Example: Fermi model: $[G_\mu] = -2$

$\Rightarrow$ Fermi model is only physically meaningful for energies $E \ll M_\mathrm{W}, M_\mathrm{Z}$, violates probability conservation for higher en.

$\Rightarrow$ Fermi model is an effective low-energy theory, only valid below a certain cutoff energy

Renormalisable and non-renormalisable theories

Further examples for non-renormalisable theories:

– chiral perturbation theory

– heavy quark effective theory

– many models of new physics: extra dim., Little Higgs models, . . .

$\Rightarrow$ Effective low-energy theories, have cutoff energy

Renormalisable theories: no energy cutoff, can in principle be valid up to arbitrarily high energies

But: SM doesn't include gravity, cannot be valid beyond M_{Pl}

Renormalisation of the electroweak Standard Model

Bare input parameters: $e_0, M_{\mathrm{W},0}, M_{\mathrm{Z},0}, M_{\mathrm{H},0}, m_{f,0}, V_{ij,0}$

renormalisation transformation:

- parameter renormalisation:

$$
\begin{aligned}
e_0 &= Z_e e \\
M^2_{\mathrm{W},0} &= M^2_{\mathrm{W}} + \delta M^2_{\mathrm{W}}, \qquad & M^2_{\mathrm{Z},0} &= M^2_{\mathrm{Z}} + \delta M^2_{\mathrm{Z}}, \\
M^2_{\mathrm{H},0} &= M^2_{\mathrm{H}} + \delta M^2_{\mathrm{H}}, \\
m_{f,0} &= m_f + \delta m_f, \qquad & V_{ij,0} &= V_{ij} + \delta V_{ij},
\end{aligned}
$$

(both $V_{ij,0}, V_{ij}$ unitary)

Ren. of $c_{\mathrm{W}}, s_{\mathrm{W}}$ fixed by on-shell condition: $c_{\mathrm{W}} = M_{\mathrm{W}}/M_{\mathrm{Z}}$

$\Rightarrow s_{\mathrm{W}}$ is not a free parameter if M_{W}, M_{Z} are used as input

Renormalisation of the electroweak Standard Model

- field renormalisation: (physical fields)

$$W_0^\pm = \sqrt{Z_W}\, W^\pm, \qquad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix},$$

$$H_0 = \sqrt{Z_H}\, H$$

$$\psi^{\mathrm{L}}_{f,0} = \sqrt{Z^{\mathrm{L}}_{ff'}}\, \psi^{\mathrm{L}}_{f'}, \qquad \psi^{\mathrm{R}}_{f,0} = \sqrt{Z^{\mathrm{R}}_{ff'}}\, \psi^{\mathrm{R}}_{f'}$$

Matrix ren. necessary to account for loop-induced mixing

From 2-loop order on: renormalisation of gauge-fixing sector enters the calculation of predictions for physical observables (gauge-parameter independence is a highly non-trivial check)

Renormalisation conditions for the Standard Model

- Mass renormalisation:

  on-shell definition: mass^2 is location of pole in propagator

  $\Rightarrow \delta M_\mathrm{W}^2 = \mathrm{Re}\{\Sigma_\mathrm{T}^W(M_\mathrm{W}^2)\}$ at 1-loop, ... similar for $\delta M_\mathrm{Z}^2, \delta M_\mathrm{H}^2, \delta m_f$

- Field renormalisation: (bosons and leptons)

  $Z_i = 1 + \delta Z_i, \quad Z_{ii} = 1 + \delta Z_{ii}, \quad Z_{ij} = \delta Z_{ij}, \;\; i \neq j$

  - residues of propagators (diagonal, transv. parts) normalised to 1

    $\Rightarrow \delta Z_W = -\mathrm{Re}\{\Sigma_\mathrm{T}^{W\,\prime}(M_\mathrm{W}^2)\}$, sim. for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{\mathrm{L/R}}$

  - mixing propagators on particle poles:

    physical on-shell particles do not mix

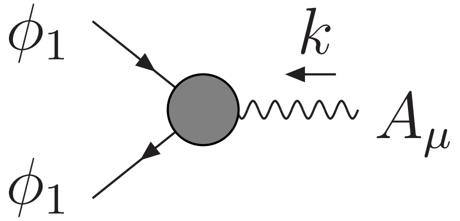
    i.e. $\Gamma_\mathrm{T}^{AZ}(0) = 0 = \Gamma_\mathrm{T}^{AZ}(M_\mathrm{Z}^2)$; fixes non-diagonal constants:

    $\delta Z_{AZ} = -2\,\mathrm{Re}\{\Sigma_\mathrm{T}^{AZ}(M_\mathrm{Z}^2)\}/M_\mathrm{Z}^2, \quad \delta Z_{ZA} = 2\{\Sigma_\mathrm{T}^{AZ}(0)\}/M_\mathrm{Z}^2$

    similar expressions for $\delta Z_{ff'}^{\mathrm{L/R}} \;\; (f \neq f')$

Charge renormalisation and CKM matrix renormalisation

- Charge renormalisation: define e in Thomson limit



$\xrightarrow{k \to 0}$ $ie\gamma_\mu$ for on-shell electrons

$\Rightarrow e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

gauge invariance relates δZ_e to photon wave-function renormalisation:

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_{\mathrm{W}}}{2c_{\mathrm{W}}}\delta Z_{ZA} \qquad \text{(at one loop)}$$

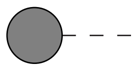
- CKM-matrix renormalisation $\to$ fixes δV_{ij}

Renormalisation of vacuum expectation value, tadpole

Renormalisation of spontaneously broken theories requires in addition renormalisation of vacuum expectation value (vev) of scalar field

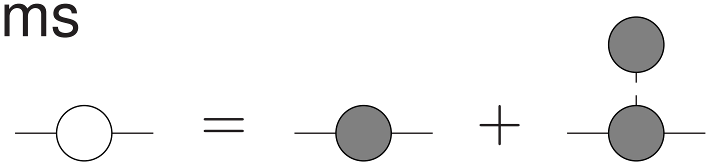
- elimination of divergences $\Rightarrow$ renormalise v as ϕ: $v_0 = Z_\phi v$

- spontaneous symmetry breaking gives rise to tadpole diagrams

  $\Gamma^H(0) =$ 

  tadpoles contribute to all Green functions and matrix elements

  $\Rightarrow$ many more diagrams

  e.g. 2-point function 

  convenient ren. cond. for vev: vanishing of ren. tadpole

  $\Rightarrow$ introduce counterterm $\delta t\, H(x)$ in $\mathcal{L}$

  require $i\delta t + \Gamma^H(0) = 0 \quad \Leftrightarrow \quad$ renormalised v is true vev

  connection between tadpole and vev: $\delta t = v_0\left(\mu_0^2 - \frac{1}{4}\lambda_0 v_0^2\right)$

What is the mass of an unstable particle?

Particle masses are not directly physical observables

Can only measure cross sections, branching ratios, kinematical distributions, ...

$\Rightarrow$ masses are “pseudo-observables”

Need to define what is meant by M_{Z}, M_{W}, m_{t}, ...:

$\overline{\mathrm{MS}}$ mass, pole mass (real pole, real part of complex pole, Breit–Wigner shape with running or constant width), ...

$\Rightarrow$ Determination of M_{Z}, M_{W}, m_{t}, ... involves deconvolution procedure (unfolding)

Mass obtained from comparison data – Monte Carlo

$\Rightarrow$ M_{Z}, M_{W}, m_{t}, ... are not strictly model-independent

What is the mass of an unstable particle?

The experimental value of M_{Z} is (slightly) model-dependent

In the SM: Dependence on the value of the Higgs mass

$\delta M_{\mathrm{Z}} = \pm 0.2$ MeV for $100 \text{ GeV} \leq M_{\mathrm{H}} \leq 1 \text{ TeV}$

$\Rightarrow M_{\mathrm{Z}}^{\mathrm{SM}}$ (slightly) differs from $M_{\mathrm{Z}}^{\mathrm{MSSM}}$

Differences could be much bigger, in principle, for $\alpha_s(M_{\mathrm{Z}})$

Numerical differences between the various mass definitions for M_{W}, M_{Z} are relatively large

(Running / constant width in Breit–Wigner $\Rightarrow \Delta M_{\mathrm{W}} = 27 \text{ MeV}$)

$\Rightarrow$ Important to always refer to the same definition

Renormalisation of M_{W}, M_{Z} at the two-loop level

Expansion of amplitude around complex pole:

$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_{\mathrm{Z}}^2} + S + (s - \mathcal{M}_{\mathrm{Z}}^2)\, S' + \cdots$$

$$\mathcal{M}_{\mathrm{Z}}^2 = \overline{M}_{\mathrm{Z}}^2 - i\overline{M}_{\mathrm{Z}}\,\overline{\Gamma}_{\mathrm{Z}}$$

Expanding up to $\mathcal{O}(\alpha^2)$ using $\mathcal{O}(\overline{\Gamma}_{\mathrm{Z}}/\overline{M}_{\mathrm{Z}}) = \mathcal{O}(\alpha)$

From 2-loop order on:

real part of complex pole, $\overline{M}_{\mathrm{Z}} \neq$ pole of real part, $\widetilde{M}_{\mathrm{Z}}^2$

$$\delta\overline{M}^2_{(2)} = \delta\widetilde{M}^2_{(2)} + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\}\ \mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}_{\text{gauge-parameter dependent!}}$$

Renormalisation of M_W, M_Z at the two-loop level

$\Rightarrow$ Only the complex pole is gauge-invariant

Expansion around the complex pole leads to a Breit–Wigner shape with constant width

For historical reasons, the experimental values of M_Z, M_W are defined according to a Breit–Wigner shape with running width

$\Rightarrow$ Need to correct for the difference in definition when comparing theory with experiment

What is the mass of the top quark?

