Electroweak Physics: Preparing for TESLA

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DESY Academic Training Lectures

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Lecture 2:

- Limitations of the Standard Model
- Supersymmetric extensions of the Standard Model
- Electroweak precision tests: SM vs. MSSM

The SM cannot be the ultimate theory

– doesn't contain gravity

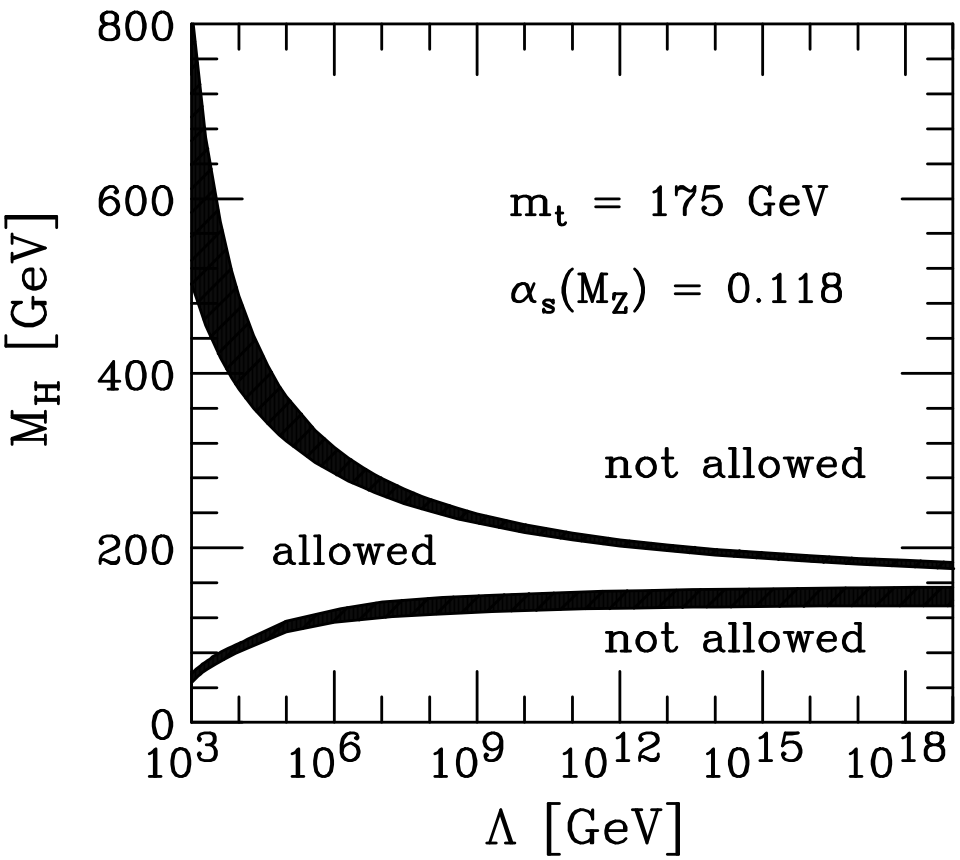
– further problems: "hierarchy problem", ...

Up to which energy scale Λ can it be valid?

– $\Lambda < M_{\mathrm{P}} \approx 10^{19}$ GeV
(inclusion of gravity effects necessary)

– hierarchy problem: Higgs mass unstable w.r.t. large quantum corrections
$\Rightarrow \delta M_{\mathrm{H}}^2 \sim \Lambda^2$

– stability of Higgs potential:



[*T. Hambye, K. Riesselmann '97*]

Higher-order corrections to Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger\Phi\right| + \lambda \left|\Phi^\dagger\Phi\right|^2$$

radiative corrections $\Rightarrow$ running of λ

$$\beta_\mathsf{H} \equiv \mu\frac{\partial}{\partial\mu}\lambda = \frac{1}{16\pi^2}\left(4\lambda^2 - 36g_\mathsf{t}^4 + \frac{27}{4}g^4 + \frac{9}{4}g'^4 + \frac{9}{2}g^2g'^2\right) + 2\lambda\gamma_\mathsf{H}$$

Conditions on Higgs potential:

– has to be bounded from below
  $\lambda > 0$: vacuum stability bound

– $\lambda < \infty$: "triviality bound"

With increasing μ:

$4\lambda^2 \sim M_\mathsf{H}^4 \Rightarrow$ increase of λ

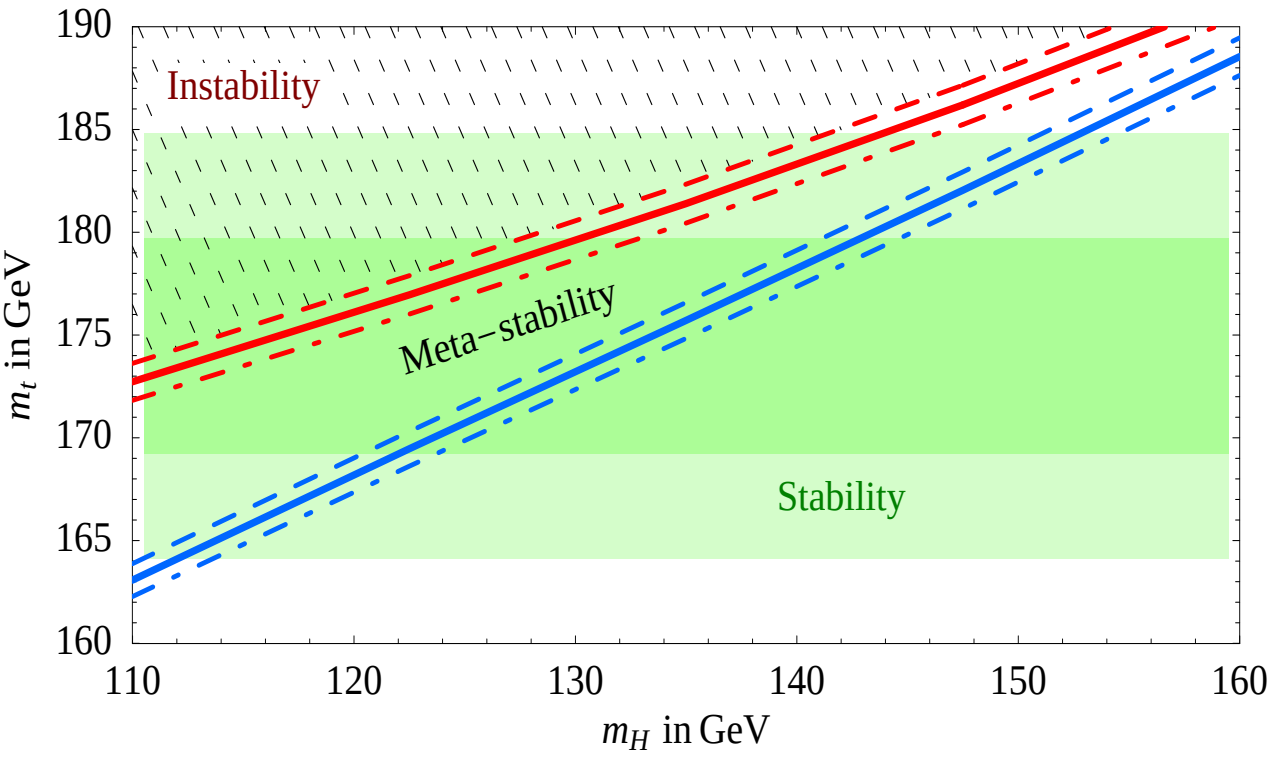
$-36g_\mathsf{t}^4 \sim m_\mathsf{t}^4 \Rightarrow$ decrease of λ

$\Rightarrow$ within SM:
  triviality bound yields upper bound on M_H
  vacuum stability bound yields lower bound on M_H

What if the Higgs were "just around the corner", $M_{\mathrm{H}} \approx 115$ GeV?

Regions of instability, meta-stability and stability in M_{H}–m_{t} plane for $\Lambda \approx 10^{19}$ GeV:

[*G. Isidori, G. Ridolfi, A. Strumia '01*]



$\Rightarrow$ vacuum stability problematic

Can vacuum be stabilized via new physics contributions?

