Lectures on Phenomenology

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Lecture 2:

- Electroweak precision physics
- Quantum corrections, renormalization and running couplings
- Observables, pseudo-observables and electroweak parameters
- Constraints on the SM Higgs from electroweak precision tests

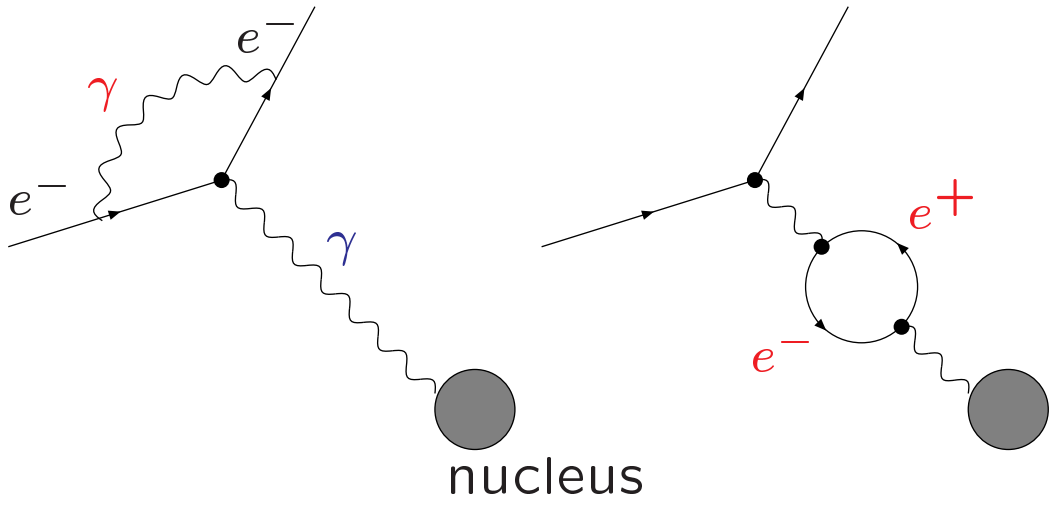
Electroweak precision physics

Precision measurements

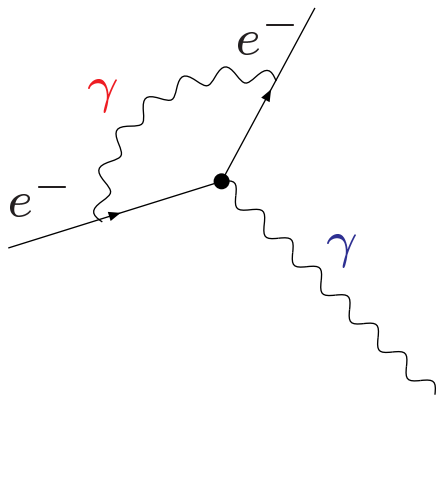
$\Rightarrow$ test quantum effects of the theory

Historical: QED

Lamb shift:



$g-2$:



$$a = \tfrac{1}{2}(g-2)$$

$$\begin{aligned} a_{\text{exp}} &= 1\,159\,652\,188(\pm 4) \times 10^{-12} \\ a_{\text{theo}} &= 1\,159\,652\,157(\pm 28) \times 10^{-12} \end{aligned}$$

Electroweak precision measurements:

- LEP1/SLC: e^+e^- annihilation at the Z-boson resonance
  LEP1: $\sim 4 \times 10^6$ events/experiment
- LEP2: $e^+e^- \to W^+W^-$
- Tevatron: $q\bar{q}' \to W \to l\bar{\nu},\ q\bar{q}'$
  $q\bar{q} \to t\bar{t},\ t \to W^+b \to \ldots$
- low-energy experiments

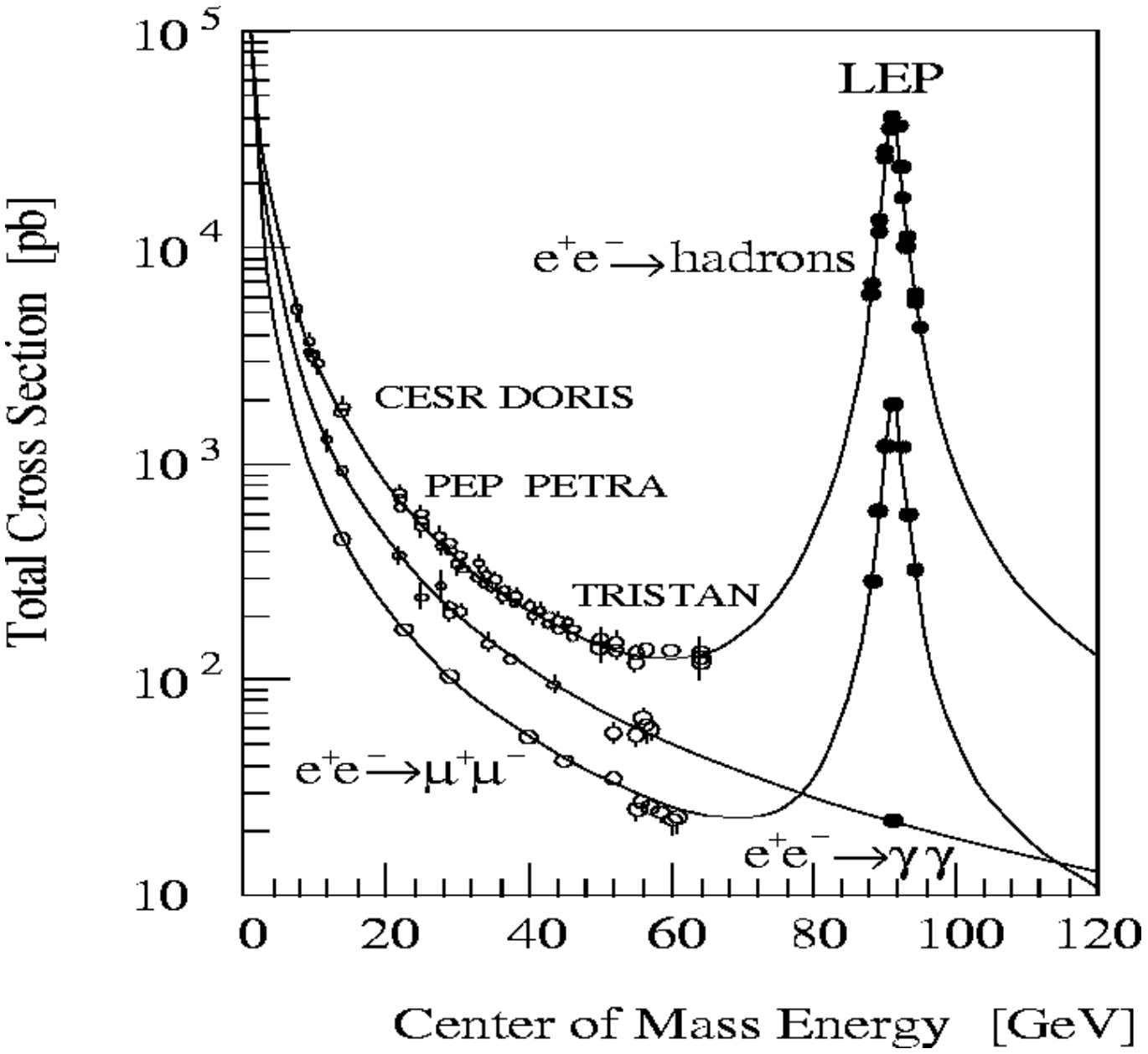
$M_{\mathrm{Z}}[\,\mathrm{GeV}]$	=	91.1875 ± 0.0021	0.002%
$\Gamma_{\mathrm{Z}}[\,\mathrm{GeV}]$	=	2.4952 ± 0.0023	0.09%
$\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$	=	0.23148 ± 0.00017	0.07%
$M_{\mathrm{W}}[\,\mathrm{GeV}]$	=	80.450 ± 0.034	0.04%
$m_{\mathrm{t}}[\,\mathrm{GeV}]$	=	174.3 ± 5.1	2.9%
$G_\mu[\,\mathrm{GeV}^{-2}]$	=	$1.16637(1)\,10^{-5}$	0.0009%

Necessary to take into account at LEP:
tides, water level of Lake Léman,
train schedule TGV Geneva – Paris

Quantum effects of the theory:

$\sim \mathcal{O}(1\%)$: loop corrections

$e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q}$:

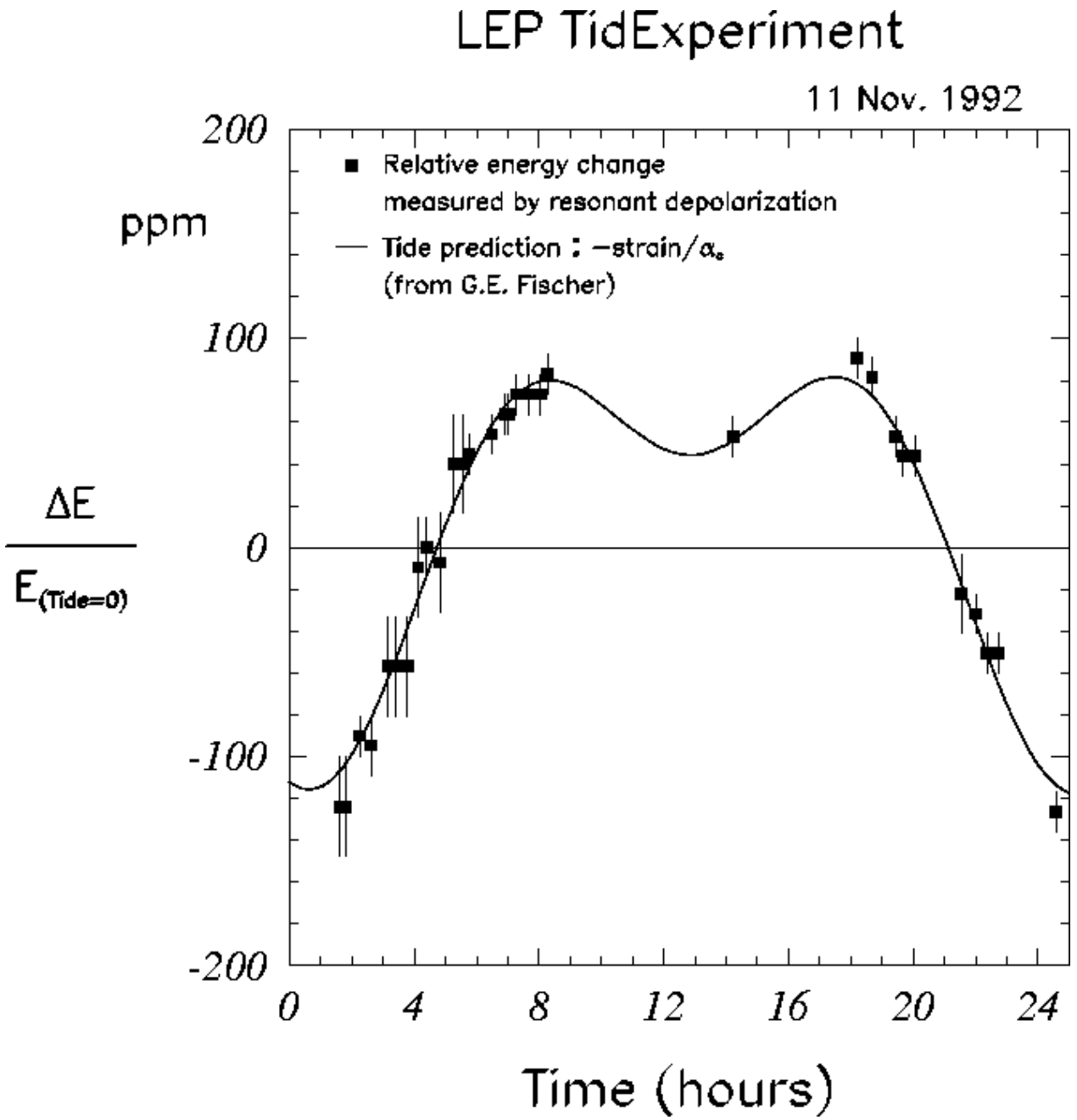


Z-propagator in lowest order:

