Lectures on Phenomenology

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Outline

- The Standard Model of the electroweak and strong interactions
- Brief tour of collider physics
- Higgs search at present and future colliders
- Electroweak precision physics
- Quark mixing, neutrino masses and mixing
- Limitations of the Standard Model
- SUSY phenomenology
- Prospects for SUSY searches at the next generation of colliders

Further reading

Standard Model phenomenology:

- D.H. Perkins
  Introduction to High Energy Physics (Addison-Wesley, 2000)

- M. Böhm, A. Denner and H. Joos
  Gauge theories of the strong and electroweak
  interaction (Teubner, 2001)

- D. Bardin and G. Passarino
  The Standard Model in the Making : Precision Study of the Electroweak
  Interactions (Oxford University Press, 1999)

Higgs physics:

- J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson
  The Higgs Hunter's Guide (Perseus Books, 2000)

- M. Carena and H.E. Haber
  Higgs boson theory and phenomenology
  Prog. Part. Nucl. Phys. **50** (2003) 63, hep-ph/0208209

SUSY phenomenology:

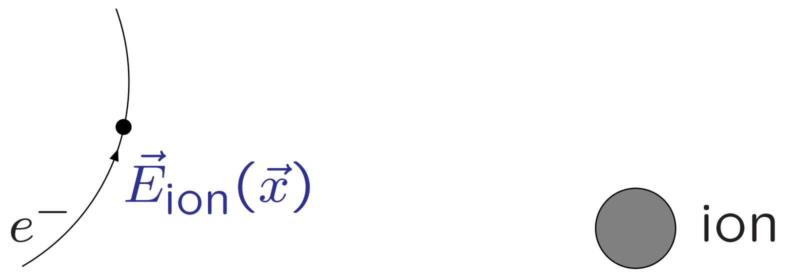
- S.P. Martin
  A Supersymmetry Primer
  hep-ph/9709356
  (In: Perspectives on Supersymmetry, ed. G. Kane, World Sci., 1998)

The Standard Model of the electroweak and strong interactions

Standard Model (SM): quantum field theory

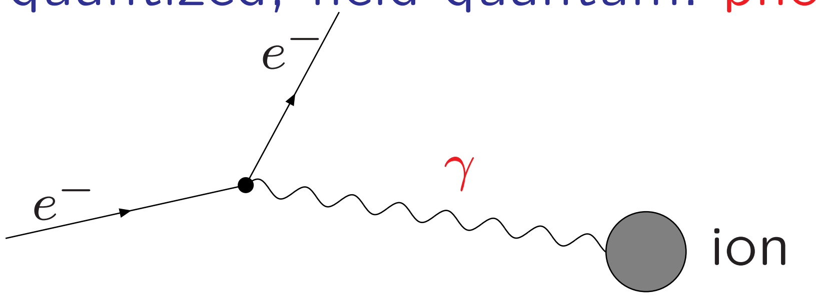
Classical field theory:

Classical Electrodynamics (Maxwell theory, unification of electrostatics and magnetism)



Quantum field theory:

QED; field is quantized, field quantum: photon



Interaction: exchange of field quanta

Electroweak interaction:

Fermion fields:

$$\text{quarks: } \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R, \quad \text{leptons: } \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, e_R$$

3 generations:

$$\begin{matrix} u, d, & s, c & t, b \\ \nu_e, e & \nu_\mu, \mu & \nu_\tau, \tau \end{matrix}$$

gauge bosons: γ, Z, W^+, W^-

Gauge group: $\mathrm{SU(2)_I \times U(1)_Y \supset U(1)_{em}}$

Strong interaction: QCD

quarks: q_r, q_g, q_b

gauge bosons: $g_1, \ldots g_8$: gluons, $\mathrm{SU(3)_C}$

All postulated fermions and gauge bosons experimentally verified

Construction principle of SM: gauge invariance

Example: Quantum electrodynamics (QED)

free electron field: $\mathcal{L}_{\text{Dirac}} = i\overline{\Psi}\gamma_\mu\partial^\mu\Psi - m\overline{\Psi}\Psi$

invariant under global gauge transformation: $\Psi \to e^{i\theta}\Psi$

Requirement of local gauge invariance:

gauge field A_μ introduced, $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$

gauge transformation: $\Psi \to e^{i\mathrm{e}\lambda(x)}\Psi$, $A_\mu \to A_\mu + \partial_\mu\lambda(x)$

⇒ Lagrangian with interaction term:

$$\mathcal{L}_{\text{QED}} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{free photon field}} + \underbrace{\overline{\Psi}(i\gamma_\mu\partial^\mu - m)\Psi}_{\text{free electron field}} + \underbrace{\mathrm{e}\overline{\Psi}\gamma_\mu\Psi A^\mu}_{\text{interaction}}$$

invariant under local gauge transformation

mass term, $m^2A^\mu A_\mu$: not gauge-invariant ⇒ A_μ: massless gauge field

Problem:

gauge fields Z, W^+, W^- are massive

Explicit mass terms for gauge bosons Z, W^+, W^- break gauge invariance of the Lagrangian

$\Rightarrow$ Higgs mechanism, spontaneous electroweak symmetry breaking:

Scalar field postulated, gauge-invariant mass terms from coupling to Higgs field

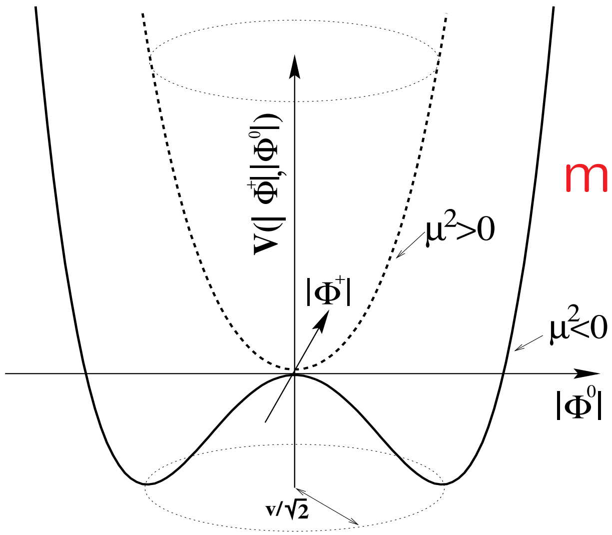
Higgs sector of the Standard Model:

$$\text{scalar SU(2) doublet:} \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$$

$\mu^2 < 0$: spontaneous symmetry breaking



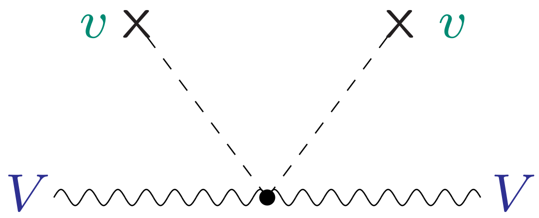
minimum of the potential at $\langle \Phi \rangle = \frac{1}{\sqrt{2}}\sqrt{\frac{-\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}}$

Unitary gauge: $\Phi = \begin{pmatrix} 0 \\ v + H \end{pmatrix}$

Gauge-invariant interaction with gauge fields:

$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad \Rightarrow$ mass terms

$VV\Phi\Phi$ coupling:

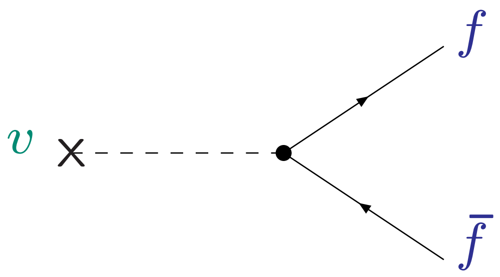


$\Rightarrow$ VV mass terms: $\quad \frac{1}{2} g_2^2 v^2 \equiv M_{\text{W}}^2, \quad \frac{1}{2}(g_1^2 + g_2^2) v^2 \equiv M_{\text{Z}}^2$

3 components of Higgs doublet $\longrightarrow$ longitudinal components of $W^\pm$, Z

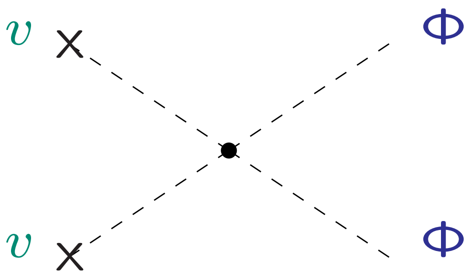
H: elementary scalar field, Higgs boson

Fermion mass terms: Yukawa couplings



$m_f = v\, g_f$ free parameters

Mass of the Higgs boson: self-interaction



$M_{\mathsf{H}} = v\sqrt{\lambda}$ free parameter

$\Rightarrow$ Higgs couplings proportional to masses of the particles

Gauge-invariant Lagrangian:

$$\mathcal{L}_{\mathsf{EW}}(\underbrace{g_2, g_1, v}_{M_{\mathsf{W}},\, M_{\mathsf{Z}},\, \alpha},\ \underbrace{\lambda}_{M_{\mathsf{H}}},\ \underbrace{g_f}_{m_f}) + \mathcal{L}_{\mathsf{QCD}}(\alpha_{\mathsf{s}})$$

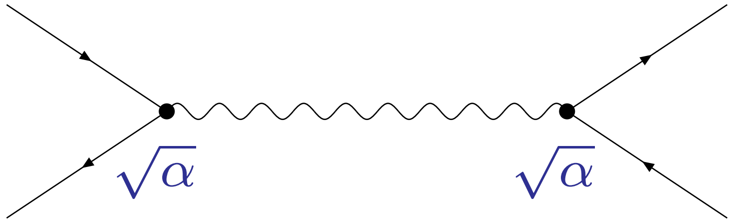
gauge invariance $\Rightarrow$ **theory is renormalizable**

[*G. 't Hooft '71*] [*G. 't Hooft, M. Veltman '72*] *Nobel price '99*

$\Rightarrow$ quantum field theory: quantum effects calculable

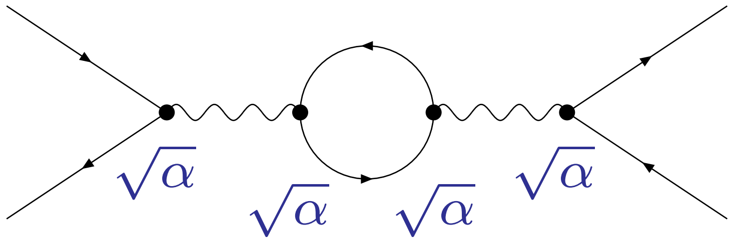
Expansion in coupling constant:

lowest order, classical limit



$$\alpha \approx \frac{1}{137}$$

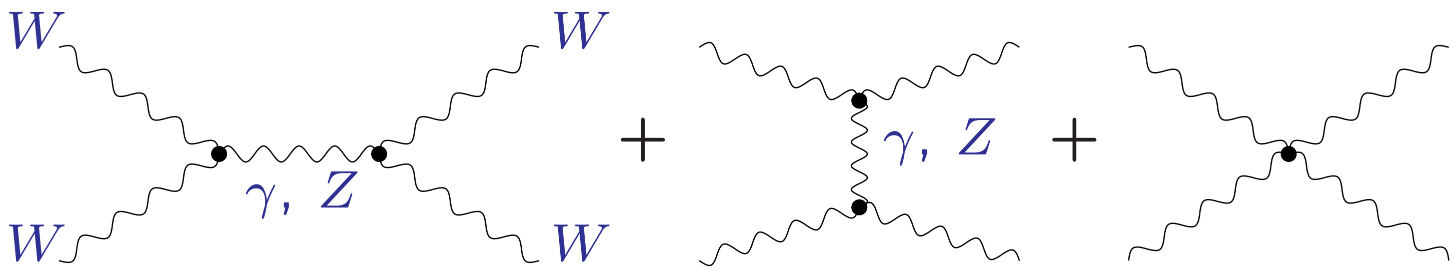
quantum corrections:
loop diagrams



Further effect of the Higgs field:

Scattering of longitudinally polarized W bosons: $W_L W_L \to W_L W_L$

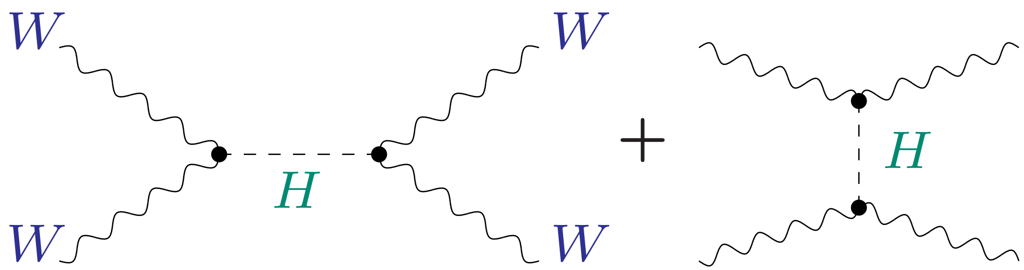
$\mathcal{M}_V =$



$= -g^2 \frac{E^2}{M_\mathrm{W}^2} + \mathcal{O}(1)$ for $E \to \infty$ $\Rightarrow$ violation of probability conservation

Extra contribution from scalar particle:

$\mathcal{M}_S =$



$= g_{WWH}^2 \frac{E^2}{M_\mathrm{W}^4} + \mathcal{O}(1)$ for $E \to \infty$

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_V + \mathcal{M}_S$$

$\Rightarrow$ terms with bad high-energy behavior cancel for

$$g_{WWH} = g\, M_{\text{W}}$$

$\Rightarrow$ Upper limit on M_{H} (unitarity bound): $M_{\text{H}} \lesssim 1$ TeV

If no fundamental Higgs field exists:

strong interaction in gauge-boson sector for $E \gtrsim 1$ TeV

probability conservation via non-perturbative effects:

dynamical symmetry breaking, "Technicolor", . . .

[*S. Weinberg '79*] [*L. Susskind '79*]

$\Rightarrow$ Higgs boson is last missing ingredient of ew SM

M_H: free parameter of SM

Theoretical arguments (unitarity, ...): $M_\mathrm{H} \lesssim 1$ TeV

$\Rightarrow$ Higgs search (Higgs physics) is one of the main goals of collider physics:

≤ 2000: LEP:

e^+e^- collider, $E_\mathrm{CM} \lesssim 206$ GeV

≥ 2001: Tevatron, Run II:

$p\bar{p}$ collider, $E_\mathrm{CM} \approx 2$ TeV

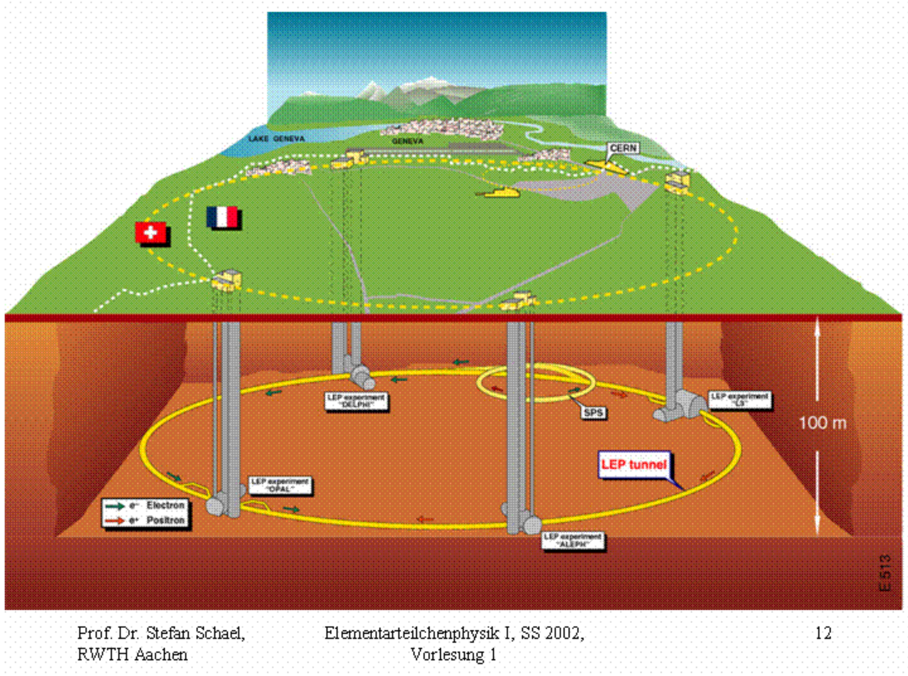
$\gtrsim 2007$: LHC:

pp collider, $E_\mathrm{CM} \approx 14$ TeV

$\gtrsim 201x$: ? LC:

e^+e^- collider, $E_\mathrm{CM} \approx 500$–$1000$ GeV

LEP: circular accelerator, ≈ 28 km long



Energy loss due to synchrotron radiation:

$$\Delta E \sim \frac{E^4}{m^4\, r}$$

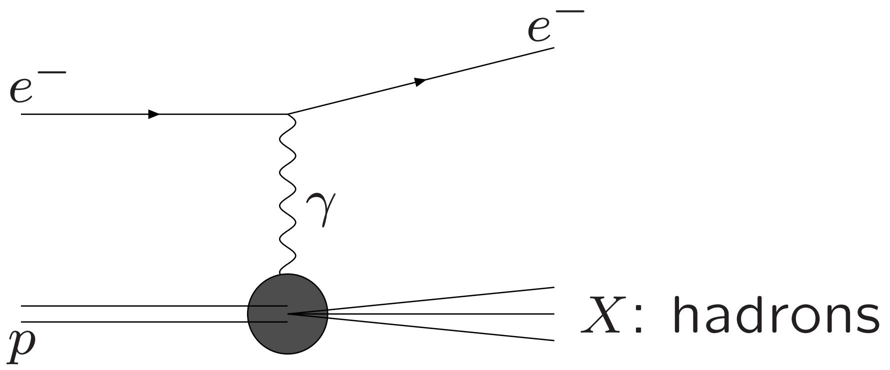
$\Rightarrow$ High energy e^+e^- collider can only be realized as Linear Collider (LC)

Synchrotron radiation loss smaller for proton by factor $(m_\mathrm{e}/m_\mathrm{p})^4 \approx 10^{-13}$

Electron–proton collider: HERA

e beam probes structure of the proton

$ep \to eX$: "deep inelastic scattering"



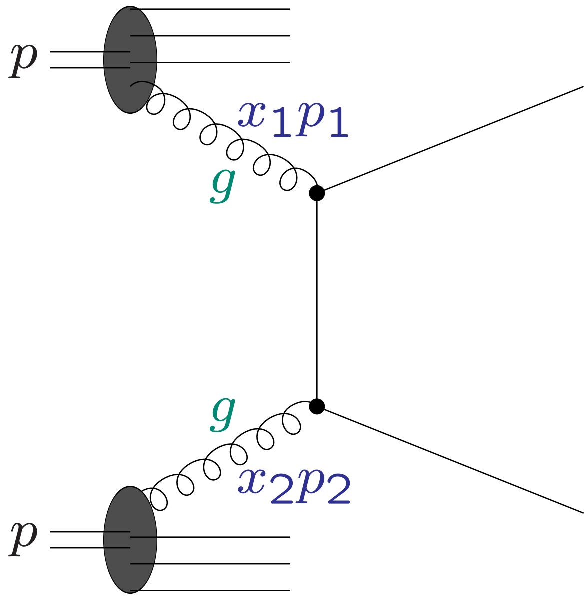
Relation between ep scattering cross section and eq "partonic" sub-process:

$$\sigma(ep) = \int_0^1 dx \sum_{q,\bar{q} \text{ in proton}} \underbrace{q^p(x)}_{\text{parton density function}} \sigma(eq)$$

$q^h(x)\,dx$: probability of finding parton of type q inside hadron h carrying fraction of hadron momentum between x and $x + dx$

Proton – proton scattering:

pp scattering contains "hard" collision process of partons q, $\bar{q}$, g, e.g.:



Available (energy)^2 for partonic sub-process: $\hat{s} = x_1 x_2 s$

LHC: $\sqrt{s} = 14$ TeV; $\sqrt{\hat{s}}$ up to several TeV

most of available energy lost in beam pipe as proton remnant

Comparison: lepton vs. hadron collider physics

Lepton colliders (LEP, SLC, LC, . . .):

- "Clean" experimental environment
- Lepton collider kinematics:
  momentum conservation can be used to analyze decay products
- well-defined initial state, beam polarization, tunable c.m. energy
- good signal–to–background ratio
- practicably all events in the detector can be stored and analyzed

$\Rightarrow$ high-precision physics

discovery reach limited by available c.m. energy

Hadron colliders (Tevatron, LHC, . . .):

- Easier to reach high c.m. energy

- Hadron collider kinematics:
  can in general only use conservation of transverse momentum p_T

- Composite nature of colliding protons $\Rightarrow$ "underlying event"

- Strongly interacting particles in initial state (LHC: mostly gluons)
  $\Rightarrow$ huge QCD backgrounds
  (Higgs, SUSY, . . . : weakly interacting phenomena)
  $\Rightarrow$ signal–to–background ratios in general much worse

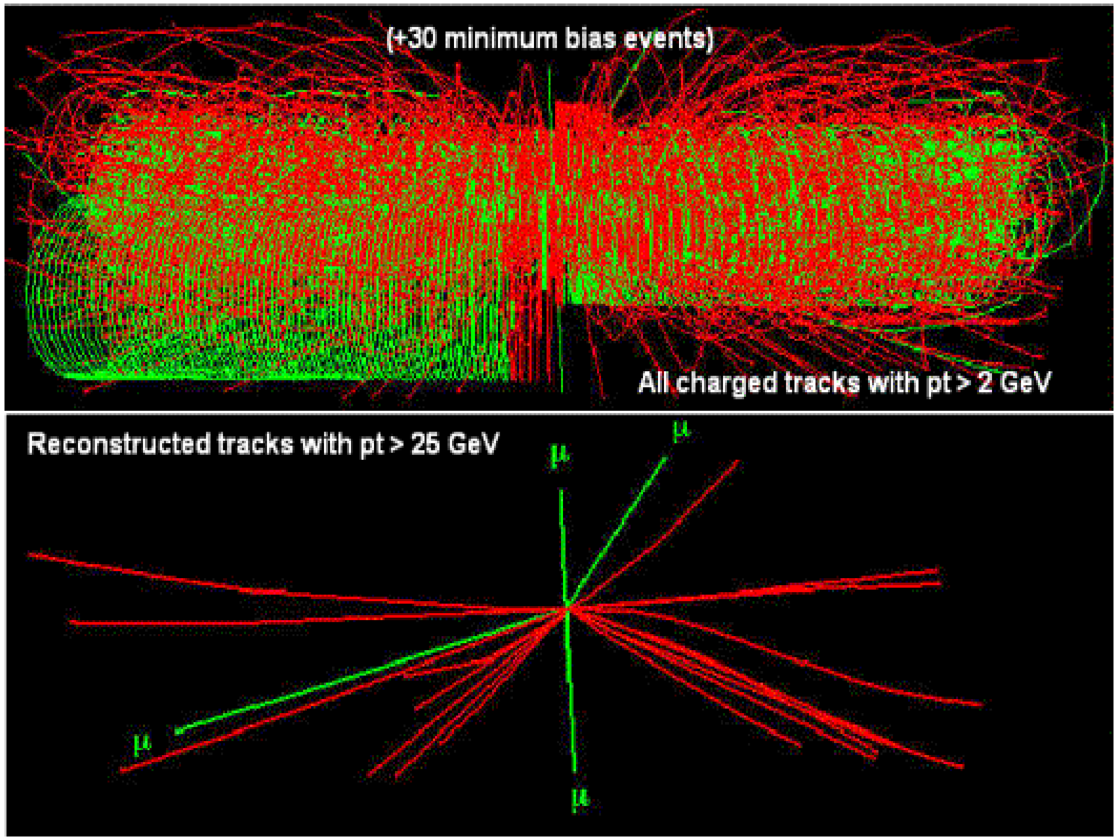
- Only small fraction of the events can be recorded
  $\Rightarrow$ highly selective triggers needed, LHC: rejection factor 10^7

$\Rightarrow$ in general bigger mass reach for direct discovery

difficult experimental environment

Simulation of an LHC event: $gg \to H \to ZZ \to \mu^+\mu^-\mu^+\mu^-$

Computing in LHC experiments



1 Higgs decay in 4 muons in 10^{13} events

1000 charged tracks/event, most of them p<0.5 GeV

Traces of produced particles in collider detectors:

$e^{\pm}$: charged $\Rightarrow$ leaves track in inner tracking chambers, energy deposit in calorimeter