What kind of new physics?
(should preferably also stabilize gauge hierarchy)

Composite Higgs models:

technicolor, . . .

[*E. Farhi, L. Susskind '81*]

H produced in $e^+e^- \to ZH$ expected to have larger mass than 115 GeV

$\Rightarrow$ preference for elementary Higgs

Analysis of stability of effective Higgs potential:

[*J. Ellis, D. Ross '00*]

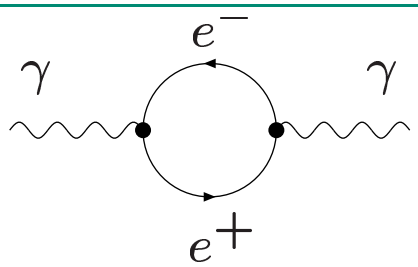
- Need extra bosonic degrees of freedom
- Fine-tuning (symmetry relations) between couplings
- Splitting bosonic/fermionic partners: 10^2–10^5 GeV
- Small mass splitting in scalar isomultiplets

$\Rightarrow$ Features of SUSY theories

The hierarchy problem

Consider loop corrections to propagators
$\Leftrightarrow$ corrections to particle masses

Photon self-energy in QED:

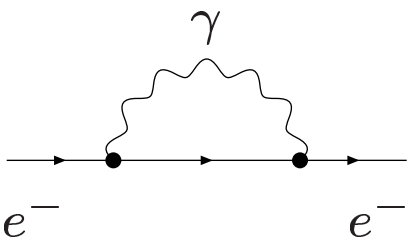


$$\Sigma^{\gamma\gamma}(0) = 0$$

consequence of U(1) gauge invariance of QED

$\Rightarrow$ photon stays massless

Electron self-energy in QED:



for $\Lambda \to \infty$:

$$\Sigma^{ee} \sim m_e \int^{\Lambda} \frac{dk}{k} \to \ln \Lambda$$

$\Rightarrow$ logarithmically divergent

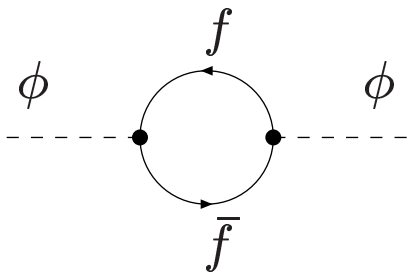
For $\Lambda = M_P$: $\delta m_e \approx 2\frac{\alpha}{\pi} m_e \ln \frac{M_P}{m_e} \approx 0.24\, m_e$

$\Rightarrow$ modest correction, proportional to m_e

reason: chiral symmetry in limit $m_e \to 0$,
breaking proportional to m_e

$\Rightarrow$ symmetry "protects" m_e

Contribution of heavy fermions to Higgs self-energy:



$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

for $\Lambda \to \infty$:

$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_P$: $\delta M_\phi^2 \sim M_P^2$

$\Rightarrow$ $\delta M_\phi^2 \approx 10^{30}\, M_\phi^2$ ($M_\phi \lesssim 1$ TeV)

no additional symmetry for $M_\phi = 0$, no protection against large corrections

$\Rightarrow$ in general: scalar masses tend to be near highest mass scale of the theory

$\Rightarrow$ hierarchy problem

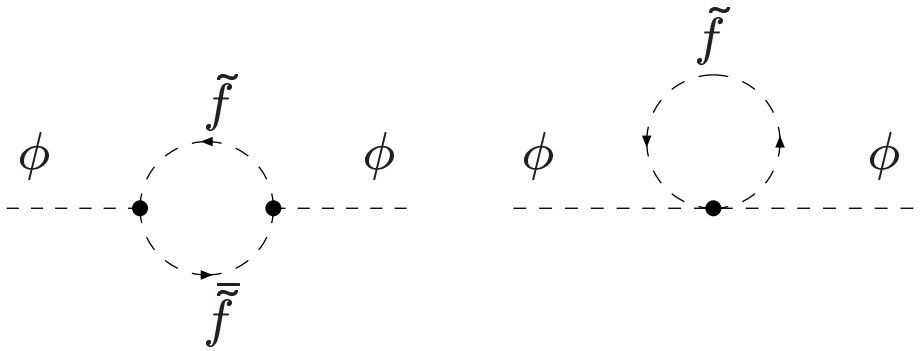
extreme fine-tuning necessary to get small M_ϕ

Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

SUSY: additional contributions from scalar fields:



$$\Sigma_{\tilde{f}}^{\phi\phi} \sim N(\tilde{f})\,\tilde{\lambda}_f \int d^4k \left(\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right)$$
$$+ \text{ terms without quadratic divergencies}$$

for $\Lambda \to \infty$:

$$\Sigma_{\tilde{f}}^{\phi\phi} \sim 2\, N(\tilde{f})\,\tilde{\lambda}_f\,\Lambda^2$$

$\Rightarrow$ quadratic divergencies cancel for

$$N(\tilde{f}_L) = N(\tilde{f}_R) = N(f)$$
$$\tilde{\lambda}_f = \lambda_f^2$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

For $m_{\tilde{f}}^2 = m_f^2 + \Delta^2$, $\tilde{\lambda}_f = \lambda_f^2$, "soft SUSY breaking"

$$\Rightarrow \Sigma^{\phi\phi}_{f+\tilde{f}} \sim N(f)\, \lambda_f^2\, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\ \text{TeV}$$

$\Rightarrow$ low-energy SUSY provides attractive solution of hierarchy problem

Other recent proposals for solving hierarchy problem in models with extra spatial dimensions:

E.g.: Gravity in large extra dimensions

[*N. Arkani-Hamed, S. Dimopoulos, G. Dvali '98*]

Gravity propagates in δ large extra spatial dimensions

fundamental Planck scale in $4+\delta$-dimensional space, M_*, is of order 1 TeV

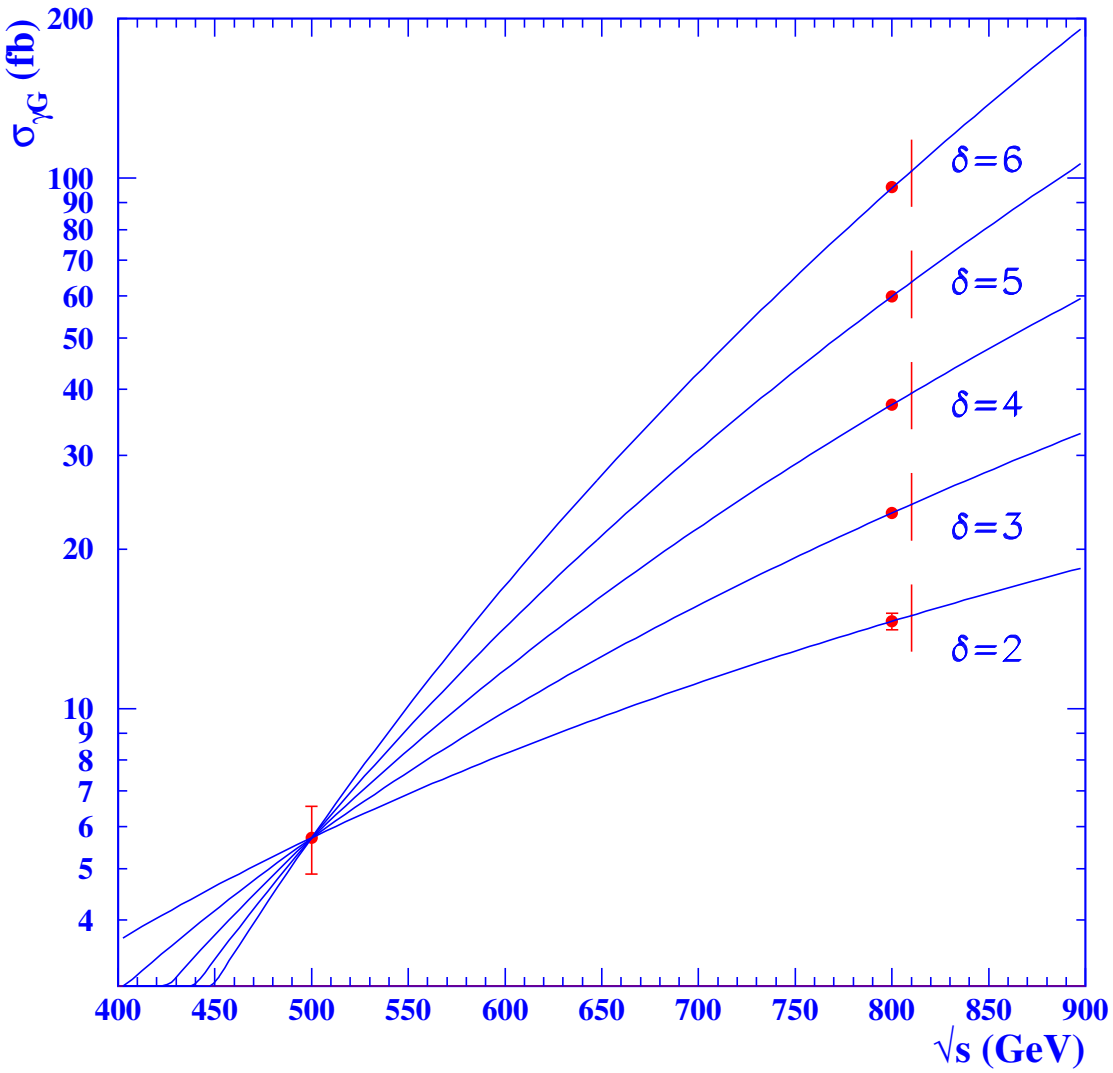
$$M_{\mathrm{P}}^2 = V_\delta \, M_*^{2+\delta}$$