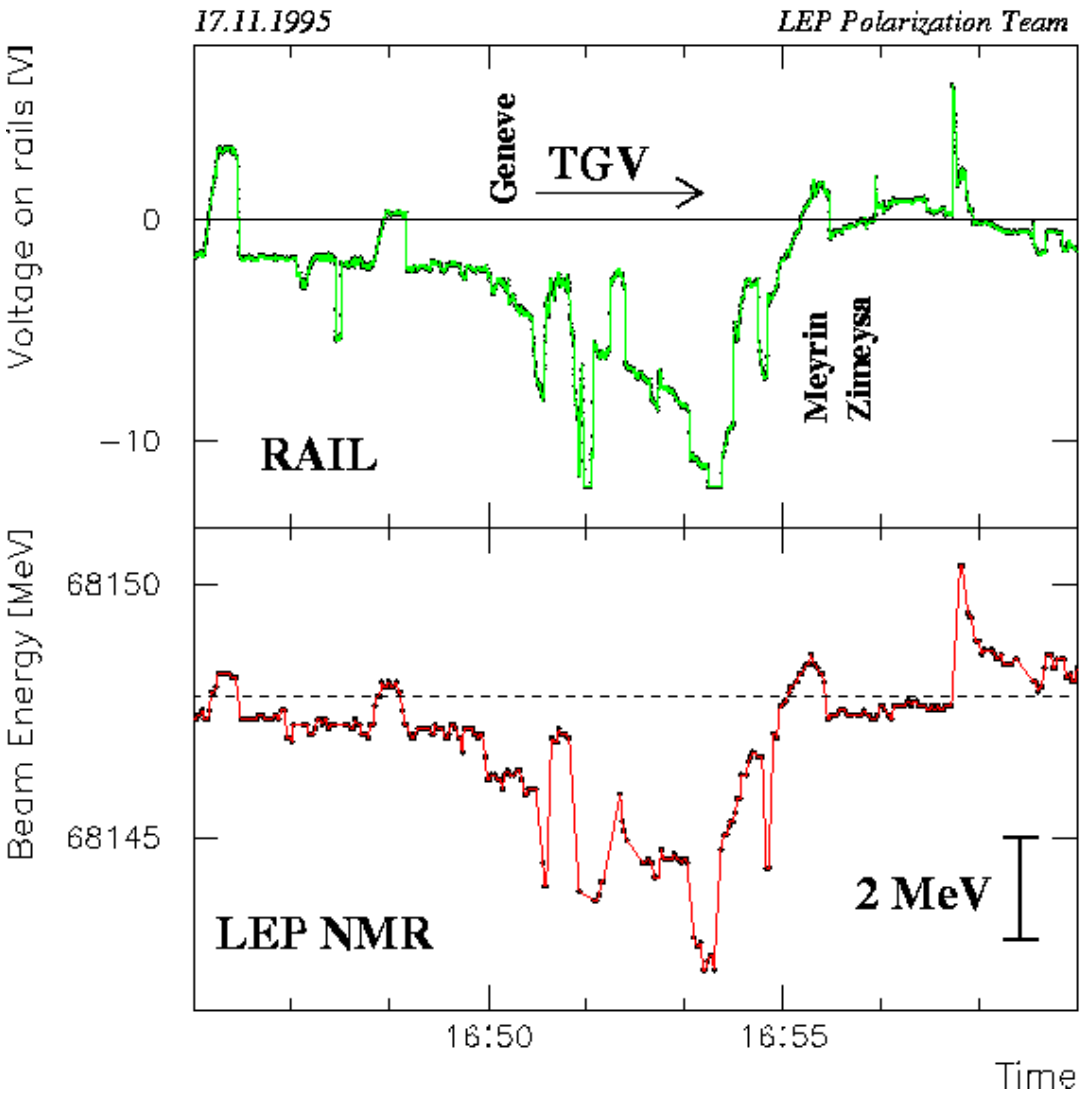
$$\Delta_{ZZ}(s) \sim \frac{1}{(s - M_Z^2)}$$

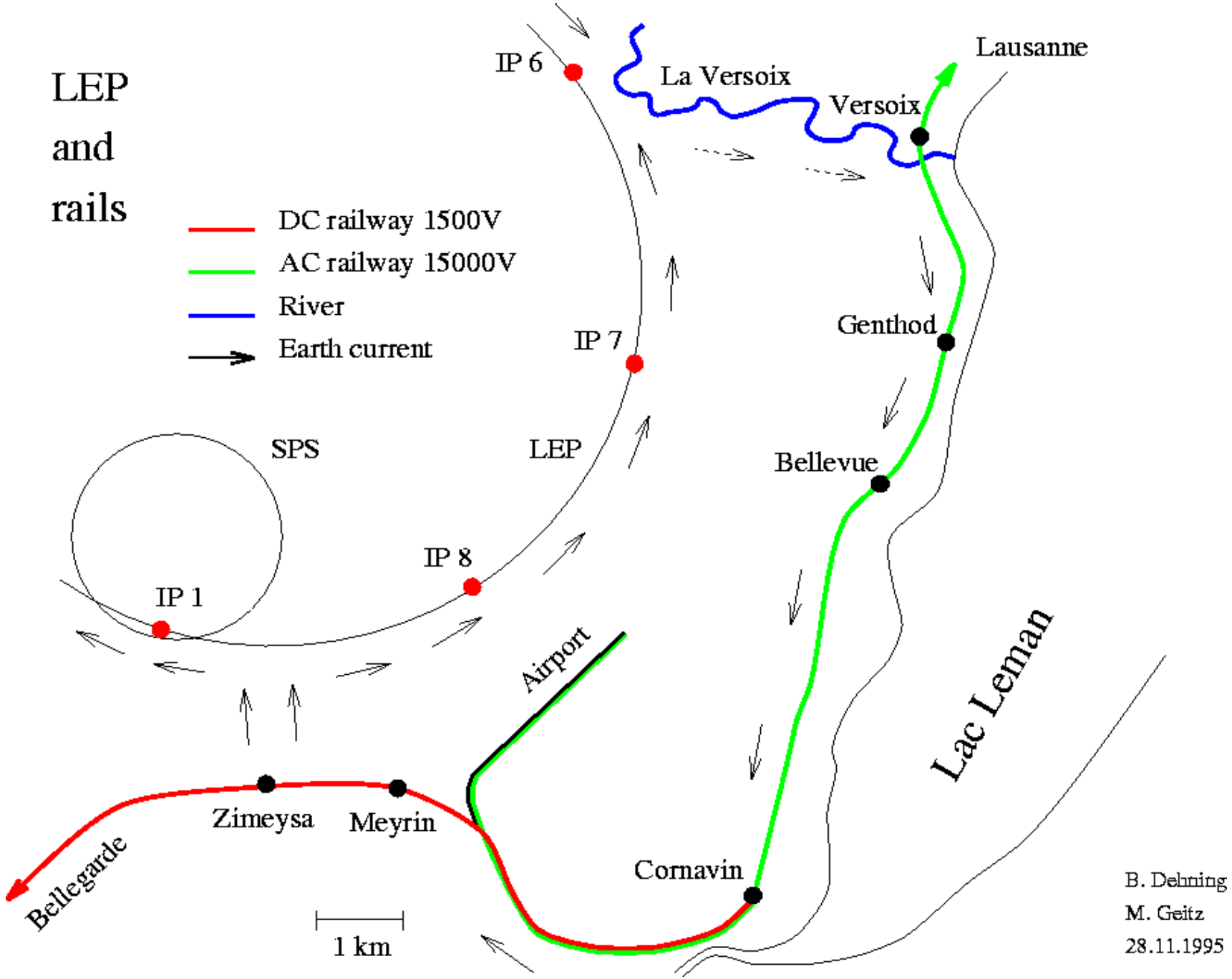
$\Rightarrow$ resonance enhancement of cross section for $s \approx M_Z^2$:

$$\sigma \sim \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$


LEP TidExperiment
11 Nov. 1992
Relative energy change
measured by resonant depolarization
Tide prediction : −strain/α_e
(from G.E. Fischer)
ppm
$\frac{\Delta E}{E_{(Tide=0)}}$
200
100
0
-100
-200
0
4
8
12
16
20
24
Time (hours)

Correlation between trains and LEP energy




LEP
and
rails
DC railway 1500V
AC railway 15000V
River
Earth current
IP 6
IP 7
IP 8
IP 1
SPS
LEP
La Versoix
Versoix
Lausanne
Genthod
Bellevue
Airport
Lac Leman
Zimeysa
Meyrin
Cornavin
Bellegarde
1 km
B. Dehning
M. Geitz
28.11.1995

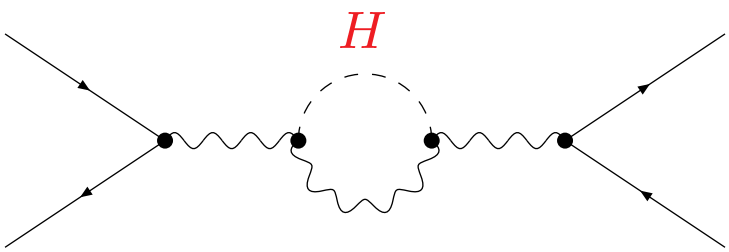
Comparison of ew precision data with theory:

EW precision data:	Theory:
$M_{\mathrm{Z}}, M_{\mathrm{W}}, \sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}, \ldots$	SM, MSSM, ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



All parameters of the model enter

$\Downarrow$

Improve indirect constraints on unknown parameters: M_{H}, $m_{\tilde{\mathrm{t}}}$, ...

effects of "new physics"?

Indirect determination of m_t from precision data:

$$m_t = 180.3^{+11.7}_{-9.2} \text{ GeV}$$

Direct measurement:

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$

Leading corrections to precision observables:

$$\sim m_t^2$$

$$\sim \ln M_H$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

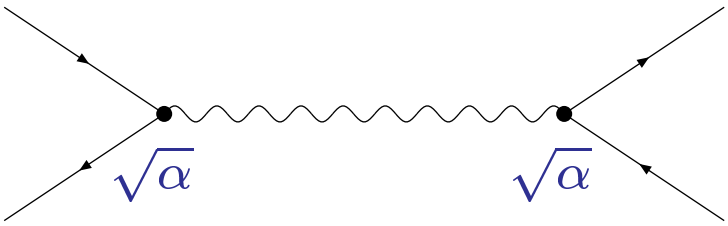
– unknown higher-order corrections

– exp. error of input parameters: m_t, $\Delta\alpha_{had}$, ...

Perturbative evaluation of QFTs based on Feynman diagrams:

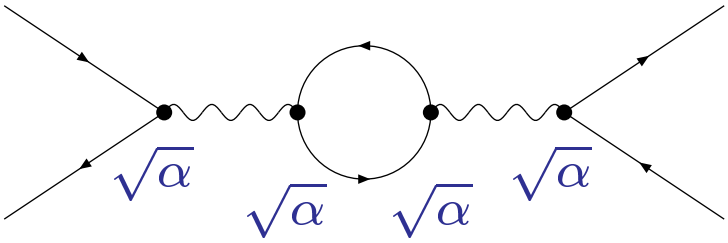
Example: $2 \to 2$ process

Lowest order (tree level):



$\alpha \approx \frac{1}{137}$

One-loop diagrams:



$+ \quad \ldots$

$\mathcal{O}(\alpha)$ relative to lowest order

...