$\mu^{\pm}$: track similar to $e^{\pm}$, but leads to signal in muon chambers

γ: calorimeter signal as for $e^{\pm}$, but no charged track

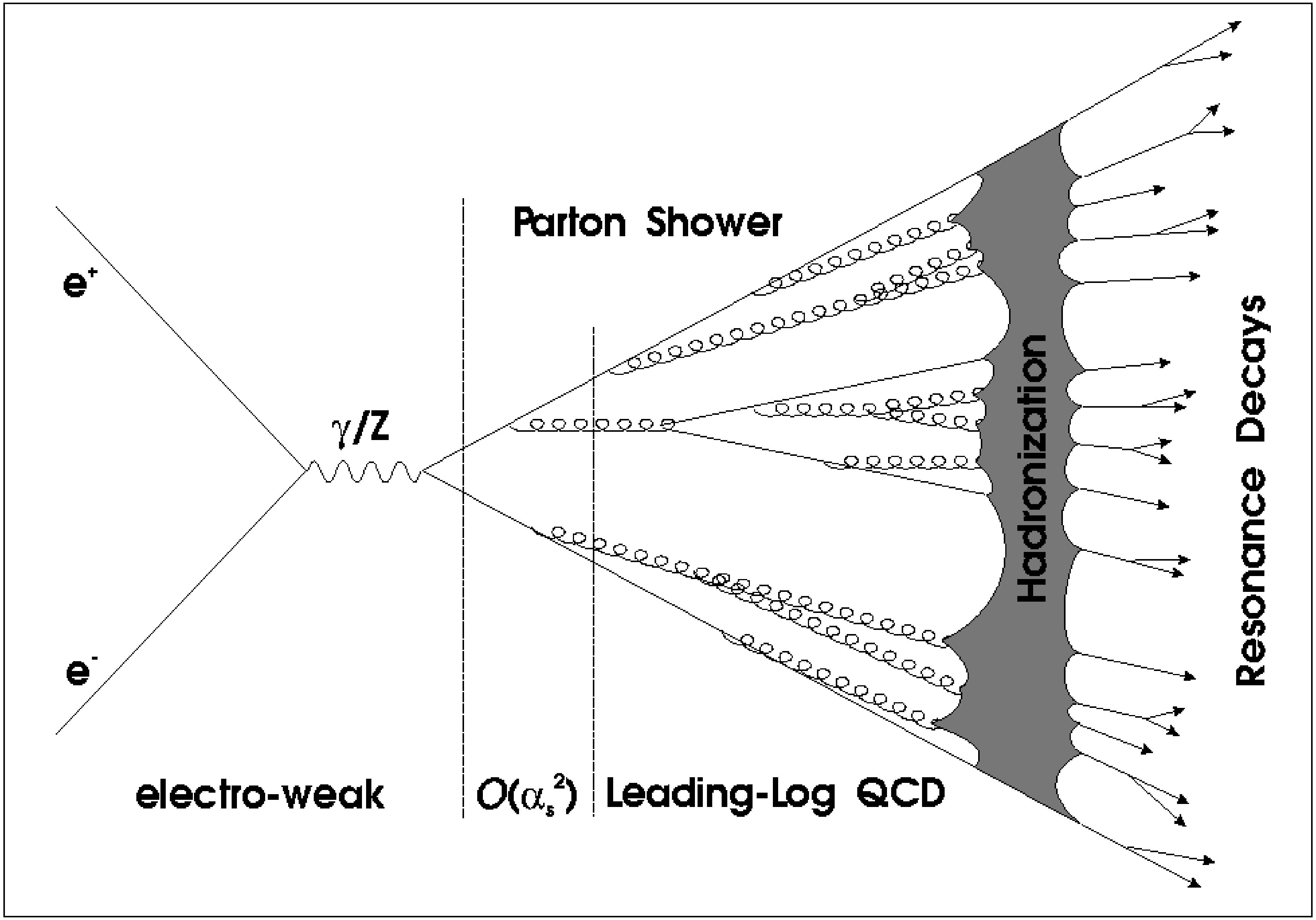
neutrinos, other weakly interacting particles: do not interact inside detector $\Rightarrow$ "missing momentum", "missing energy"

quarks: hadronization $\Rightarrow$ "jets"; direction of high-energetic jets related to parton momentum

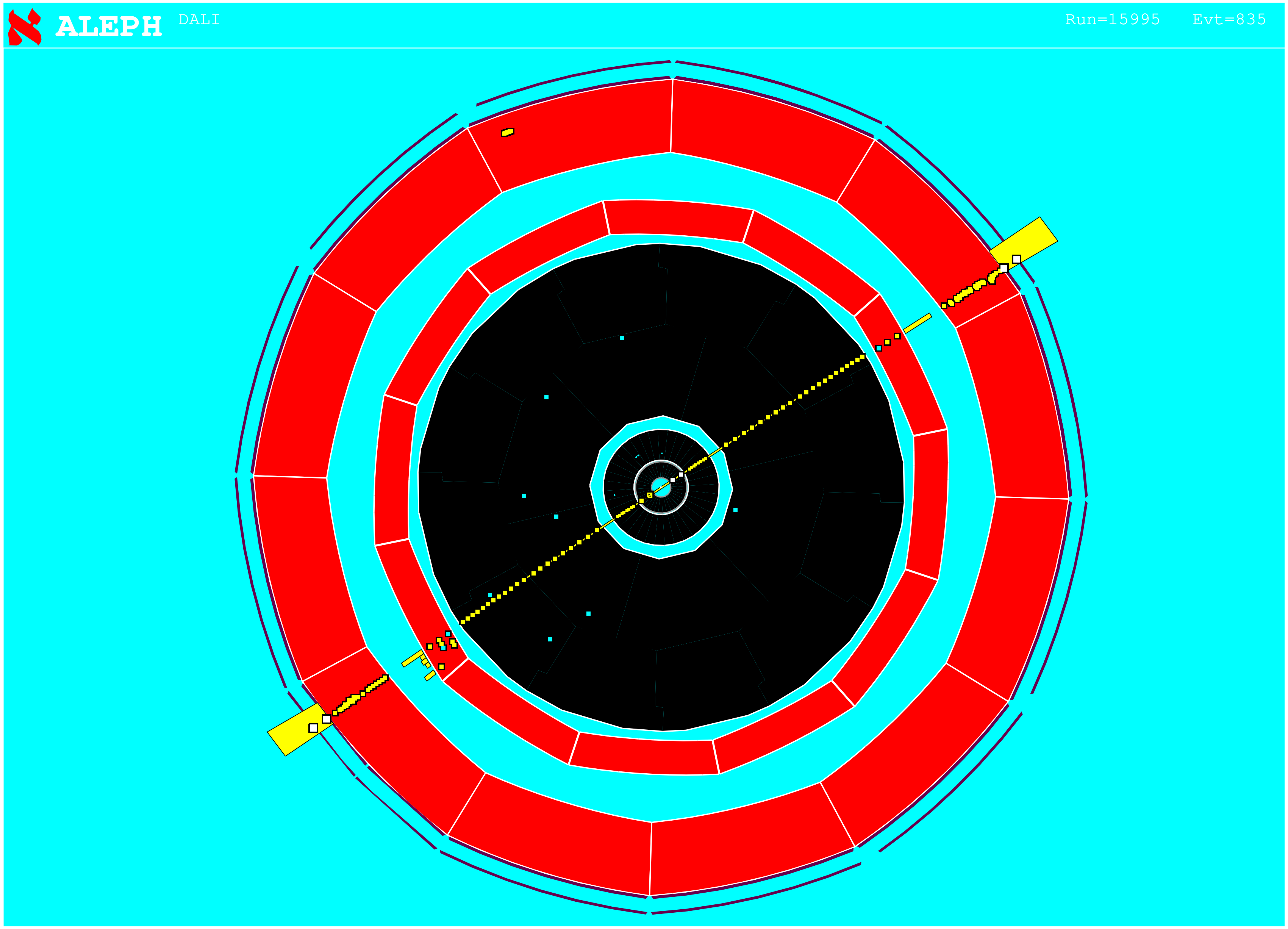
heavy quarks or leptons: reconstruction of decay vertex in micro vertex detector; "b tagging" with displaced vertices, etc.

"Cuts" $\Rightarrow$ improve signal over background ratio

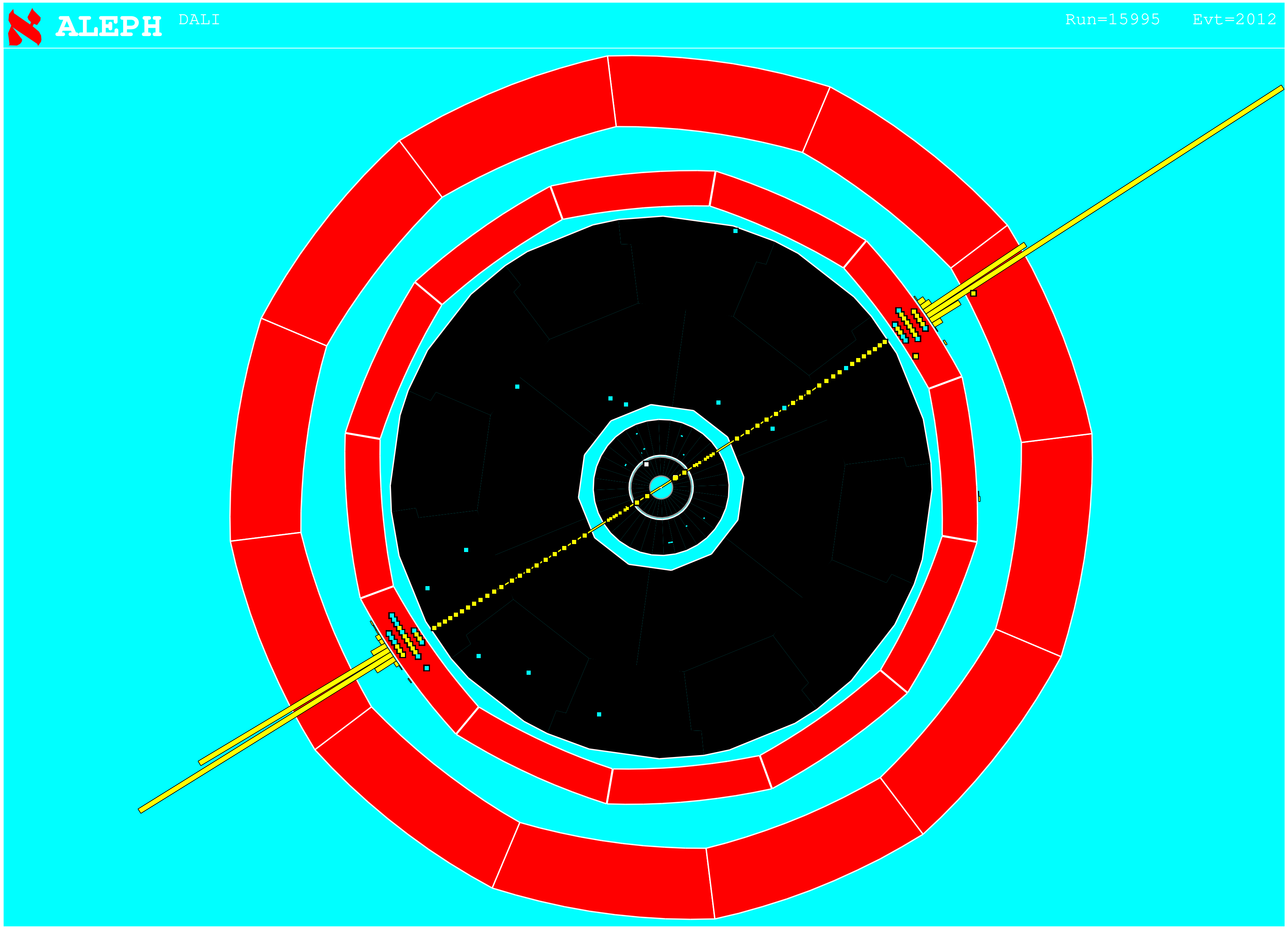
important to have good understanding of backgrounds


e+
e-
γ/Z
Parton Shower
Hadronization
Resonance Decays
electro-weak
O(αs2)
Leading-Log QCD

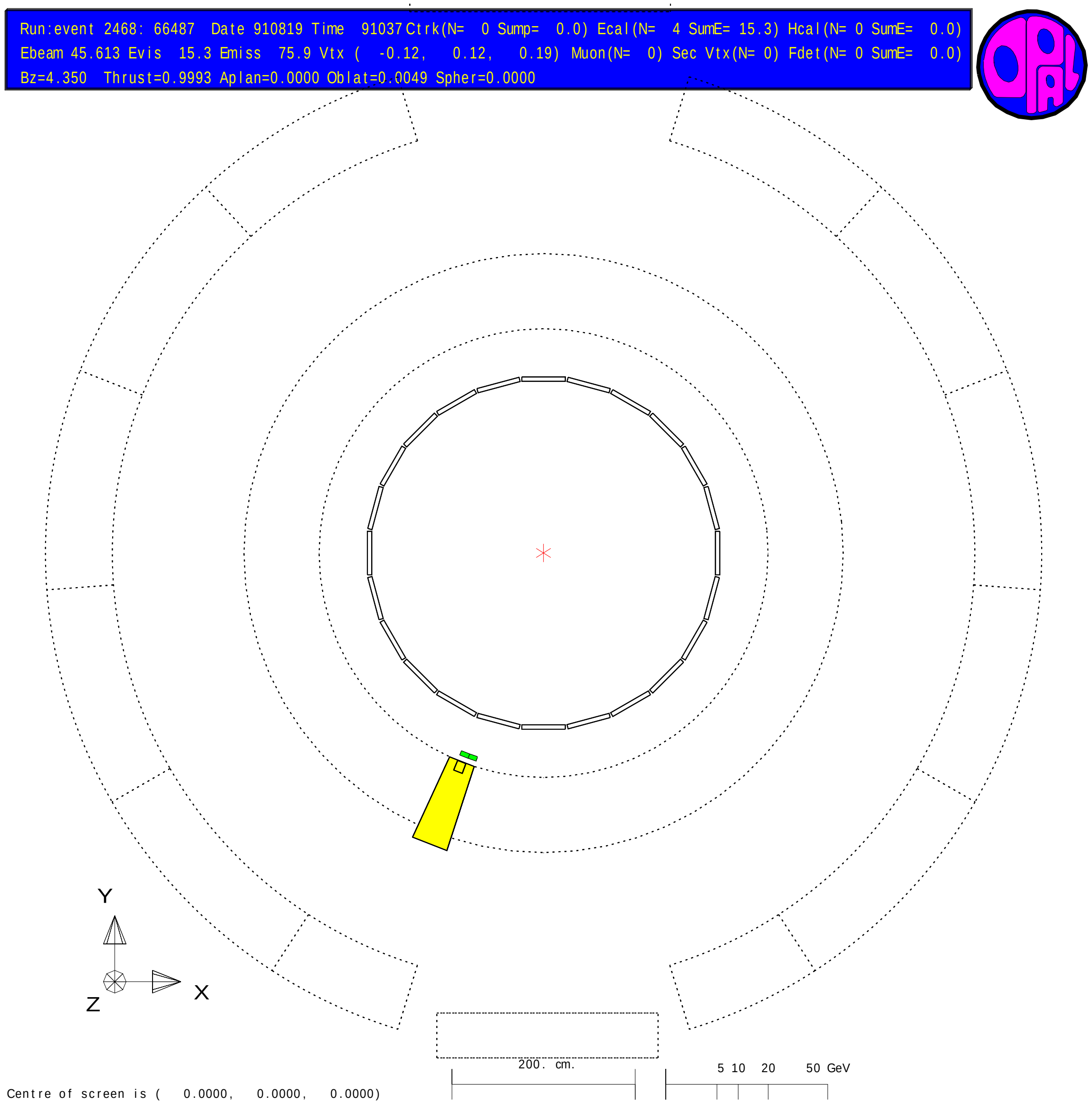
$e^+e^- \to \mu^+\mu^-$:



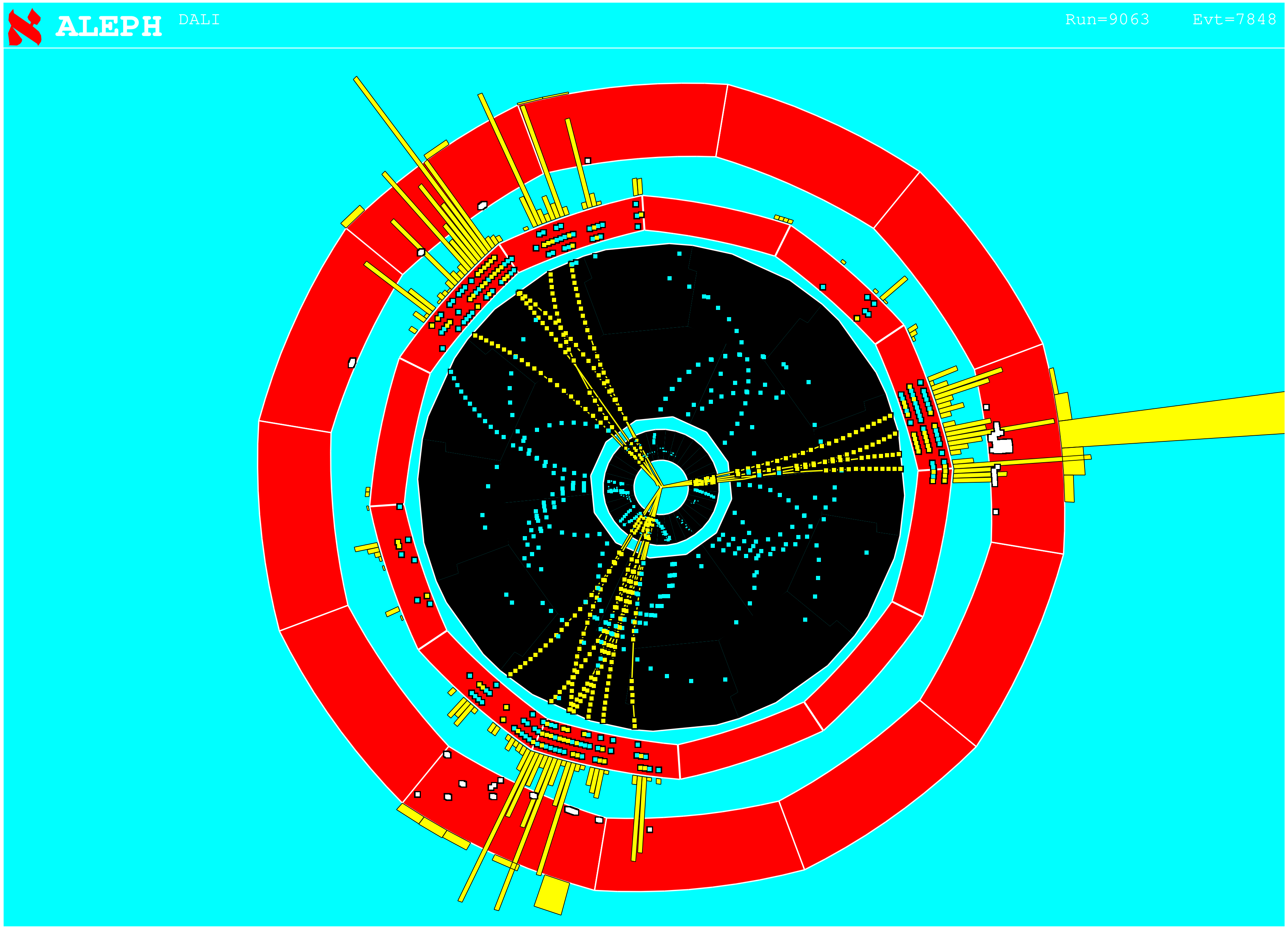
$$e^+e^- \to e^+e^-:$$



$e^+e^- \to \nu\bar{\nu}\gamma$:

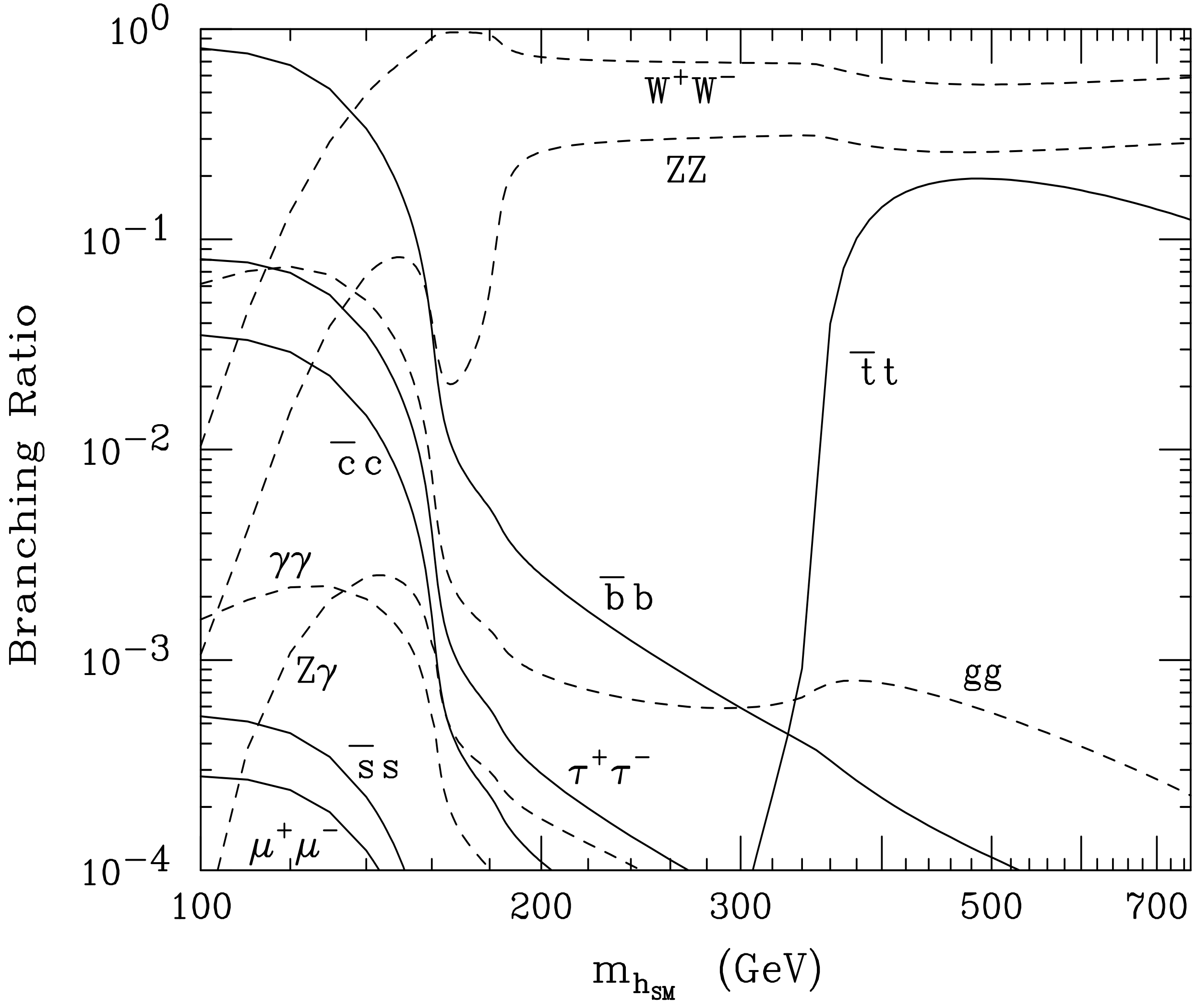


$e^+e^- \rightarrow$ 3 jets:



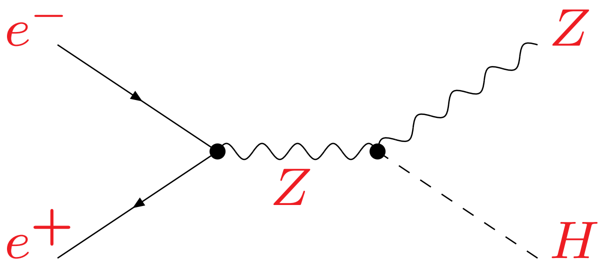
Search for the Standard Model Higgs

Branching ratios of the SM Higgs:

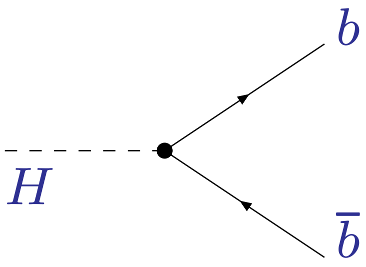


Search for the Standard Model Higgs at LEP

Dominant production process: $e^+e^- \to ZH$



Dominant decay process: $H \to b\bar{b}$

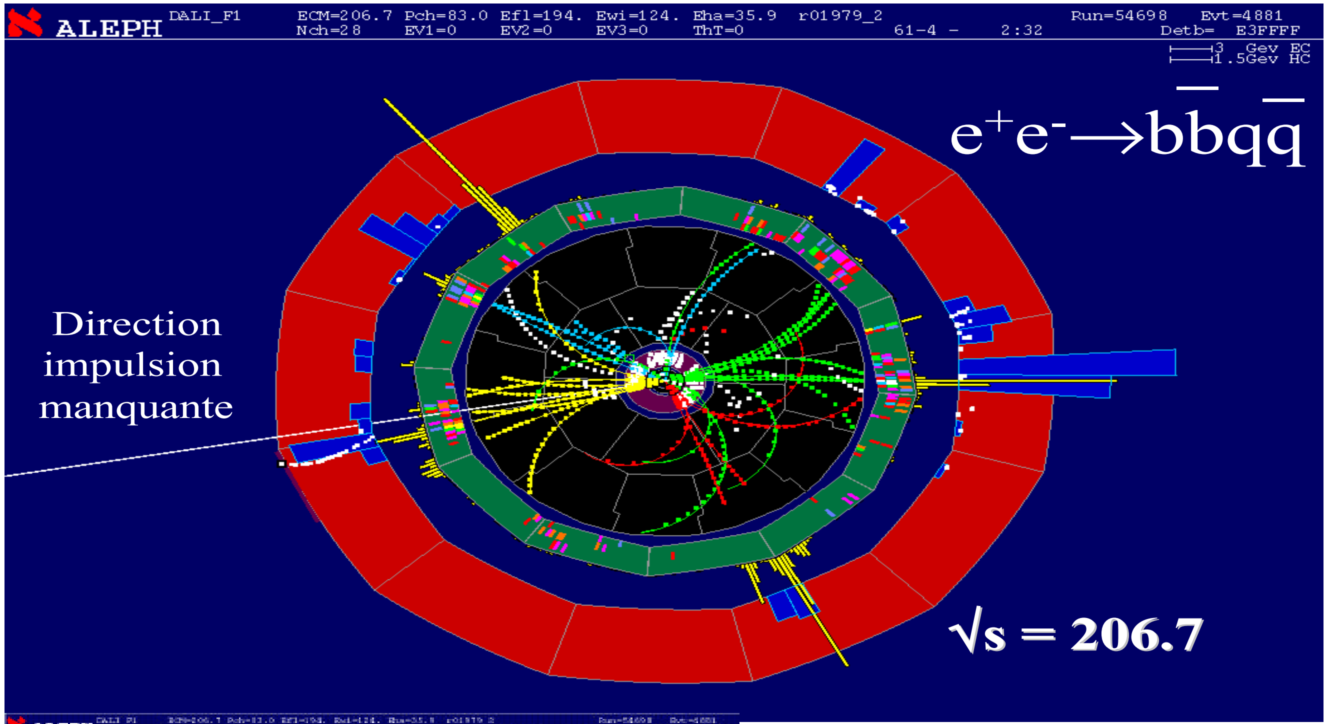


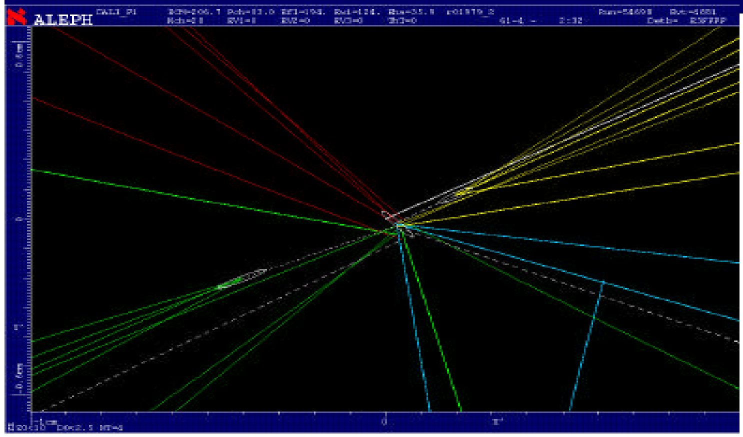
Highest energy: $E_{\text{CM}} \approx 206$ GeV

