Lectures on Phenomenology
Problem sheet 1

1) (a) In its first phase the e^+e^- collider LEP was running at a CM energy of $\sqrt{s} \approx M_Z$, where it produced Z bosons "on resonance". The highest energies that were reached later on at LEP were about $\sqrt{s} = 206$ GeV. Explain why the exclusion bound on the Higgs mass obtained in this way is only about 114 GeV, i.e. far below the CM energy of 206 GeV.

(b) The photon (which mediates the electromagnetic interaction) couples in lowest order perturbation theory only to particles that are electrically charged, while the gluon (which mediates the strong interaction) couples in lowest order only to particles that are coloured (i.e. carry the charge of the strong interaction). Applying this to the respective couplings of these particles to the Higgs boson, it follows immediately that the photon and the gluon have to be massless. Explain why.

One of the most promising channels for the Higgs search at the LHC is Higgs production via gluon fusion, $gg \to H$, and the subsequent decay of the Higgs boson into a pair of photons, $H \to \gamma\gamma$. Draw the Feynman diagram that you expect to yield the biggest contribution for the process $gg \to H$ and the two diagrams that you expect to yield the biggest contributions for the process $H \to \gamma\gamma$. Indicate at which order the coupling constants α and α_s appear in the corresponding cross sections for the production and the decay [Note that the couplings of the Higgs boson contain a factor $\sqrt{\alpha}$, as usual for vertices of the electroweak interaction].

(c) The dominant decay mode of a Higgs boson with a mass of about 120 GeV is the decay into b quarks, $H \to b\bar{b}$. Give a qualitative explanation for this fact. Taking into account only the appropriate coupling factors, give an estimate of the ratio of branching ratios

$$\frac{\mathrm{BR}(H \to \gamma\gamma)}{\mathrm{BR}(H \to b\bar{b})}.$$

[Assume that the coupling α enters like α/π in this ratio and use $\alpha = 1/128$, $M_W = 80$ GeV (mass of the W boson), $m_b = 4$ GeV (mass of the b quark).]

(d) Which decay mode will be dominant for a Higgs boson with $M_H \approx 200$ GeV? Discuss qualitatively the behaviour of the total Higgs width as a function of the Higgs mass.

(e) The LHC is designed such that two proton beams will collide with a CM energy of 14 TeV. Why will the Higgs search via $gg \to H$ be only sensitive up to Higgs masses far below 14 TeV (in fact only up to about $M_H \approx 1$ TeV)?

2) (a) Draw the dominant Feynman diagrams for the process $e^+e^- \to W^+W^-$.

(b) Since the W boson is not a stable particle, the production process $e^+e^- \to W^+W^-$ will be followed by the decay of the two W bosons. The W boson can decay either into hadrons or leptons. What are the corresponding decay modes? How will the different decay products of a W boson appear in the detector; will all of them be seen? Accordingly, what are the signatures in the detector for different combinations of hadronic and leptonic decays of the two W bosons (the three possibilities are called fully hadronic, semi-leptonic and fully leptonic)?

(c) The squared matrix element for the decay of a W boson of mass M_W into a massless fermion-antifermion pair is given by

$$|\mathcal{M}|^2 = \frac{2}{3} C_L^2 M_W^2,$$

where the coupling C_L depends neither on the electric charge of the fermions nor on any other quantum numbers of the electroweak interaction ($C_L = e/(\sqrt{2}\sin\theta_W)$). Estimate the relative rate with which the fully hadronic, the semi-leptonic and the fully leptonic final state are produced. [The masses of leptons and light quarks can be neglected.]