Feynman rules of specific model (SM, MSSM, ...) for propagators and vertices

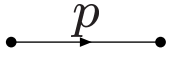
$\Rightarrow$ mathematical expression

Example: Feynman rules of QED

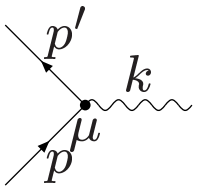
- photon propagator (Feynman gauge):

$$\Delta_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} =$$

- electron/positron propagator:

$$S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon} =$$ 

- electron–positron–photon vertex:



$$= (2\pi)^4 \delta^4(p - p' - k)(ie\gamma_\mu)$$

- ingoing and outgoing lines:

electron:	$= u(p,\sigma)$	$= \overline{u}(p,\sigma)$
Positron:	$= \overline{v}(p,\sigma)$	$= v(p,\sigma)$
Photon:	$= \epsilon_\mu(p,\lambda)$	$= \epsilon^*_\mu(p,\lambda)$

- integration over momenta of internal particles

  δ-distr. $\Rightarrow$ momentum conservation

  for loop diagrams $\Rightarrow \int \frac{d^4q}{(2\pi)^4}$ for each loop

- minus sign for fermion loops, combinat. factor

Dirac algebra, Dirac traces:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad \Rightarrow \mathrm{Tr}\,\{\gamma_\mu\gamma_\nu\} = 4g_{\mu\nu}$$

Tensor-integral decomposition:

$$\int \frac{d^Dq}{(2\pi)^D} \frac{q_\mu}{(q^2-m^2)\,[(q+k)^2-m^2]} = a\; k_\mu$$

$\Rightarrow$ scalar integrals:

$$\begin{aligned} A_0(m^2) &= \frac{1}{i\pi^2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{q^2-m^2} \\ B_0(p^2,m_1^2,m_2^2) &= \frac{1}{i\pi^2}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2-m_1^2)\left[(q+k)^2-m_2^2\right]} \\ &\dots \end{aligned}$$

Evaluation of standard scalar integrals:

One-loop: reducible to A_0, B_0, C_0, D_0:

$$A_0, B_0 : \log(\sqrt{\dots}); \quad C_0, D_0 : \log, \quad \mathrm{Li}_2(x) = \int_0^1 dt \frac{\ln t}{t-1/x}$$

Two-loop: in general not expressible in terms of polylogarithmic functions [*R. Scharf '91*]

$\Rightarrow$ numerical evaluation, asymptotic expansions, ...

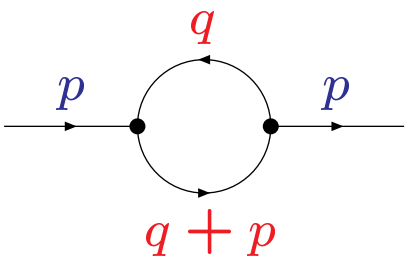
On-shell conditions for external legs

$\Rightarrow$ S-Matrix elements

$|\mathcal{M}^2|$, phase-space integration

$\Rightarrow$ cross sections, life times

Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2)\left[(q+k)^2 - m_2^2\right]}$$

$$q \to \infty : \quad \sim \int_0^\infty \frac{q^3 dq}{q^4} = \int_0^\infty \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q!

$\Rightarrow$ theory in this form not physically meaningful

$\Rightarrow$ further concept needed: renormalization

Renormalizable theories: infinities can consistently be absorbed into parameters of theory

Two step procedure:

Regularization:

theory modified such that expressions become mathematically meaningful

$\Rightarrow$ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \;\; \to \int_0^\Lambda d^4q; \quad \Lambda \to \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \;\; \to \int d^Dq, \quad D = 4 - \varepsilon; \quad D \to 4 \text{ at the end}$$

Renormalization:

original "bare" parameters replaced by renormalized parameters + counterterms

reparameterization:

$$\underbrace{g_0}_{\substack{\text{bare}\\ \text{parameter}}} = \underbrace{g}_{\substack{\text{renormalized}\\ \text{parameter}}} + \underbrace{\delta g}_{\text{counterterm}}$$

Renormalizable theory:
divergencies compensated by counterterms

Renormalization:

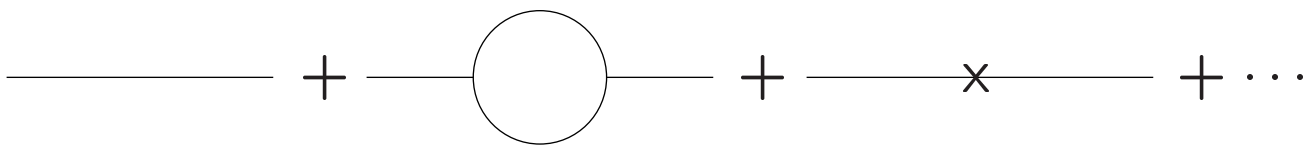
- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

Example:

mass renormalization, $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:



$p^2 - m^2$ $\quad$ $\Sigma(p^2)$ $\quad$ $-\delta m^2$

pole of the propagator:

$$p^2 - m^2 + \Sigma(p^2) - \delta m^2 = 0$$

If renormalization such that

$$\delta m^2 = \Sigma(p^2)\Big|_{p^2=m^2} \quad \text{“on-shell renormalization”}$$

$\Rightarrow$ pole of propagator for $p^2 = m^2$

$\Rightarrow$ m corresponds to physical mass

Counterterm contains divergence:

expansion in $\varepsilon \equiv 4 - D$, 1-loop:

$$\delta m^2 = \underbrace{a\frac{1}{\varepsilon}}_{\text{divergent for } D \to 4} + \underbrace{b\varepsilon^0}_{\text{finite}} + \underbrace{c\varepsilon}_{\to 0 \text{ for } D \to 4} + \ldots$$

Other renormalization prescription:

$$\delta m^2 = a\frac{1}{\varepsilon} \quad \text{“minimal subtraction” (MS)}$$

Different renormalization prescriptions $\Leftrightarrow$ different parameterizations of the result

Physical result must be independent of renormalization prescription, e.g.

$$\Sigma^{\text{ren}}(p^2 = \underbrace{\mu^2}) \equiv \Sigma(p^2 = \mu^2) - \delta m^2 = 0$$

renormalization scale

$\Rightarrow$ for physical observable σ:

$$\mu \frac{d}{d\mu} \sigma \left(p_i, g(\mu), m(\mu), \mu\right) = 0$$

"renormalization group equation"

$\Rightarrow$ explicit dependence of σ on μ compensated by μ-dependence of $g(\mu)$, $m(\mu)$

$$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right), \quad \beta = \text{const.}$$

$g(\mu)$: "running coupling"

$m(\mu)$: "running mass"

$\Rightarrow$ effective coupling $\alpha(Q)$ varies with scale Q

QED:

$$\alpha(Q) = \frac{\alpha(Q_0)}{1 + 4\pi\beta\,\alpha(Q_0)\,\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$\beta < 0$ in QED:

$\beta \equiv b/(16\pi^2)$, $b = -4/3$ (1-loop)

$\Rightarrow$ $\alpha(Q)$ increases for increasing Q $(Q > Q_0)$

$\alpha(m_e) \approx 1/137$, $\alpha(M_Z) \approx 1/128$

scale Λ_{QED} where QED coupling becomes strong:

$$\alpha(Q) = \frac{1}{4\pi\beta\,\ln\left(\frac{Q^2}{\Lambda_{QED}^2}\right)}$$

$\Lambda_{QED} \approx 10^{274}$ GeV $\gg M_P (\approx 10^{19}$ GeV)

$\Rightarrow$ QED is weakly coupled up to the Planck-scale

QCD:

$$b^{\mathrm{QCD}} = \frac{1}{3}(11N_C - 2n_f) > 0 \text{ for } n_f = 6$$

$\Rightarrow \alpha_s(Q) \to 0$ for $Q \to \infty$: "asymptotic freedom"

For large Q: perturbation theory applicable, description in terms of quarks and gluons useful

$\alpha_s(Q)$ becomes strong for small Q:

quarks and gluons bound together to form colorless hadrons

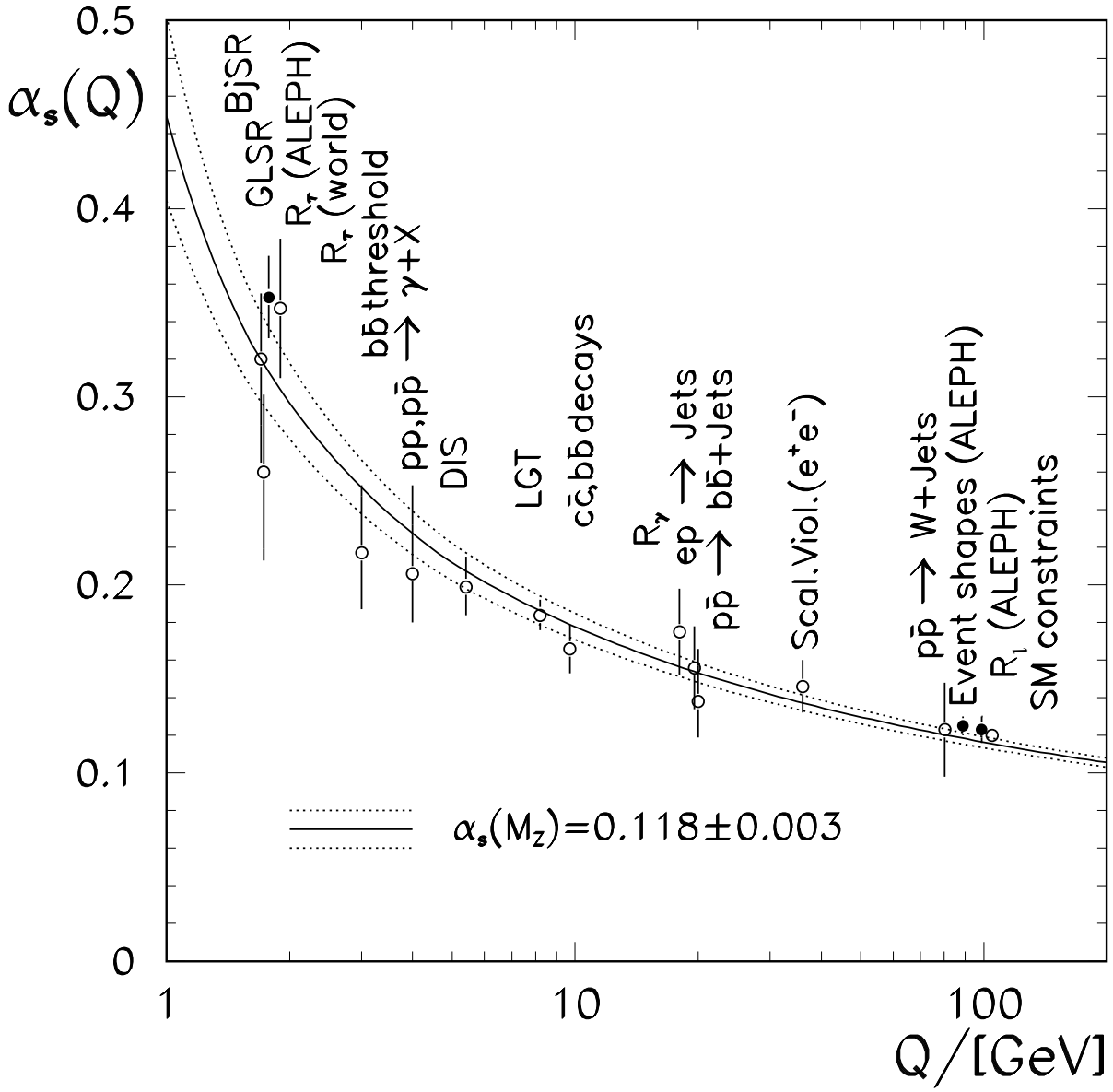
$\Rightarrow$ "confinement"

$$\alpha_s(Q) = \frac{12\pi}{(11N_C - 2n_f)\ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}$$

For scales of $\mathcal{O}(\Lambda_{\mathrm{QCD}}) \approx 100$ MeV:

$\alpha_s(Q)$ so large that perturbation theory not applicable

Running of $\alpha_s(Q)$:



$\Rightarrow$ Very good agreement with QCD prediction

Observables, pseudo-observables and electroweak parameters

Precision measurements of cross sections, BRs at next generation of colliders

⇒ potential for parameter determination with accuracy 10^{-2}–10^{-3}

⇒ careful treatment and precise definition needed

Note: couplings, masses, mixing angles, etc. are not (directly) physical observables

⇒ "pseudo-observables"

Actual observables: σ, BRs, asymmetries

Need deconvolution procedure (unfolding) in order to determine masses, partial widths, etc. from measured cross sections

⇒ pseudo-observables are not strictly model-independent

What is a suitable definition of model parameters?

need some compromise between

– simple interpretation within given model

– model independence

In principle possible (extreme case):

fit model with n free parameters to n observables

$\Rightarrow M_{\mathrm{W}}^{\mathrm{exp,SM}} = M_{\mathrm{W}}^{\mathrm{exp,SM}}(m_{\mathrm{t}}, M_{\mathrm{H}}, \ldots)$

However: want to define particle masses such that they are (approximately) model-independent, easy to relate to measurable quantities

– reconstruction of decay products

– from cross section near threshold

...