Exclusion limit, 95% C.L.:

$$M_{\text{H}} > 114.4\,\text{GeV (expected: } 115.3\,\text{GeV)}$$

Candidate -d- (14/06/2000)

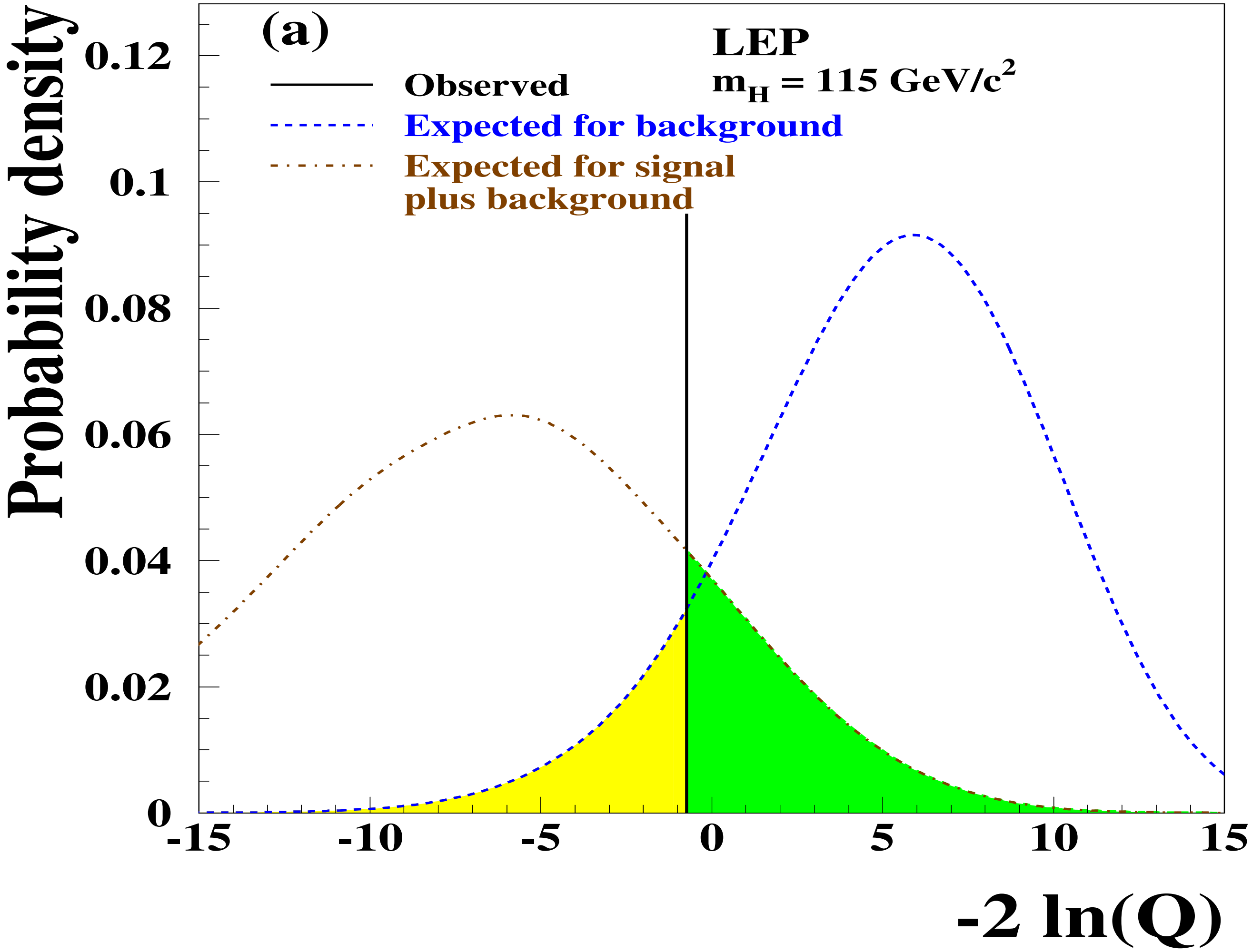




M_{rec}=114±3 GeV/c²

Two well b-tagged jets
with two reconstructed
displaced vertices

Comparison of observed data with expected background and expected signal + background for $M_H = 115$ GeV:
[*LEP Higgs Working Group '03*]

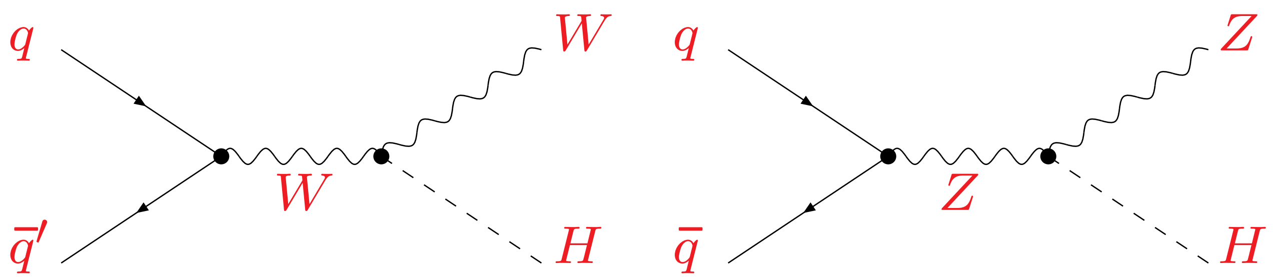


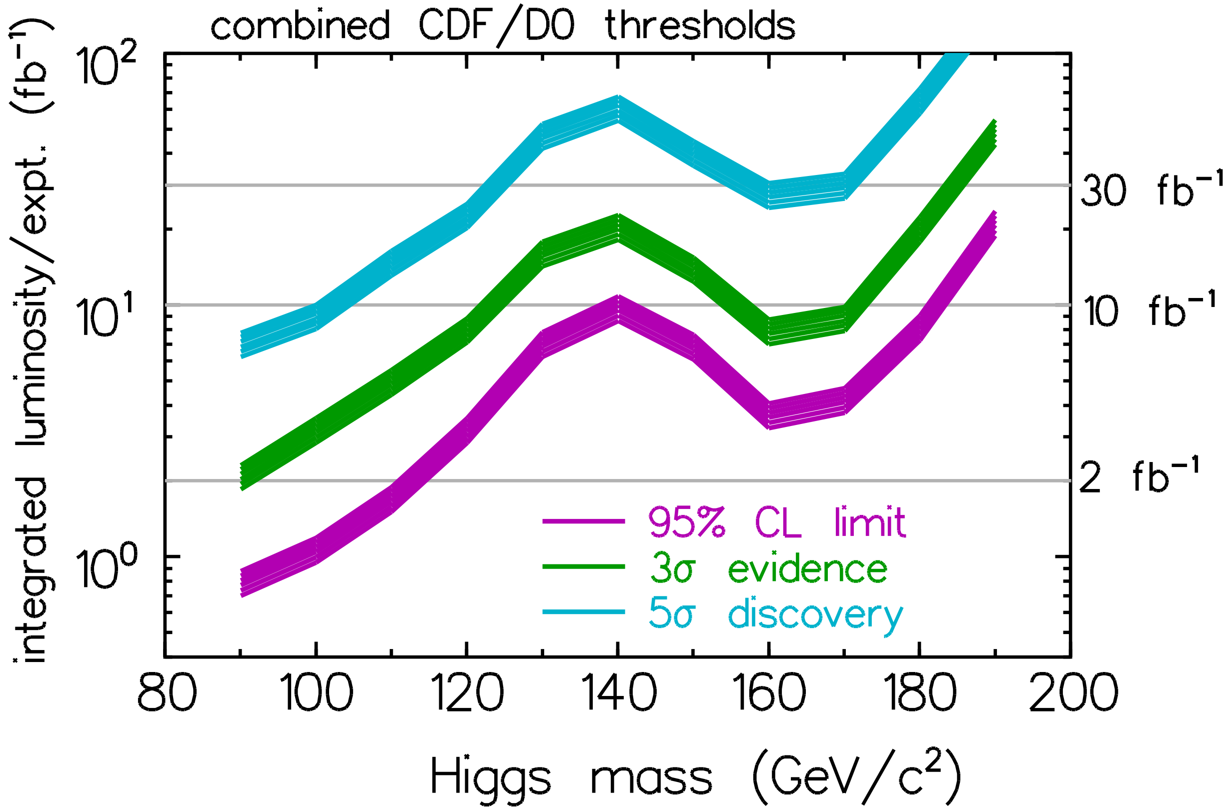
$\Rightarrow$ 1.7σ excess over background expectation

Search for SM Higgs at Tevatron, Run II:

$p\bar{p}$ collider with $E_{\mathrm{CM}} \approx 2$ TeV

Dominant production processes:



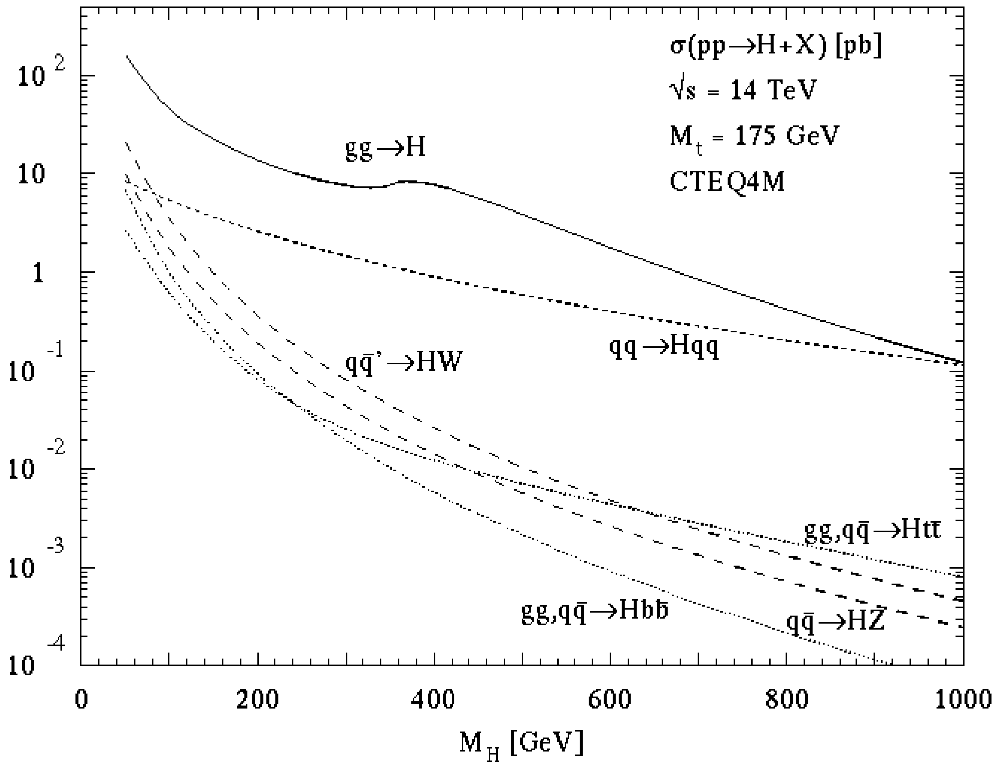


Run IIA, $\approx$ 2005:
2 fb^{-1} per experiment

Run IIB, $\approx$ 2009:
4–8 fb^{-1} per experiment

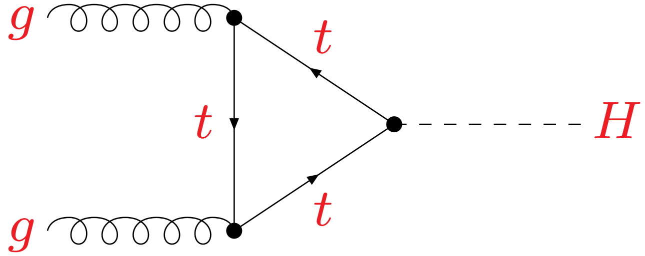
Search for the SM Higgs at the LHC:

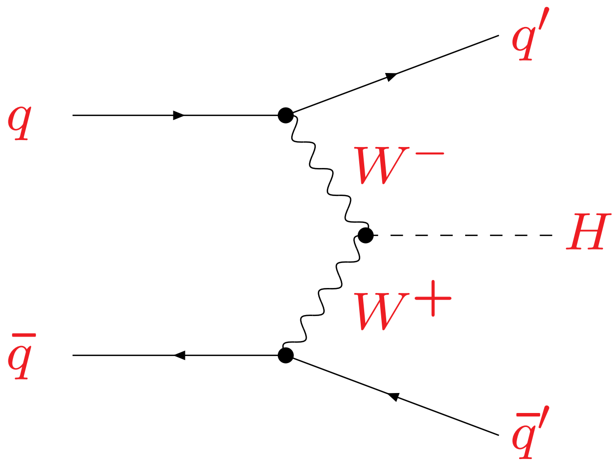
SM Higgs production at the LHC:



Dominant production processes:

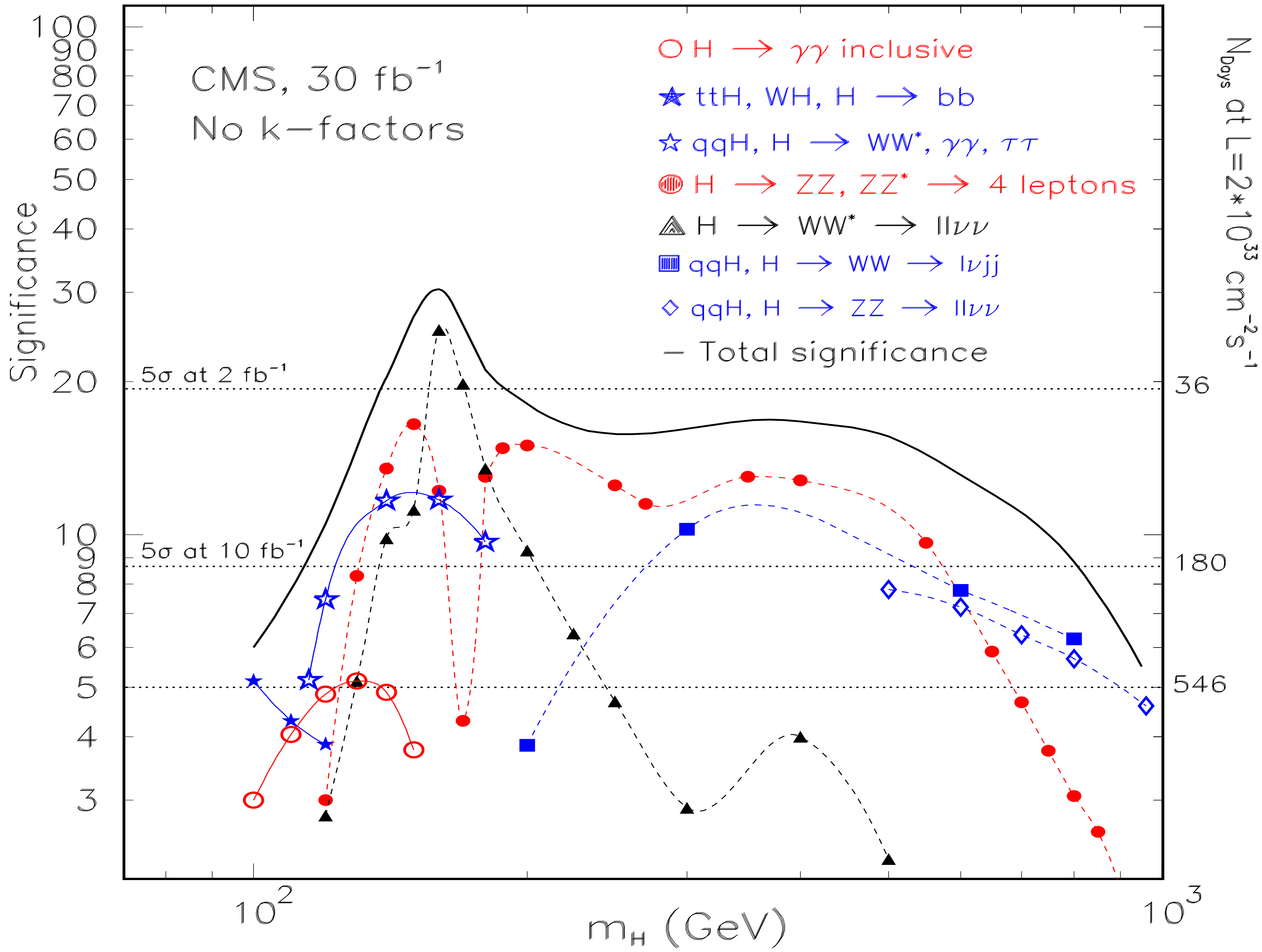
gluon fusion $gg \to H$, weak boson fusion (WBF) $q\bar{q} \to q'\bar{q}'H$:





The LHC will discover a SM-like Higgs, covers whole range $M_{\rm H} \lesssim 1$ TeV

[*CMS '03*]



For light SM-like Higgs, $H \to \gamma\gamma$ decay mode:

$\Rightarrow$ LHC can do first precision measurement in Higgs sector:

$$\delta M_{\rm H}^{\rm exp} \approx 0.2 \text{ GeV}$$

Higgs physics at an e^+e^- LC ($E_{\mathrm{CM}} \approx 500$–$1000$ GeV)

"Golden" production channel: $e^+e^- \to ZH$, $Z \to e^+e^-, \mu^+\mu^-$

Higgs discovery possible independent of decay modes
(from recoil against Z boson)

$\Delta\sigma_{\mathrm{HZ}}/\sigma_{\mathrm{HZ}} \approx 2\%$ ($E_{\mathrm{CM}} = 350$ GeV, $\int \mathcal{L}dt = 500\ \mathrm{fb}^{-1}$)

LC is a "Higgs factory"

e.g.: $E_{\mathrm{CM}} = 800$ GeV, $1000\ \mathrm{fb}^{-1}$, $M_{\mathrm{H}} = 120$ GeV:

$\Rightarrow$ ≈ 160000 Higgs events in "clean" experimental environment

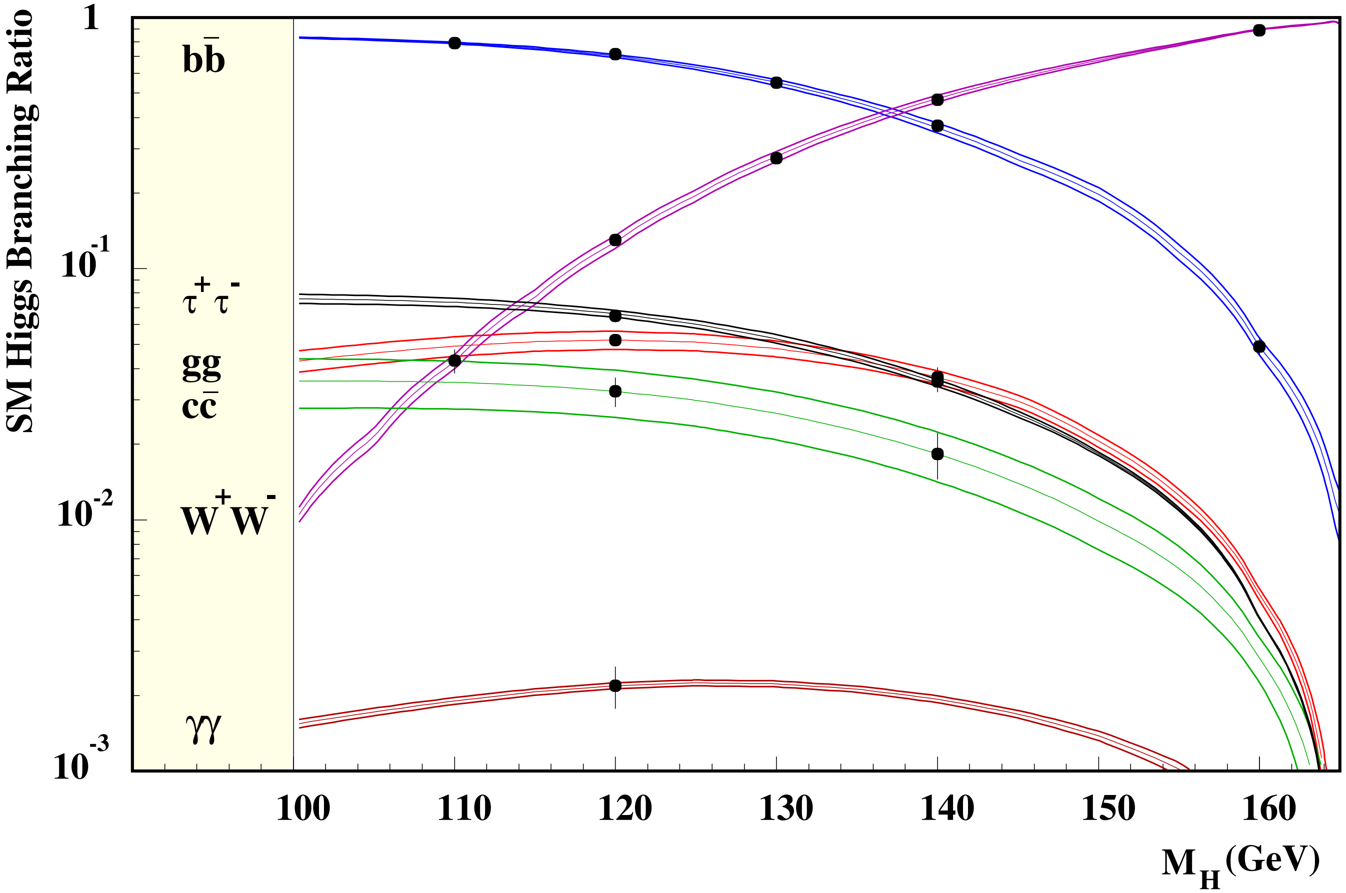
$\Rightarrow$ Precise measurement of Higgs mass ($\approx 4 \times 10^{-4}$) and couplings, determination of Higgs spin and quantum numbers, . . .

$\Rightarrow$ Verification of Higgs mechanism

Precision measurements of

Higgs mass: $\Delta M_{\rm H}/M_{\rm H} \approx 3 - 5 \times 10^{-4}$

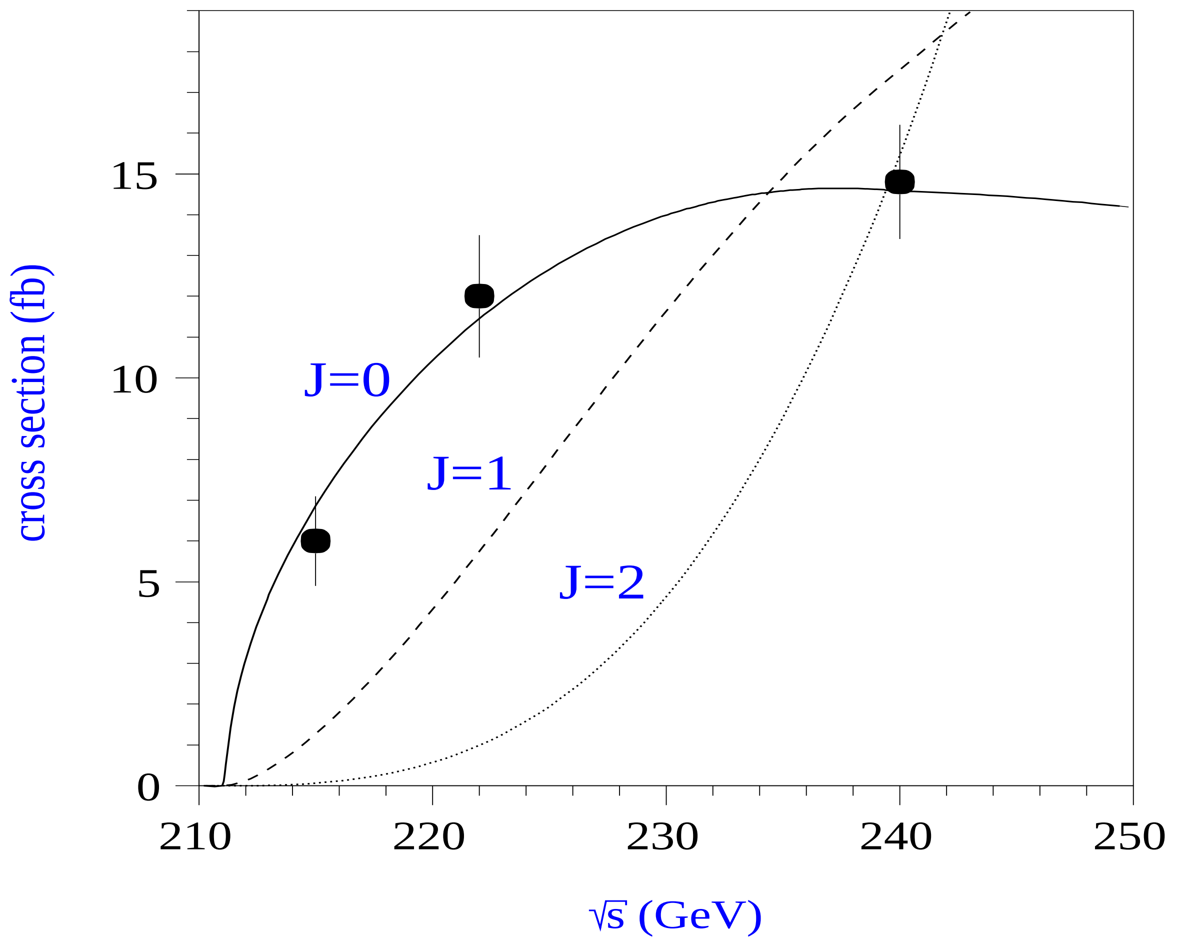
Higgs branching ratios, $H \to f\bar{f}, VV$: % level
[*TESLA TDR '01*]



Coupling to gauge bosons: % level; Higgs width: % level; . . .