Additional problem for m_t: mass of a coloured object?

Top pole mass is not “IR safe” (affected by large long-distance contributions), cannot be determined to better than $\mathcal{O}(\Lambda_\mathrm{QCD})$

Measurement of m_t:

At Tevatron, LHC: kinematic reconstruction, fit to invariant mass distribution

LHC: $\delta m_\mathrm{t} = 1\text{–}2 \text{ GeV}$

How large are the theoretical uncertainties?

At the ILC: Measurement of ‘threshold mass’ with high precision: $\lesssim 20 \text{ MeV}$ $+$ transition to suitably defined (short-distance) top mass, e.g. $\overline{\mathrm{MS}}$ mass

ILC: $\delta m_\mathrm{t} \lesssim 100 \text{ MeV}$

What is a suitable choice of renormalisation for the Lagrangian parameters?

Lagrangian parameters are not directly physical observables

$\Rightarrow$ can be defined (renormalised) in various ways

What is a suitable definition?

need some compromise between

– simple interpretation within given model

– model independence

Particle masses: relatively small model dependence

Couplings, mixing angles, etc.: relatively large model dependence

The Standard Model can only be an effective theory

SM does not include gravity, cannot be valid beyond M_{Pl}

Renormalisable theory $\Rightarrow$ cutoff scale does not enter the predictions for physical observables

What is the impact of the cutoff?

Predicting M_{H}: the hierarchy problem

Expect that in more fundamental theory the Higgs mass can be predicted

$\Rightarrow$ M_{H} in this theory is no longer a free parameter that can be independently renormalised

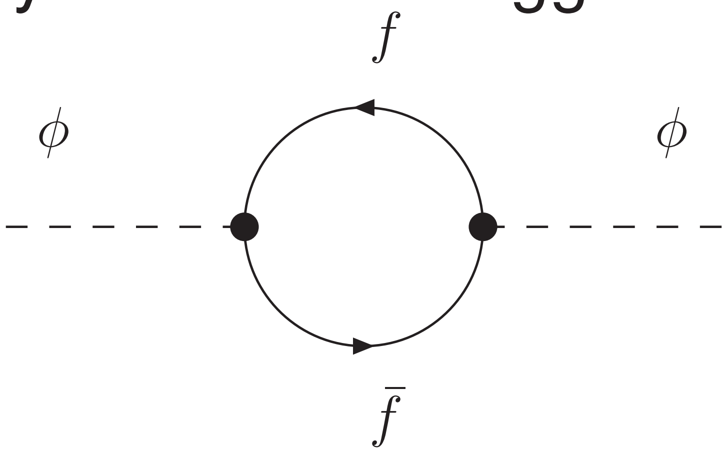
Physical value of M_{H}^2 is obtained as the sum of lowest-order contribution $+$ higher-order corrections

$$M_{\mathrm{H}}^2 = M_{\mathrm{H},0}^2 + \Delta M_{\mathrm{H},1}^2 + \Delta M_{\mathrm{H},2}^2 + \ldots$$

$\Rightarrow$ Calculation of corrections to M_{H}^2 in SM with cutoff Λ (effective theory)

Quadratic divergences

Contribution of heavy fermions to Higgs self-energy



$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

for $\Lambda \to \infty$: $\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_\mathrm{f}^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$

$\Rightarrow$ quadratically divergent!

The hierarchy problem

For $\Lambda = M_{\mathrm{Pl}}$: $\delta M_\phi^2 \sim M_{\mathrm{Pl}}^2 \Rightarrow \delta M_\phi^2 \approx 10^{30}\, M_\phi^2$ ($M_\phi \lesssim 1$ TeV)

In general: scalar masses tend to be near highest mass scale of the theory

$\Rightarrow$ Hierarchy problem, extreme fine-tuning necessary to get small M_ϕ

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

The hierarchy problem

Example: Grand Unified Theory (GUT): $\delta M_\phi^2 \approx \lambda \langle v_{\rm GUT} \rangle^2$

Even if $\lambda = 0$ at tree level: non-zero coupling regenerated by radiative corrections: $\delta M_\phi^2 \approx \mathcal{O}(\alpha/\pi) \langle v_{\rm GUT} \rangle^2$
(cf. 'Little Higgs' models)

Would need a symmetry to suppress many orders of perturbation theory

Hierarchy problem is not just a problem of the Higgs mass;

problem: why is $M_{\rm W} \ll M_{\rm GUT}, M_{\rm Pl}$, why is $V_{\rm Coulomb} \gg V_{\rm Newton}$?

Hierarchy problem is a strong argument that there should be new physics far below the Planck scale

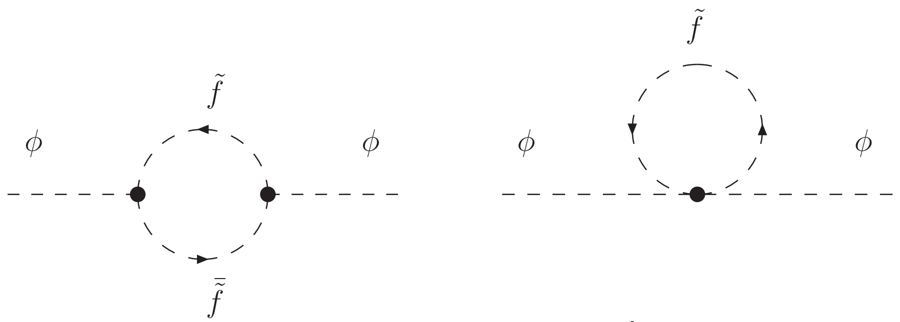
Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

Supersymmetry

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

<u>SUSY: additional contributions from scalar fields:</u>



$$\Sigma_{\tilde{f}}^{\phi\phi} \sim N(\tilde{f})\,\tilde{\lambda}_f \int d^4k \left(\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right)$$

$+$ terms without quadratic div.

for $\Lambda \to \infty$: $\Sigma_{\tilde{f}}^{\phi\phi} \sim 2\,N(\tilde{f})\,\tilde{\lambda}_f\,\Lambda^2$

Cancellation of quadratic divergences

$\Rightarrow$ quadratic divergences cancel for

$$\begin{aligned} N(\tilde{f}_L) = N(\tilde{f}_R) &= N(f) \\ \tilde{\lambda}_f &= \lambda_f^2 \end{aligned}$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

For $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \tilde{\lambda}_f = \lambda_f^2,$ "soft SUSY breaking"

$$\Rightarrow \; \Sigma^{\phi\phi}_{f+\tilde{f}} \sim N(f)\, \lambda_f^2\, \Delta^2 + \dots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners $M_{\mathrm{SUSY}} \lesssim 1\,\mathrm{TeV}$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Other approach: extra dimensions of space

Other recent proposals for solving hierarchy problem in models with extra spatial dimensions:

E.g.: Gravity in large extra dimensions
[*N. Arkani-Hamed, S. Dimopoulos, G. Dvali '98*]

Gravity propagates in δ large extra spatial dimensions

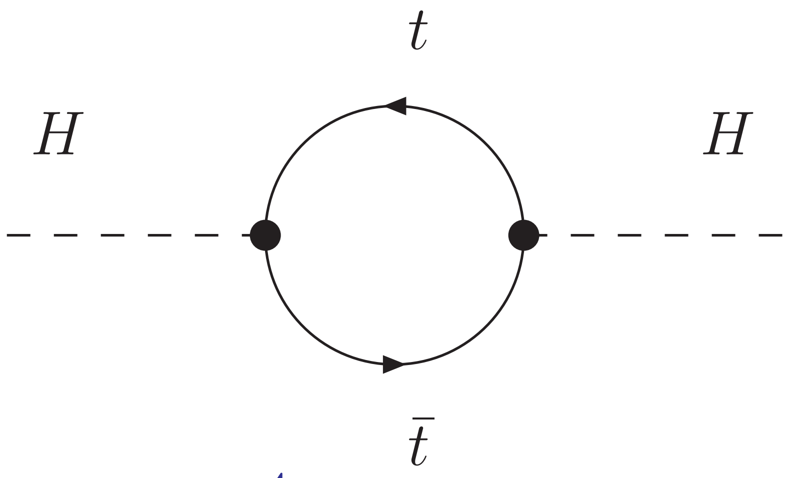
fundamental Planck scale in $4 + \delta$-dimensional space, M_*, is of order 1 TeV

$$M_{\mathrm{Pl}}^2 = V_\delta \, M_*^{2+\delta}$$

$\Rightarrow$ large value of M_{Pl} related to relatively large volume V_δ ($10^{-3}\,\mathrm{m}$–$10^{-15}\,\mathrm{m}$ for $\delta = 2$–6) of extra dimensions

Precision Higgs physics

Large coupling of Higgs to top quark



One-loop correction $\sim G_\mu m_\mathrm{t}^4$

$\Rightarrow$ M_H depends sensitively on m_t in all models where M_H can be predicted (SM: M_H is free parameter)

SUSY as an example: $\Delta m_\mathrm{t} \approx \pm 3$ GeV $\Rightarrow$ $\Delta m_\mathrm{h} \approx \pm 3$ GeV

$\Rightarrow$ Precision Higgs physics needs precision top physics (ILC: $\Delta m_\mathrm{t} \lesssim 0.1$ GeV)

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles:

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin } 1 \text{ / Spin } 0$$

$$\tilde{g} \qquad \tilde{\chi}^{\pm}_{1,2} \qquad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: two Higgs doublets, physical states:
$h^0, H^0, A^0, H^{\pm}$

SUSY breaking

MSSM: no particular SUSY breaking mechanism assumed, parametrization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged

$\Rightarrow$ no quadratic divergencies (hierarchy problem)

Most general case: 105 new parameters

Good phenomenological description for universal breaking terms

Specific models for soft-SUSY breaking:

"Hidden sector": SUSY breaking	$\longrightarrow$	Visible sector: MSSM

"Gravity-mediated": mSUGRA

"Gauge-mediated": GMSB

"Anomaly-mediated": AMSB

...