Example: what is M_{W}?

Theorist point of view:

Mass determined by pole of propagator

Which pole?

– real part of complex pole, $\mathcal{M}^2 = \overline{M}^2 - i\overline{M}\,\overline{\Gamma}$, or

– pole of real part, $\widetilde{M}^2$

From 2-loop order on:

real part of complex pole $\neq$ pole of real part

$$\delta\overline{M}^2_{(2)} = \delta\widetilde{M}^2_{(2)} + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\}\,\mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}_{\text{gauge-parameter dependent!}}$$

S-matrix theory

[*A. Sirlin '91*], [*R.G. Stuart '91*], [*H. Veltman '94*]

[*P. Gambino, P.A. Grassi '99*]

Explicit 2-loop calculation in general R_ξ-gauge:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00*]

$\Rightarrow$ Complex pole gauge-invariant

How does one measure the pole of a propagator?

Experimentalist point of view:

In practice: comparison data – Monte Carlo

$\Rightarrow$ "M_{W} is what is implemented in Monte Carlo program"

Comparison theory prediction – experimental result:

Definition via real part of complex pole:

$$\Rightarrow \sigma(s) \sim \frac{s}{(s - \overline{M}^2)^2 + \overline{M}^2\overline{\Gamma}^2}$$

$\Rightarrow$ Breit–Wigner shape with constant width

Experimental determinations at LEP/Tevatron use:

$$\sigma(s) \sim \frac{s}{(s - M^2)^2 + s^2\Gamma^2/M^2}$$

$\Rightarrow$ Breit–Wigner shape with running width

Relation:

$$M^2 = \overline{M}^2 + \overline{\Gamma}^2 \quad \Rightarrow \underbrace{\Delta M_{\mathrm{W}} \approx 27\,\mathrm{MeV}}_{\text{formally a 2-loop effect!}}$$

$\Rightarrow$ For theoretical prediction of M_{W}:
need prediction for $\overline{M}_{\mathrm{W}}$ and $\overline{\Gamma}_{\mathrm{W}}$

$\Rightarrow$ (Slightly) larger theoretical uncertainty for M_{W} than for $\overline{M}_{\mathrm{W}}$

Quark masses: on-shell (pole) masses affected by large QCD uncertainties

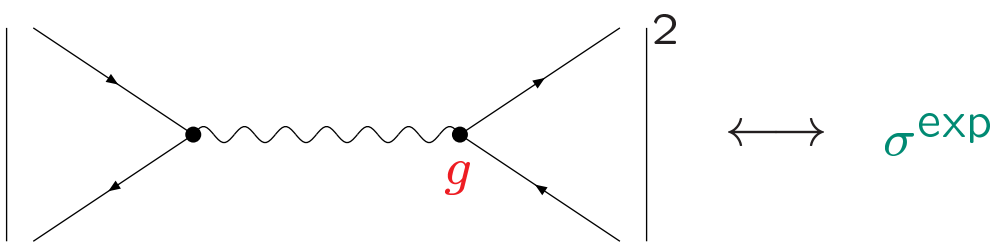
$\Rightarrow$ $\overline{\mathrm{MS}}$ masses usually preferable

Particle masses: relatively small model dependence

Couplings, mixing angles, etc.:
relatively large model dependence

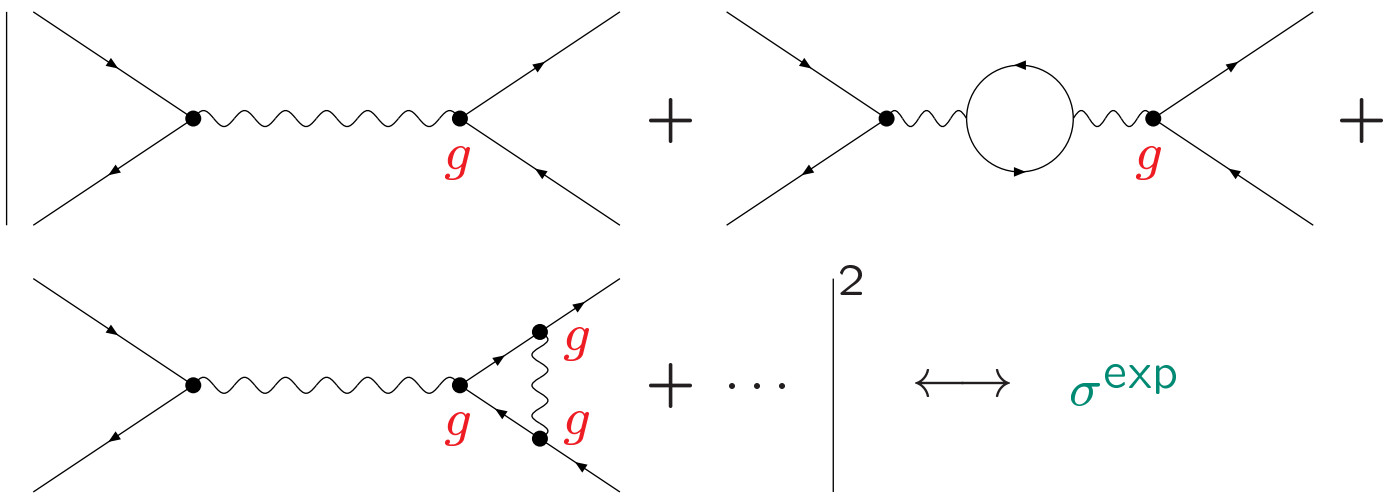
E.g.: determination of coupling g from cross section measurement

accuracy of $\mathcal{O}(10\%)$:



accuracy $\lesssim 1\%$:

need instead



$\Rightarrow$ need precise and unambigous definition of parameters

used by both experimentalists and theorists

Constraints on the Standard Model Higgs from electroweak precision tests

Theoretical prediction for M_{W} in the SM:

Definition of Fermi constant G_μ via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_{\mathrm{e}}^2}{m_\mu^2}\right)\left(1+\frac{3}{5}\frac{m_\mu^2}{M_{\mathrm{W}}^2}\right)(1+\Delta q)$$

Δq: QED corrections in Fermi Model, included in definition

[*R. Behrends, R. Finkelstein, A. Sirlin '56*]

[*T. van Ritbergen, R. Stuart '99*]

Comparison with SM prediction for muon decay:

$$\Rightarrow \quad M_{\mathrm{W}}^2\left(1-\frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1+\Delta r)\,,$$

$\Updownarrow$

loop corrections

$\Rightarrow$ Theoretical prediction for M_{W} in terms of M_{Z}, α, G_μ

SM prediction for Δr, one-loop:

[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

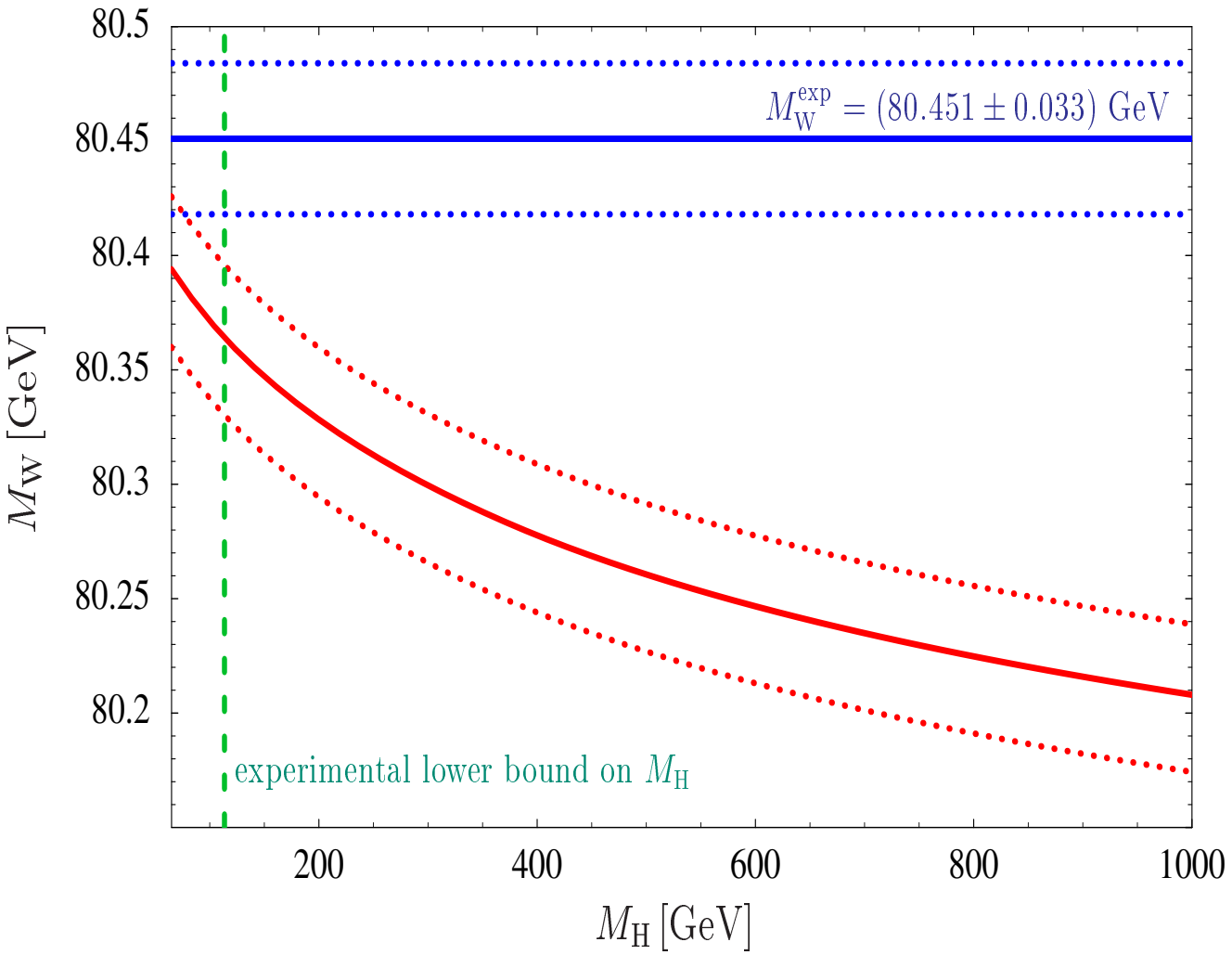
$\Delta\alpha$: running of α, $\alpha(M_Z^2) = \alpha(0)/(1-\Delta\alpha)$

$\Delta\rho$: leading m_t contributions to precision observables enter quadratically

$\Delta r_{\text{rem}}(M_H)$: leading M_H dependence is logarithmic

SM prediction for M_W vs. experimental result:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]