Determination of spin and $\mathcal{CP}$ quantum numbers:

Energy dependence of σ_{HZ} near threshold: [*TESLA TDR '01*]



$\Rightarrow$ Higgs spin

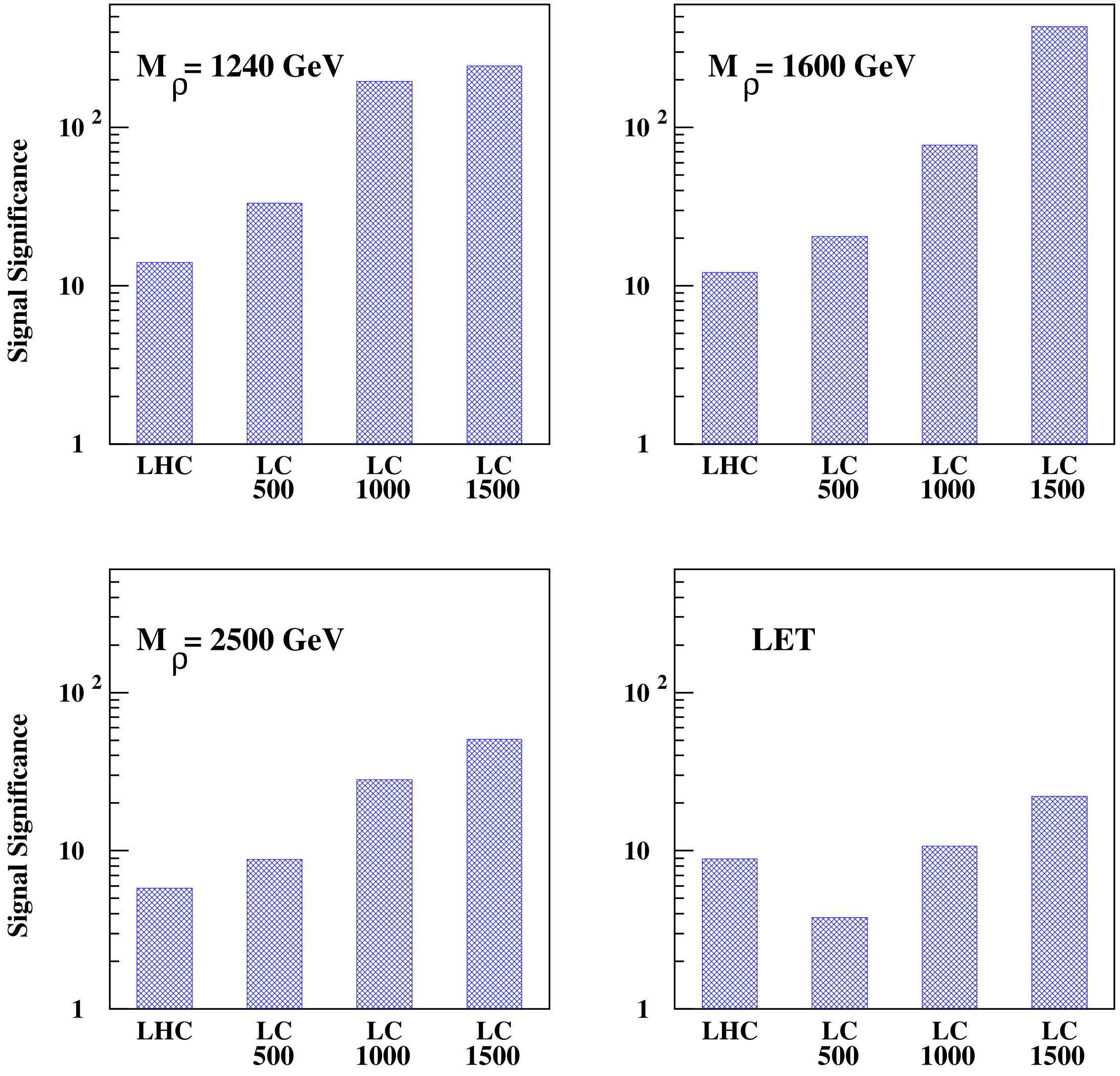
Angular distributions of fermions in $ZH \to f\bar{f}H \Rightarrow \mathcal{CP}$ quantum numbers

Higgs self coupling: $\Delta\lambda_{HHH}/\lambda_{HHH} \approx 20\%$

$\Rightarrow$ Reconstruction of Higgs potential

Sensitivity of LHC and LC measurements to signals of strong electroweak symmetry breaking: [*American LC Working Group '01*]

Signal significance in σ for various masses M_ρ of vector resonance in $W_\mathrm{L} W_\mathrm{L}$ scattering:



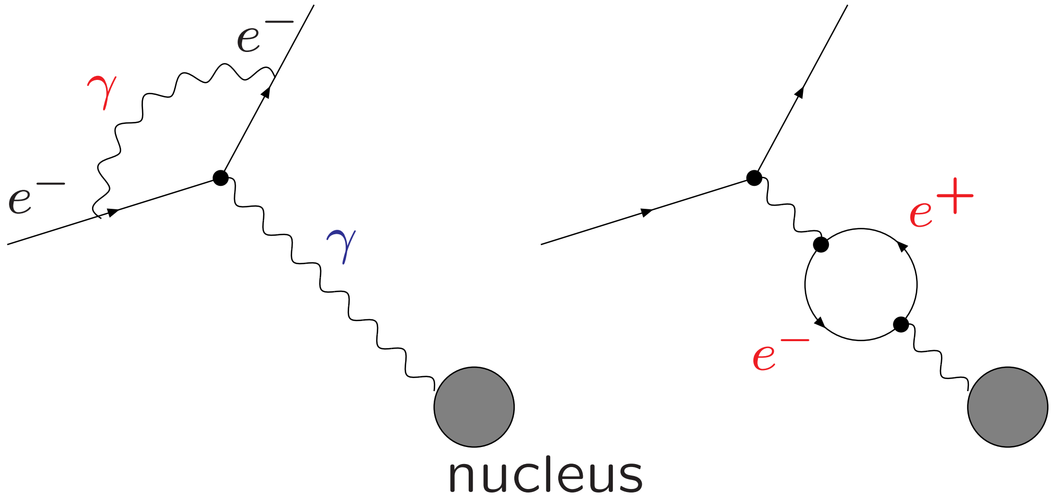
$\Rightarrow$ Strong electroweak symmetry breaking scenarios can be probed in detail

Electroweak precision physics

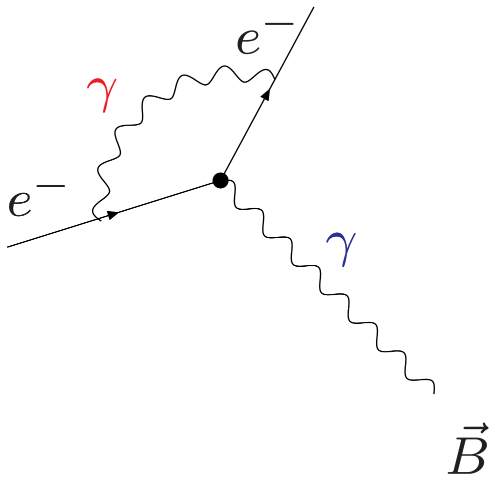
Precision measurements $\Rightarrow$ test quantum effects of the theory

Historical: QED

Lamb shift:



$g-2$:



$$a = \frac{1}{2}(g-2)$$

$$a_{\text{exp}} = 1\,159\,652\,188(\pm 4)\times 10^{-12}$$

$$a_{\text{theo}} = 1\,159\,652\,157(\pm 28)\times 10^{-12}$$

Electroweak precision measurements:

- LEP1/SLC: e^+e^- annihilation at the Z-boson resonance
  LEP1: $\sim 4 \times 10^6$ events/experiment

- LEP2: $e^+e^- \to W^+W^-$

- Tevatron: $q\bar{q}' \to W \to l\bar{\nu},\ q\bar{q}'$
  $q\bar{q} \to t\bar{t},\ t \to W^+b \to \ldots$

- low-energy experiments

$$
\begin{array}{lcll}
M_Z[\,\mathrm{GeV}] & = & 91.1875 \pm 0.0021 & 0.002\% \\
G_\mu[\,\mathrm{GeV}^{-2}] & = & 1.16637(1)\,10^{-5} & 0.0009\% \\
m_t[\,\mathrm{GeV}] & = & 174.3 \pm 5.1 & 2.9\% \\
 & & & \\
M_W[\,\mathrm{GeV}] & = & 80.426 \pm 0.034 & 0.04\% \\
\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} & = & 0.23150 \pm 0.00016 & 0.07\% \\
\Gamma_Z[\,\mathrm{GeV}] & = & 2.4952 \pm 0.0023 & 0.09\%
\end{array}
$$

Quantum effects of the theory: $\sim \mathcal{O}(1\%)$: loop corrections

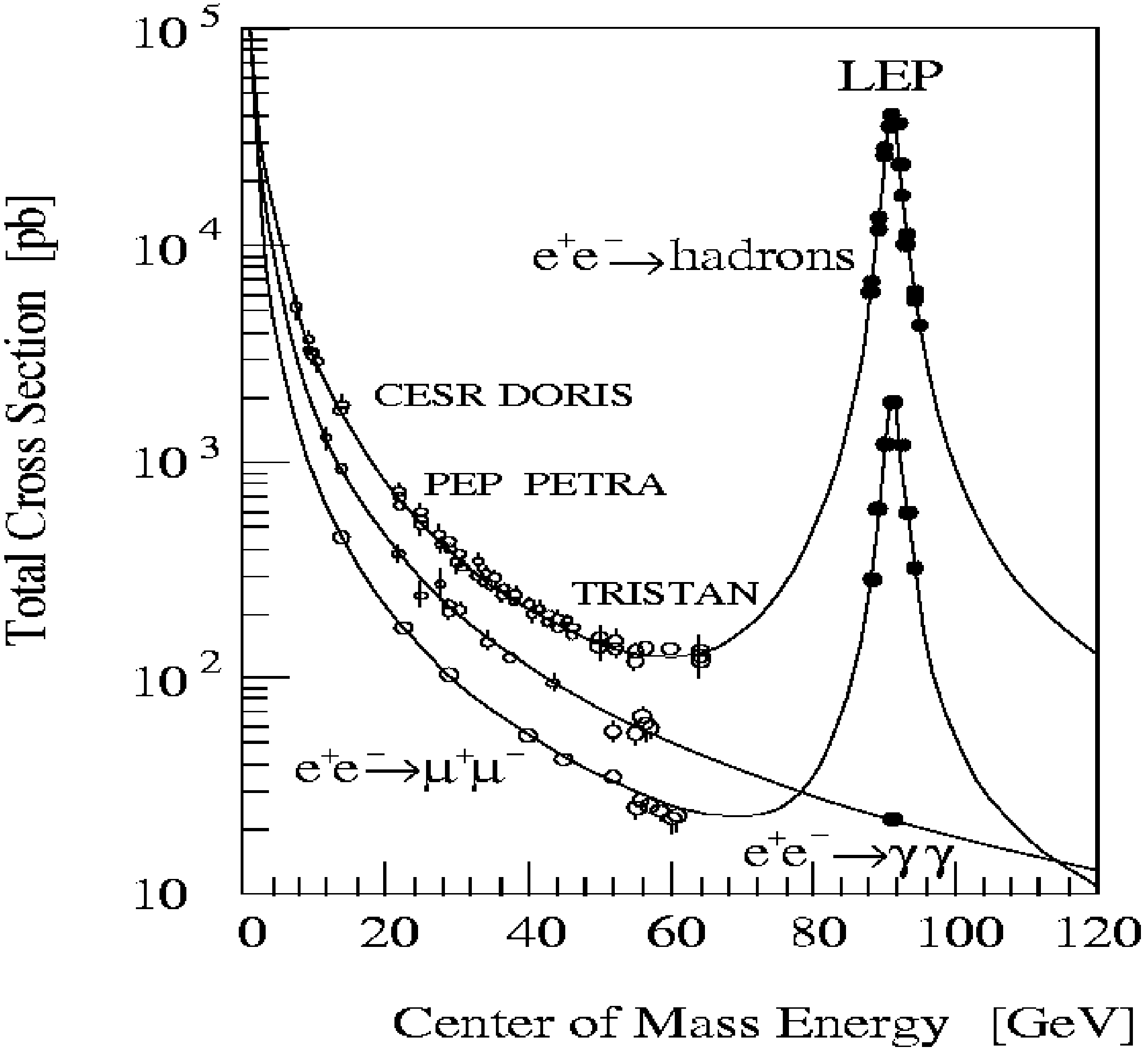
$e^+e^- \to \mu^+\mu^-$, $e^+e^- \to q\bar{q}$:

Z-propagator in lowest order:

$$\Delta_{ZZ}(s) \sim \frac{1}{(s - M_Z^2)}$$

$\Rightarrow$ resonance enhancement of cross section for $s \approx M_Z^2$:

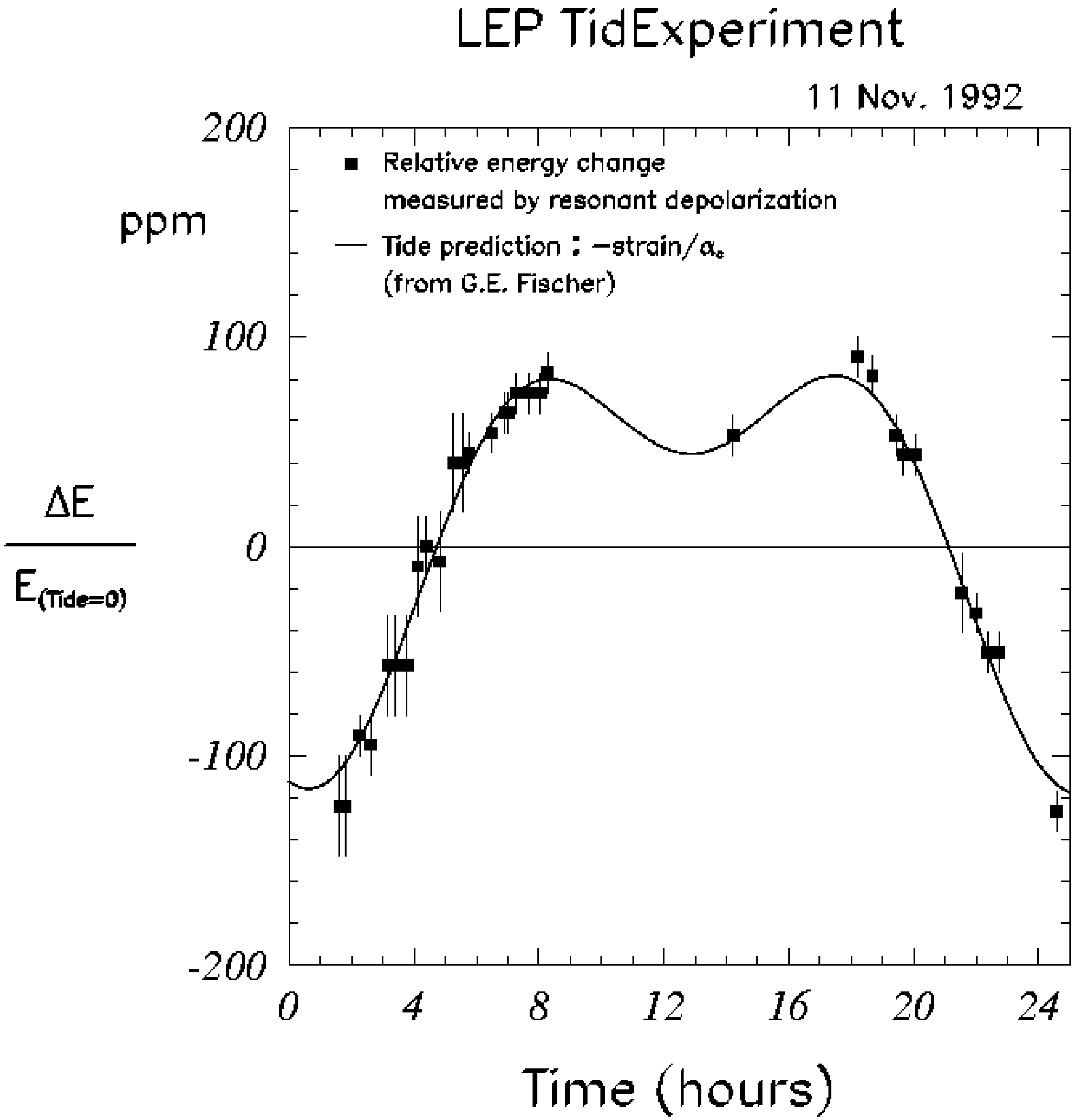
$$\sigma \sim \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2}$$



Necessary to take into account at LEP:

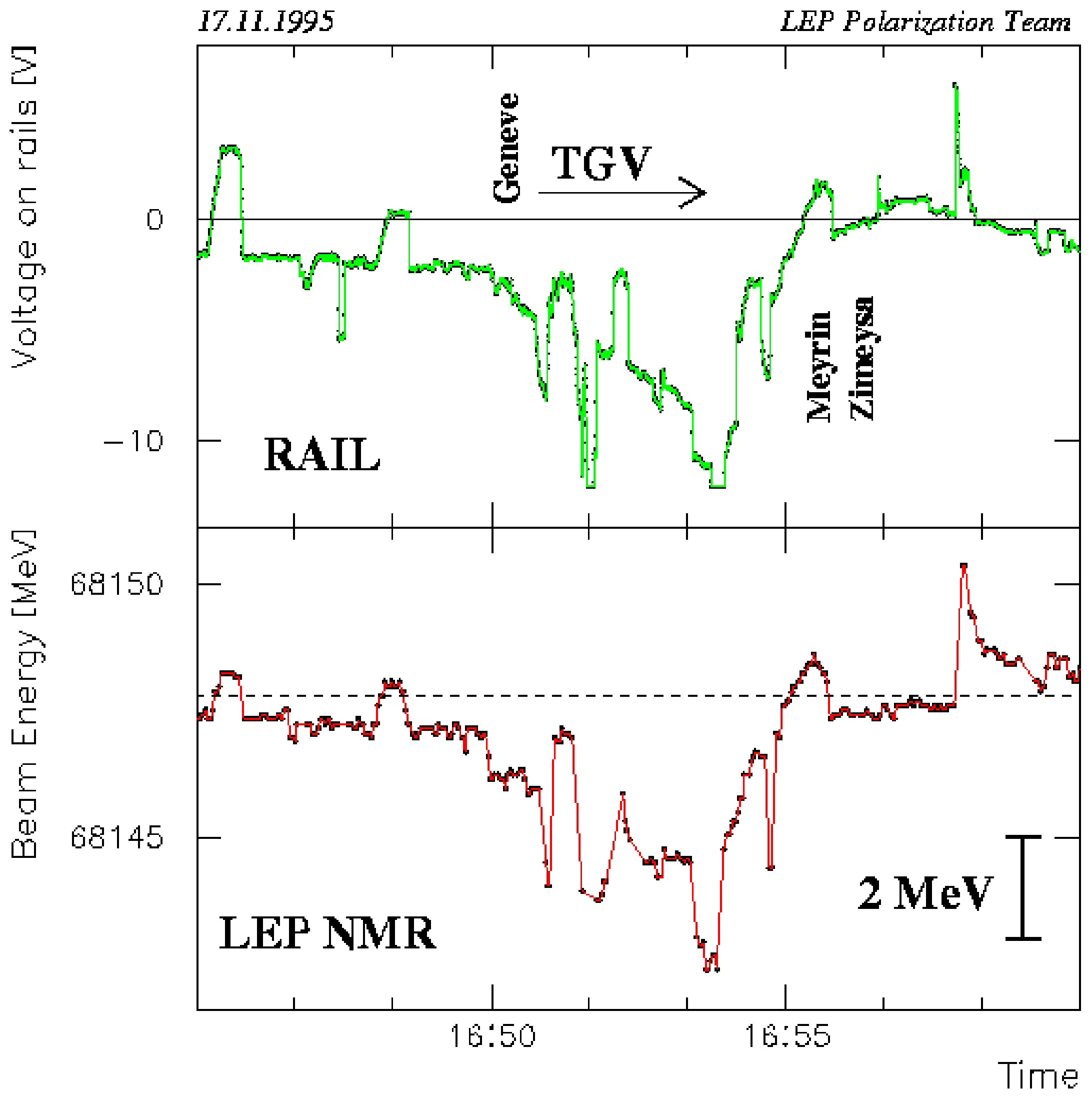
tides,

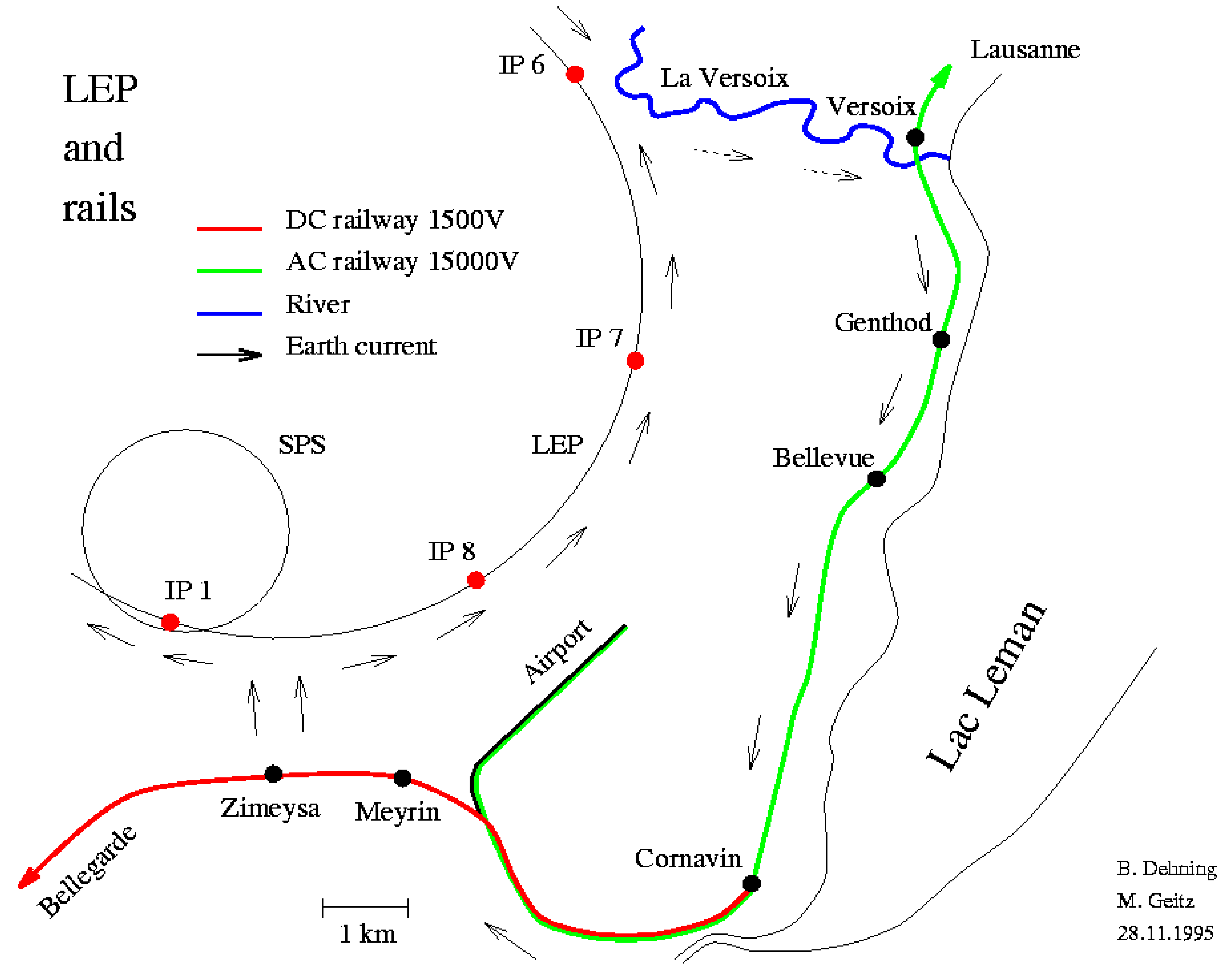
water level of Lake Léman



train schedule TGV
Geneva – Paris

Correlation between trains and LEP energy





LEP
and
rails
DC railway 1500V
AC railway 15000V
River
Earth current
IP 6
IP 7
IP 8
IP 1
SPS
LEP
La Versoix
Versoix
Lausanne
Genthod
Bellevue
Lac Leman
Airport
Cornavin
Zimeysa
Meyrin
Bellegarde
1 km
B. Dehning
M. Geitz
28.11.1995

Observables and "pseudo-observables" :

Couplings, masses, mixing angles, etc. are not (directly) physical observables

$\Rightarrow$ "pseudo-observables"

Actual observables: σ, BRs, asymmetries, . . .

Need deconvolution procedure (unfolding) in order to determine masses, partial widths, etc. from measured cross sections

$\Rightarrow$ pseudo-observables are not strictly model-independent

What is a suitable definition of model parameters?

need some compromise between

– simple interpretation within given model

– model independence

Particle masses: relatively small model dependence

Couplings, mixing angles, etc.: relatively large model dependence

Example: what is the mass of an unstable particle (M_W, M_Z, ...)?

In QFT: mass determined by pole of propagator

Which pole?