$\Rightarrow$ large value of M_{P} related to relatively large volume V_δ (10^{-3} m–10^{-15} m for $\delta = 2$–6) of extra dimensions

$\Rightarrow$ should have observable signatures in TeV-scale experiments

E.g.: graviton emission, $e^+e^- \to \gamma/Z + G_n$

determination of δ from LC measurements of single photon cross section at $\sqrt{s} = 500$ GeV and $\sqrt{s} = 800$ GeV with polarized e^- and e^+ beams:
[*TESLA TDR '01*]



The SUSY algebra

Unification of fundamental interactions?

electroweak and strong interactions:

described by gauge theories: internal symmetries

γ, Z, $W^{\pm}$: spin 1

gravity:

described by general relativity:
invariance under space–time transformations

graviton G: spin 2

General "no go theorems" of QFT:

Coleman–Mandula theorem '67:

$\Rightarrow$ non-trivial extension of space–time symmetries by internal symmetries only for "graded Lie algebra", i.e. "superalgebra"

$$[\ldots,\ldots] \rightarrow \{\ldots,\ldots\}$$

symmetry generator: fermionic operator Q

Haag, Lopuszanski, Sohnius theorem '75:

no direct symmetry transformations between fields with different integer spins

$\Rightarrow$ unification of gravity with other interactions within QFT requires SUSY transformations

spin 2	$\rightarrow$	spin $\frac{3}{2}$	$\rightarrow$	spin 1
graviton		gravitino		photon

symmetry generator Q needs to have spin 1/2

SUSY algebra ($N = 1$), global SUSY transformation:

$$
\begin{aligned}
[Q, P_\mu] &= 0 \\
\{Q, Q\} = \{Q^\dagger, Q^\dagger\} &= 0 \\
\{Q, Q^\dagger\} &= 2\sigma^\mu \underbrace{P_\mu}_{\substack{\text{constant translation} \\ \text{in space–time}}}
\end{aligned}
$$

Invariance under local SUSY transformation:

$\Rightarrow$ invariance under local coordinate change

$\Rightarrow$ general relativity

$\Rightarrow$ local SUSY includes gravity, "supergravity"

The Minimal Supersymmetric Standard Model:

Superpartners for Standard Model particles:

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } 0$$

$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0}$ Spin 1 / Spin 0

$\tilde{g} \qquad \tilde{\chi}^{\pm}_{1,2} \qquad \tilde{\chi}^{0}_{1,2,3,4}$ Spin $\dfrac{1}{2}$

Enlarged Higgs sector:

Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Exact SUSY: $m_f = m_{\tilde{f}}$, ...

$\Rightarrow$ SUSY must be broken: $M_{\text{SUSY}} \lesssim 1$ TeV

SUSY breaking mechanism not well understood

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergencies

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenological description for universal breaking terms (FCNC, . . .)

Specific models for soft-SUSY breaking:

"Hidden sector": $\longrightarrow$ Visible sector:

SUSY breaking MSSM

"Gravity-mediated": mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
. . .

Determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking

Properties of SUSY theories:

Gauge coupling unification:

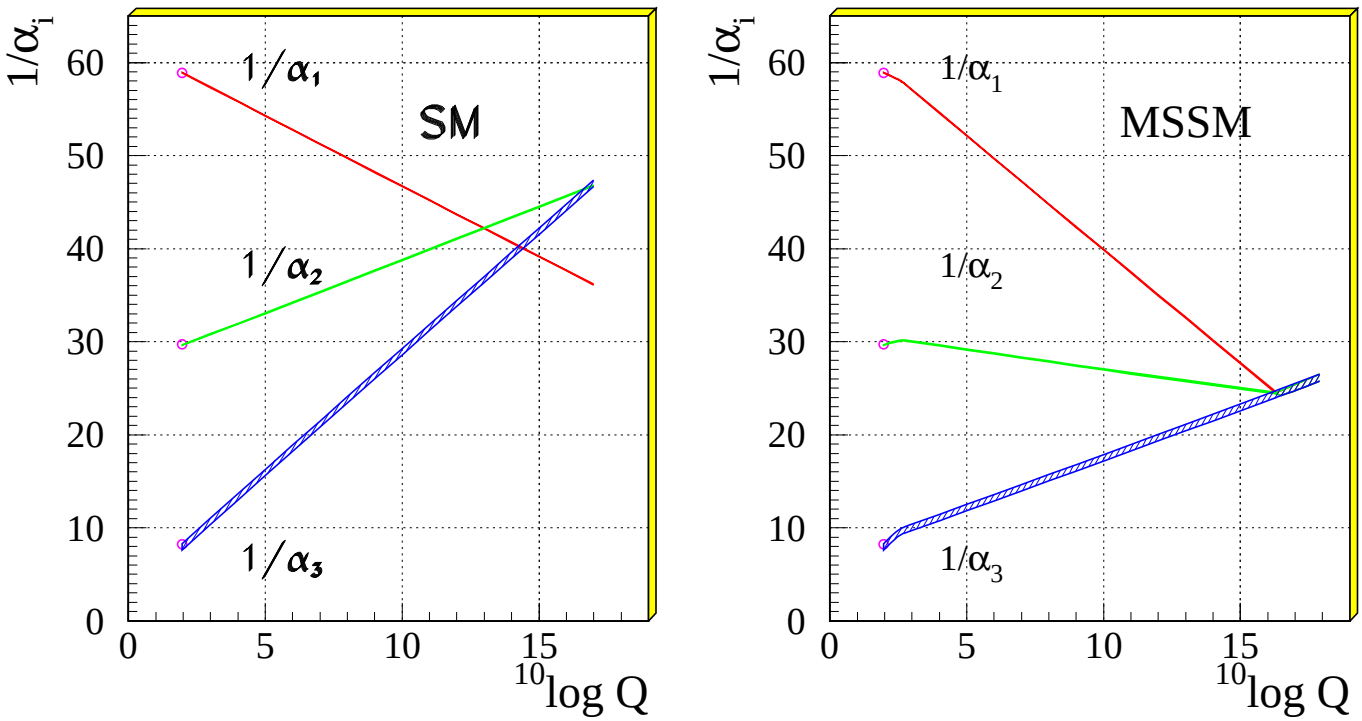
Running of gauge couplings:

$$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

Unification of couplings at high scale

$\Leftrightarrow$ "Grand unified theories" (GUTs)

