Lectures on Phenomenology

Problem sheet 4

1) The potential in the neutral Higgs sector of the MSSM is given by

$$
\begin{aligned}
V &= \left(|\mu|^2 + m_{H_u}^2\right)\left|h_u^0\right|^2 + \left(|\mu|^2 + m_{H_d}^2\right)\left|h_d^0\right|^2 \\
&\quad - \left(b\,h_u^0 h_d^0 + \text{h.c.}\right) + \frac{g^2 + g'^2}{8}\left(\left|h_u^0\right|^2 - \left|h_d^0\right|^2\right)^2 ,
\end{aligned}
$$

where m_{H_u}, m_{H_d}, b are soft SUSY-breaking parameters, μ is the coefficient of the mixing term between the two Higgs doublets in the superpotential, and g, g' are the gauge couplings. The neutral components of the two Higgs doublets, h_u^0, h_d^0, can be decomposed into the vacuum expectation values v_u, v_d and the real and imaginary parts of the fields,

$$
h_u^0 = v_u + \frac{1}{\sqrt{2}}(\phi_2^0 + i\chi_2^0), \quad h_d^0 = v_d + \frac{1}{\sqrt{2}}(\phi_1^0 + i\chi_1^0).
$$

a) Interpret the Higgs self-coupling term in V in comparison with the corresponding term in the Higgs potential of the Standard Model.

   How many parameters does V contain (besides the ones that are experimentally known) and which kind of conditions can be derived for them? Thus, why are only two independent SUSY parameters necessary to describe V (one usually chooses $\tan\beta \equiv v_u/v_d$ and the mass of the $\mathcal{CP}$-odd Higgs boson, $M_A^2 = b(\tan\beta\ + \cot\beta\ )$)?

b) For the two $\mathcal{CP}$-even fields ϕ_1, ϕ_2 the Higgs potential yields the following mass matrix

$$
\mathcal{L} = \frac{1}{2}\left(\phi_1^0, \phi_2^0\right)\ \mathcal{M}_H^2 \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix},
$$

   where

$$
\begin{aligned}
\mathcal{M}_H^2 &= \begin{pmatrix} m_{\phi_1} & m_{\phi_1\phi_2} \\ m_{\phi_1\phi_2} & m_{\phi_2} \end{pmatrix} \\
&= \begin{pmatrix} \cos^2\beta\, M_Z^2 + \sin^2\beta\, M_A^2 + T_{11} & -\sin\beta\cos\beta\,(M_Z^2 + M_A^2) + T_{12} \\ -\sin\beta\cos\beta\,(M_Z^2 + M_A^2) + T_{12} & \sin^2\beta\, M_Z^2 + \cos^2\beta\, M_A^2 + T_{22} \end{pmatrix}.
\end{aligned}
$$

   Here the T_{xy} denote combinations of tadpole contributions, which vanish in lowest order (why?). They are only needed for the renormalization (see below).

   Determine the masses m_H, m_h ($m_H > m_h$) of the physical $\mathcal{CP}$-even Higgs bosons of the MSSM in lowest order. Which region of mass values is possible for m_h (investigate in particular the limit $M_A \gg M_Z$)?

c) Because of the large value of the top-quark mass ($m_t \approx 175$ GeV) the one-loop corrections to m_h arising from contributions of the top quark and its superpartners are particularly important. The biggest contribution comes from the corrections to the $\phi_2\phi_2$ term,

$$m_{\phi_2} \to m_{\phi_2} - \hat{\Sigma}_{\phi_2}, \quad \hat{\Sigma}_{\phi_2} = \Sigma_{\phi_2} - \delta V_{\phi_2}.$$

Here $\hat{\Sigma}_{\phi_2}$ denotes the renormalized ϕ_2 self-energy, Σ_{ϕ_2} denotes the unrenormalized self-energy, and δV_{ϕ_2} is the counterterm from the mass matrix. In order to obtain the leading contributions, only the following renormalization transformations are necessary

$$M_A^2 \to M_A^2 + \delta M_A^2, \quad t_i \to t_i + \delta t_i,$$

where t_i, $i = 1, 2$, denote the tadpole contributions of the fields ϕ_i.

Use

$$T_{22} = -g/(2M_W)t_2 \sin\beta(1 + \cos^2\beta\ ) + g/(2M_W)t_1 \sin^2\beta\ \cos\beta$$

to derive the counterterm δV_{ϕ_2}.

The relevant Feynman-rules which are needed for the actual computation are

$$\begin{aligned}
\phi_2 t\bar{t} &: \frac{igm_t}{2M_W \sin\beta}(\omega_+ + \omega_-) \\
\phi_2 \tilde{t}_L \tilde{t}_L &: \frac{igM_Z}{c_W}\left(\frac{1}{2} - \frac{2}{3}s_W^2\right)\sin\beta - \frac{igm_t^2}{M_W \sin\beta} \\
\phi_2 \tilde{t}_R \tilde{t}_R &: \frac{igM_Z}{c_W}\left(\frac{2}{3}s_W^2\right)\sin\beta - \frac{igm_t^2}{M_W \sin\beta} \\
A t\bar{t} &: \frac{g\cos\beta\, m_t}{2M_W \sin\beta}(\omega_- - \omega_+).
\end{aligned}$$

Which structure do you expect for the leading (logarithmic) term (no calculation necessary)? Obtain a rough estimate of the relative size of the correction compared to the lowest-order result.