– real part of complex pole, $\mathcal{M}^2 = \overline{M}^2 - i\overline{M}\,\overline{\Gamma}$, or

– pole of real part, $\widetilde{M}^2$

From 2-loop order on:

real part of complex pole $\neq$ pole of real part

$$\delta\overline{M}^2_{(2)} = \delta\widetilde{M}^2_{(2)} + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\}\,\mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}_{\text{gauge-parameter dependent!}}$$

$\Rightarrow$ Only the complex pole is gauge-invariant

Quark masses: on-shell (pole) masses affected by large QCD uncertainties
$\Rightarrow$ $\overline{\mathrm{MS}}$ masses usually preferable

Once a suitable definition is adopted, how does one measure this mass?

In practice: comparison data – Monte Carlo

$\Rightarrow$ dependence on the other parameters of the model

$\Rightarrow$ The experimental value of M_{Z} (slightly) depends on the value of the Higgs mass

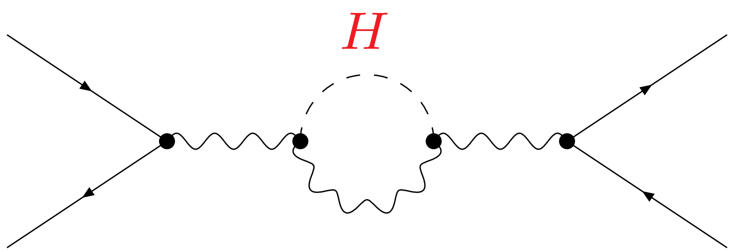
($\delta M_{\mathsf{Z}} = \pm 0.2$ MeV for 100 GeV $\leq M_{\mathsf{H}} \leq$ 1 TeV)

Comparison of electroweak precision data with theory predictions:

EW precision data:	Theory:
$M_Z, M_W, \sin^2\theta_{eff}^{lept}, \ldots$	SM, MSSM, . . .

$\Downarrow$

Test of theory at quantum level: sensitivity to loop corrections



$\Downarrow$

Indirect constraints on unknown parameters: M_H, . . .

Effects of “new physics”?

Indirect determination of m_{t} from precision data:

$$m_{\mathrm{t}} = 180.3^{+11.7}_{-9.2}\ \mathrm{GeV}$$

Direct measurement:

$$m_{\mathrm{t}} = 174.3 \pm 5.1\ \mathrm{GeV}$$

Leading corrections to precision observables:

$$\sim m_{\mathrm{t}}^2$$
$$\sim \ln M_{\mathrm{H}}$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

– experimental error of input parameters: m_{t}, $\Delta\alpha_{\mathrm{had}}$, ...

Comparison of current experimental errors with anticipated precision at Run IIA of the Tevatron, the LHC, and a LC with and without low-energy running mode (GigaZ):

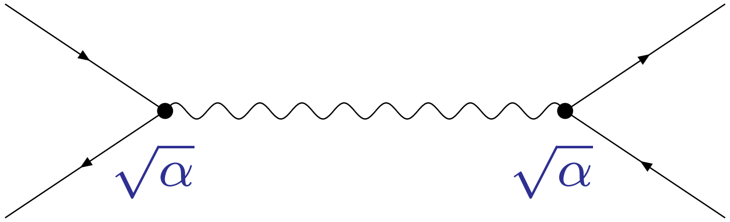
	now	Tev. Run IIA	LHC	LC	GigaZ
$\delta \sin^2\theta_{\text{eff}}(\times 10^5)$	16	78	14–20	—	1.3
δM_{W} [MeV]	34	27	15	10	7
δm_{t} [GeV]	5.1	2.7	1–2	0.1	0.1
δm_{h} [MeV]	—	—	200	50	50

$\Rightarrow$ Large improvement at next generation of colliders

Perturbative evaluation of QFTs based on Feynman diagrams:

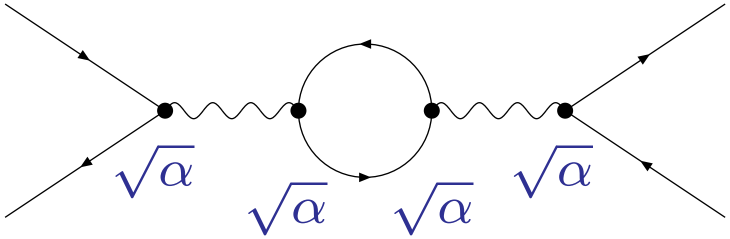
Example: $2 \to 2$ process

Lowest order (tree level):



$\alpha \approx \frac{1}{137}$

One-loop diagrams:



$+ \quad \ldots$

$\mathcal{O}(\alpha)$ relative to lowest order

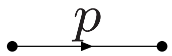
...

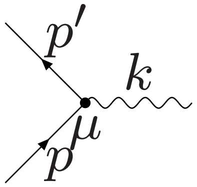
Feynman rules of specific model (SM, MSSM, . . .) for propagators and vertices

$\Rightarrow$ mathematical expression

Example: Feynman rules of QED

- photon propagator (Feynman gauge): $\Delta_{\mu\nu}(k) = \dfrac{-ig_{\mu\nu}}{k^2 + i\varepsilon} =$ 

- electron/positron propagator: $S_F(p) = \dfrac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon} =$ 

- electron–positron–photon vertex: 

$= (2\pi)^4 \delta^4(p - p' - k)(ie\gamma_\mu)$

- ingoing and outgoing lines:

electron:	 $= u(p,\sigma)$	 $= \overline{u}(p,\sigma)$
positron:	 $= \overline{v}(p,\sigma)$	 $= v(p,\sigma)$
photon:	 $= \epsilon_\mu(p,\lambda)$	 $= \epsilon^*_\mu(p,\lambda)$

- minus sign for fermion loops, combinatorial factor

- integration over momenta of internal particles

  δ-distr. $\Rightarrow$ momentum conservation

  for loop diagrams $\Rightarrow \int \frac{d^4q}{(2\pi)^4}$ for each loop

Calculation of loop diagrams:

Evaluation of Dirac algebra, Dirac traces:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \qquad \Rightarrow \mathrm{Tr}\,\{\gamma_\mu \gamma_\nu\} = 4g_{\mu\nu}$$

Tensor-integral decomposition:

$$\int \frac{d^Dq}{(2\pi)^D} \frac{q_\mu}{(q^2 - m^2)\,[(q+k)^2 - m^2]} = a\; k_\mu$$

$\Rightarrow$ scalar integrals:

$$\begin{aligned}
A_0(m^2) &= \frac{1}{i\pi^2} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2 - m^2} \\
B_0(p^2, m_1^2, m_2^2) &= \frac{1}{i\pi^2} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{(q^2 - m_1^2)\left[(q+k)^2 - m_2^2\right]} \\
\ldots &
\end{aligned}$$

Evaluation of standard scalar integrals:

One-loop: reducible to A_0, B_0, C_0, D_0:

$$A_0, B_0 : \log(\sqrt{\ldots}); \quad C_0, D_0 : \log, \;\; \mathrm{Li}_2(x) = \int_0^1 dt \frac{\ln t}{t - 1/x}$$

Two-loop: in general not expressible in terms of polylogarithmic functions [*R. Scharf '91*]

$\Rightarrow$ numerical evaluation, asymptotic expansions, ...

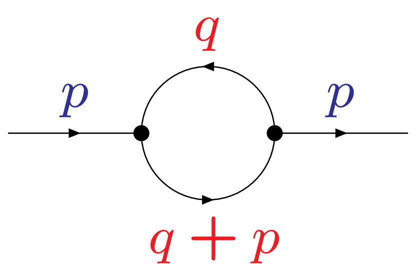
On-shell conditions for external legs

$\Rightarrow$ S-Matrix elements

$|\mathcal{M}^2|$, phase-space integration

$\Rightarrow$ cross sections, life times

Example of loop integral:



$$\sim \int d^4q \frac{1}{(q^2 - m_1^2)\left[(q+k)^2 - m_2^2\right]}$$

$$q \to \infty: \quad \sim \int_0^\infty \frac{q^3 dq}{q^4} = \int_0^\infty \frac{dq}{q} \quad \to \infty$$

$\Rightarrow$ integral diverges for large q!

$\Rightarrow$ theory in this form not physically meaningful

$\Rightarrow$ further concept needed: renormalization

Renormalizable theories: infinities can consistently be absorbed into parameters of theory

Two step procedure:

Regularization:

theory modified such that expressions become mathematically meaningful

$\Rightarrow$ "regulator" introduced, removed at the end

e.g. cut-off in loop integral

$$\int_0^\infty d^4q \;\to \int_0^\Lambda d^4q; \quad \Lambda \to \infty \text{ at the end}$$

technically more convenient: dimensional regularization

$$\int d^4q \;\to \int d^Dq, \quad D = 4 - \varepsilon; \quad D \to 4 \text{ at the end}$$

Renormalization:

original "bare" param. replaced by renormalized param. + counterterms

reparameterization:

$$\underbrace{g_0}_{\substack{\text{bare}\\ \text{parameter}}} = \underbrace{g}_{\substack{\text{renormalized}\\ \text{parameter}}} + \underbrace{\delta g}_{\text{counterterm}}$$

Renormalizable theory: divergencies compensated by counterterms

Renormalization:

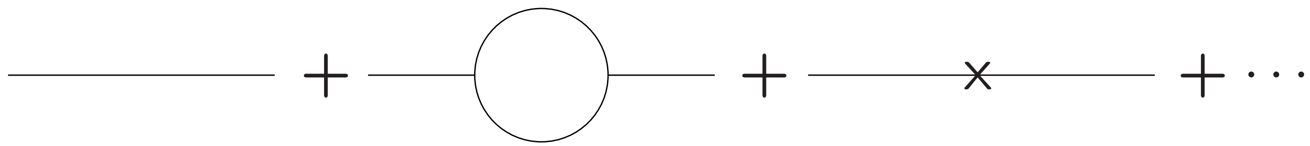
- absorption of divergencies
- determination of physical meaning of parameters order by order in perturbation theory

<u>Example:</u>

mass renormalization, $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:



$$p^2 - m^2 \qquad\qquad \Sigma(p^2) \qquad\qquad - \, \delta m^2$$

pole of the propagator: $p^2 - m^2 + \Sigma(p^2) - \delta m^2 = 0$

If renormalization such that

$$\delta m^2 = \Sigma(p^2)\Big|_{p^2=m^2} \qquad \text{“on-shell renormalization”}$$

$\Rightarrow$ pole of propagator for $p^2 = m^2 \Rightarrow m$ corresponds to physical mass

Counterterm contains divergence:

expansion in $\varepsilon \equiv 4 - D$, 1-loop:

$$\delta m^2 = \underbrace{a\frac{1}{\varepsilon}}_{\substack{\text{divergent}\\ \text{for } D \to 4}} + \underbrace{b\varepsilon^0}_{\text{finite}} + \underbrace{c\varepsilon}_{\substack{\to 0\\ \text{for } D \to 4}} + \ldots$$

Other renormalization prescription:

$$\delta m^2 = a\frac{1}{\varepsilon} \qquad \text{“minimal subtraction” (MS)}$$

Different renormalization prescriptions $\Leftrightarrow$ different parametrizations of result

Physical result must be independent of renormalization prescription, e.g.

$$\Sigma^{\text{ren}}(p^2 = \underbrace{\mu^2}_{\text{renormalization scale}}) \equiv \Sigma(p^2 = \mu^2) - \delta m^2 = 0$$

$\Rightarrow$ for physical observable σ:

$$\mu \frac{d}{d\mu} \sigma \left(p_i, g(\mu), m(\mu), \mu \right) = 0$$

"renormalization group equation"

$\Rightarrow$ explicit dependence of σ on μ compensated by μ-dep. of $g(\mu)$, $m(\mu)$

$$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln \left(\frac{\mu^2}{\mu_0^2} \right), \quad \beta = \text{const.}$$

$g(\mu)$: "running coupling", $m(\mu)$: "running mass"

$\Rightarrow$ effective coupling $\alpha(Q)$ varies with scale Q

<u>QED:</u>

$$\alpha(Q) = \frac{\alpha(Q_0)}{1 + 4\pi\beta\,\alpha(Q_0)\,\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$\beta < 0$ in QED: $\beta \equiv b/(16\pi^2)$, $b = -4/3$ (1-loop)

$\Rightarrow$ $\alpha(Q)$ increases for increasing Q ($Q > Q_0$), $\alpha(m_e) \approx 1/137$, $\alpha(M_Z) \approx 1/128$

scale Λ_{QED} where QED coupling becomes strong:

$$\alpha(Q) = \frac{1}{4\pi\beta\,\ln\left(\frac{Q^2}{\Lambda_{QED}^2}\right)}$$

$\Lambda_{QED} \approx 10^{274}$ GeV $\gg M_P (\approx 10^{19}$ GeV$)$

$\Rightarrow$ QED is weakly coupled up to the Planck-scale

QCD:

$$b^{\mathrm{QCD}} = \frac{1}{3}(11N_C - 2n_f) > 0 \text{ for } n_f = 6$$

$\Rightarrow$ $\alpha_{\mathrm{s}}(Q) \to 0$ for $Q \to \infty$: "asymptotic freedom"

For large Q: perturbation theory applicable,
description in terms of quarks and gluons useful

$\alpha_{\mathrm{s}}(Q)$ becomes strong for small Q:

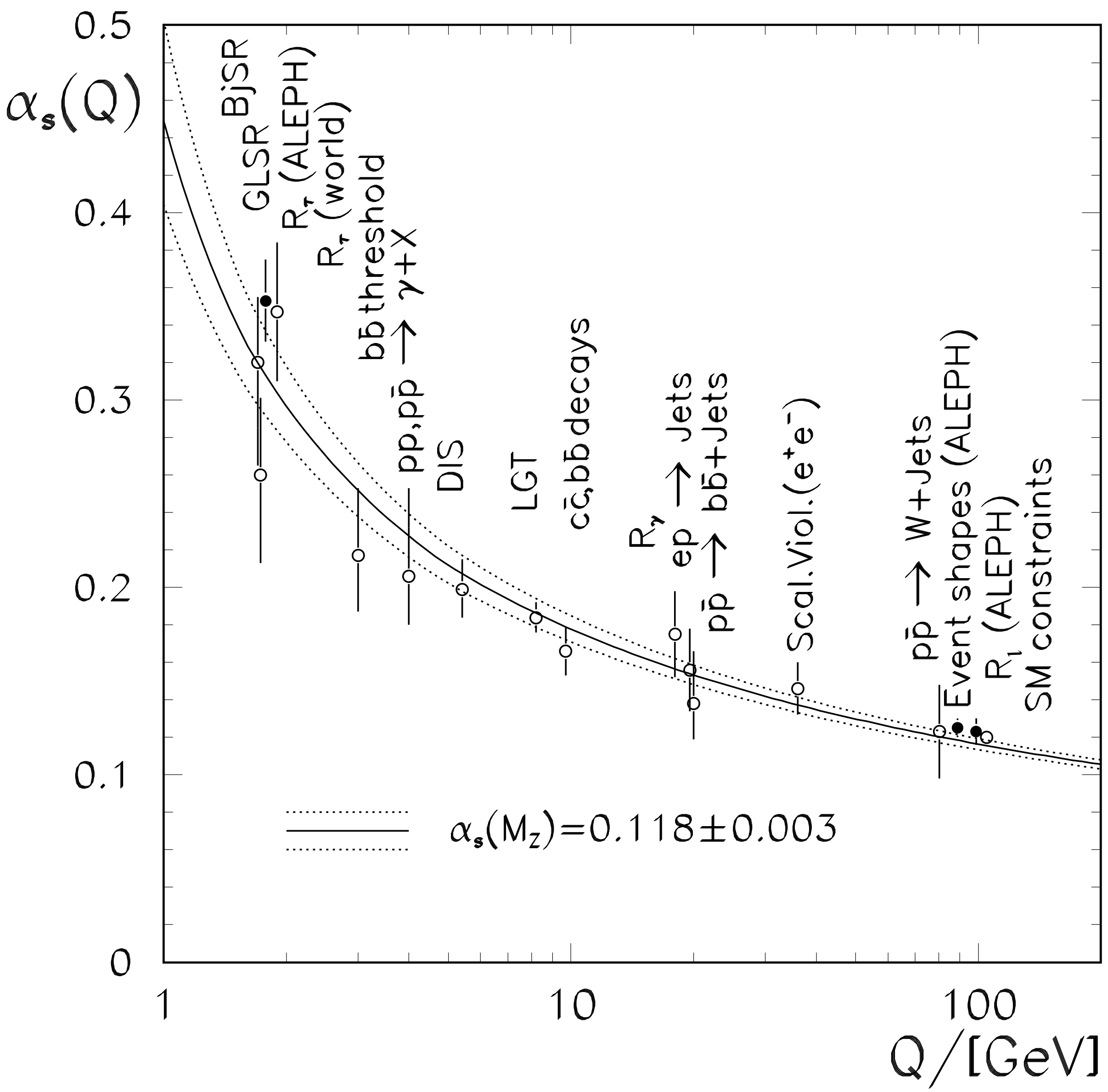
quarks and gluons bound together to form colorless hadrons

$\Rightarrow$ "confinement"

$$\alpha_{\mathrm{s}}(Q) = \frac{12\pi}{(11N_C - 2n_f)\ln\left(\frac{Q^2}{\Lambda_{\mathrm{QCD}}^2}\right)}$$

For scales of $\mathcal{O}(\Lambda_{\mathrm{QCD}}) \approx 100$ MeV: $\alpha_{\mathrm{s}}(Q)$ so large that perturbation theory not applicable

Running of $\alpha_s(Q)$:



⇒ Very good agreement with QCD prediction

Constraints on the Standard Model Higgs from electroweak precision tests

Theoretical predictions for M_{W}, $\sin^2\theta_{\mathrm{eff}}$, ... in the SM:

Definition of Fermi constant G_μ via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_{\mathrm{e}}^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_{\mathrm{W}}^2}\right)(1+\Delta q)$$

Δq: QED corrections in Fermi Model [*R. Behrends, R. Finkelstein, A. Sirlin '56*] [*T. van Ritbergen, R. Stuart '99*] [*M. Steinhauser, T. Seidensticker '99*]

Comparison with SM prediction for muon decay:

$$\Rightarrow \quad M_{\mathrm{W}}^2\left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1+\underbrace{\Delta r}_{\text{loop corrections}}),$$

$\Rightarrow$ Theoretical prediction for M_{W} in terms of M_{Z}, α, G_μ, $\Delta r(m_{\mathrm{t}}, M_{\mathrm{H}}, \ldots)$

Effective couplings at the Z resonance:

$$\Rightarrow \qquad \sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \frac{\text{Re}\, g_V}{\text{Re}\, g_A}\right) = \left(1 - \frac{M_\text{W}^2}{M_\text{Z}^2}\right)\kappa(M_\text{W}^2)$$

One-loop result for M_W in the SM:

[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \quad \Delta\alpha \quad - \quad \frac{c_\text{W}^2}{s_\text{W}^2}\Delta\rho \quad + \quad \Delta r_{\text{rem}}(M_\text{H})$$

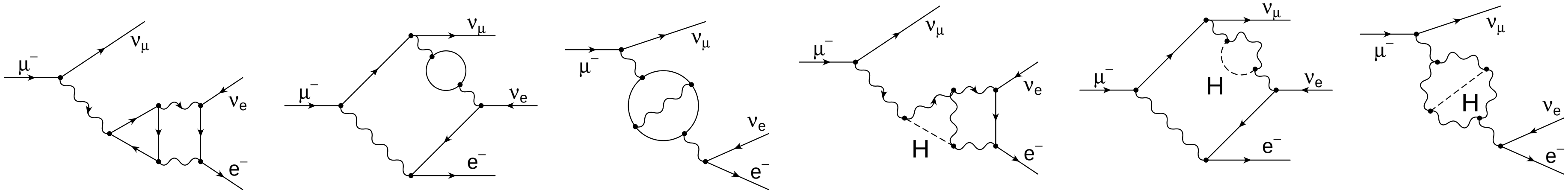
$$\sim \log\frac{M_\text{Z}}{m_f} \qquad \qquad \sim m_\text{t}^2$$

$$\sim 6\% \qquad \qquad \sim 3.3\% \qquad \qquad \sim 1\%$$

Complete electroweak two-loop result:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

[*M. Awramik, M. Czakon '02, '03*] [*A. Onishchenko, O. Veretin '02*]



Estimate of remaining uncertainties from unknown higher orders in M_W and $\sin^2\theta_\mathrm{eff}$:

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]

M_W: $\mathcal{O}(G_\mu^3 m_\mathrm{t}^4 M_\mathrm{Z}^2)$, $\mathcal{O}(G_\mu^2 \alpha_\mathrm{s} m_\mathrm{t}^2 M_\mathrm{Z}^2)$, $\mathcal{O}(\alpha\alpha_\mathrm{s}^3)$

$$\Delta M_\mathrm{W}^\mathrm{theo} \approx 4 \text{ MeV}$$

$\sin^2\theta_\mathrm{eff}$: non-leading fermionic two-loop contributions, purely bosonic two-loop contributions, $\mathcal{O}(G_\mu^3 m_\mathrm{t}^4 M_\mathrm{Z}^2)$, $\mathcal{O}(G_\mu^2 \alpha_\mathrm{s} m_\mathrm{t}^2 M_\mathrm{Z}^2)$, $\mathcal{O}(\alpha\alpha_\mathrm{s}^3)$

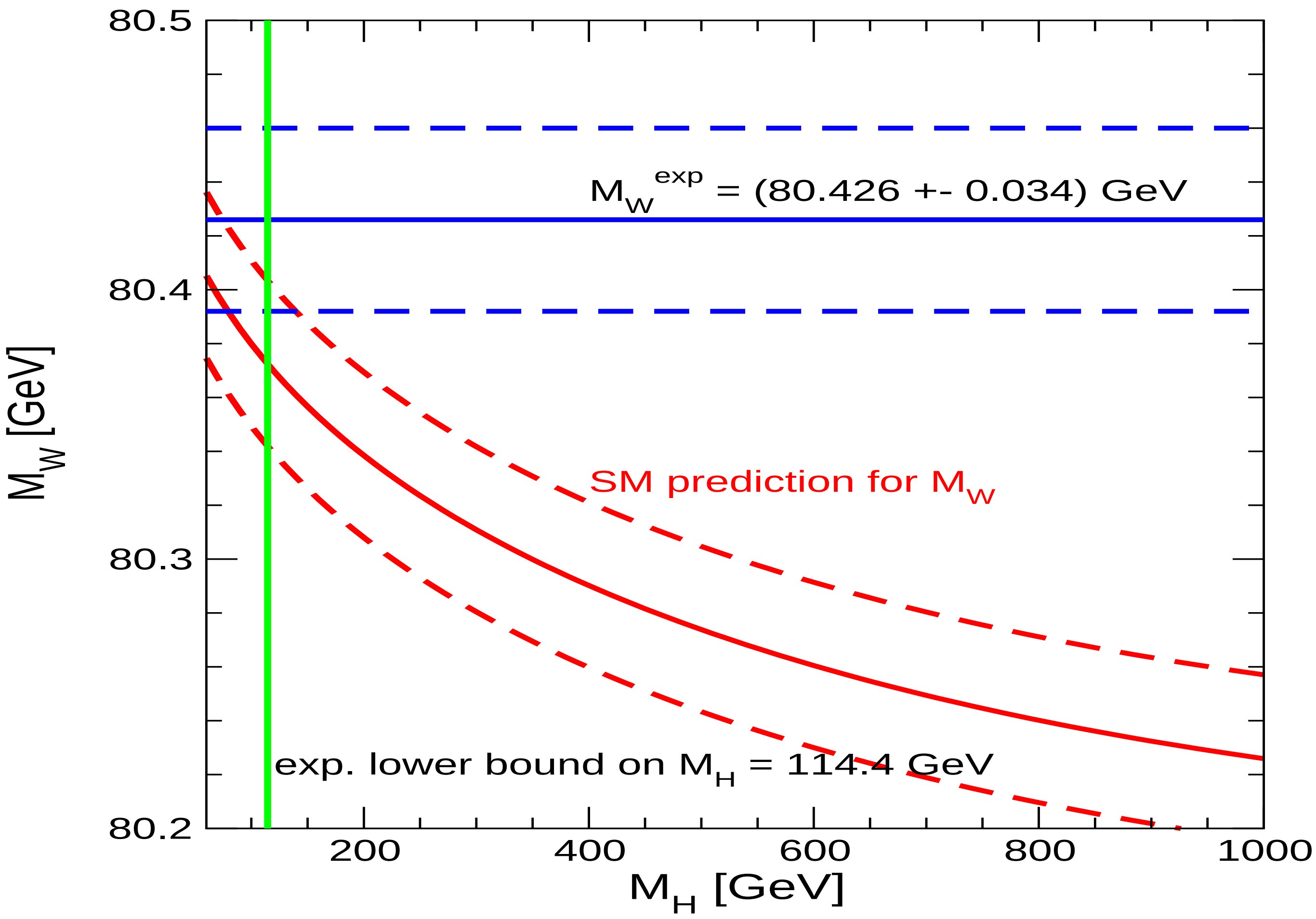
$$\Delta \sin^2\theta_\mathrm{eff}^\mathrm{theo} \approx 6 \times 10^{-5}$$

Uncertainties from experimental errors of input parameters:

$$\Delta m_\mathrm{t} = \pm 5.1 \text{ GeV} \Rightarrow \Delta M_\mathrm{W} \approx \pm 31 \text{ MeV}, \quad \Delta \sin^2\theta_\mathrm{eff} \approx \pm 16 \times 10^{-5}$$

SM prediction for M_W vs. experimental result:

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]



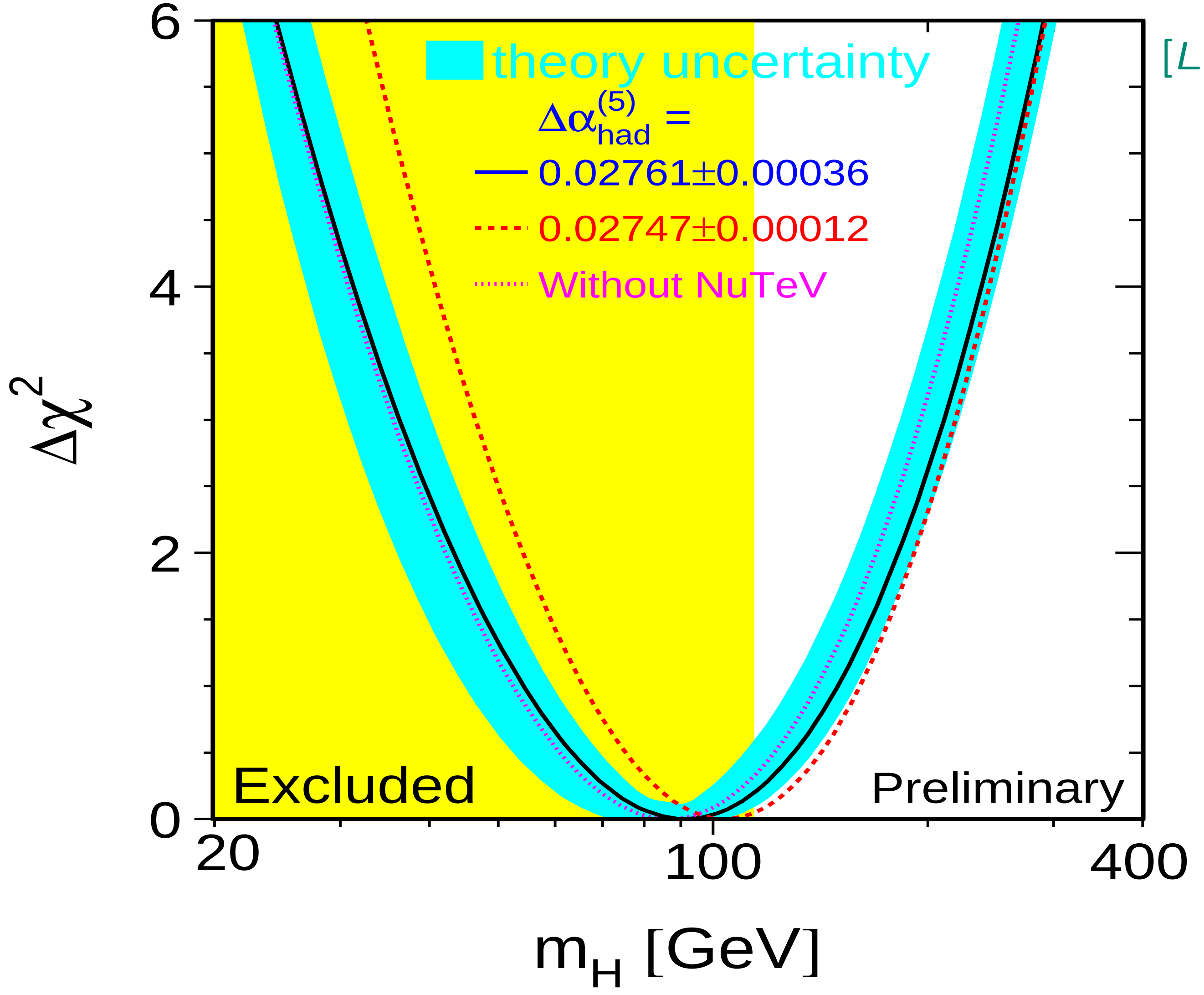
$\Rightarrow$ Light Higgs preferred

$M_W^{\text{exp}} = 80.426 \pm 0.034$ GeV, $m_t^{\text{exp}} = 174.3 \pm 5.1$ GeV,

$M_H > 114.4$ GeV, 95% C.L.