$\Rightarrow$ Light Higgs preferred

$M_W^{exp} = 80.451 \pm 0.033$ GeV, $m_t^{exp} = 174.3 \pm 5.1$ GeV

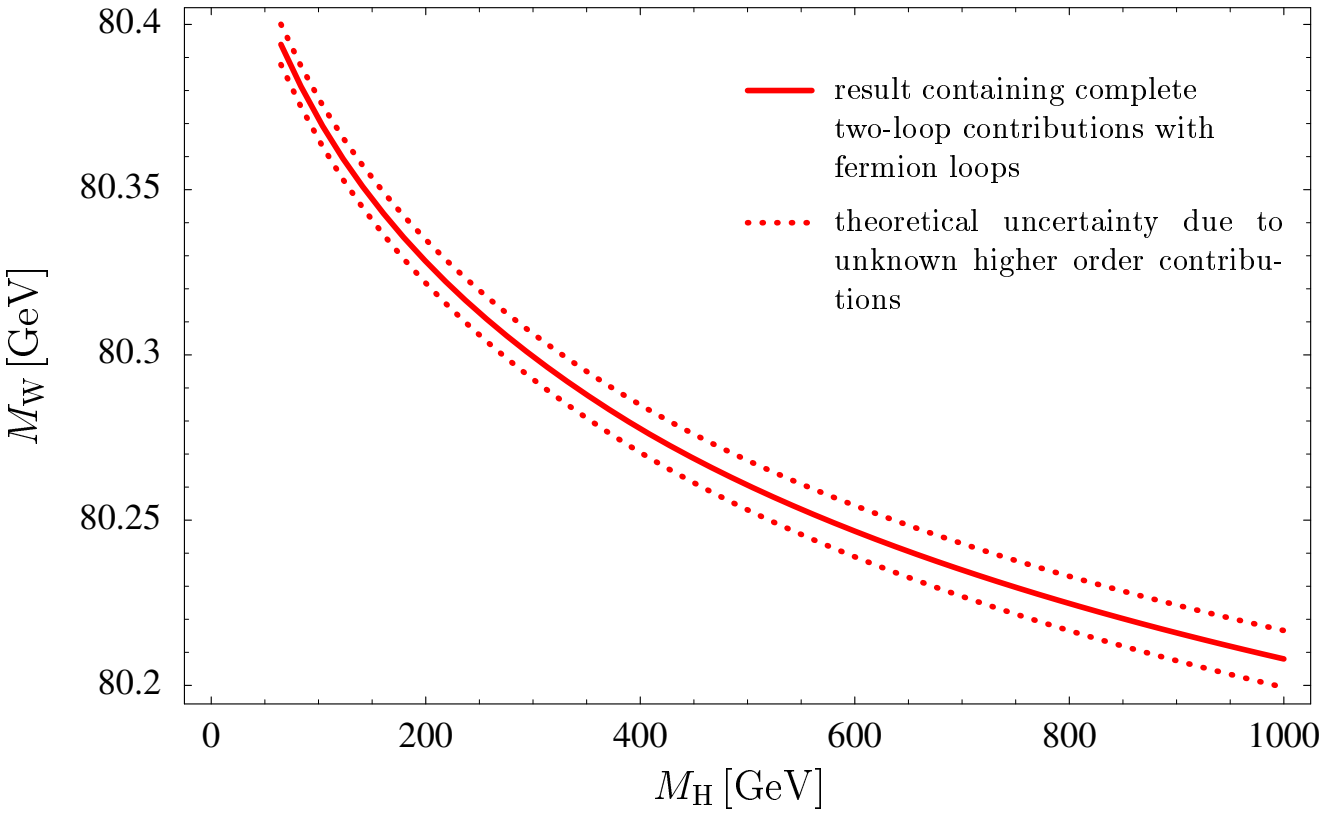
$M_H > 114.1$ GeV, 95% C.L.

$\Rightarrow$ 1σ regions do not overlap

Uncertainty from experimental error of input parameters: $\delta m_{\mathrm{t}} = \pm 5.1$ GeV $\Rightarrow \delta M_{\mathrm{W}} \approx \pm 31$ MeV

Estimate of theoretical uncertainty from unknown higher-order corrections:

- Purely bosonic $\mathcal{O}(\alpha^2)$ electroweak corrections
- Higher-order electroweak corrections: $\mathcal{O}(\alpha^3)$
- Higher-order QCD corrections: $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^3)$, $\mathcal{O}(\alpha^2\alpha_{\mathrm{s}})$

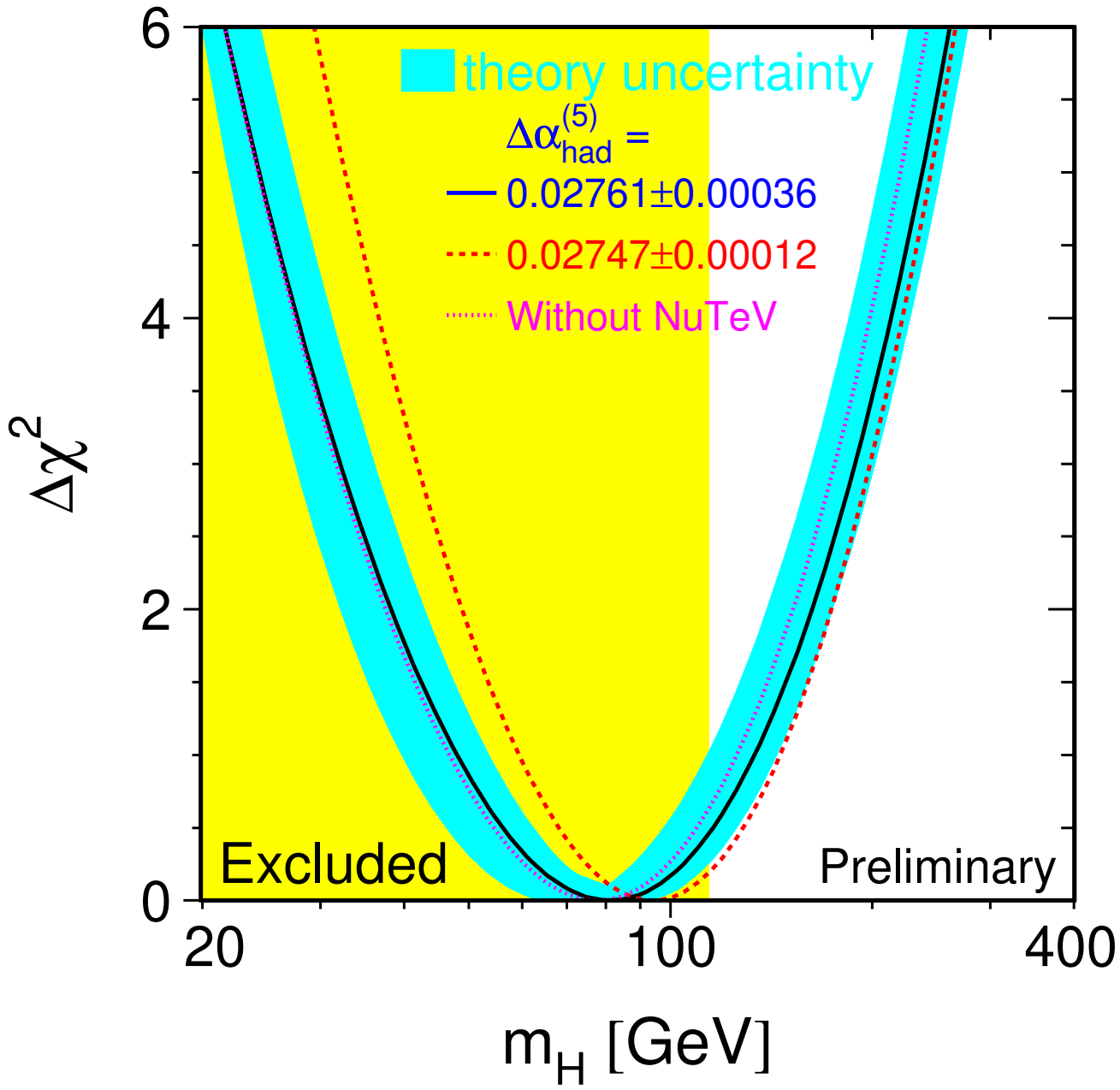


For light Higgs masses:

$$\Delta M_{\mathrm{W}}^{\mathrm{theo}} \approx \pm 6\,\mathrm{MeV}$$

Global fit to all data in the SM:

[*LEPEWWG '02*]



$\Rightarrow M_{\mathrm{H}} = 81^{+52}_{-33}$ GeV

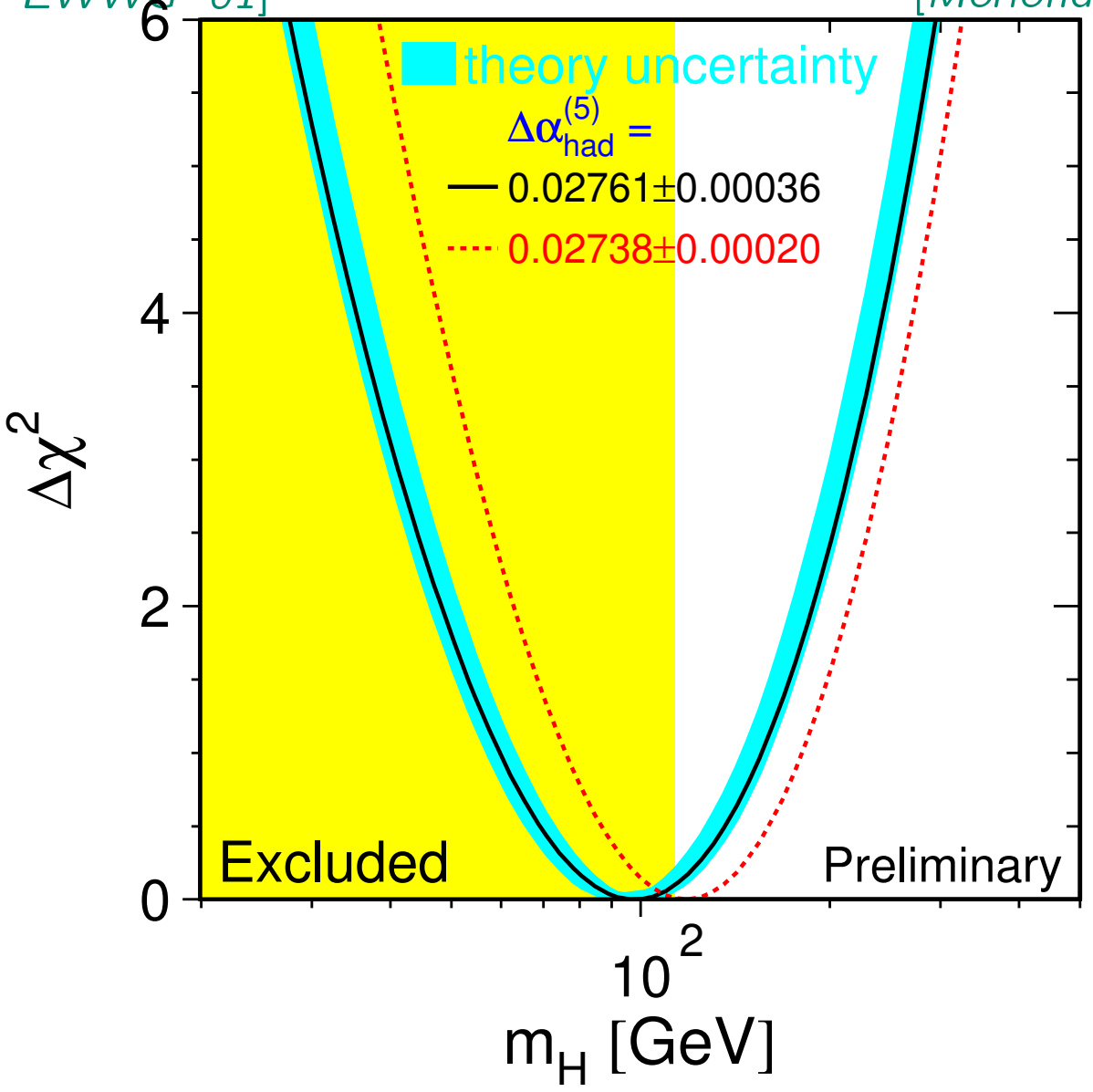
$M_{\mathrm{H}} < 193$ GeV, 95% C.L.