Unification of the Coupling Constants
in the SM and the MSSM

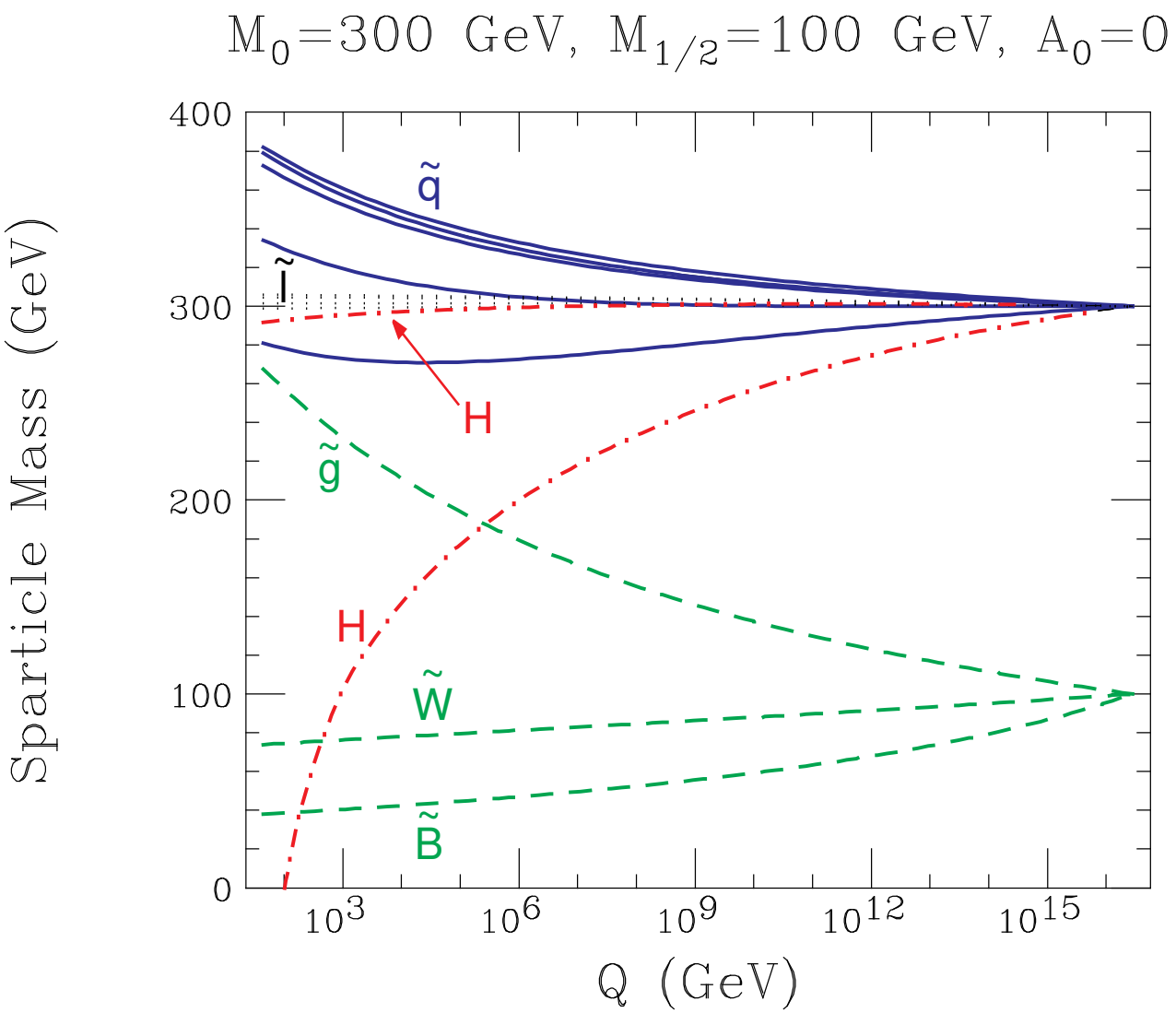


[*U. Amaldi, W. de Boer, H. Fürstenau '91*]

$\Rightarrow$ coupling constant unification in MSSM for $M_{\text{SUSY}} \lesssim 1$ TeV

Radiative electroweak symmetry breaking:

Universal boundary conditions at GUT scale, renormalization group running down to weak scale



large corrections from top-quark Yukawa coupling

$\Rightarrow$ $m^2_{H_u}$ driven to negative values

$\Rightarrow$ electroweak symmetry breaking

emerges naturally at scale $\sim 10^2$ GeV for $100\ \text{GeV} \lesssim m_\text{t} \lesssim 200\ \text{GeV}$

R parity

minimal SUSY extension of the SM contains only terms in the Lagrangian with even number of SUSY particles

$\Rightarrow$ additional symmetry: "R parity"

$\Rightarrow$ all SM particles have even R parity, all SUSY particles have odd R parity

Phenomenological consequences of R parity conservation:

- SUSY particles can only be produced in pairs, e.g. $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$

- Decay of SUSY particles into SM particles + odd number of SUSY particles

  $\Rightarrow$ Lightest SUSY particle (LSP) is stable

- All SUSY particles will eventually decay into LSP

  $\Rightarrow$ Decay cascades of heavy SUSY particles

- LSP stable

  $\Rightarrow$ some LSP must have survived from Big Bang

  $\Rightarrow$ Weakly interacting massive particle ("WIMP")

  (otherwise ruled out from bounds on exotic isotopes etc.)

  $\Rightarrow$ Candidate for cold dark matter in the Universe if $M_{\mathrm{SUSY}} \lesssim 1$ TeV

- LSP neutral, uncolored

  $\Rightarrow$ leaves no traces in collider detectors

  $\Rightarrow$ Typical SUSY signatures: "missing energy"

Relations between SUSY parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.: gauge boson–fermion coupling

=

gaugino–fermion–sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters (except M_W–M_Z interdependence)

MSSM:

- Upper bound on mass of lightest $\mathcal{CP}$-even Higgs boson
- Relations between neutralino and chargino masses
- Sfermion mass relations, e.g.

$$m^2_{\tilde{e}_L} = m^2_{\tilde{\nu}_L} - M_W^2 \cos(2\beta)$$

All relations receive corrections from loop effects

$\Leftrightarrow$ effects of soft SUSY breaking, electroweak symmetry breaking

Experimental verification of parameter relations is a crucial test of SUSY!

Specific SUSY-breaking scenarios:

mSUGRA

parameters: $m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$

m_0: universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0: universal trilinear coupling