Renormalisation of supersymmetric theories

- SM: all masses are free input parameters

  MSSM: additional symmetry (even after inclusion of soft SUSY-breaking terms)

  $\Rightarrow$ many relations between different masses

  $\Rightarrow$ not all masses can be renormalised independently

  $\Rightarrow$ need to be careful to avoid breaking of symmetries of the model

  $\Rightarrow$ Many SUSY masses can be calculated from other parameters

  $\Rightarrow$ crucial predictions of SUSY models

- Similarly: relations for couplings

Renormalisation of supersymmetric theories

- In general: interaction eigenstates $\neq$ mass eigenstates
  $\Rightarrow$ need ren. prescription for mixing matrices

- Higgs sector: very large corrections possible ($\mathrm{t}/\tilde{\mathrm{t}}$ loops, corrections to b Yukawa coupling)
  $\Rightarrow$ need to be careful to avoid unphysically large corr.
  $\Rightarrow$ large loop corrections can induce $\mathcal{CP}$-violating effects
  $\Rightarrow$ all three neutral Higgs bosons of the MSSM can mix: $h, H, A \rightarrow h_1, h_2, h_3$, unstable particles
  $\Rightarrow$ masses have to be determined as eigenvalues of complex (3×3) matrix
  $\Rightarrow$ M_{A} is no longer a convenient parameter for renormalisation $\Rightarrow$ choose $M_{\mathrm{H}^\pm}$ instead

Renormalisation of supersymmetric theories

- Higgs sector with $\mathcal{CP}$-violation:

  h_2, h_3 are almost mass-degenerate for most of the parameter space $\Rightarrow$ resonance effects

- MSSM: many parameters that are not closely related to one particular observable ($\tan\beta$, μ, $\theta_{\tilde{t}}$, $\theta_{\tilde{b}}$, …, complex phases, …)

  $\Rightarrow$ there is no obvious 'best' choice of a definition

  $\Rightarrow$ already now many different renormalisation prescriptions exist in the literature

  $\Rightarrow$ have to convert between them when comparing theoretical predictions

  $\Rightarrow$ we should better converge to few commonly accepted standards (remember in the SM the case of $\sin^2\theta_{\mathrm{W}}$ …)

Renormalisation of supersymmetric theories

- Parameter determination in the MSSM:

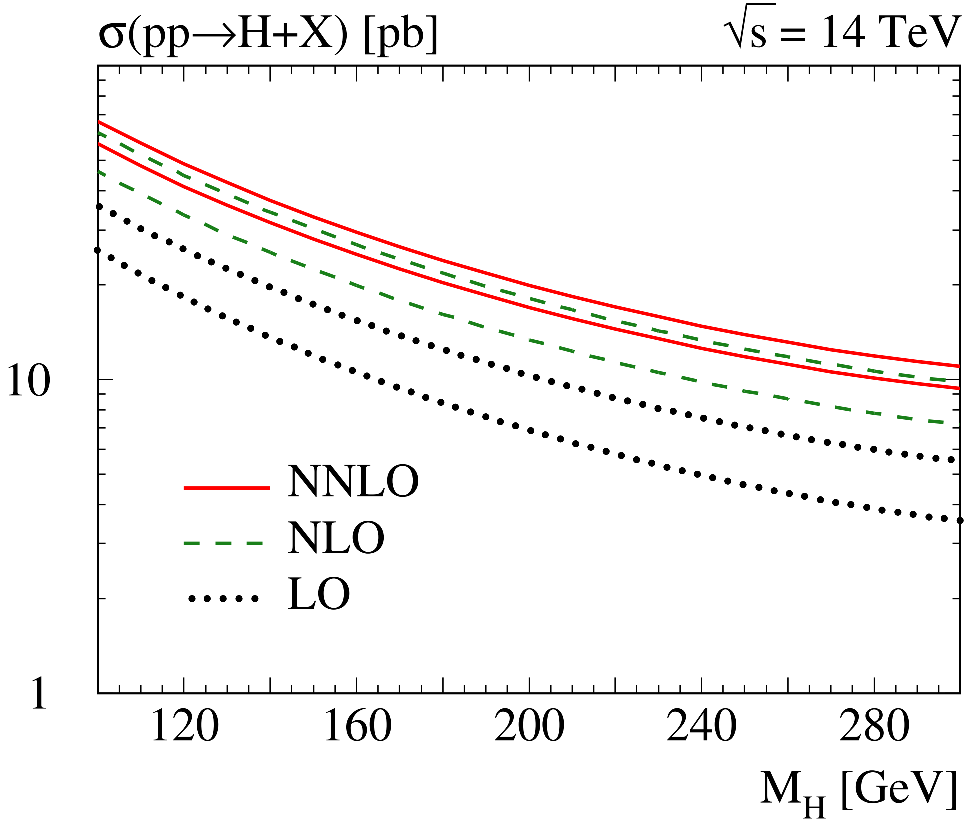
  If parameter a is determined from observable A, it will in general sensitively depend on parameters b, c, d, $\ldots$

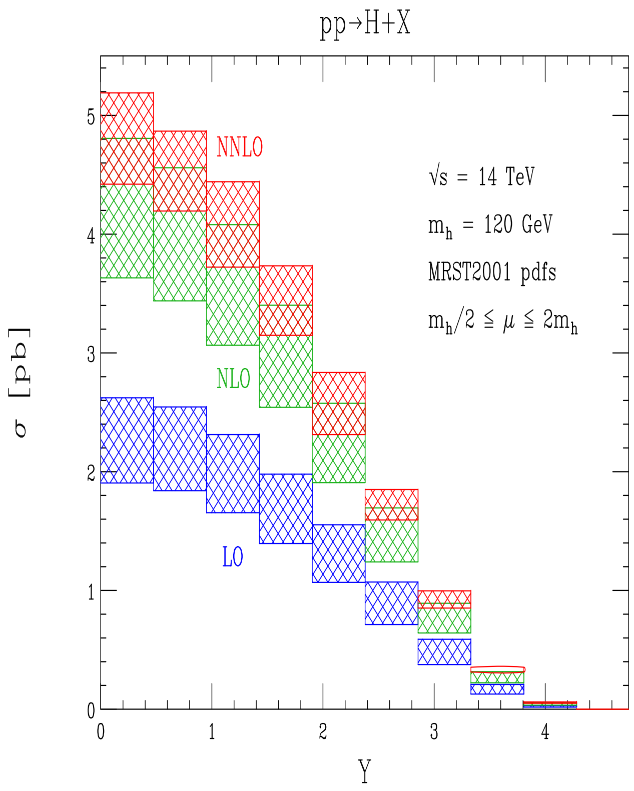
  $\Rightarrow$ Global fit necessary

- Extrapolation to high scales:

  all parameters, whole structure of model enter

Precision physics in the Standard Model: SM Higgs production at the LHC



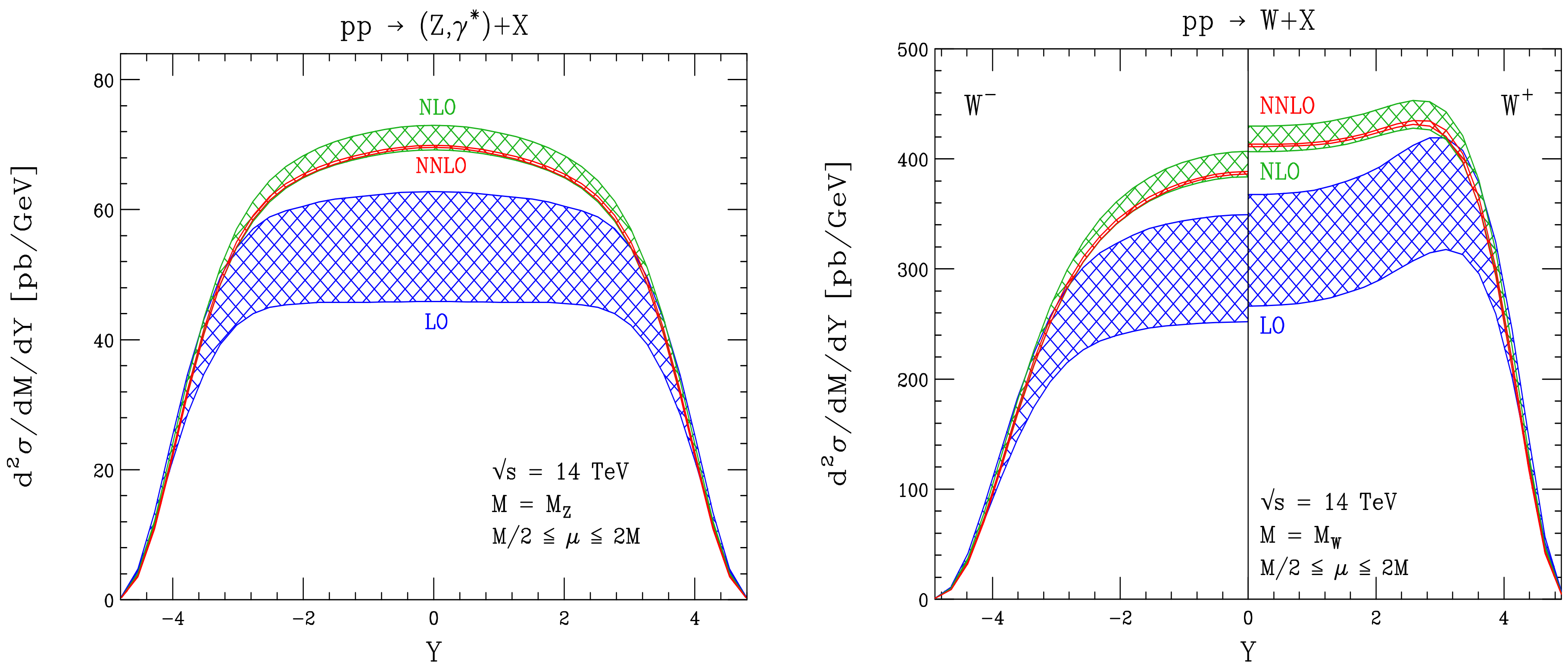


Total cross section: [*R. Harlander, W. Kilgore '02*]