$\Rightarrow$ Preference for light Higgs

Uncertainties from unknown higher-order corrections:

Comparison: global fit results one year ago

[*LEPEWWG '01*] [*Moriond '01*]



$\Rightarrow\ M_{\mathrm{H}} = 98^{+58}_{-38}$ GeV, $M_{\mathrm{H}} < 212$ GeV, 95% C.L.

"Blue band" has widened w.r.t. one year ago

Old estimate (up to Moriond '01):

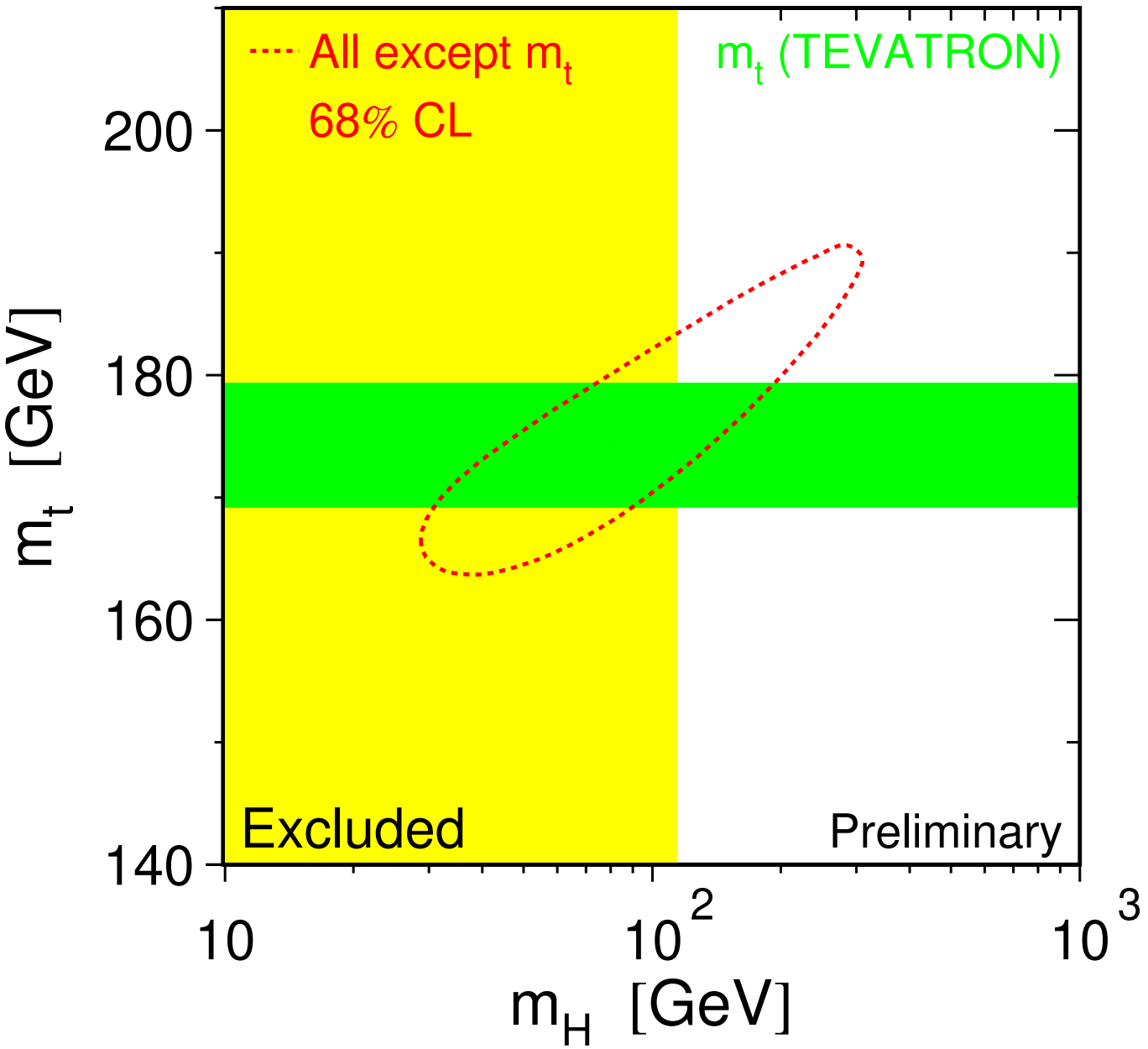
$\Delta \sin^2\theta^{\mathrm{theo}}_{\mathrm{eff}} \approx \pm 3 \times 10^{-5}, \quad \Delta M^{\mathrm{theo}}_{\mathrm{W}} \approx \pm 3$ MeV

New estimate of higher-order uncertainties:

$\Delta \sin^2\theta^{\mathrm{theo}}_{\mathrm{eff}} \approx \pm 7 \times 10^{-5}, \quad \Delta M^{\mathrm{theo}}_{\mathrm{W}} \approx \pm 6$ MeV

Interpretation of fit result:

Strong correlation between M_{H} and m_{t} in the fit:
[*LEPEWWG '02*]



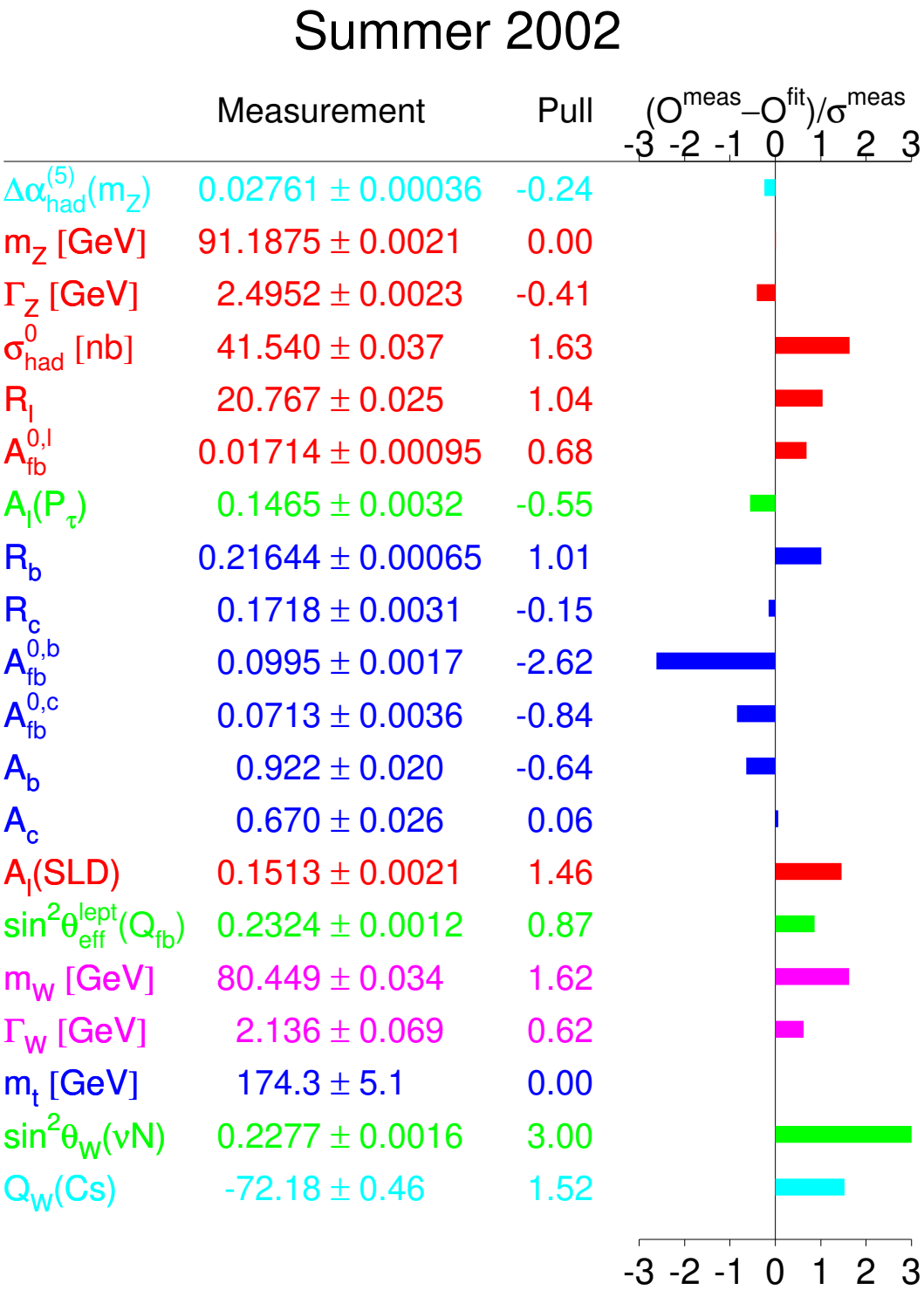
$\Rightarrow$ Upward shift of m_{t} by 1σ would shift M_{H} bound by 35%

Basic assumption in global fit:

SM provides correct description of experimental data

Comparison of SM prediction with the data:

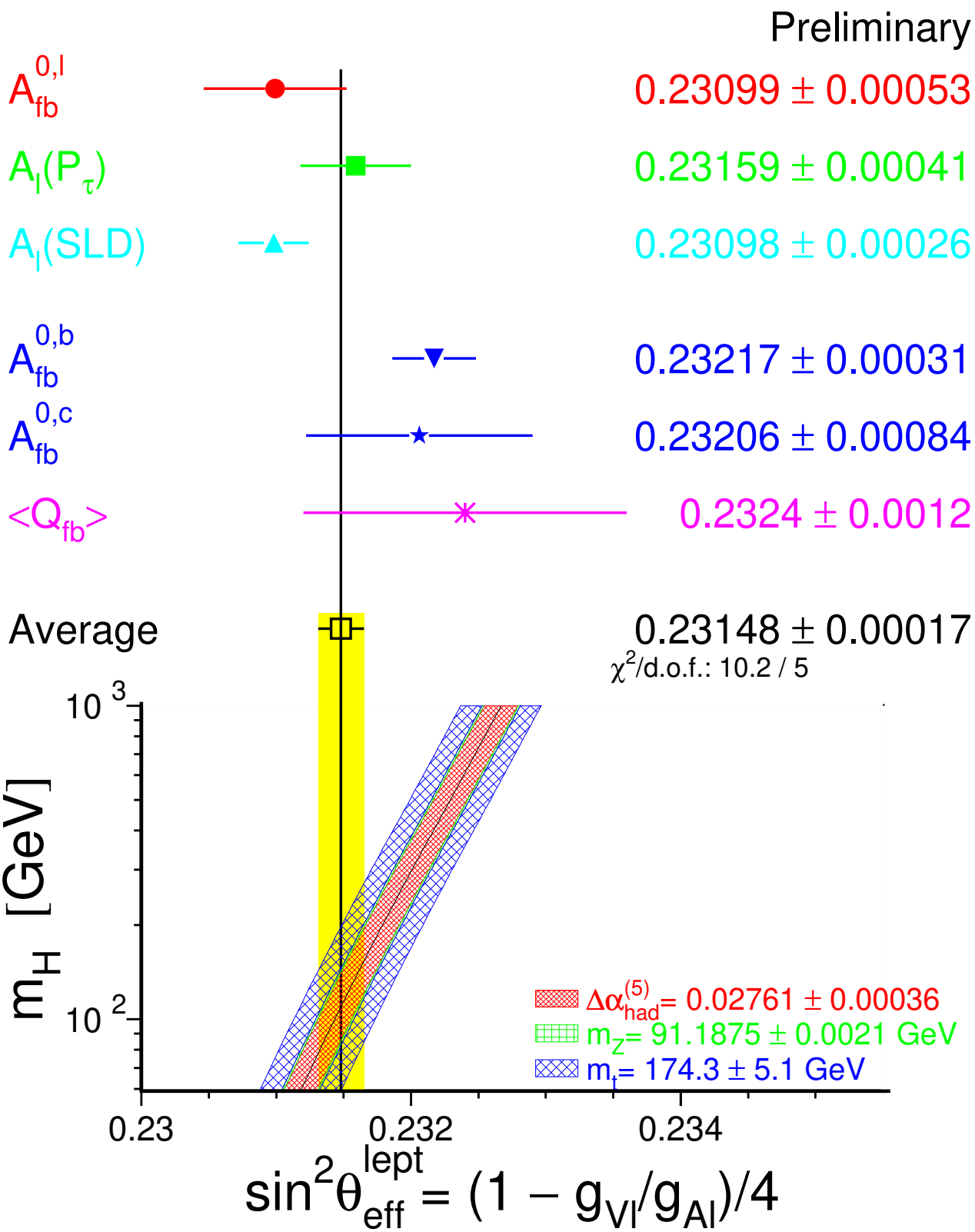
[*LEPEWWG '02*]



