Lectures on Phenomenology
Problem sheet 2

1) a) Draw all the Feynman diagrams contributing to the process

$$e^+e^- \to e^+e^-$$

in the electroweak Standard Model in lowest order in perturbation theory.

b) Draw some typical diagrams that contribute to the above process at $\mathcal{O}(\alpha)$ relative to the leading order.

c) At which order of perturbation theory would the first effect of the strong interaction appear in this process? Draw some typical diagrams.

2) The squared matrix element for the decay of the Z boson to a fermion-antifermion pair is given by

$$|\mathcal{M}|^2 = \frac{2}{3}(C_L^2 + C_R^2)M_Z^2,$$

where C_L and C_R are the coupling of the Z to left and right handed particles, respectively.

(a) Taking the known values for M_Z and the fermion masses into account, which decays are kinematically possible (i.e. allowed by the conservation of energy and 3-momentum) within the Standard Model?

Calculate the partial width for the decay into fermions assuming the fermion masses can be ignored.

(b) A more detailed analysis of the Standard Model reveals that the couplings of the Z boson to fermion f are given by,

$$\begin{aligned} C_L &= g_Z(T_3 - Q\sin^2\theta_W) \\ C_R &= g_Z(-Q\sin^2\theta_W), \end{aligned}$$

where Q is the electric charge of fermion f, and $T_3 = 1/2$ for $f = u, c, t, \nu_e, \nu_\mu$ and ν_τ, and $T_3 = -1/2$ for $f = d, s, b, e, \mu$ and τ. The weak mixing angle is defined by,

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \approx 0.222,$$

while the coupling g_Z is given by,

$$g_Z = \frac{e}{\sin\theta_W \cos\theta_W} \approx 0.729.$$

Calculate the partial decay widths for $Z \to \mu^+\mu^-$, $Z \to \nu_e\bar{\nu}_e$, $Z \to u\bar{u}$, $Z \to d\bar{d}$ (the fermion masses can be neglected). Use these results to estimate the total Z width.

(c) Collider detectors can only "see" particles which interact via the strong or the electromagnetic interaction. As a consequence, which of the decays that you just calculated will actually be observed in a collider detector?

(d) The cross section for e^+e^- collisions near the Z resonance, i.e. for $\sqrt{s} \approx M_Z$, is given by

$$\sigma \sim \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$

up to a constant factor, where Γ_Z is the total Z width. Thus, the total Z width (and also M_Z) can be determined from measuring the cross section near the Z resonance. Which important information can be obtained by comparing the experimental result on the total Z width with the sum of the measured partial decay widths?