Fully differential: [*C. Anastasiou, K. Melnikov, F. Petriello '04*]

$\Rightarrow$ Next-to-next-to-leading order needed for reliable predictions

Gauge boson production at the LHC



[*C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello ’03*]

⇒ “Gold-plated” processes:
NNLO perturbative accuracy better than 1%

⇒ use to determine parton-parton luminosities at the LHC

Electroweak precision physics

Theoretical predictions for M_W, $\sin^2\theta_\mathrm{eff}$:

Comparison of prediction for muon decay with experiment (Fermi constant G_μ)

$$\Rightarrow \qquad M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2} G_\mu} \left(1 + \underset{\substack{\Updownarrow \\ \text{loop corrections}}}{\Delta r}\right),$$

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, M_\mathrm{H}, \ldots)$

Effective couplings at the Z resonance:

$$\Rightarrow \qquad \sin^2\theta_\mathrm{eff} = \frac{1}{4}\left(1 - \mathrm{Re}\,\frac{g_V}{g_A}\right) = \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) \mathrm{Re}\,\kappa(M_\mathrm{W}^2)$$

Theoretical predictions for M_W, $\sin^2\theta_\mathrm{eff}$:

Current status in the SM: complete two-loop result known for M_W, fermionic two-loop corrections known for $\sin^2\theta_\mathrm{eff}$

Necessary diagrams for evaluation of 2-loop corrections to $\sin^2\theta_\mathrm{eff}$:

- Renormalisation: $\delta M_\mathrm{W}^2 = \mathrm{Re}\Sigma^\mathrm{W}_{\mathrm{T}(2)}(M_\mathrm{W}^2) + \ldots$

  $\Rightarrow$ 2-loop self-energies with arbitrary momentum and masses

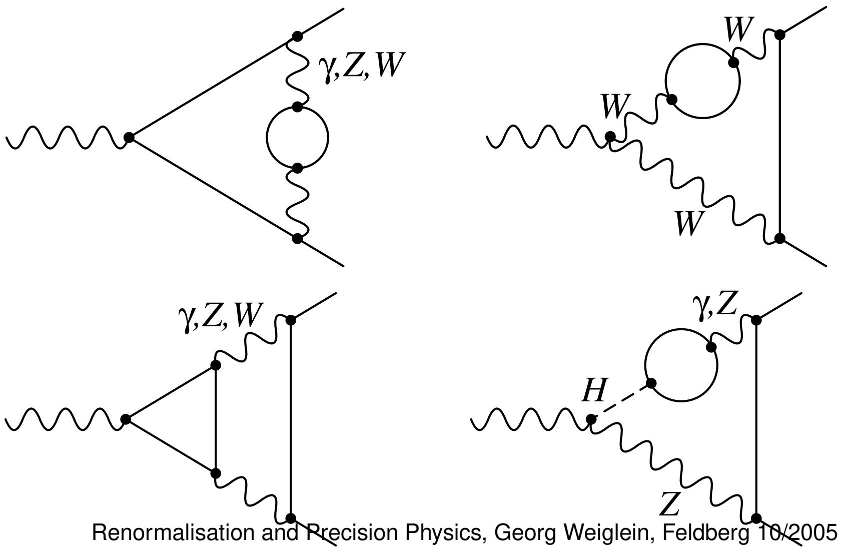
  No analytical results available for general case

  $\Rightarrow$ numerical integration

- Two-loop vertex diagrams:

  top, light fermion loops

  $\Rightarrow$ analytical + num. results



Leading m_{t} contributions

One-loop result for M_{W} in the SM:

[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\begin{array}{ccccccc} \Delta r_{1-\mathrm{loop}} = & \Delta\alpha & - & \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho & + & \Delta r_{\mathrm{rem}}(M_{\mathrm{H}}) \\ & \sim \log\frac{M_{\mathrm{Z}}}{m_f} & & \sim m_{\mathrm{t}}^2 & & \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{array}$$

Leading m_{t} contribution to $\sin^2\theta_{\mathrm{eff}}$:

$$\Delta\sin^2\theta_{\mathrm{eff}} \approx -\frac{c_{\mathrm{W}}^2 s_{\mathrm{W}}^2}{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}\Delta\rho \sim m_{\mathrm{t}}^2$$

Sources of theoretical uncertainties

- From unknown higher-order corrections (“intrinsic”)
  [*M. Awramik, M. Czakon, A. Freitas, G.W. ’03, ’04*]

$$\Delta M_{\mathrm{W}}^{\mathrm{intr}} \approx 4 \ \mathrm{MeV}, \quad \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{intr}} \approx 5 \times 10^{-5}$$

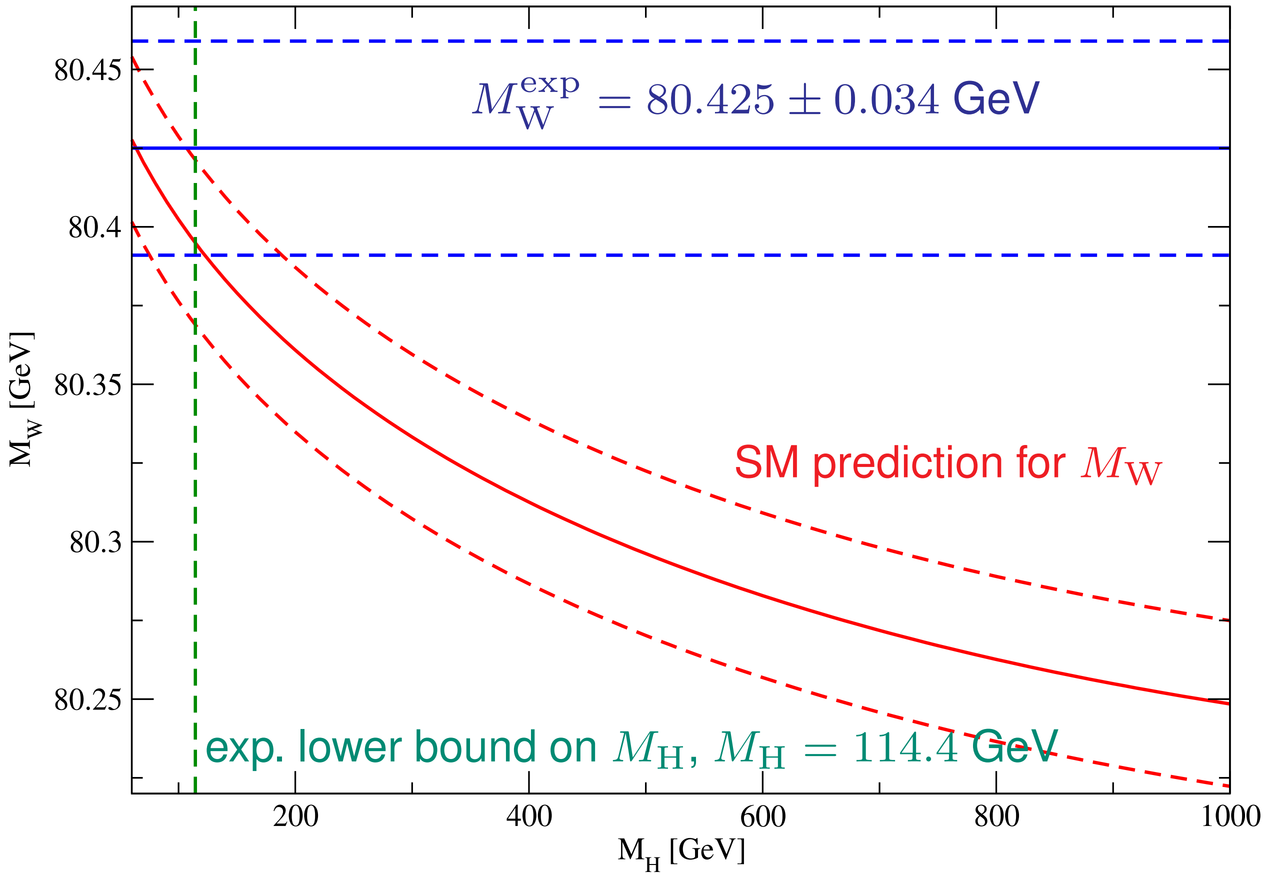
- From experimental errors of input parameters (“parametric”)

$$\delta m_{\mathrm{t}} = 2.7 \ \mathrm{GeV} \ \Rightarrow \ \Delta M_{\mathrm{W}}^{\mathrm{para}} \approx 18 \ \mathrm{MeV}, \quad \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{para}} \approx 9 \times 10^{-5}$$