Overall fit probability (quality of the fit): 1.3%

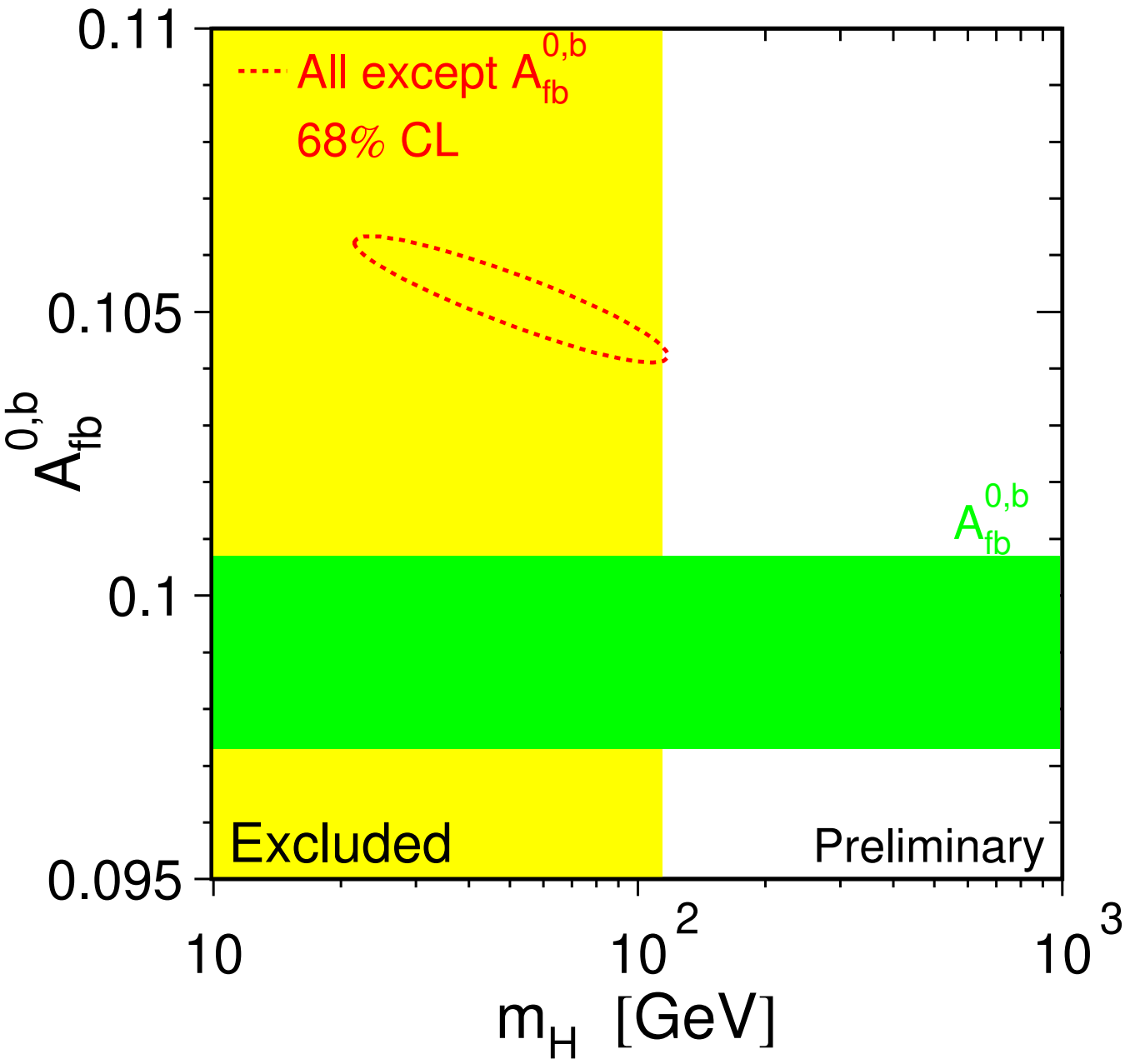
The effective weak mixing angle $\sin^2\theta_{\text{eff}}$:

[*LEPEWWG '02*]



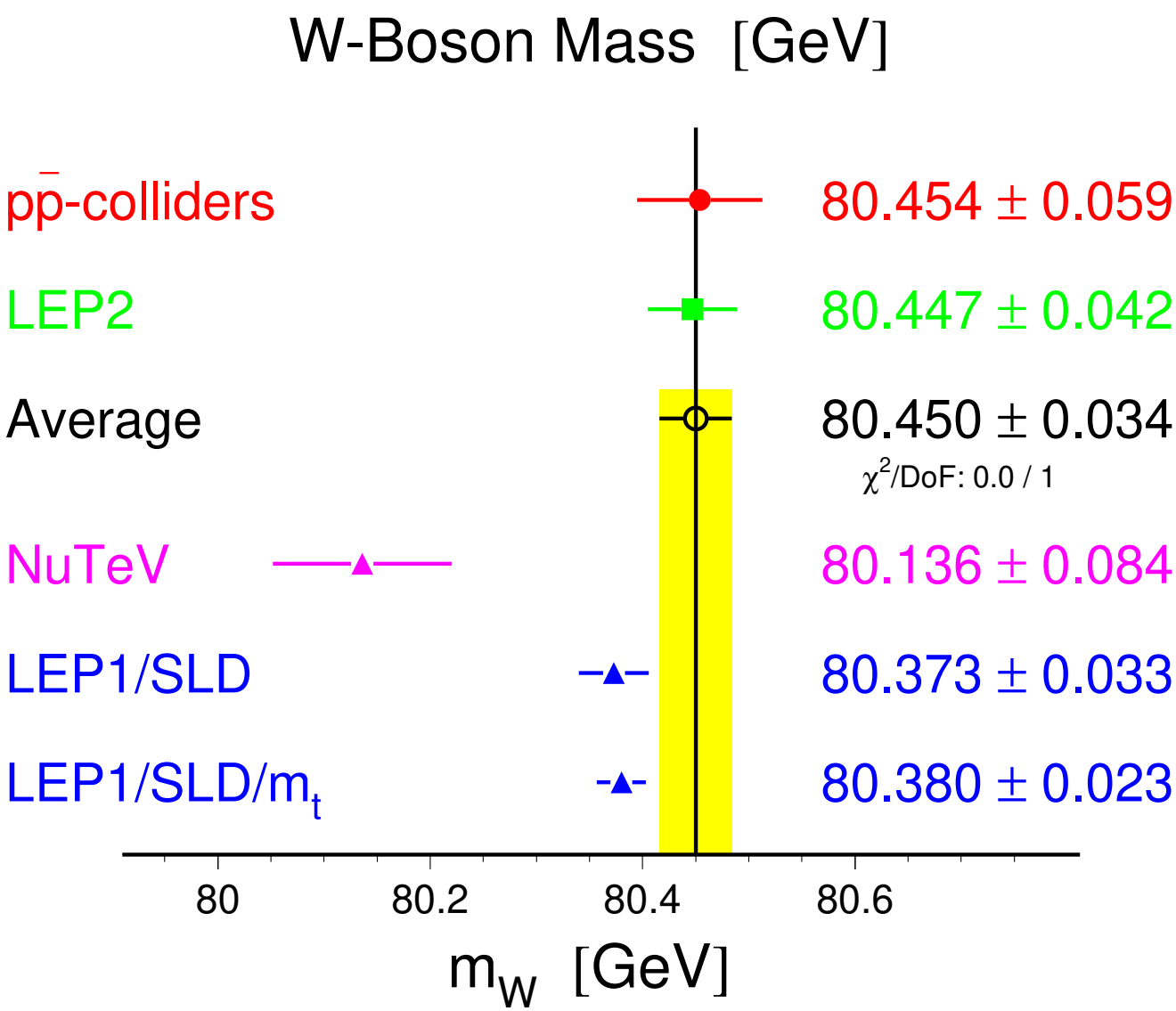
Forward–backward asymmetry of b quarks:

[*LEPEWWG '01*]



$\Rightarrow$ the "$A_{\mathrm{fb}}^{0,b}$ crisis"

Direct measurement of M_{W} at LEP2, Tevatron vs. indirect determinations from LEP1, SLD, NuTeV: [*LEPEWWG '02*]



No convincing new physics model for "explaining" deviations from SM

Note, however:

if NuTeV result interpreted as deviation in $Z\nu\bar{\nu}$ coupling:

(ν neutral current rate)/prediction:

0.995 ± 0.003 LEP1 lineshape (invisible Z width)

0.988 ± 0.004 NuTeV

Need some patience until experimental clarification:

A_{LR}, A_{FB}: LC with GigaZ option?

ν–nucleon scatt.: ν factory?

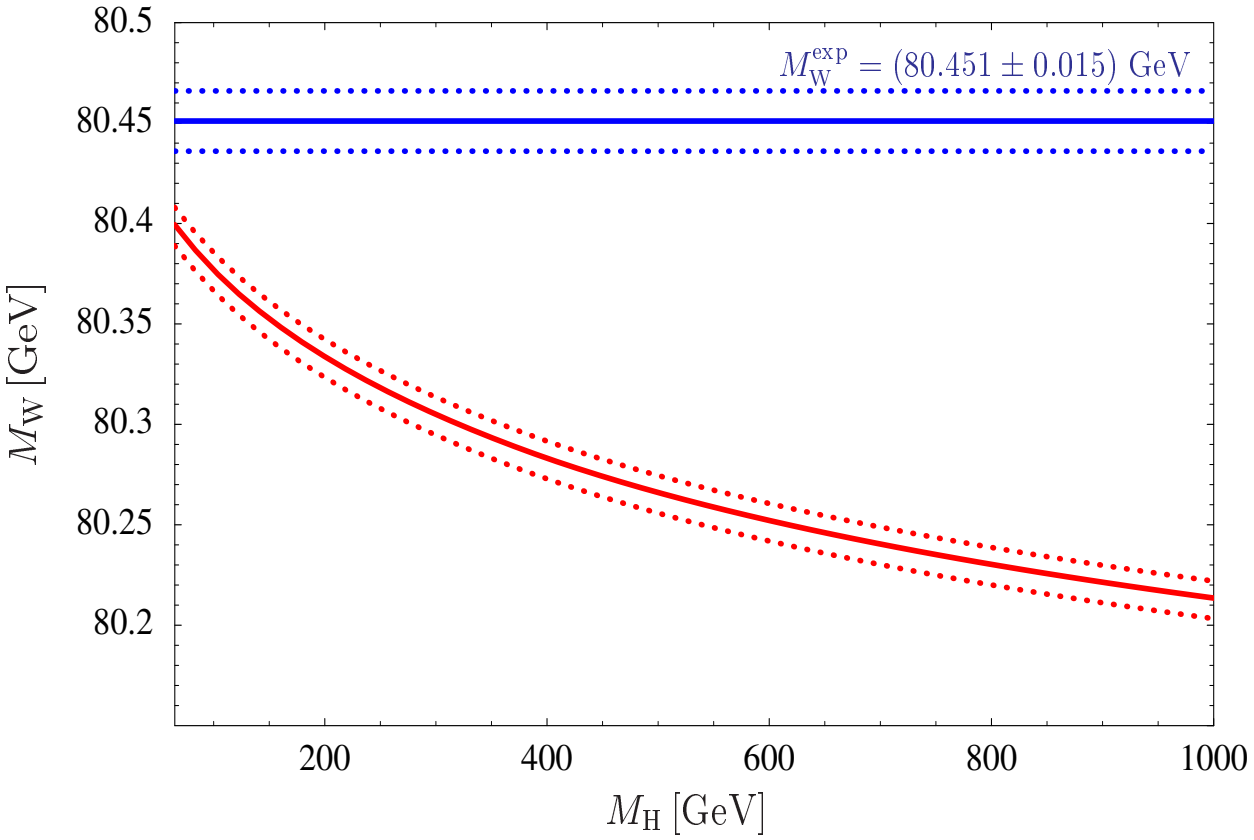
If hadronic data on A_{FB} and NuTeV data on ν–nucleon scattering were removed from fit:

much better fit quality, probability $\approx 70\%$

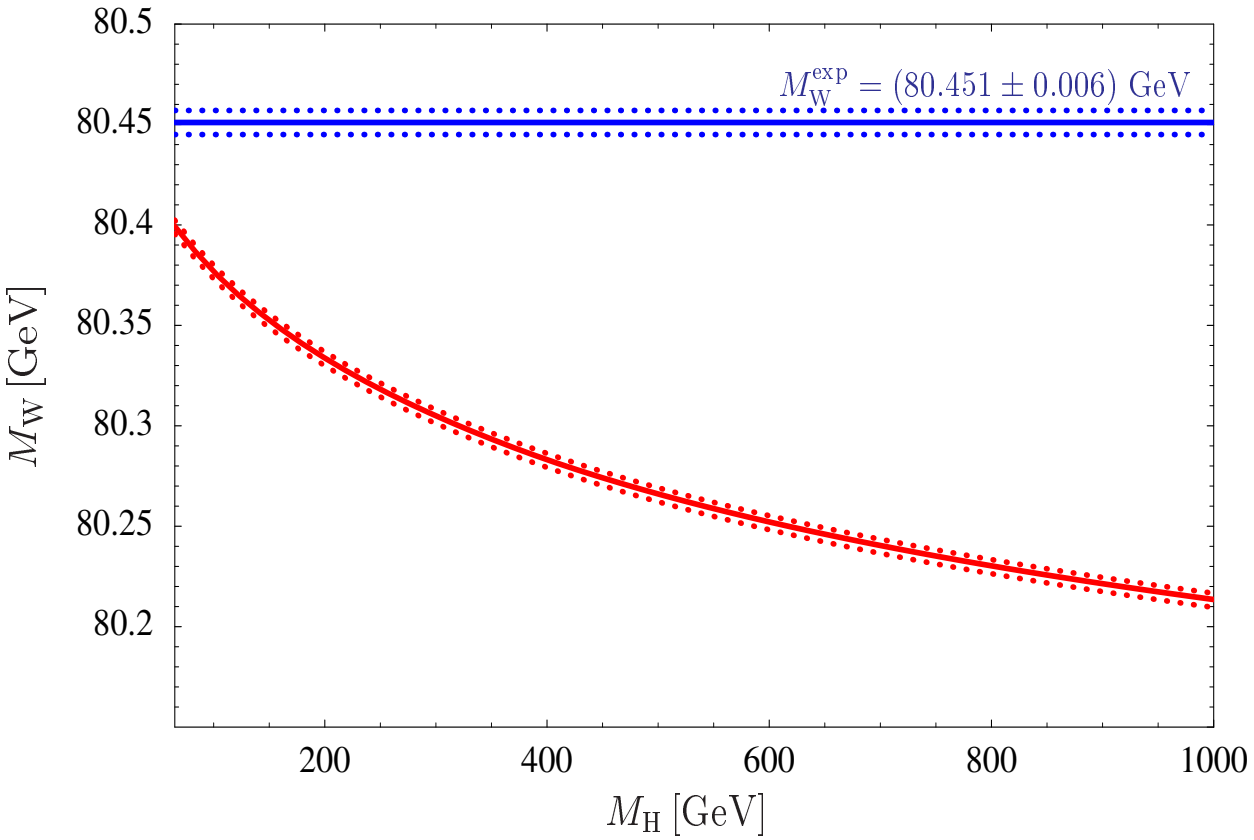
stronger preference for light Higgs:

$M_{\mathrm{H}} \lesssim 117$ GeV, 95% C.L.

Prospects for Tevatron Run II, LHC:

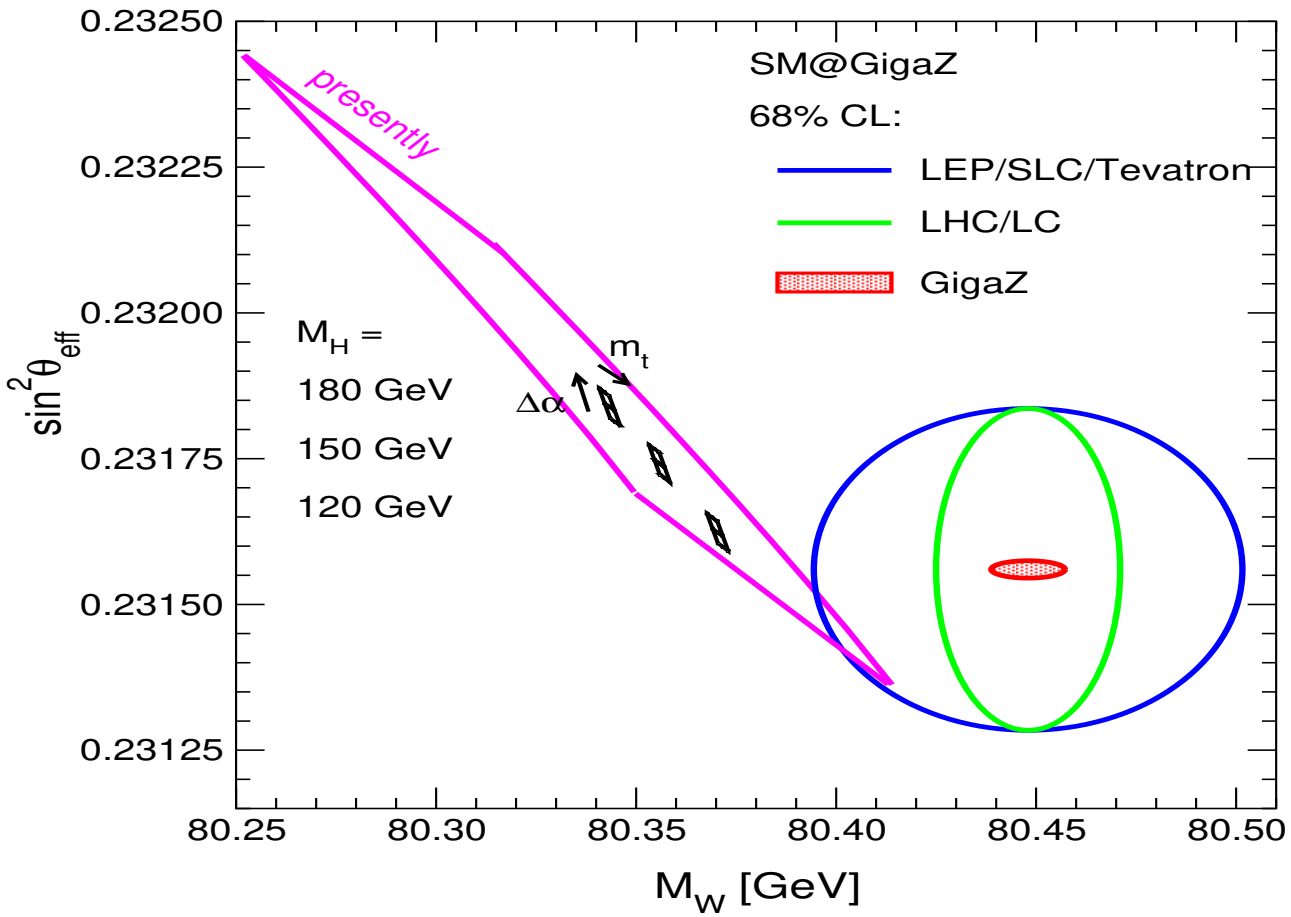


LC with high luminosity in low-energy mode: GigaZ

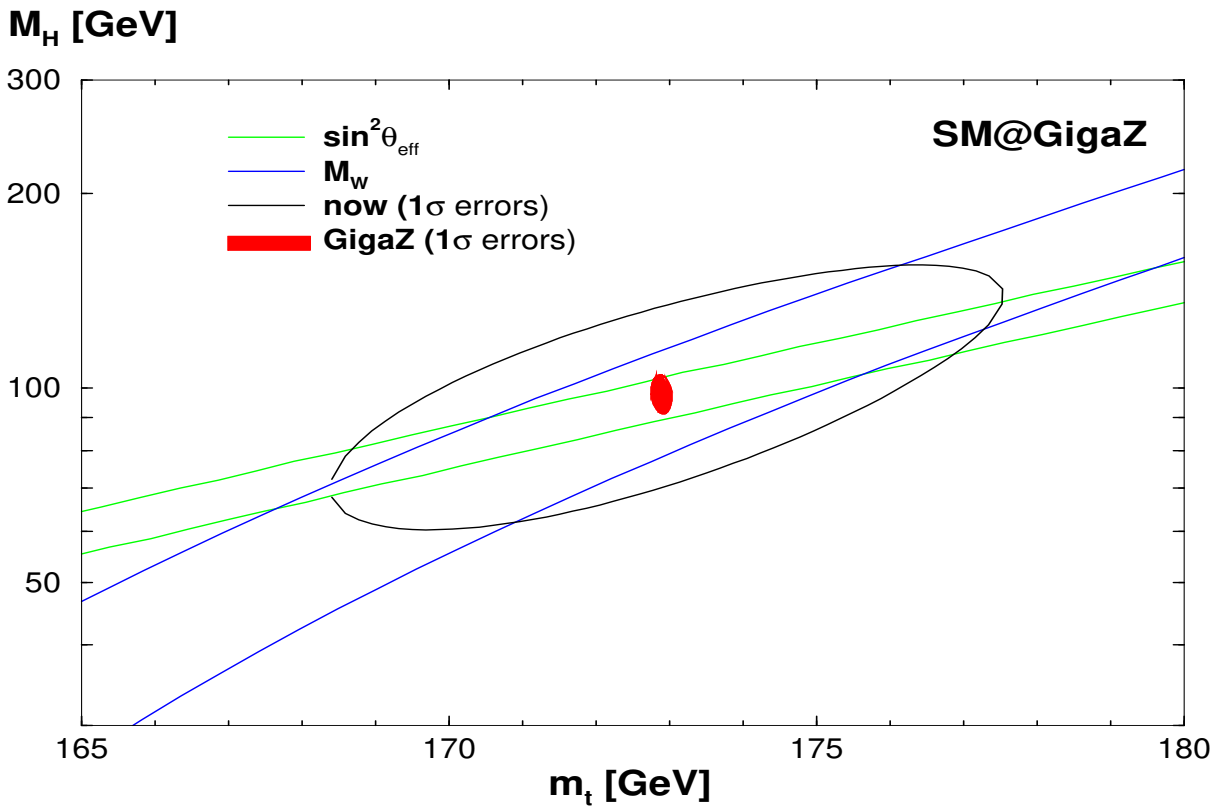


SM prediction for M_W and $\sin^2\theta_{\rm eff}$ vs. prospective accuracies at LEP2/Tevatron, Tev./LHC/LC, GigaZ:

[J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00]



Indirect determination of m_t, M_H at GigaZ:



Summary of Lecture 2:

- Electroweak precision physics

  $\Rightarrow$ Sensitivity to quantum effects of the theory

  $\Rightarrow$ test consistency of the model

  constraints on unknown parameters

- Precision tests of the SM

  $\Rightarrow$ light Higgs preferred, $M_{\mathrm{H}} \lesssim 200$ GeV

  preference for light Higgs is not an artefact of observables deviating by $\approx 3\sigma$ from SM prediction

- Prospects for next generation of colliders:

  improved accuracy of precision observables
  M_{W}, $\sin^2\theta_{\mathrm{eff}}$, m_{h}, ... and
  input parameters m_{t}, ...

  $\Rightarrow$ Highly sensitive test of electroweak theory