$\tan\beta$: ratio of Higgs vacuum expectation values

$\text{sign}(\mu)$: sign of supersymmetric Higgs parameter

m_0, $m_{1/2}$, A_0: GUT scale parameters

$\Rightarrow$ particle spectra from renormalization group running to weak scale

LSP is usually lightest neutralino

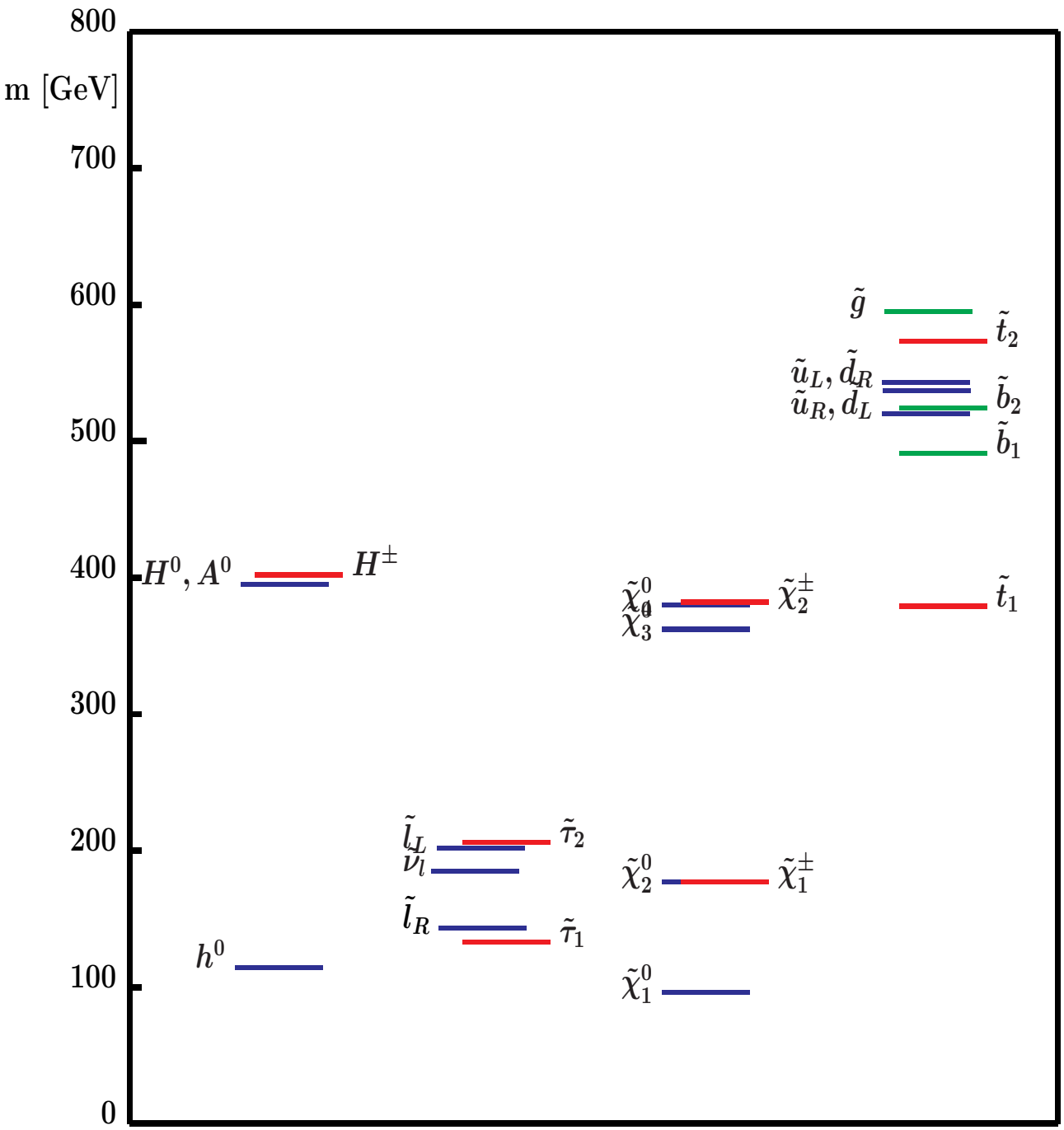
gaugino masses run in same way as gauge couplings

$\Rightarrow$ gluino heavier than charginos, neutralinos

The "Snowmass Points and Slopes" (SPS): benchmarks for SUSY searches

www.ippp.dur.ac.uk/∼georg/sps

SPS 1a: "typical" mSUGRA scenario



GMSB

parameters: $M_{\text{mes}}, N_{\text{mes}}, \Lambda, \tan\beta, \text{sign}(\mu)$

M_{mes}: messenger mass scale

N_{mes}: messenger index (number of messenger multiplets)

Λ: universal soft SUSY breaking mass scale felt by low-energy sector

LSP is always the gravitino, $m_{\frac{3}{2}} \approx 10^{-9}$ GeV

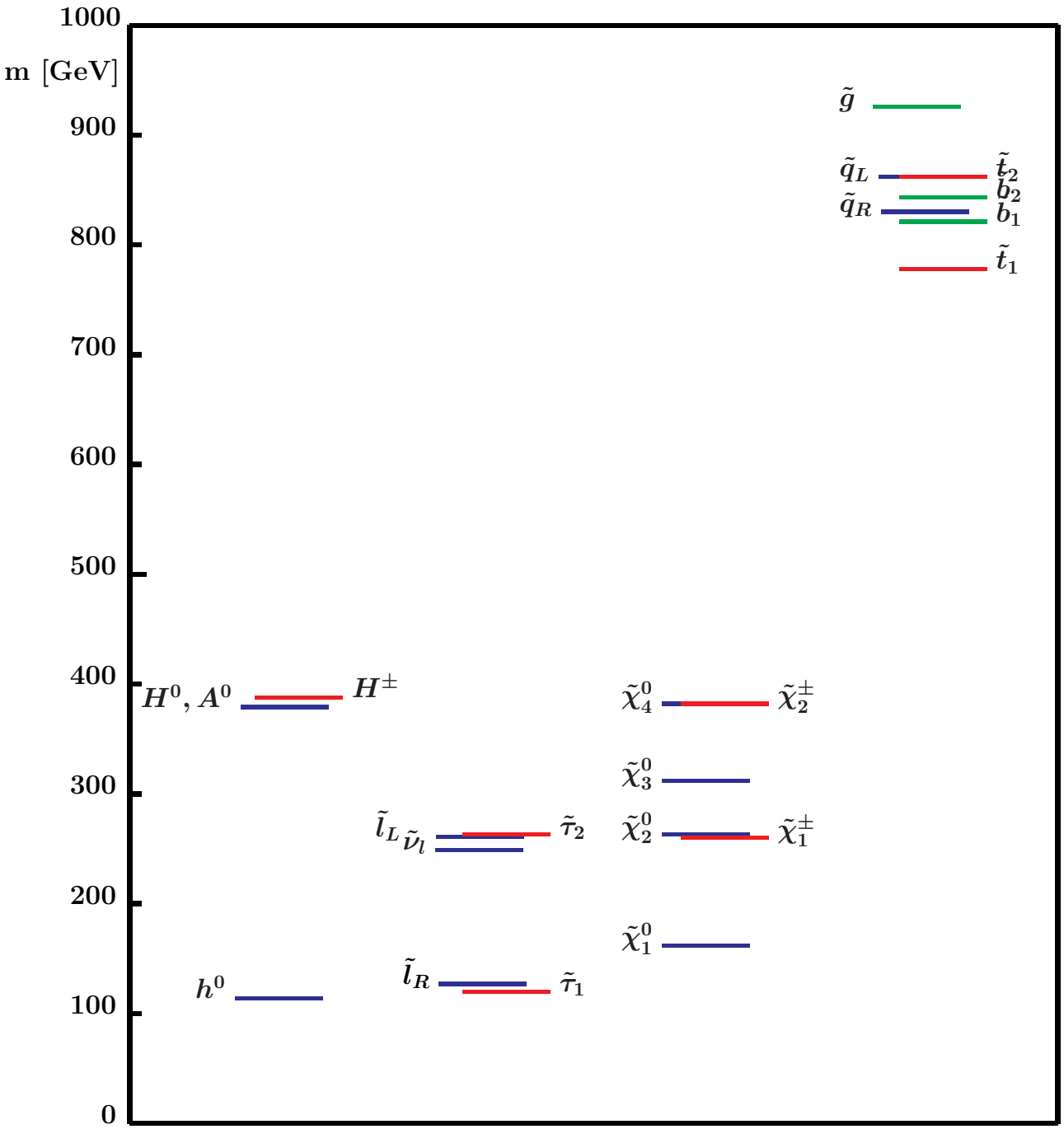
Next-to-lightest SUSY particle (NLSP): $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

can decay into LSP inside or outside the detector

masses of SUSY particles from loop diagrams involving messenger fields, couple to MSSM fields via ordinary gauge interactions

$\Rightarrow$ colored particles usually heavier than weakly interacting particles