$\Rightarrow$ Experimental error of m_{t} is dominant source of theoretical uncertainties

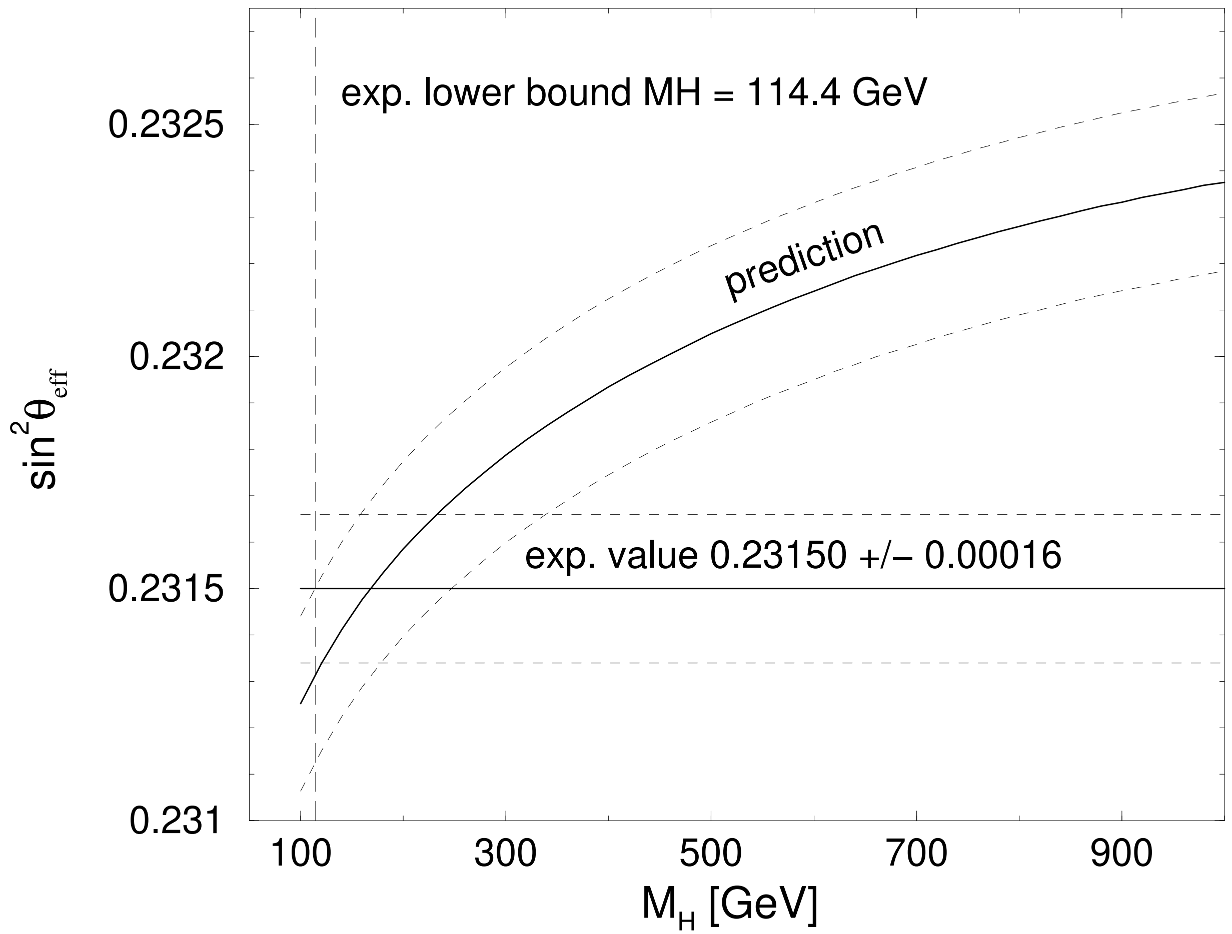
SM prediction for M_W (complete 2-loop result) vs. experimental result

[M. Awramik, M. Czakon, A. Freitas, G.W. '04]



$\Rightarrow$ Preference for light Higgs

SM prediction for $\sin^2\theta_{\rm eff}$ (fermionic 2-loop result) vs. experimental result

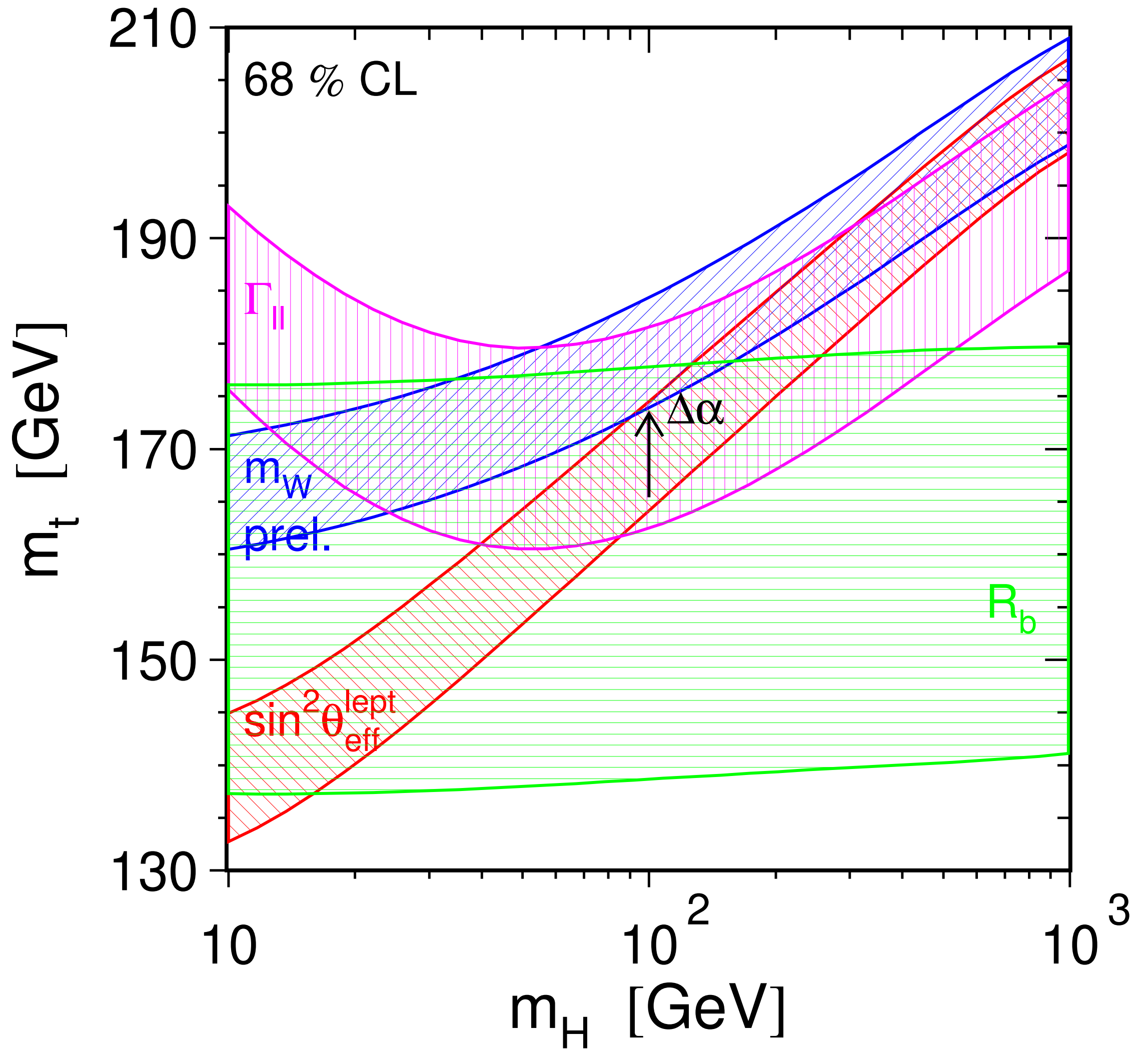


[*M. Awramik, M. Czakon, A. Freitas, G.W. '04*]

$\Rightarrow$ Preference for light Higgs

However: experimental value for $\sin^2\theta_{\rm eff}$ contains average over $A_{\rm l} = 0.23098 \pm 0.00026$ (SLD) and $A_{\rm fb}^{0,\rm b} = 0.23210 \pm 0.00030$ (LEP)