$\Rightarrow$ 1σ regions hardly overlap

Global fit to all data in the SM:



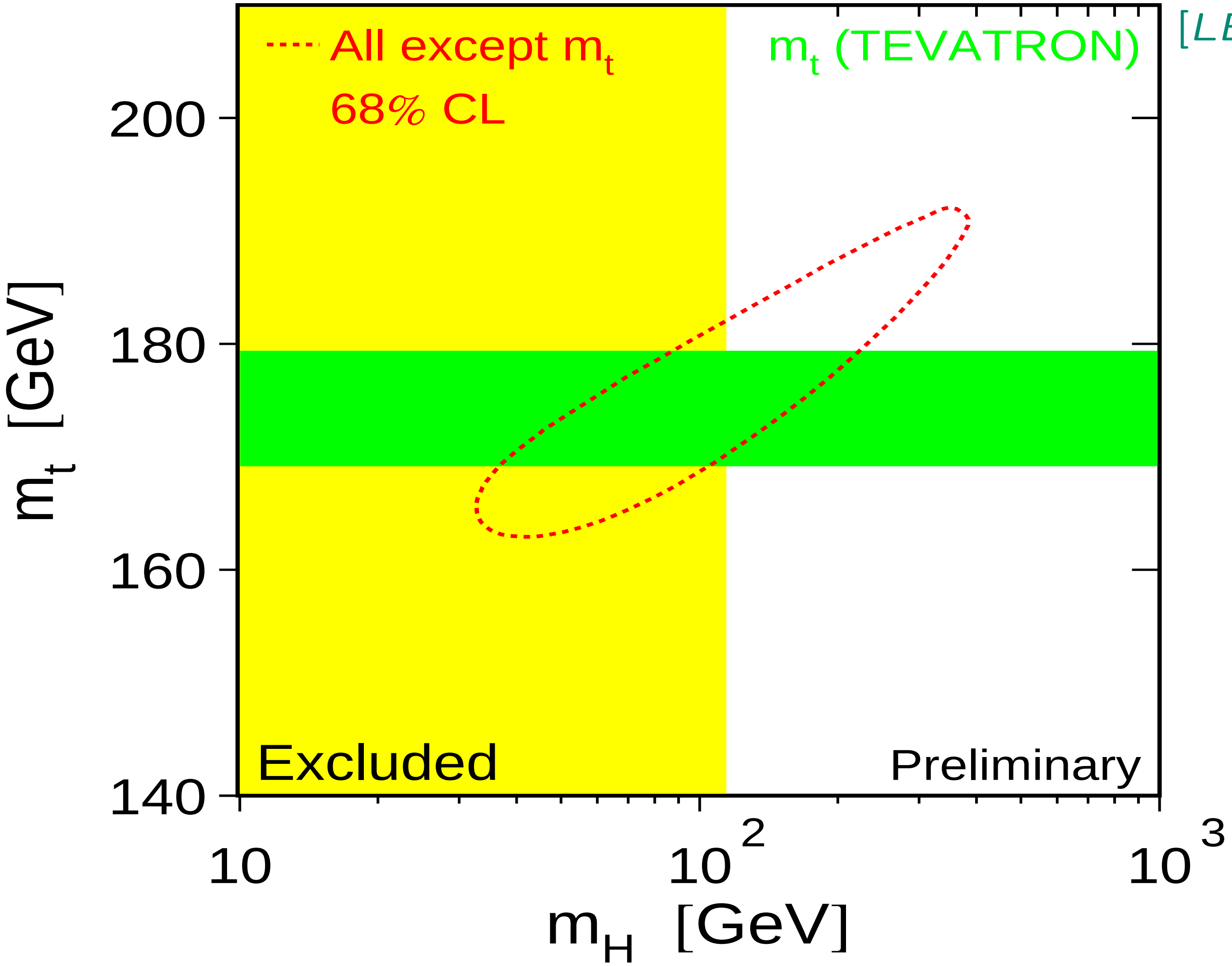
[*LEPEWWG '03*]

$\Rightarrow$ $M_{\rm H} = 91^{+58}_{-37}$ GeV, $\quad M_{\rm H} < 211$ GeV, 95% C.L.

$\Rightarrow$ Preference for light Higgs

Interpretation of fit result:

Strong correlation between M_H and m_t in the fit:



[*LEPEWWG '03*]

$\Rightarrow$ Upward shift of m_t by 1σ would shift M_H bound by 35%

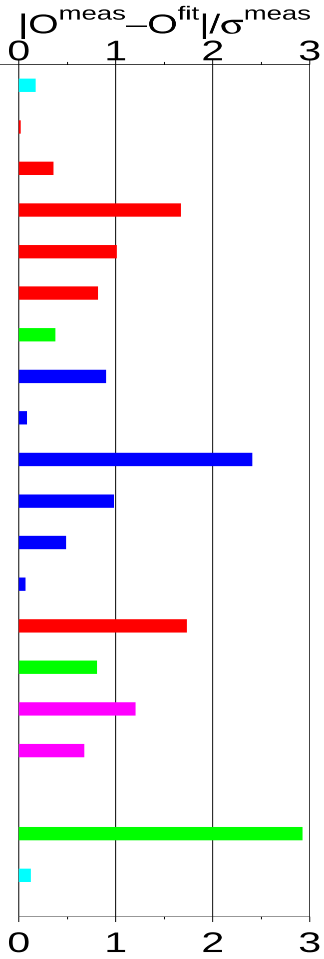
Basic assumption in global fit:

SM provides correct description of experimental data

Comparison of SM prediction with the data: [*LEPEWWG '03*]

Summer 2003

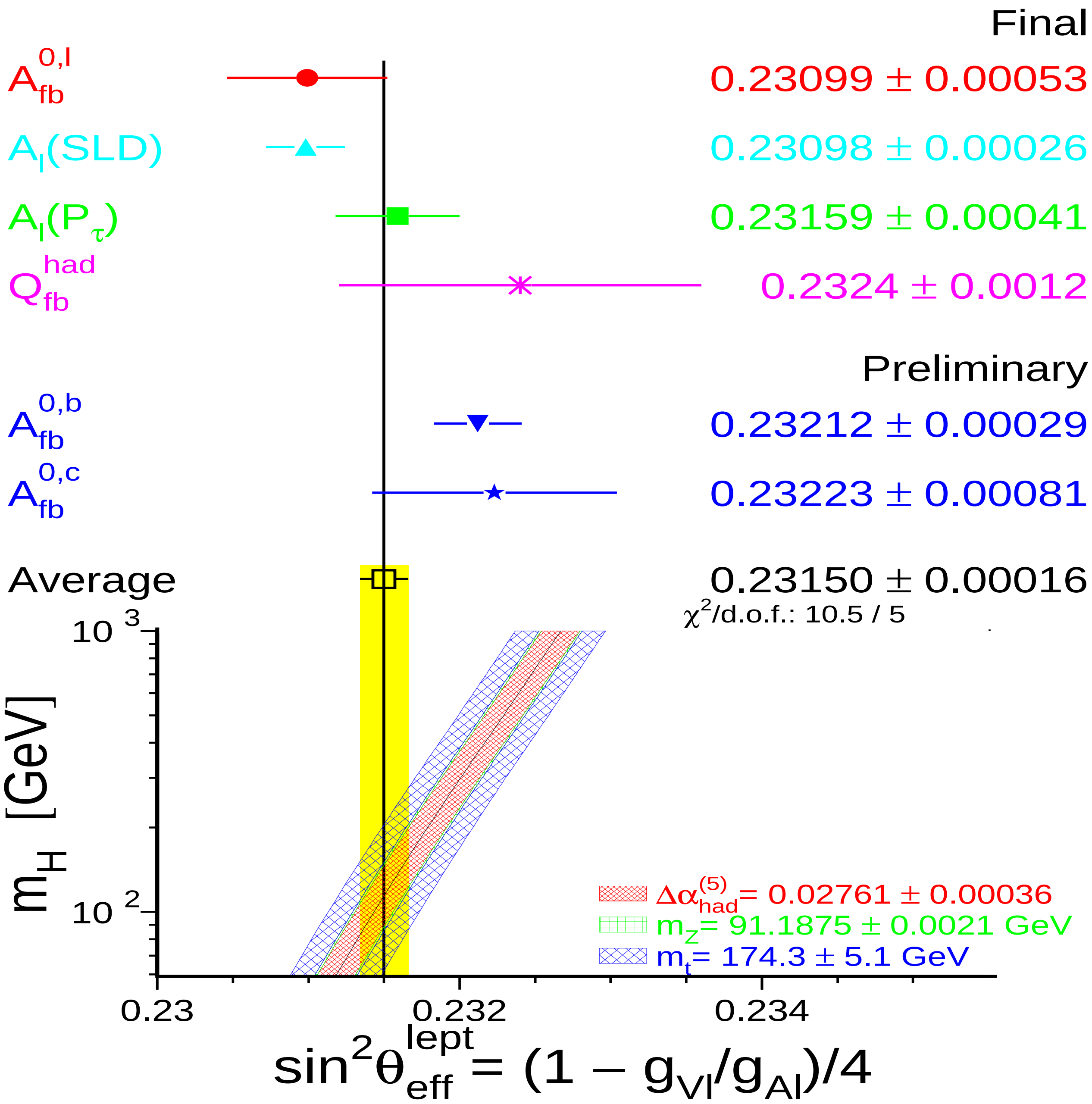
	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4960
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01636
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1477
R_b	0.21638 ± 0.00066	0.21579
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1036
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0740
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1477
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.426 ± 0.034	80.385
Γ_W [GeV]	2.139 ± 0.069	2.093
m_t [GeV]	174.3 ± 5.1	174.3
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	0.2229
$Q_W(Cs)$	-72.84 ± 0.46	-72.90



Overall fit probability (quality of the fit): 4.4% (27%) with (without) NuTeV

The effective weak mixing angle $\sin^2\theta_{\mathrm{eff}}$:

[*LEPEWWG '03*]



No convincing new physics model for “explaining” deviations from SM in $\sin^2\theta_{\text{eff}}$ and ν–nucleon scattering from NuTeV

Note, however:

if NuTeV result interpreted as deviation in $Z\nu\bar{\nu}$ coupling:

(ν neutral current rate)/prediction:

0.995 ± 0.003	LEP1 lineshape (invisible Z width)
0.988 ± 0.004	NuTeV

Need some patience until experimental clarification:

A_{LR}, A_{FB}:	LC with GigaZ option?
ν–nucleon scatt.:	ν factory?

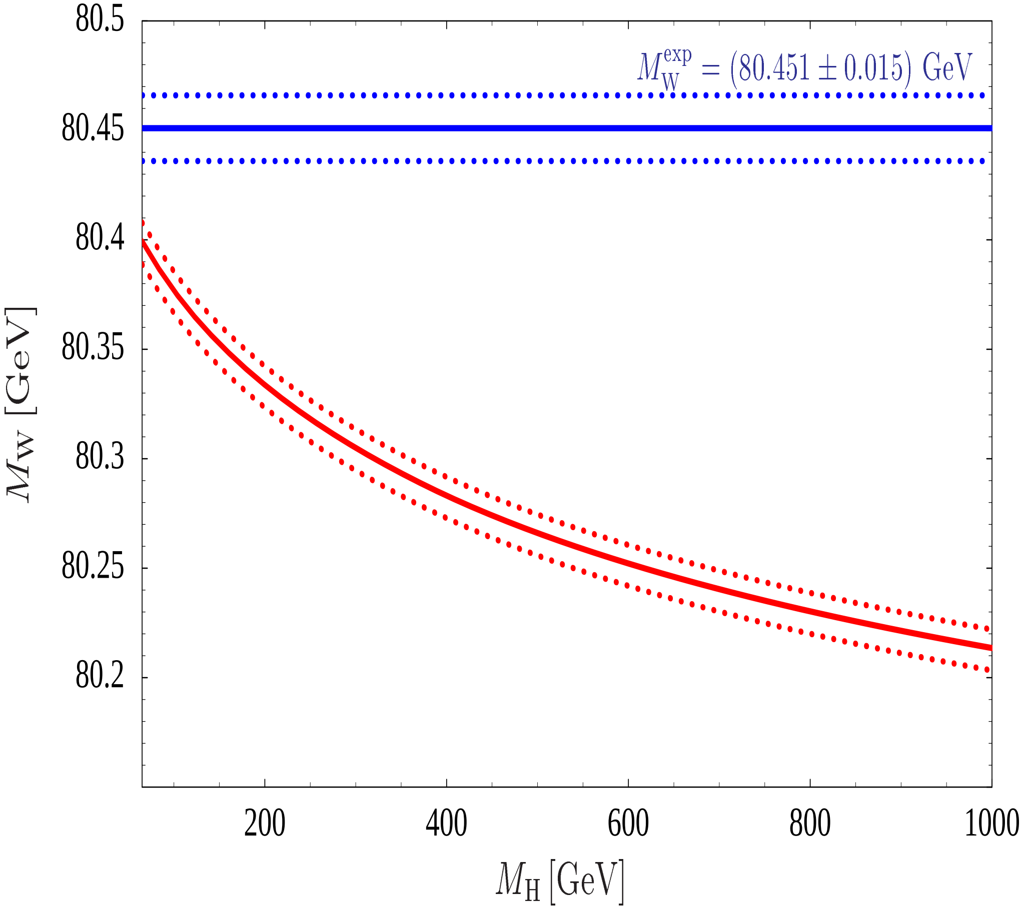
If hadronic data on A_{FB} and NuTeV data on ν–nucleon scattering were removed from fit:

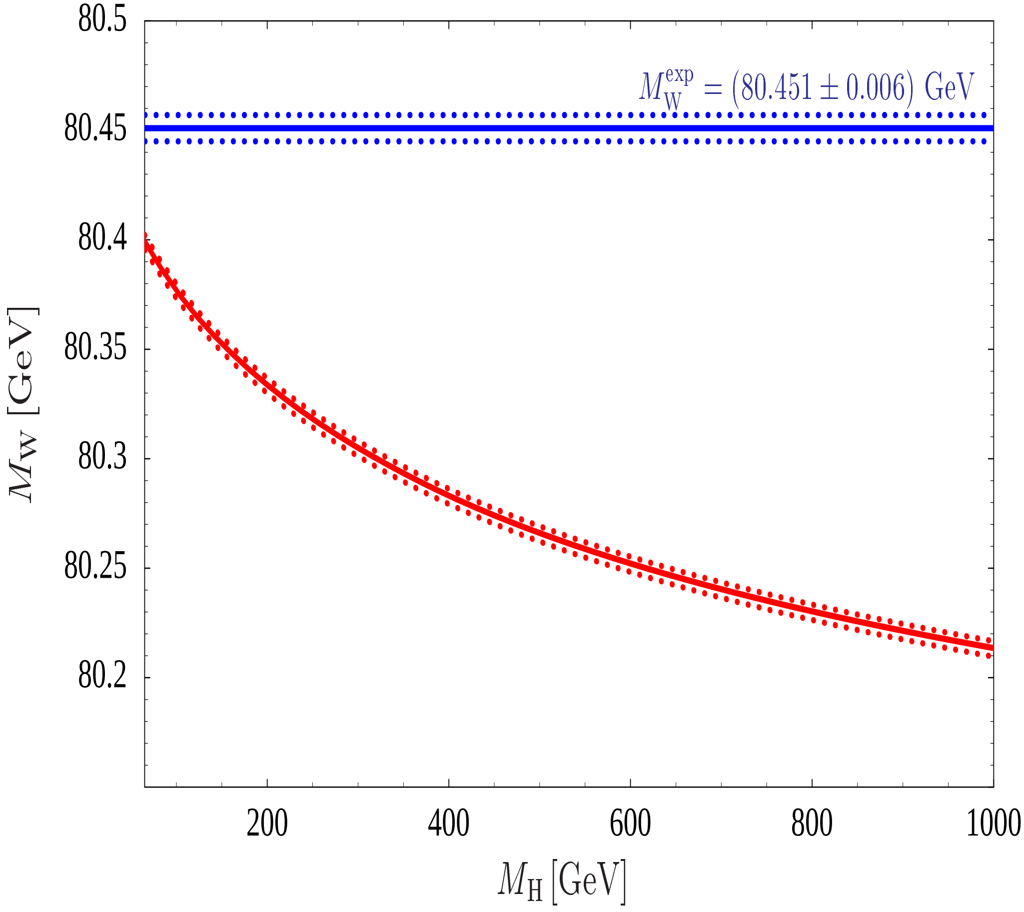
much better fit quality, probability $\approx 70\%$

stronger preference for light Higgs: $M_{\text{H}} \lesssim 150$ GeV, 95% C.L.

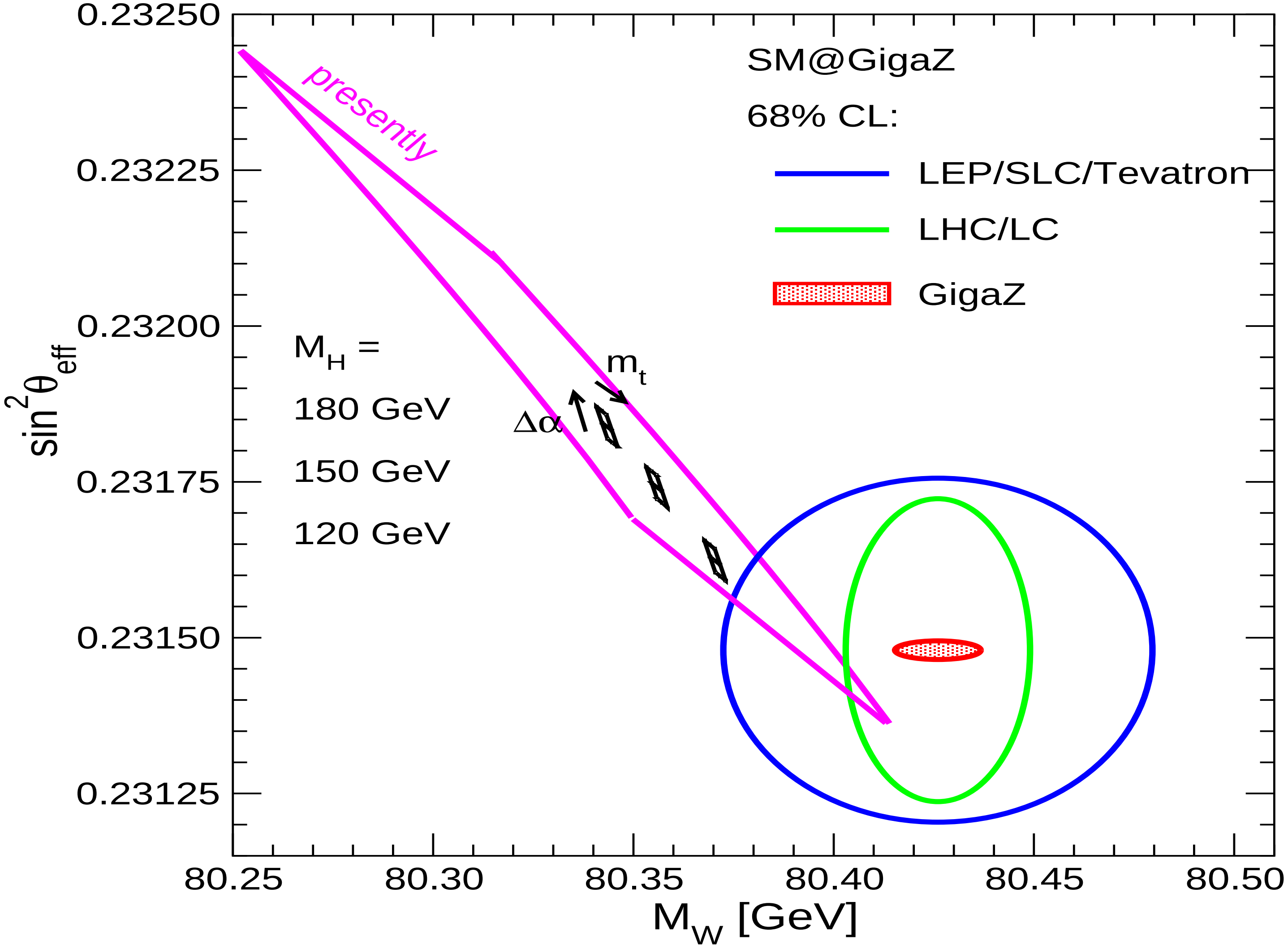
Prospects for Tevatron Run II, LHC (left), LC with high luminosity in low-energy mode, GigaZ (right):

[*A. Freitas, W. Hollik, W. Walter, G.W. '02*]





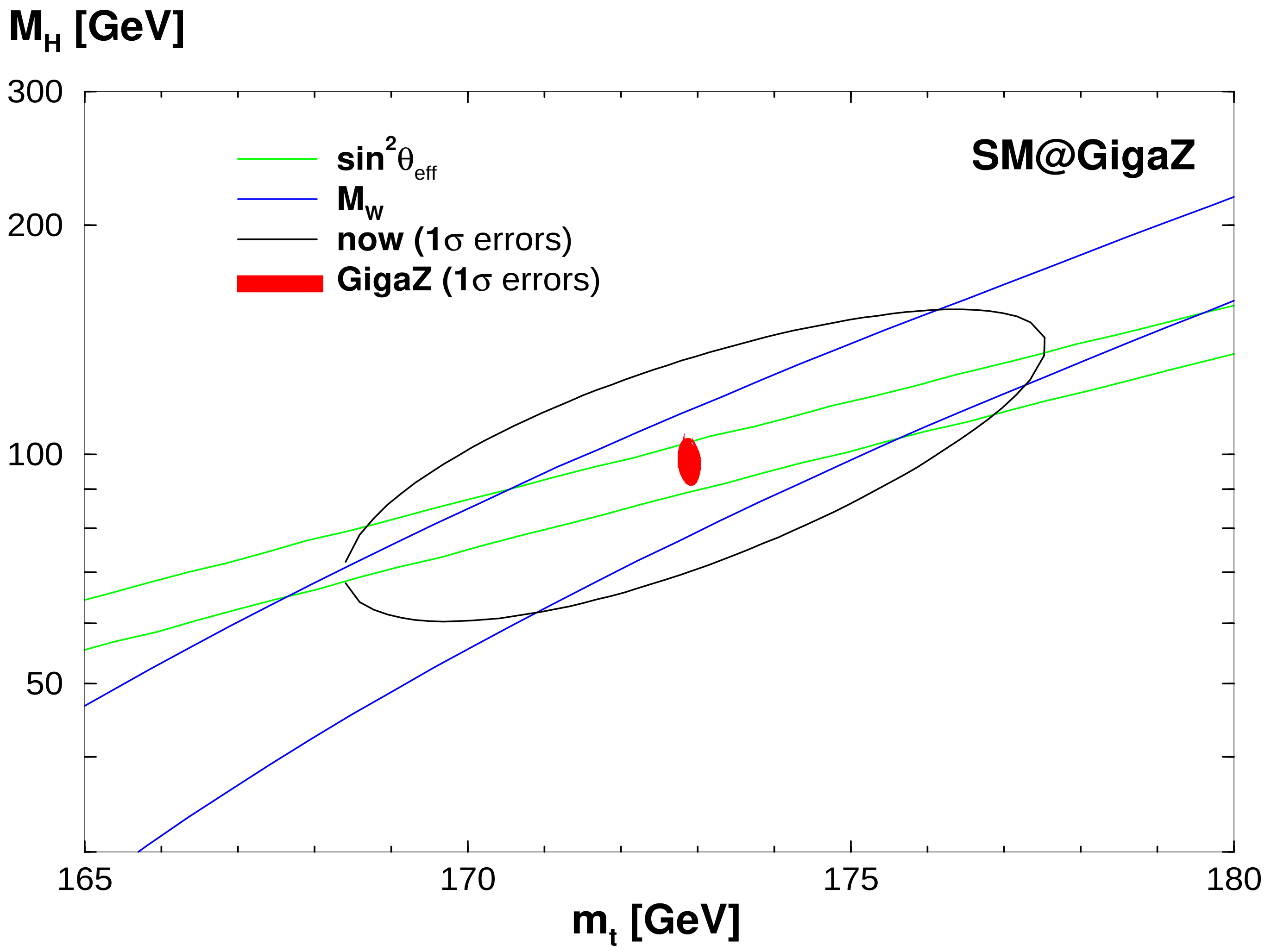
SM prediction for M_W and $\sin^2\theta_{\text{eff}}$ vs. current experimental result (LEP2/Tevatron) and prospective accuracies at the LHC and a LC with low-energy option (GigaZ): [*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]