SPS 7: GMSB scenario with $\tilde{\tau}$ NLSP



AMSB

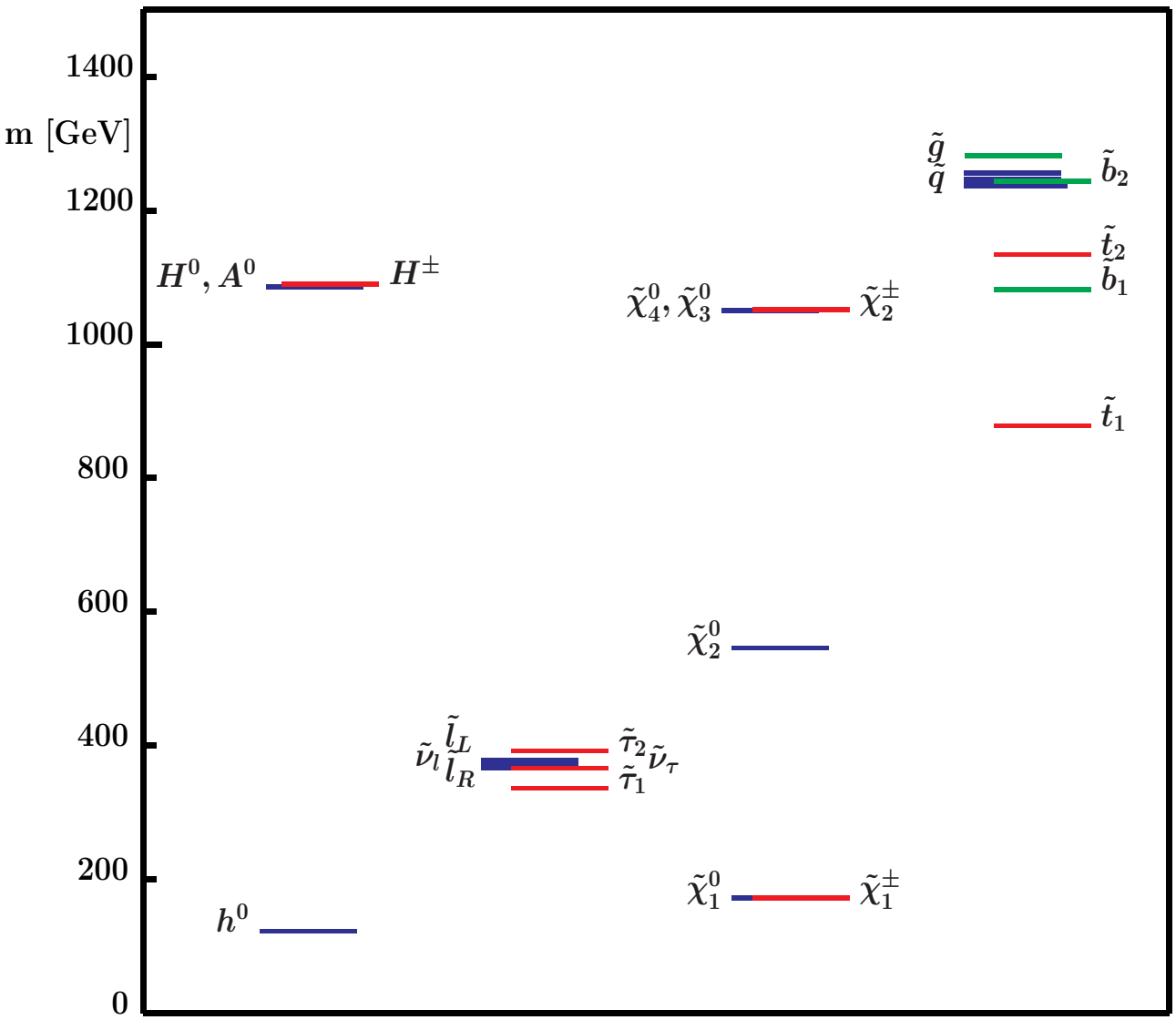
parameters: $m_{\text{aux}}, m_0, \tan\beta, \text{sign}(\mu)$

m_{aux}: overall scale of SUSY particle masses

m_0: phenomenological parameter: universal scalar mass term

typical feature: very small neutralino–chargino mass difference

SPS 9: AMSB scenario

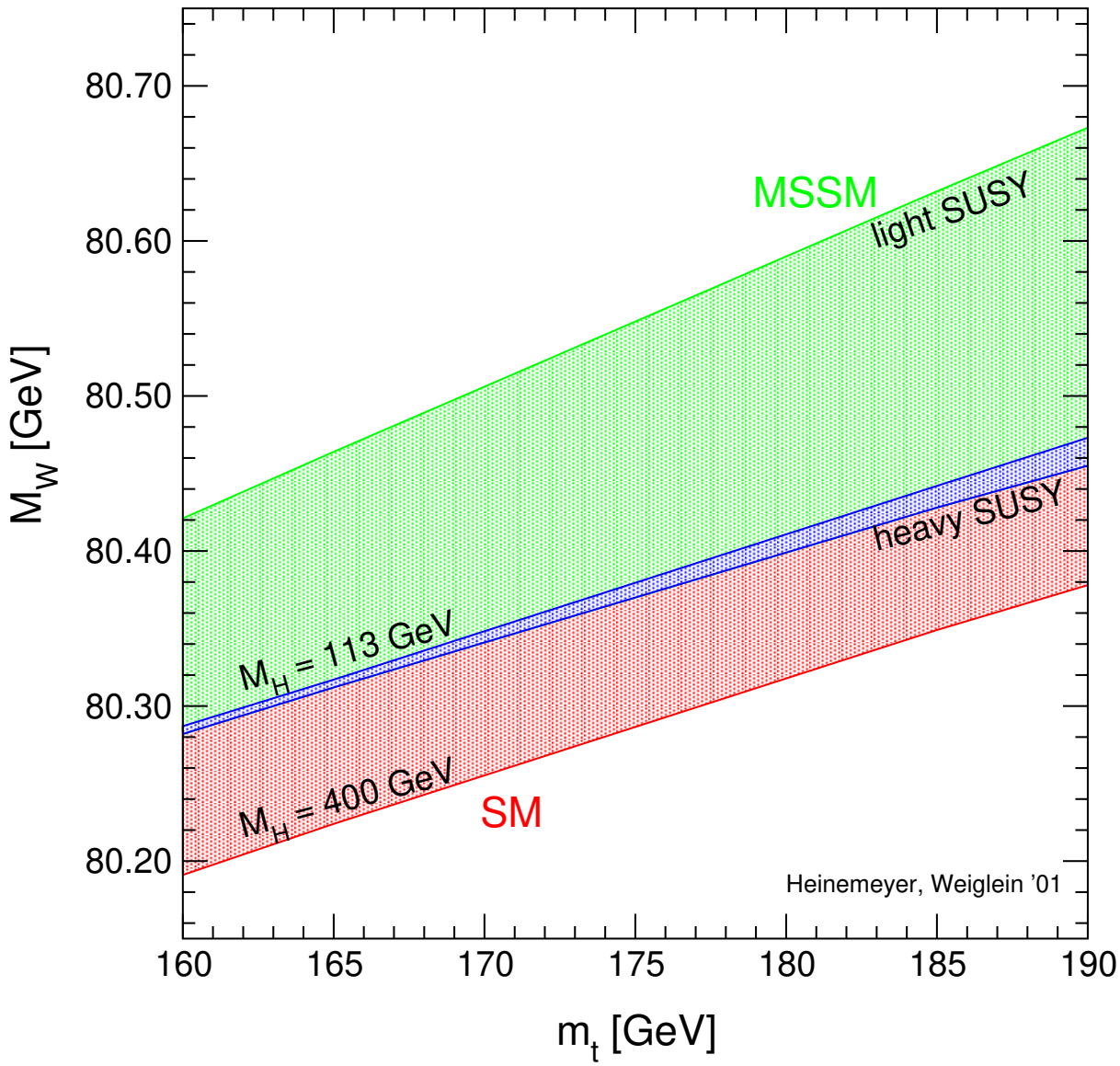


Electroweak precision tests: SM vs. MSSM

Prediction for M_W in the SM and the MSSM:

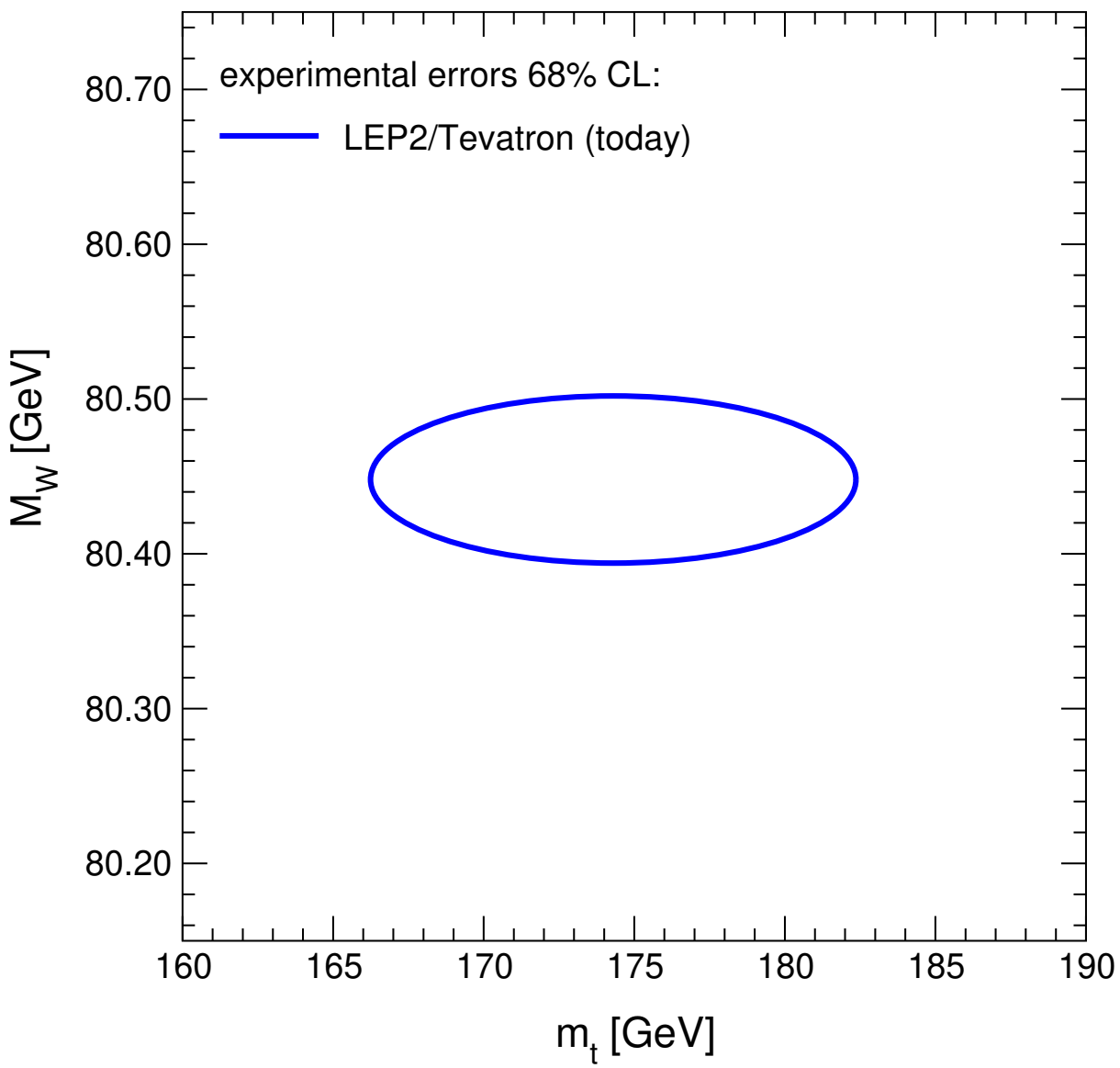
[*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]

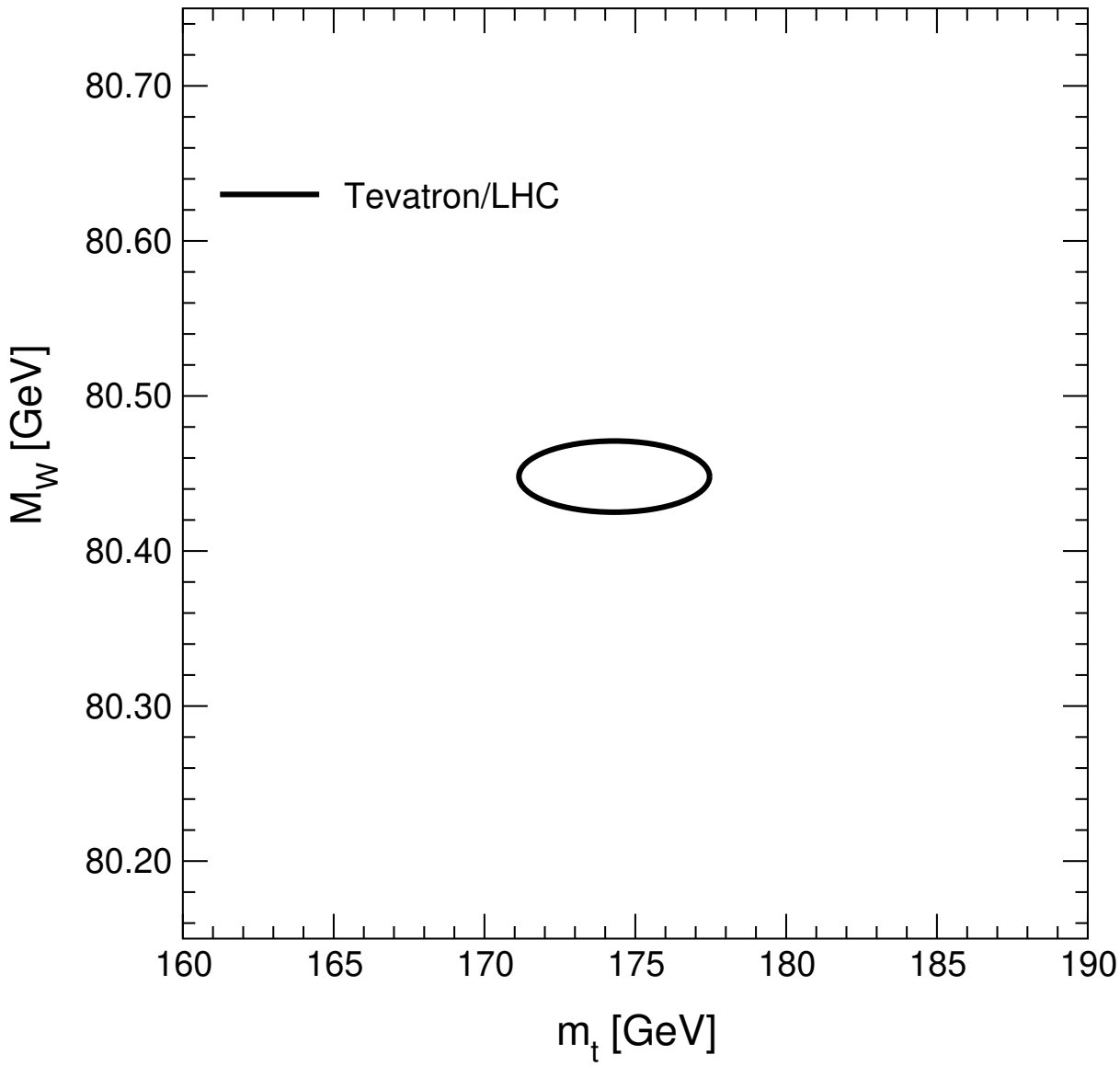
[*S. Heinemeyer, W. Hollik, G. W. '98*]



SM: M_H varied

MSSM: SUSY parameters varied


experimental errors 68% CL:
LEP2/Tevatron (today)
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]