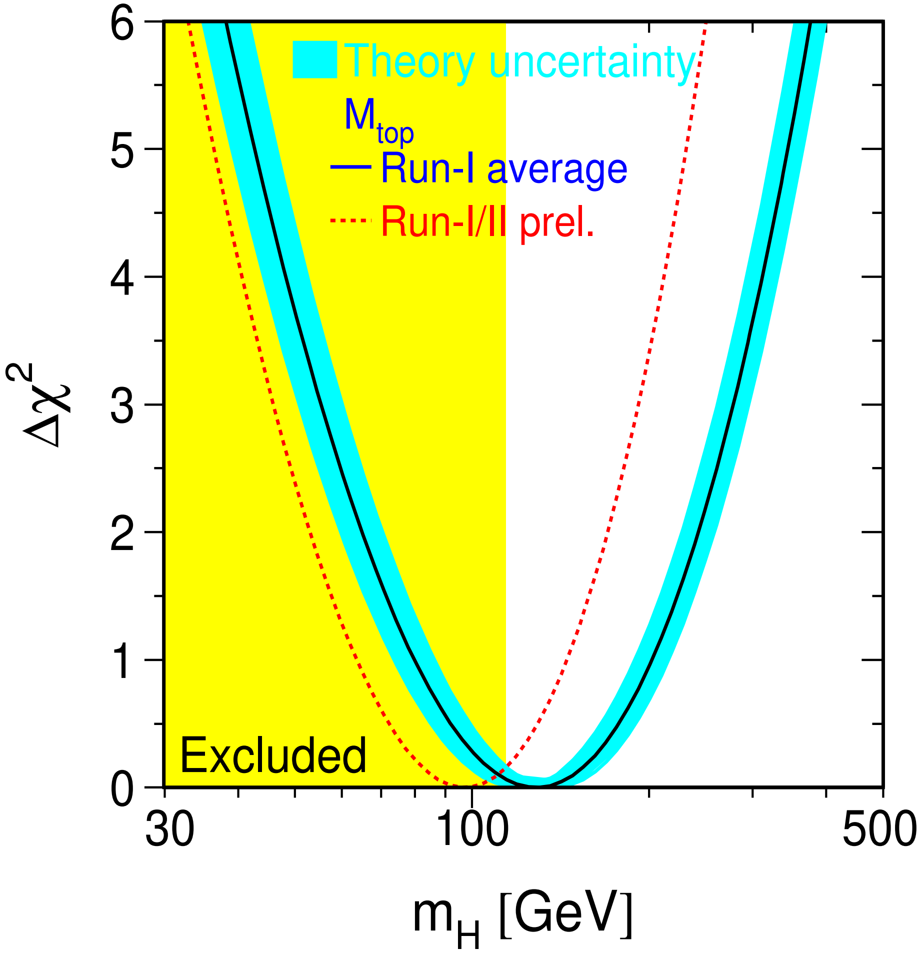
Sensitivity of different (pseudo-)observables to the Higgs mass in the SM



[*LEPEWWG '05*]

$\Rightarrow$ Highest sensitivity from $\sin^2\theta_{\text{eff}}$ and M_W

Electroweak precision physics: Global fit to all data in the SM



[*LEPEWWG '05*]

Theoretical uncertainties:

– exp. error of input parameters:

$m_t, \Delta\alpha_{had}, \ldots$

$\Rightarrow$ large m_t–M_H correlation

– unknown higher-order corrections

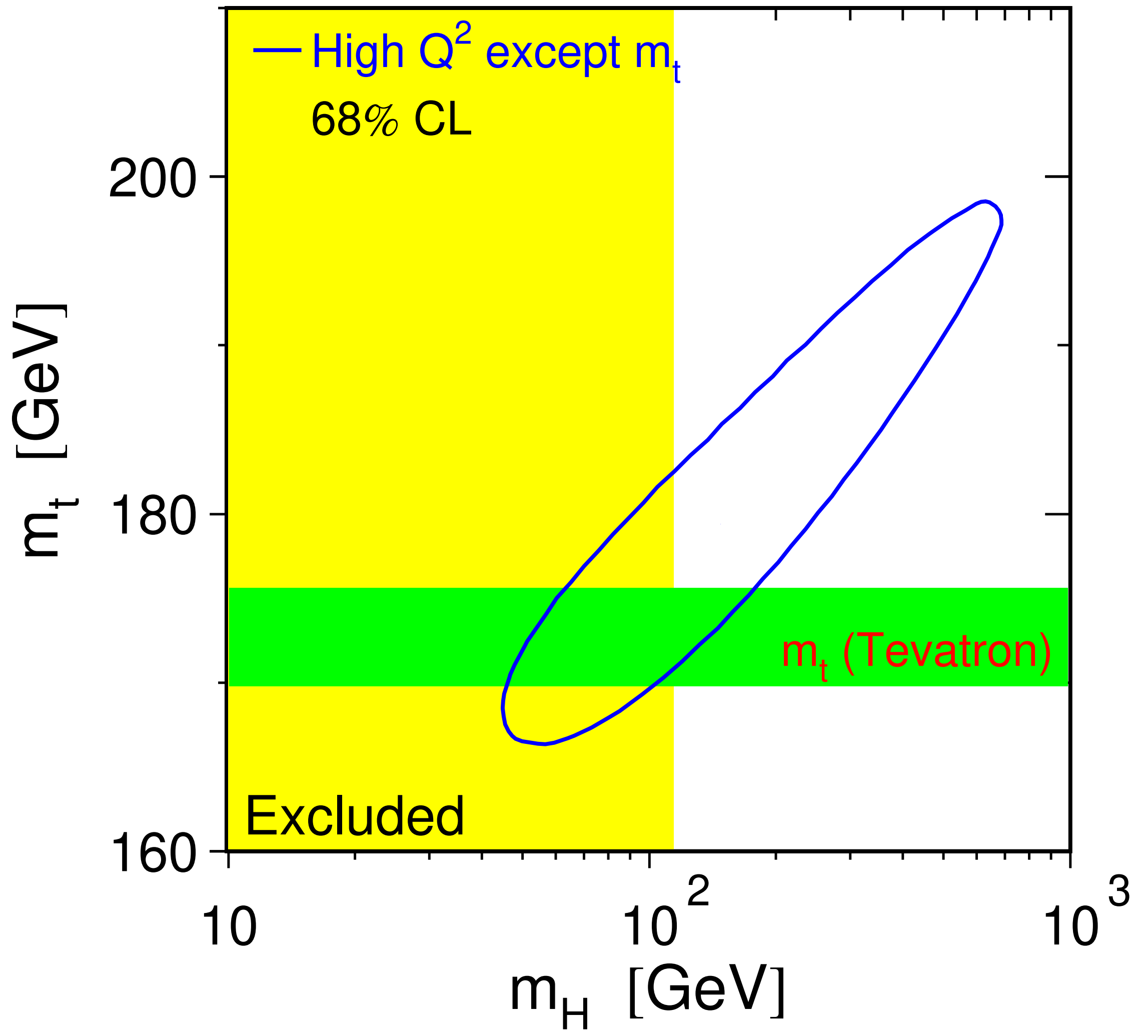
$\Rightarrow$ "blue band"

New m_t value, $m_t = 172.7 \pm 2.9$ GeV

new result for $\sin^2\theta_{eff}$ [*M. Awramik, M. Czakon, A. Freitas, G.W. '04*]

$\Rightarrow M_H = 91^{+45}_{-32}$ GeV, $M_H < 186$ GeV, 95% C.L.

Correlation between M_{H} and m_{t} in the fit:



[*LEPEWWG '05*]

$\Rightarrow$ Precise knowledge of m_{t} crucial for constraining M_{H}

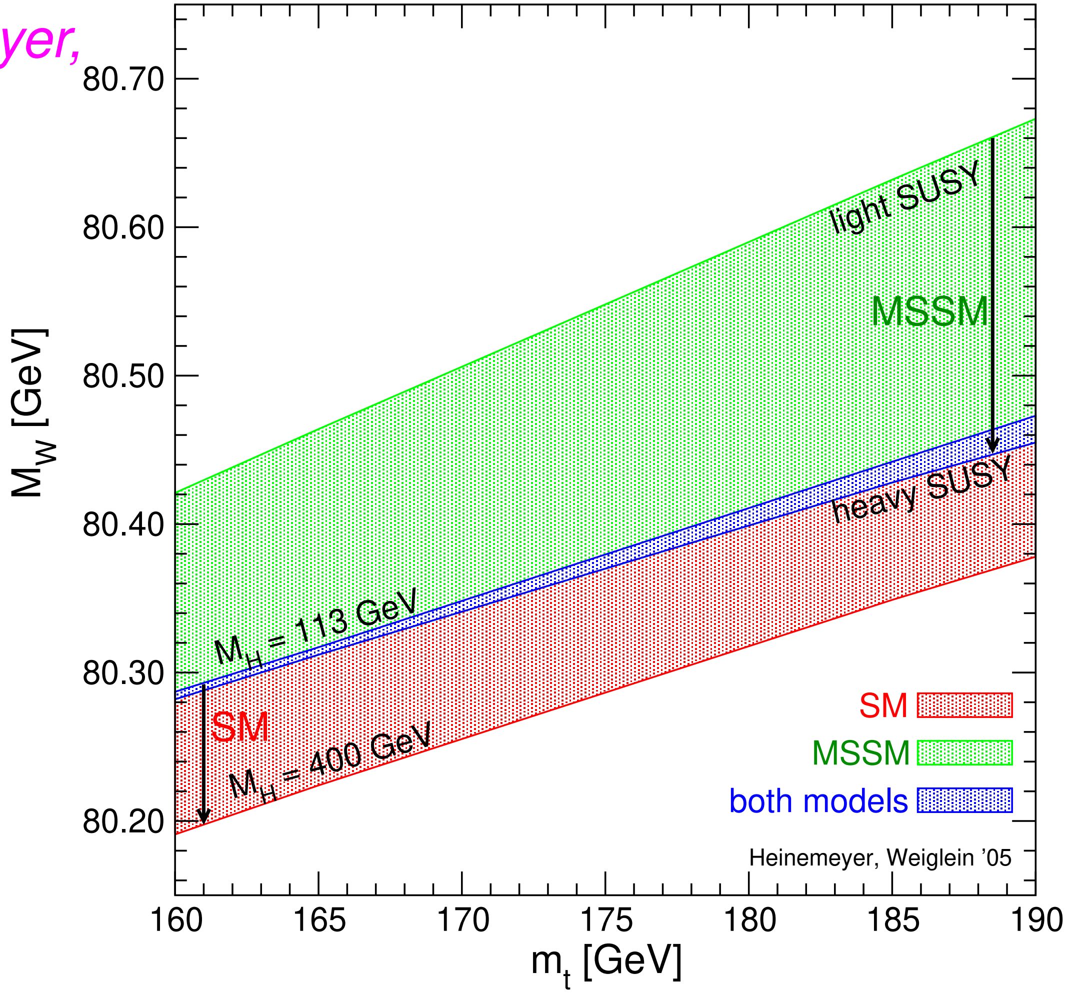
Electroweak precision tests: SM vs. MSSM