⇒ Sensitive test of the electroweak theory

Indirect determination of m_t, M_H at GigaZ:

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]



$\Rightarrow$ At GigaZ: indirect sensitivity to M_H similar to todays indirect sensitivity to m_t

Quark mixing and $\mathcal{CP}$-violation

Weak eigenstates (d', s', b') of quarks differ from mass eigenstates d, s, b:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V_{\mathsf{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} .$$

V_{CKM}: unitary transformation, Cabibbo-Kobayashi-Maskawa (CKM) matrix

GIM Mechanism:

unitarity of CKM-matrix, $V^\dagger_{\mathsf{CKM}} V_{\mathsf{CKM}} = 1$
$\Rightarrow$ no flavor changing neutral current transitions at tree level

$b \to s\gamma$, ... are loop-induced in SM $\Rightarrow$ high sensitivity to new physics effects

$V^\dagger_{\mathsf{CKM}} V_{\mathsf{CKM}} = 1 \Rightarrow$ parameterized by 3 angles $\underbrace{+\ 1\ \text{phase}}_{\text{gives rise to } \mathcal{CP}\text{-violation in SM}}$

$\Rightarrow$ weak interaction violates $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{CP}$

SM: CKM phase is only source of $\mathcal{CP}$-violation

almost all models of new physics give new sources of $\mathcal{CP}$-violation

cosmology: baryogenesis ($\longrightarrow$ matter – antimatter imbalance in universe) requires further sources of $\mathcal{CP}$-violation beyond CKM phase

$\mathcal{CP}$-violation discovered in neutral kaon system (1964)

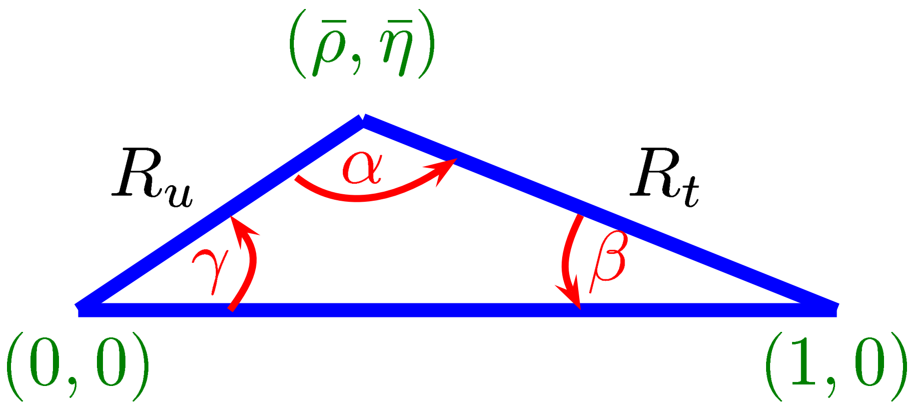
Recently ("b factories") also established in B-meson decays

Unitarity triangles:

$V_{\mathsf{CKM}}^{\dagger} V_{\mathsf{CKM}} = 1 \Rightarrow V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$, etc.

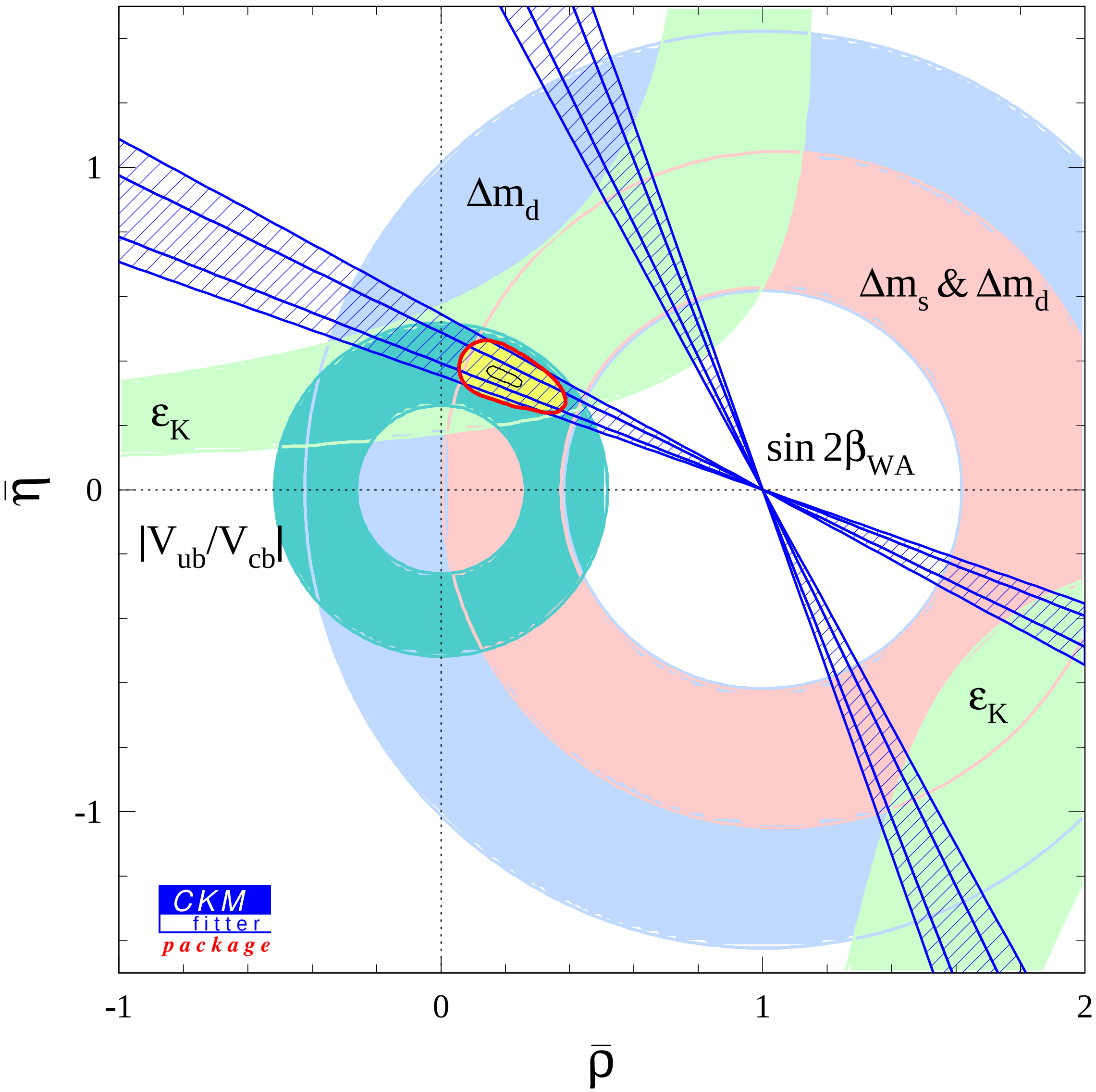
$\Rightarrow$ triangle relation in complex plane

after reparametrization:



Constraints on the parameters of the unitarity triangle from all observables ($\mathcal{CP}$-violating and $\mathcal{CP}$-conserving): *[ICHEP '02]*

⇒ first precision test of CKM picture



⇒ CKM matrix provides consistent description of flavor and $\mathcal{CP}$-violating processes

large ($\gtrsim$ 20%) new physics contributions to $\mathcal{CP}$-violation in flavor changing processes disfavored

The Standard Model cannot be the ultimate theory

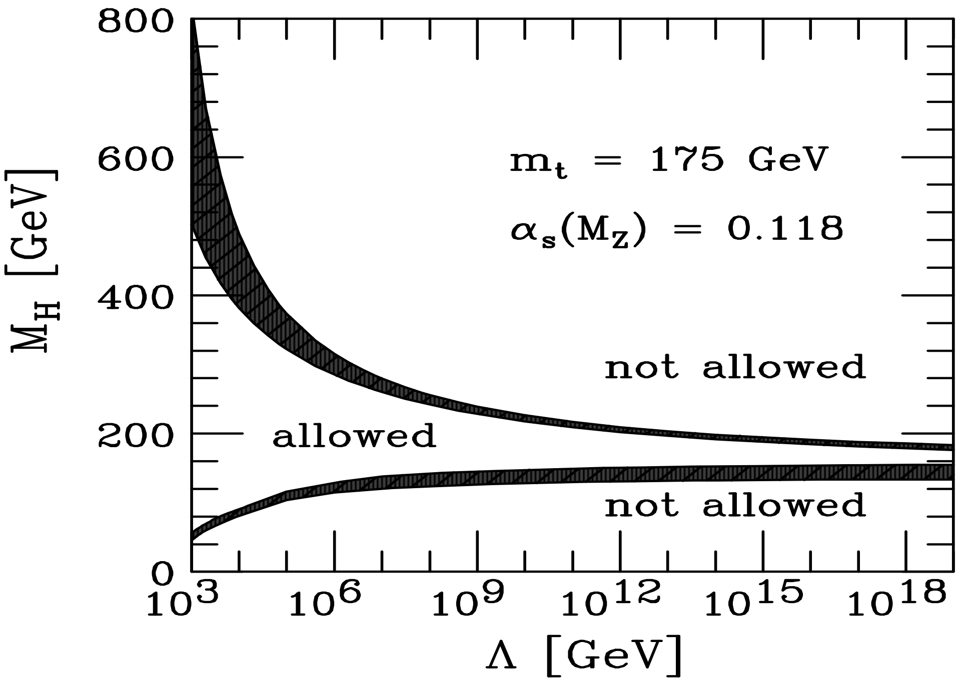
– doesn't contain gravity

– further problems: "hierarchy problem", . . .

Up to which energy scale Λ can it be valid?

– $\Lambda < M_{\mathrm{Pl}} \approx 10^{19}$ GeV (inclusion of gravity effects necessary)

– stability of Higgs potential:
[*T. Hambye, K. Riesselmann '97*]



– hierarchy problem: Higgs mass unstable w.r.t. large quantum corrections
$\Rightarrow \delta M_{\mathrm{H}}^2 \sim \Lambda^2$

Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Prediction of Supersymmetric extensions of the SM:

– superpartners for all SM fields, spins differ by $\frac{1}{2}$

– superpartners have same couplings as SM particles

– same number of degrees of freedom

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots \Rightarrow$ SUSY must be broken

$\Rightarrow$ solution of hierarchy problem:

quantum corrections of fermions and bosons compensate each other

Works because of coupling relations, same number of degrees of freedom

if mass splitting between SM particles and superpartners is not too large

$\Rightarrow$ need mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

$\Rightarrow$ Low-energy SUSY provides attractive solution of hierarchy problem

Further properties of SUSY theories

- Gauge coupling unification:

  Running of gauge couplings: unify at high scale $M_{\text{GUT}} \approx 10^{16}$ GeV (for $M_{\text{SUSY}} = \mathcal{O}(1\ \text{TeV})$)

  $\Leftrightarrow$ "Grand unified theories" (GUTs)

- Local SUSY $\Leftrightarrow$ gravity

- Radiative electroweak symmetry breaking:

  emerges naturally at scale $\sim 10^2$ GeV for 100 GeV $\lesssim m_{\text{t}} \lesssim$ 200 GeV

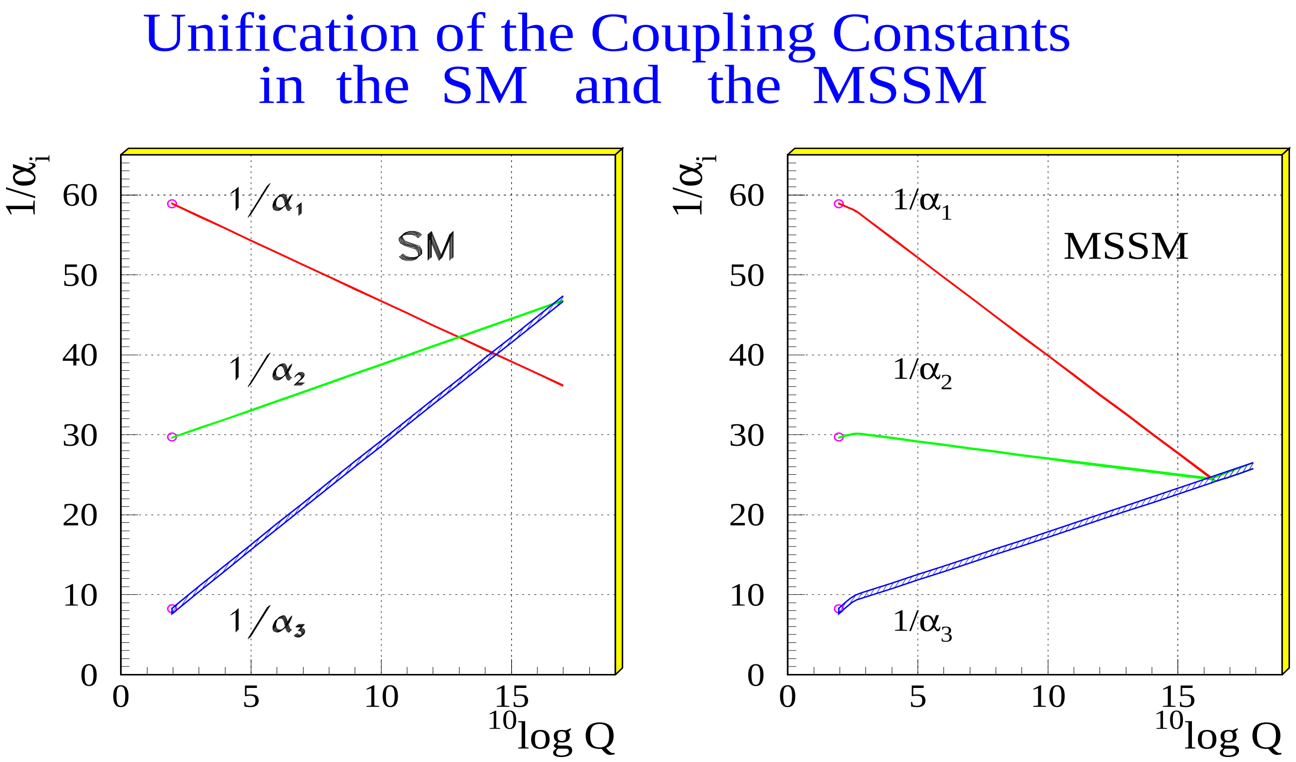
Gauge coupling unification:

Running of gauge couplings: $$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

Unification of couplings at high scale ⇔ "Grand unified theories" (GUTs)

E.g.: SO(10) GUTs, can naturally accomodate right-handed neutrinos

[U. Amaldi, W. de Boer, H. Fürstenau '91]



⇒ coupling constant unification in MSSM for $M_{\text{SUSY}} \lesssim 1$ TeV

Minimal SUSY extension of the SM contains only terms in the Lagrangian with even number of SUSY particles

$\Rightarrow$ additional symmetry: "R parity"

$\Rightarrow$ all SM particles have even R parity, all SUSY particles have odd R parity

Phenomenological consequences of R parity conservation:

- SUSY particles can only be produced in pairs, e.g. $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$
- Decay of SUSY particles into SM particles
  + odd number of SUSY particles
  $\Rightarrow$ Lightest SUSY particle (LSP) is stable
- All SUSY particles will eventually decay into LSP
  $\Rightarrow$ Decay cascades of heavy SUSY particles

- LSP stable

  $\Rightarrow$ some LSP must have survived from Big Bang

  $\Rightarrow$ Weakly interacting massive particle ("WIMP")
  (otherwise ruled out from bounds on exotic isotopes etc.)

  $\Rightarrow$ Candidate for cold dark matter in the Universe (if $m_{\mathrm{LSP}} \lesssim 1$ TeV)

- LSP neutral, uncolored $\Rightarrow$ leaves no traces in collider detectors

  $\Rightarrow$ Typical SUSY signatures: "missing energy"

Other recent proposals for solving hierarchy problem in models with extra spatial dimensions:

E.g.: Gravity in large extra dimensions

[*N. Arkani-Hamed, S. Dimopoulos, G. Dvali '98*]

Gravity propagates in δ large extra spatial dimensions

fundamental Planck scale in $4+\delta$-dimensional space, M_*, is of order 1 TeV

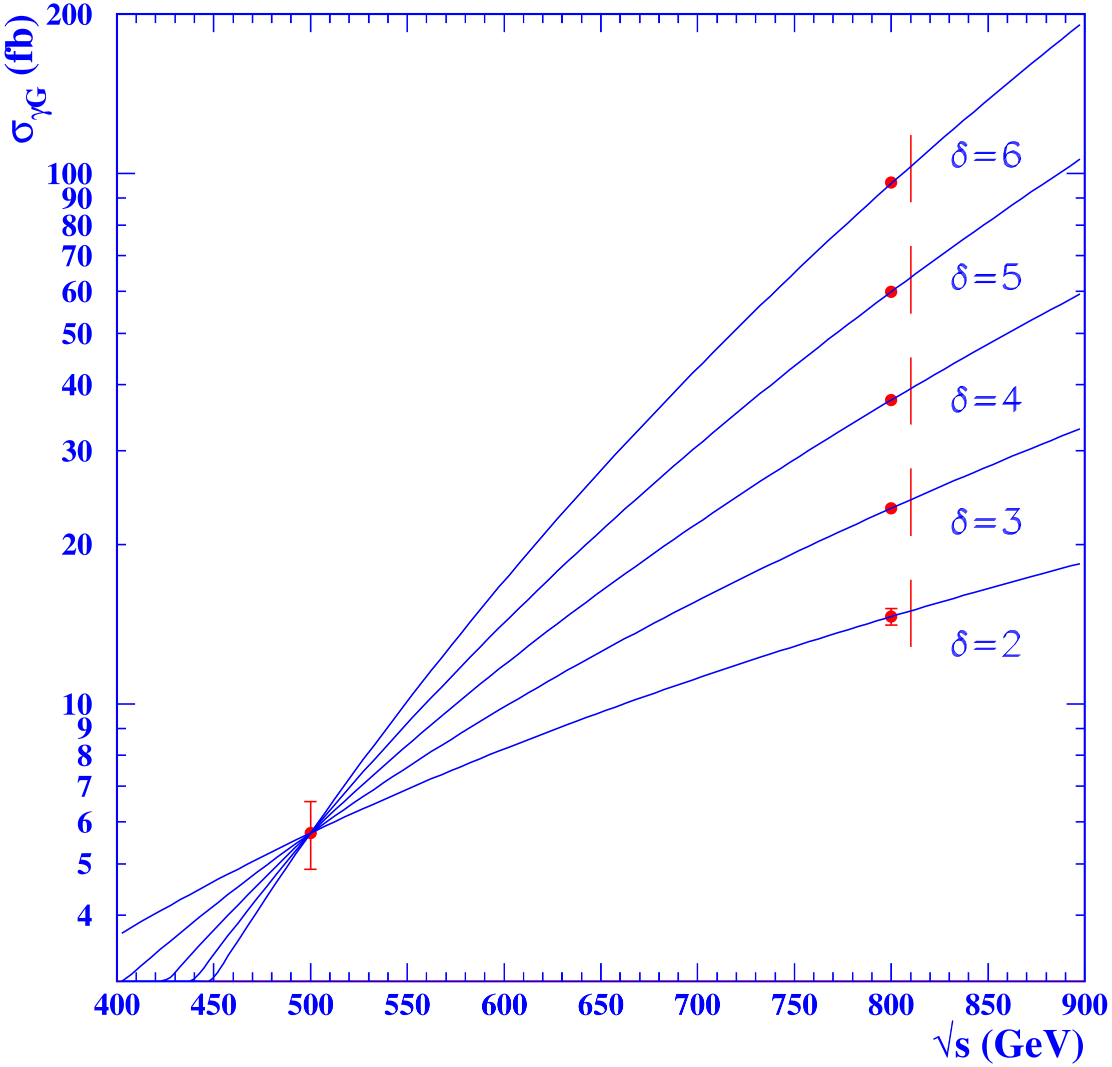
$$M_{\mathrm{Pl}}^2 = V_\delta \, M_*^{2+\delta}$$

$\Rightarrow$ large value of M_{Pl} related to relatively large volume V_δ (10^{-3} m–10^{-15} m for $\delta = 2$–6) of extra dimensions

E.g.: graviton emission, $e^+e^- \to \gamma/Z + G_n$

determination of δ from LC measurements of single photon cross section at $\sqrt{s} = 500$ GeV and $\sqrt{s} = 800$ GeV with polarized e^- and e^+ beams:

[*TESLA TDR '01*]



Neutrino masses and mixing

SM requires massless neutrinos, contains no right-handed neutrinos

We now have compelling experimental evidence that neutrinos have non-zero mass

⇒ **Experimental evidence for physics beyond the Standard Model**

Evidence from

atmospheric neutrinos:
disappearance of ν_μ at $> 15\ \sigma$, most likely to ν_τ ($> 99\%$ C.L.)

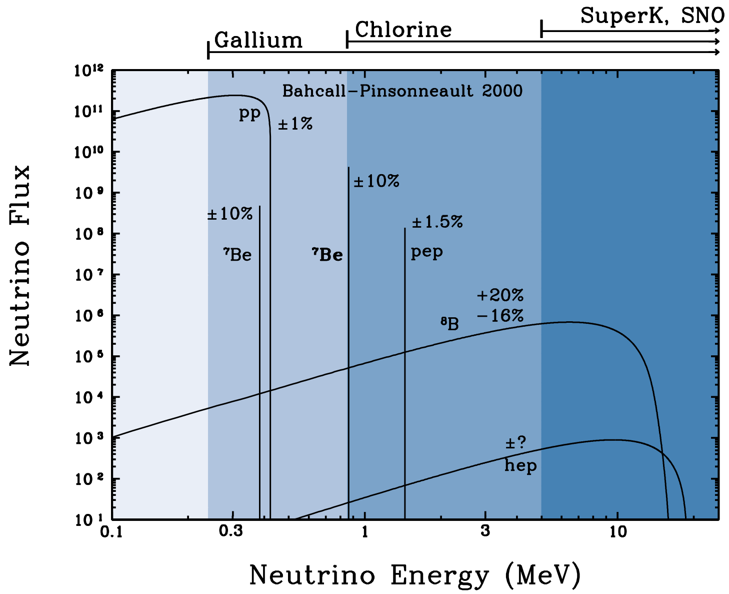
neutrinos emitted from the sun (solar neutrinos):
verification of flavor conversion, $\nu_e \to \nu_\mu$ or ν_τ at $> 5\sigma$

reactor neutrinos:
disappearance of $\bar{\nu}_e$ verified (99.95% C.L.)

⇒ Strong evidence for neutrino oscillations

II. Global Fits

Solar Neutrinos



- The sun emits $\nu_e' s$.