80.70
80.60
80.50
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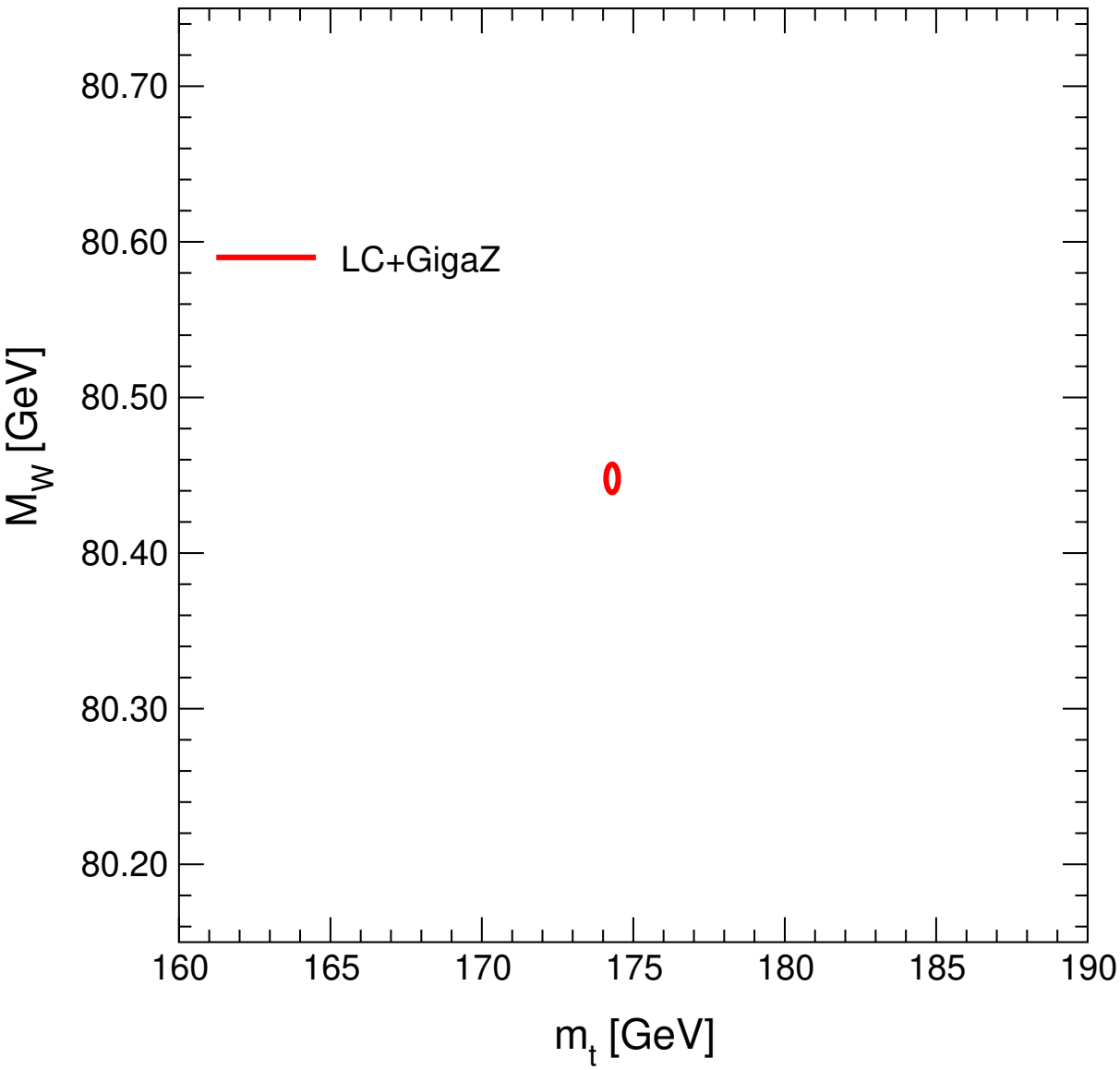
80.30
80.20
M_W [GeV]
Tevatron/LHC
160
165
170
175
180
185
190
m_t [GeV]


80.70
80.60
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80.40
80.30
80.20
M_W [GeV]
LC+GigaZ
160
165
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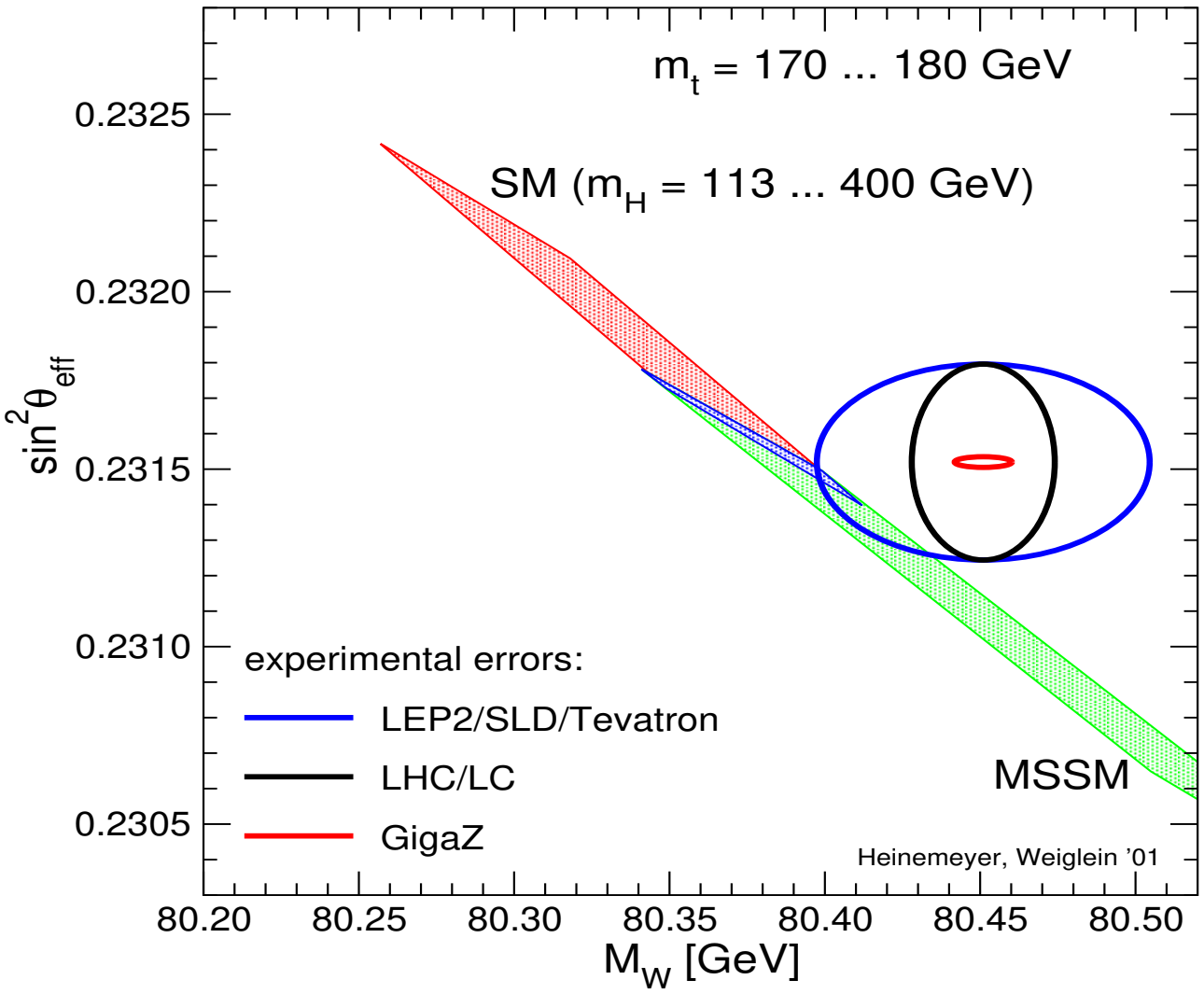
185
190
m_t [GeV]

Prediction for M_W, $\sin^2\theta_{\rm eff}$ in SM and MSSM:



$\Rightarrow$ Large improvement in experimental accuracy LEP/TevI $\rightarrow$ TevII/LHC/LC $\rightarrow$ GigaZ:

$\Rightarrow$ Very sensitive test of theory

Summary of Lecture 2:

- Standard Model cannot be the ultimate theory

  Hierarchy problem, ...
  $\Rightarrow$ strong motivation for low-energy SUSY

- Properties of SUSY theories:
  - superpartners for SM particles required
  - allow gauge coupling unification, radiative electroweak symmetry breaking
  - LSP: attractive candidate for cold dark matter in Universe
  - local SUSY includes gravity
  - SUSY parameter relations
    $\Rightarrow$ crucial test of SUSY

- Electroweak precision tests:

  MSSM yields equally complete description as SM

  Comparison SM/MSSM prediction for M_W:

  Preference for non-zero SUSY contribution