Prediction for M_W in the SM and the MSSM:

[*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]

[*S. Heinemeyer, W. Hollik, G. W. '98*]

[*S. Heinemeyer, G. W. '02*]



SM: M_{H} varied

MSSM: SUSY parameters varied

Electroweak precision tests: SM vs. MSSM

Prediction for M_{W} in the SM and the MSSM:

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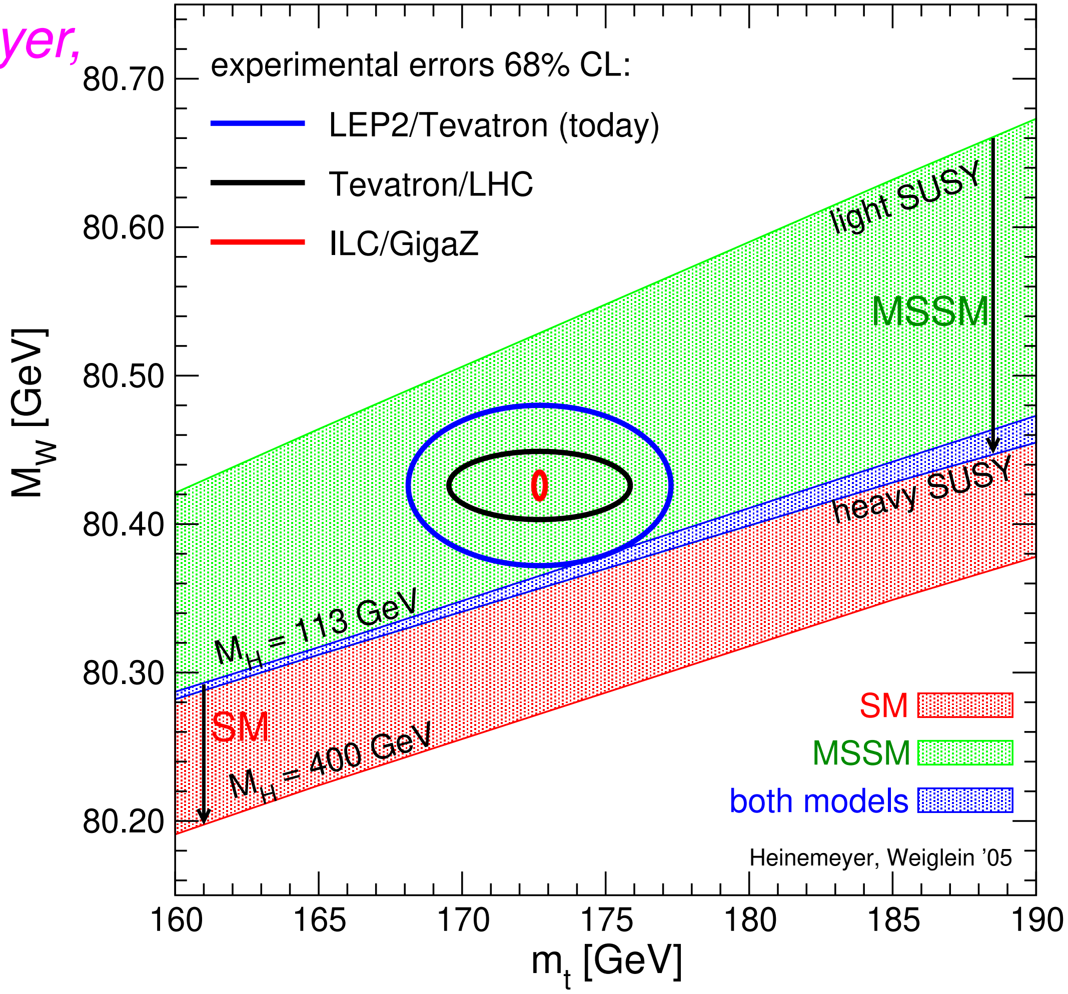


SM: M_{H} varied

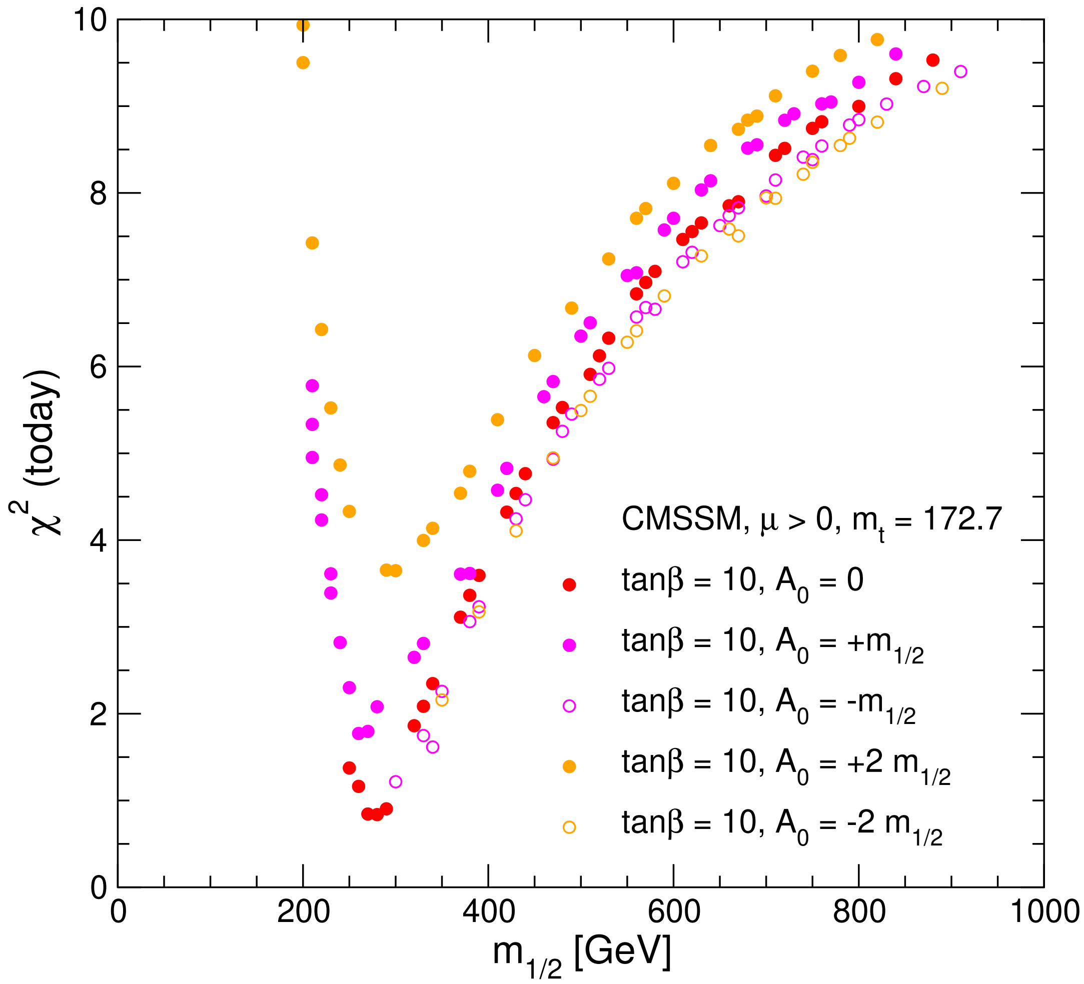
MSSM: SUSY parameters varied

χ^2 *fit in CMSSM with dark matter constraints:*

M_W, $\sin^2\theta_{\mathrm{eff}}$, $(g-2)_\mu$, $\mathrm{BR}(b \to s\gamma)$

[J. Ellis, S. Heinemeyer, K. Olive, G. W. '04]

$\tan\beta = 10$:



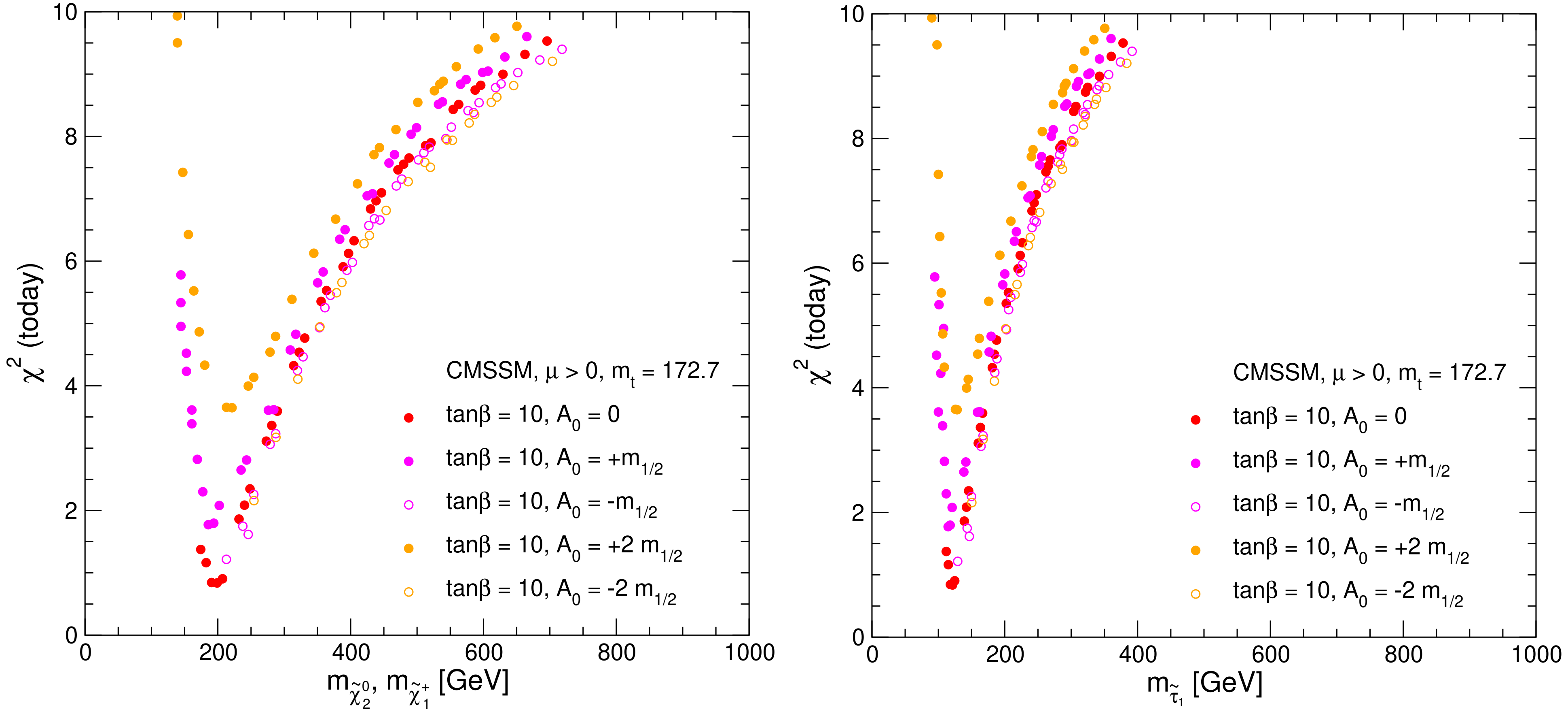
$\Rightarrow$ very good description of the data

preference for relatively small mass values

Fit results for particle masses, $\tan\beta = 10$*:*

$$m_{\tilde{\chi}_1^+} \approx m_{\tilde{\chi}_2^0}, \quad m_{\tilde{\tau}_1}$$

[J. Ellis, S. Heinemeyer, K. Olive, G. W. '04]



$\Rightarrow$ Good prospects for the LHC and ILC

Comparison of current and anticipated future experimental errors

Present errors vs. Run II of Tevatron, LHC, and ILC with and without low-energy running mode (GigaZ):

	now	Tevatron	LHC	ILC	ILC + GigaZ
$\delta \sin^2\theta_{\mathrm{eff}}(\times 10^5)$	16	—	??	—	1.3
δM_{W} [MeV]	32	25	$\lesssim$ 15	10	7
δm_{t} [GeV]	2.9	2	1–2	0.1	0.1
δm_{h} [MeV]	—	—	200	50	50

Higgs physics in the MSSM

'Simplest' extension of the minimal Higgs sector:

Minimal Supersymmetric Standard Model (MSSM)

- Two doublets to give masses to up-type and down-type fermions (extra symmetry forbids to use same doublet for both)
- Supersymmetry imposes relations between the parameters

$\Rightarrow$ Two parameters instead of one:

$$\tan\beta \equiv \frac{v_2}{v_1}, \qquad M_\mathrm{A} \text{ (or } M_{\mathrm{H}^\pm})$$

Higgs potential of the MSSM

MSSM Higgs potential contains two Higgs doublets:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, M_A

$\Rightarrow m_\text{h}$, m_H, mixing angle α, $m_{\text{H}^\pm}$: no free parameters

Higgs mass bound in the MSSM:

$\Rightarrow$ Prediction for $m_{\mathrm{h}}, m_{\mathrm{H}}, \ldots$

Tree-level result for $m_{\mathrm{h}}, m_{\mathrm{H}}$:

$$m^2_{\mathrm{H,h}} = \frac{1}{2}\left[M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2 \pm \sqrt{(M_{\mathrm{A}}^2 + M_{\mathrm{Z}}^2)^2 - 4M_{\mathrm{Z}}^2 M_{\mathrm{A}}^2 \cos^2 2\beta}\right]$$

$\Rightarrow$ $m_{\mathrm{h}} \leq M_{\mathrm{Z}}$ at tree level

MSSM tree-level bound (gauge sector): excluded by LEP!

Large radiative corrections (Yukawa sector, ...):

Yukawa couplings: $\frac{\mathrm{e}\, m_{\mathrm{t}}}{2 M_{\mathrm{W}} s_{\mathrm{W}}}, \frac{\mathrm{e}\, m_{\mathrm{t}}^2}{M_{\mathrm{W}} s_{\mathrm{W}}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_{\mathrm{t}}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right)$, $\quad \mathcal{O}(100\%)$!

Higgs mass bound in the MSSM:

Present status of m_{h} prediction in the MSSM:

Complete one-loop $+$ "almost complete" two-loop result available

Upper bound on m_{h} (*FeynHiggs*):

'Unconstrained' MSSM: M_{A}, $\tan\beta$, 5 param. in $\tilde{\mathrm{t}}$-$\tilde{\mathrm{b}}$ sector, μ, $m_{\tilde{\mathrm{g}}}$, M_2

$$m_{\mathrm{h}} \lesssim 131\,\mathrm{GeV}$$

[*S. Heinemeyer, W. Hollik, G. W. '99*], [*M. Frank, S. Heinemeyer, W. Hollik, G. W. '02*] [*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]

for $m_{\mathrm{t}} = 172.7\,\mathrm{GeV}$, no theoretical uncertainties included

Bound reduced by $\approx 6, 11, 8$ GeV in mSUGRA, GMSB, AMSB scenarios [*S. Ambrosanio, A. Dedes, S. Heinemeyer, S. Su, G. W. '01*]

Determination of SUSY parameters: global fit

Use *Fittino* to compare the ability of LHC only and LHC $\oplus$ ILC for SPS1a' point

[*P. Bechtle, K. Desch, P. Wienemann '05*]

LHC input:

mass measurements and precision as in LHC / ILC report (SPS 1a: optimistic scenario)

$+$ assumption on $\tilde{t}_{1,2}$ mass measurement

$+$ ratios of Higgs branching ratios

Fittino: LHC only vs. LHC $\oplus$ ILC

Parameter	"True" value	ILC Fit value	Uncertainty (ILC+LHC)	Uncertainty (LHC only)
$\tan\beta$	10.00	10.00	0.11	6.7
μ	400.4 GeV	400.4 GeV	1.2 GeV	811. GeV
X_τ	-4449. GeV	-4449. GeV	20.GeV	6368. GeV
$M_{\tilde{e}_R}$	115.60 GeV	115.60 GeV	0.27 GeV	39. GeV
$M_{\tilde{\tau}_R}$	109.89 GeV	109.89 GeV	0.41 GeV	1056. GeV
$M_{\tilde{e}_L}$	181.30 GeV	181.30 GeV	0.10 GeV	12.9 GeV
$M_{\tilde{\tau}_L}$	179.54 GeV	179.54 GeV	0.14 GeV	1369. GeV
X_t	-565.7 GeV	-565.7 GeV	3.1 GeV	548. GeV
X_b	-4935. GeV	-4935. GeV	1284. GeV	6703. GeV
$M_{\tilde{u}_R}$	503. GeV	503. GeV	24. GeV	25. GeV
$M_{\tilde{b}_R}$	497. GeV	497. GeV	8. GeV	1269. GeV
$M_{\tilde{t}_R}$	380.9 GeV	380.9 GeV	2.5 GeV	753. GeV
$M_{\tilde{u}_L}$	523. GeV	523. GeV	10. GeV	19. GeV
$M_{\tilde{t}_L}$	467.7 GeV	467.7 GeV	3.1 GeV	424. GeV
M_1	103.27 GeV	103.27 GeV	0.06 GeV	8.0 GeV
M_2	193.45 GeV	193.45 GeV	0.10 GeV	132. GeV
M_3	569. GeV	569. GeV	7. GeV	10.1 GeV
$m_{A run}$	312.0 GeV	311.9 GeV	4.6 GeV	1272. GeV
m_t	178.00 GeV	178.00 GeV	0.050 GeV	0.27 GeV

χ^2 for unsmeared observables: 5.3×10^{-5}

$\Rightarrow$ most of the Lagrangian parameters can hardly be constrained by LHC data alone

$\Rightarrow$ precise determination of SUSY parameters only possible with LHC $\oplus$ ILC

Conclusions and outlook

- Renormalisable gauge theories provide a very powerful description of the fundamental interactios:

  High predictive power, can be extended to very high scales

- SM: tremendous success story

- MSSM: best studied model of new physics

- Testing the theory at the quantum level is challenging both from the experimental and the theoretical side

  The physics gain from the LHC (and eventually the ILC) will definitely be worth the effort