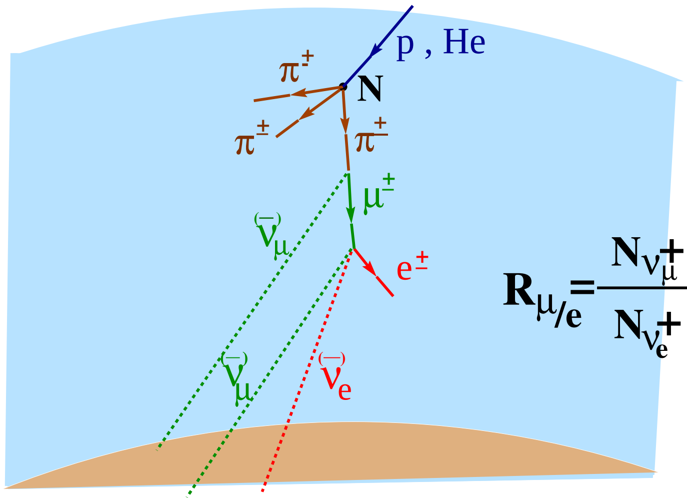
- Detected at

	Experiment	Detection	Flavour	$E_{\rm th}$ (MeV)	$\frac{\rm Data}{\rm BP00}$
radio-chemical	Homestake	$^{37}{\rm Cl}(\nu,e^-)^{37}{\rm Ar}$	ν_e	$E_\nu > 0.81$	0.35 ± 0.06
	Sage + Gallex+GNO	$^{71}{\rm Ga}(\nu,e^-)^{71}{\rm Ge}$	ν_e	$E_\nu > 0.23$	0.55 ± 0.05
real time	Kam $\Rightarrow$ SK	ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$ $\left(\frac{\sigma_{\mu\tau}}{\sigma_e} \simeq \frac{1}{6}\right)$	$E_e > 5$	0.46 ± 0.09
	SNO	CC $\nu_e d \to p p e^-$	ν_e	$T_e > 5$	$0.35 \pm 0.07^*$
		NC $\nu_x d \to \nu_x d$	$\nu_e, \nu_{\mu/\tau}$	$T_\gamma > 5$	$1.01 \pm 0.23^*$
		ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$T_e > 5$	$0.46 \pm 0.23^*$

(* For undistorted ^{8}B spectrum)

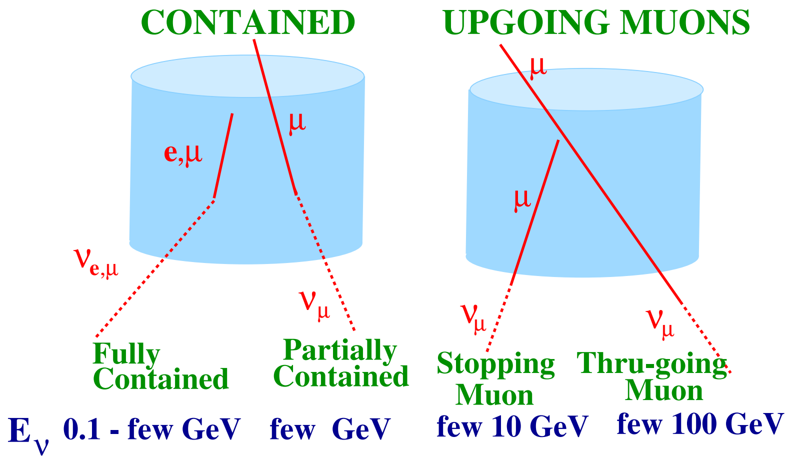
Atmospheric Neutrinos

$\nu_{e,\mu}$ are produced by interaction of cosmic rays (p, He . . .) with the atmosphere

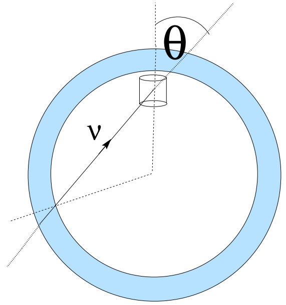


$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

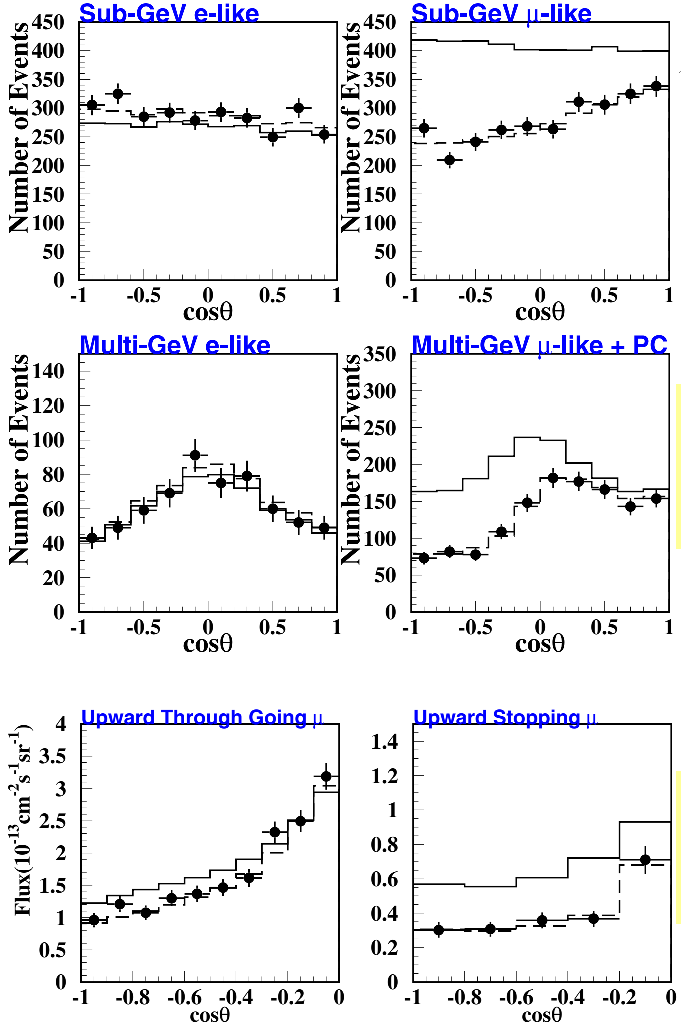
EVENT CLASSIFICATION



Angular Distribution at SK



ν_e in agreement with SM



→ Deficit grows with L

→ Decreases with E

If neutrinos have mass $\Rightarrow$ mixing as in quark sector

mixing matrix for three neutrino generations described by 3 mixing angles and 1 complex phase

LEP results $\Rightarrow$ number of light ($< M_Z/2$) active neutrinos $= 3$

Weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \rightarrow \nu_\alpha + N'$ is linear combination of mass eigenstates $|\nu_i\rangle$:

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} V_{\alpha i} |\nu_i\rangle$$

After distance L (time t): mass eigenstates evolve with $e^{-iE_i t}$

$\Rightarrow$ can be detected with different flavor β with probability $|\langle\nu_\beta|\nu(t)\rangle|^2$

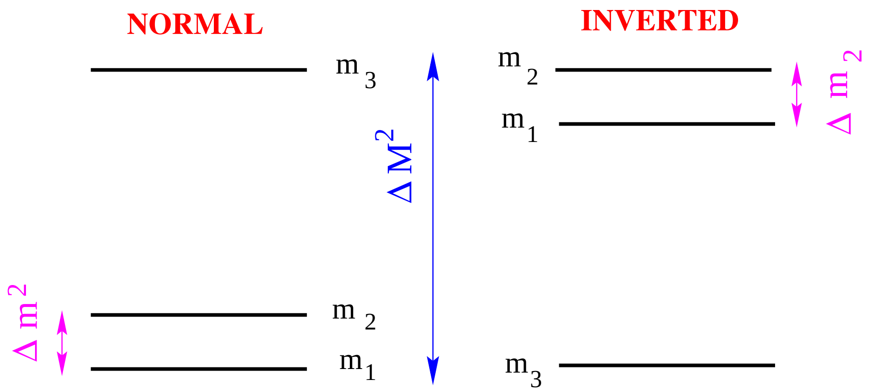
$\Rightarrow$ neutrino oscillations

Solar + Atmospheric 3ν Oscillations

U: 3 angles, 1 CP-phase
+ (2 Majorana phases)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Two mass schemes



2ν oscillation analysis $\Rightarrow \Delta m^2 = \Delta m^2_\odot \ll \Delta M^2 \simeq \pm\Delta m^2_{atm} \simeq \pm\Delta m^2_{32} \simeq \pm\Delta m^2_{31}$

$\Rightarrow$ Generic 3ν effects in ATM+SOLAR+CHOOZ oscillations :

	Potential Dependence							$\Delta m^2 \ll \Delta M^2$						
	θ_{12}	θ_{13}	θ_{23}	Δm^2	ΔM^2	δ	N/I	θ_{12}	θ_{13}	θ_{23}	Δm^2	ΔM^2	δ	N/I
Solar	X	X		X	X		X	X	X		X			
Atmos	X	X	X	X	X	X	X		X	X		X		X?
Chooz	X	X		X	X		X	X	X		X			

CP violation *Unobservable*
2 wavelengths *Unobservable*
N versus I *Below sensitivity*
θ_{13}?

Additional ingredient: neutrinos have no charge

$\Rightarrow$ right-handed neutrino ν_R is electroweak singlet

$\Rightarrow$ can have Majorana mass term $M\,\overline{\nu^c_\mathsf{R}}\nu_\mathsf{R}$

Explicit mass term, not related to electroweak symmetry breaking

$\Rightarrow$ $M \gg$ electroweak scale naturally

$\Rightarrow$ "Seesaw mechanism":

$$\begin{pmatrix} \nu_\mathsf{L} & \nu_\mathsf{R} \end{pmatrix} \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} \begin{pmatrix} \nu_\mathsf{L} \\ \nu_\mathsf{R} \end{pmatrix}$$

$\Rightarrow$ yields for $m \ll M$: $\underbrace{m^T \frac{1}{M} m}_{\text{light}}$ $\qquad$ $\underbrace{M}_{\text{heavy}}$

Works if $m \approx$ electroweak scale: $\sim 10^2$ GeV, $M \approx$ GUT scale $\sim 10^{16}$

$\Rightarrow$ neutrino mass $\sim m^2/M \sim 10^{-3}$ eV

$\Rightarrow$ Further motivation for new physics close to GUT scale

SUSY phenomenology:

The Minimal Supersymmetric Standard Model:

Superpartners for Standard Model particles:

$$\Big[u, d, c, s, t, b\Big]_{L,R} \quad \Big[e, \mu, \tau\Big]_{L,R} \quad \Big[\nu_{e,\mu,\tau}\Big]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\Big[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\Big]_{L,R} \quad \Big[\tilde{e}, \tilde{\mu}, \tilde{\tau}\Big]_{L,R} \quad \Big[\tilde{\nu}_{e,\mu,\tau}\Big]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}^{\pm}_{1,2} \quad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Exact SUSY: $m_f = m_{\tilde{f}}, \ \ldots$

$\Rightarrow$ SUSY must be broken: $M_{\text{SUSY}} \lesssim 1$ TeV

Breaking of SU(2) $\times$ U(1)$_Y$ (electroweak symmetry breaking)

$\Rightarrow$ fields with different SU(2) $\times$ U(1)$_Y$ quantum numbers can mix if they have the same SU(3)$_c$, U(1)$_{\text{em}}$ quantum numbers

Squark mixing:

Stop, sbottom mass matrices ($X_\text{t} = A_\text{t} - \mu/\tan\beta$, $X_\text{b} = A_\text{b} - \mu\tan\beta$):

$$\mathcal{M}^2_{\tilde{\text{t}}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m^2_\text{t} + DT_1 & m_\text{t} X_\text{t} \\ m_\text{t} X_\text{t} & m^2_{\tilde{t}_R} + m^2_\text{t} + DT_2 \end{pmatrix} \quad \Rightarrow \quad m_{\tilde{\text{t}}_1}, m_{\tilde{\text{t}}_2}, \theta_{\tilde{\text{t}}}$$

$$\mathcal{M}^2_{\tilde{\text{b}}} = \begin{pmatrix} m^2_{\tilde{b}_L} + m^2_\text{b} + DB_1 & m_\text{b} X_\text{b} \\ m_\text{b} X_\text{b} & m^2_{\tilde{b}_R} + m^2_\text{b} + DB_2 \end{pmatrix} \quad \Rightarrow \quad m_{\tilde{\text{b}}_1}, m_{\tilde{\text{b}}_2}, \theta_{\tilde{\text{b}}}$$

off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

$\Rightarrow$ mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

gauge invariance $\Rightarrow m_{\tilde{t}_L} = m_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{\text{t}}_1}, m_{\tilde{\text{t}}_2}, \theta_{\tilde{\text{t}}}, m_{\tilde{\text{b}}_1}, m_{\tilde{\text{b}}_2}, \theta_{\tilde{\text{b}}}$

Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

$\Rightarrow$ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

$\Rightarrow$ only one new parameter

$\Rightarrow$ MSSM predicts mass relations between neutralinos and charginos

SUSY breaking

SUSY breaking mechanism not well understood

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parametrization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergencies (hierarchy problem)

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenmenological description for universal breaking terms (avoids large flavor changing neutral currents, . . .)

Specific SUSY-breaking scenarios:

Experimental constraints give guidance concerning possible "organizing principle" for soft breaking terms in MSSM:

Strong constraints on

– lepton number violation (e.g. $\mu \to e\gamma$)

– flavor changing neutral currents (FCNC, e.g. $b \to s\gamma$)

– electric dipole moments

...

$\Rightarrow$ Constraints on flavor off-diagonal terms, $\mathcal{CP}$-violating phases

$\Rightarrow$ experimental constraints satisfied if SUSY breaking is suitably 'universal', $\approx$ diagonal in flavour space

Most promising approach:

SUSY breaking happens in a “hidden sector”, i.e. by fields which have only very small couplings to ordinary matter

SUSY-breaking terms in the MSSM arise via interaction that communicates between the hidden sector and the MSSM fields

$\Rightarrow$ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

If mediating interactions $\approx$ flavor-diagonal $\Rightarrow$ universal soft-breaking terms

“Hidden sector”:	$\longrightarrow$	Visible sector:
SUSY breaking		MSSM

“Gravity-mediated”: mSUGRA

“Gauge-mediated”: GMSB

“Anomaly-mediated”: AMSB

“Gaugino-mediated”

...

Determination of SUSY parameters $\Rightarrow$ Patterns of SUSY breaking

Gravity-mediated SUSY breaking:

communication via gravitational interactions

terms (non-renormalizable) in supergravity Lagrangian proportional to $1/M_{\mathrm{Pl}}$ and SUSY-breaking v.e.v.

$\Rightarrow$ soft SUSY-breaking terms of MSSM

minimal version (universality assumptions): mSUGRA

Gauge-mediated SUSY breaking:

"Messenger fields" couple to SUSY-breaking v.e.v. in hidden sector

interact with MSSM fields via ordinary electroweak and QCD gauge interactions

Anomaly-mediated SUSY breaking, Gaugino-mediation:

SUSY breaking happens on a different brane in a higher-dimensional theory

mSUGRA

“supergravity inspired scenario”, characterized by five parameters

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}\,\mu$$

m_0: universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0: universal trilinear coupling

$\tan\beta$: ratio of Higgs vacuum expectation values

sign(μ): sign of supersymmetric Higgs parameter

m_0, $m_{1/2}$, A_0: GUT scale parameters

$\Rightarrow$ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino

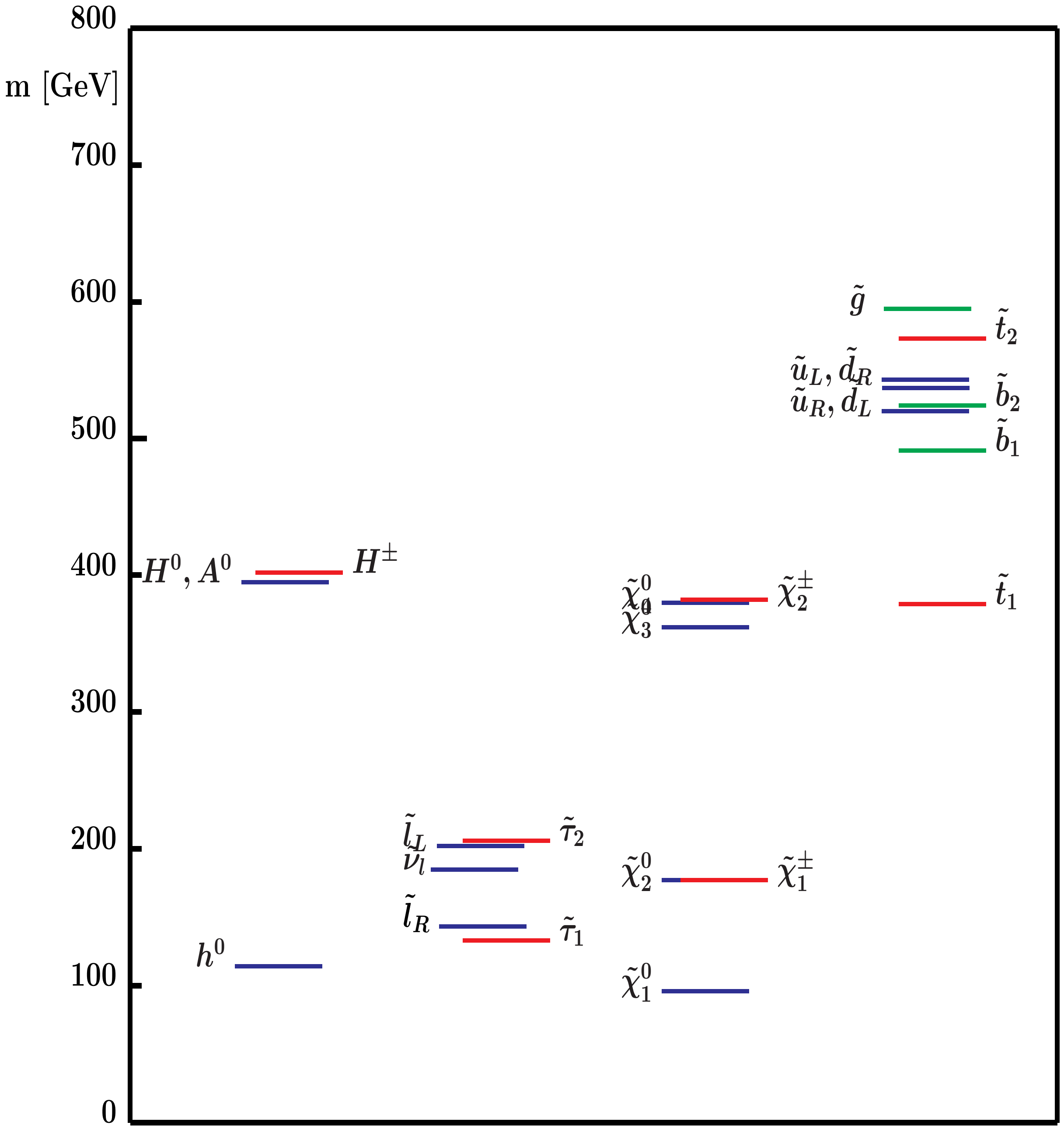
gaugino masses run in same way as gauge couplings

$\Rightarrow$ gluino heavier than charginos, neutralinos

"Typical" mSUGRA scenario

(SPS 1a benchmark scenario):

www.ippp.dur.ac.uk/∼georg/sps



GMSB

parameters: $M_{\text{mes}}, N_{\text{mes}}, \Lambda, \tan\beta, \text{sign}(\mu)$

M_{mes}: messenger mass scale

N_{mes}: messenger index (number of messenger multiplets)

Λ: universal soft SUSY breaking mass scale felt by low-energy sector

LSP is always the gravitino, $m_{\frac{3}{2}} \approx 10^{-9}$ GeV

Next-to-lightest SUSY particle (NLSP): $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

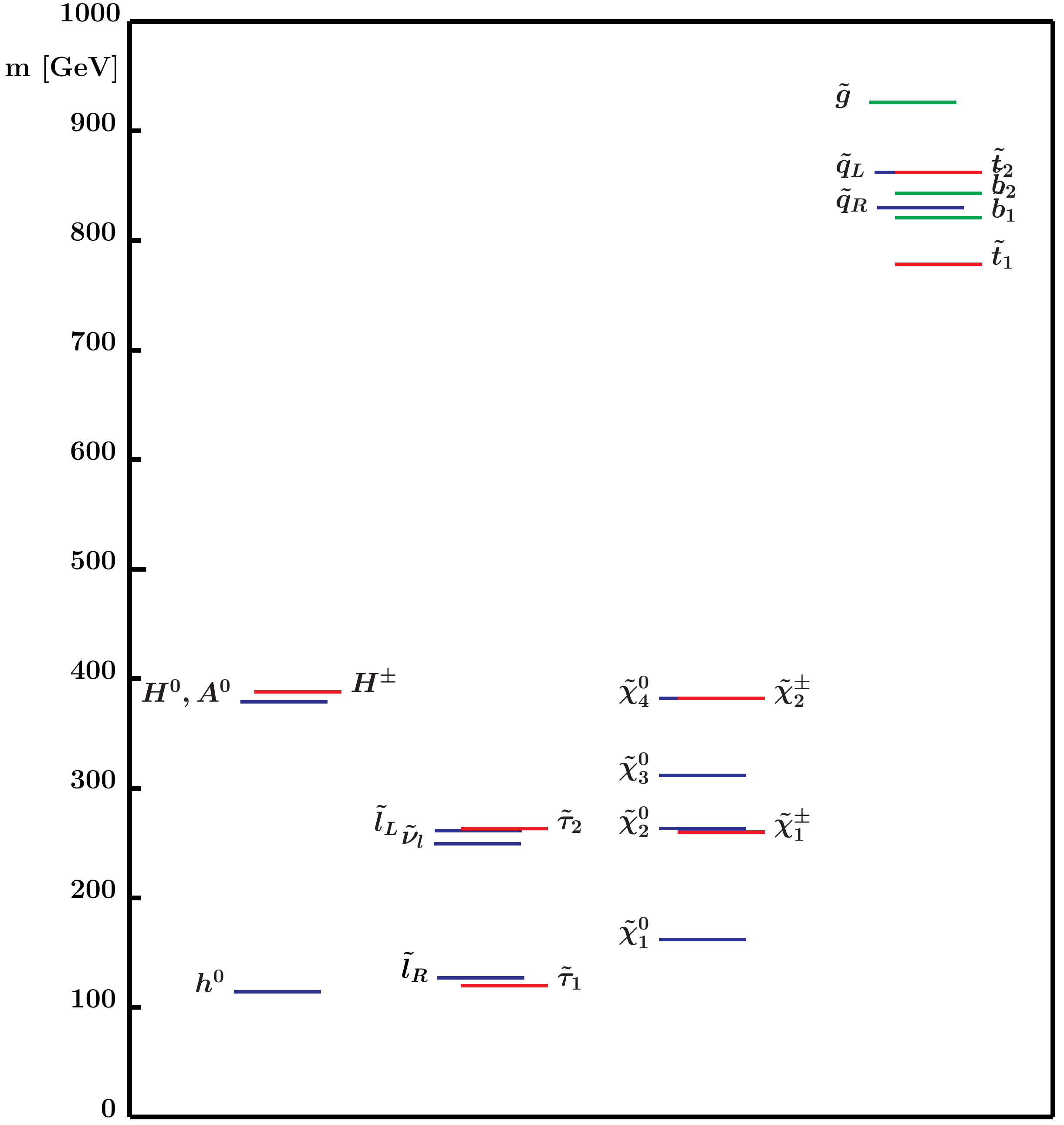
can decay into LSP inside or outside the detector

masses of SUSY particles from loop diagrams involving messenger fields, couple to MSSM fields via ordinary gauge interactions

$\Rightarrow$ colored particles usually heavier than weakly interacting particles

GMSB scenario with $\tilde{\tau}$ NLSP

(SPS 7 benchmark scenario):



AMSB scenario characterized by

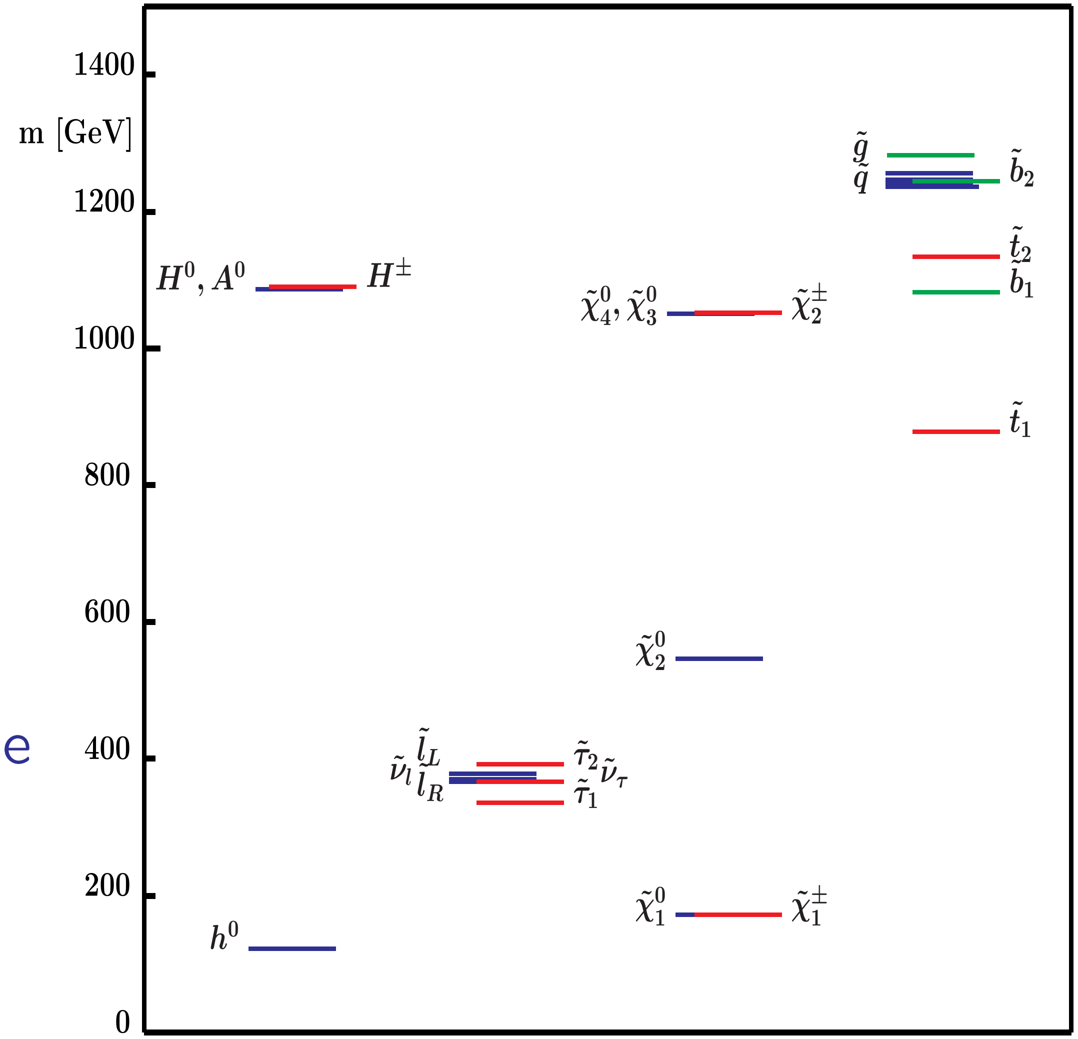
$$m_{\mathrm{aux}}, m_0, \tan\beta, \mathrm{sign}(\mu)$$

m_{aux}: overall scale of SUSY particle masses

m_0: phenomenological parameter: universal scalar mass term

AMSB spectrum (SPS 9):

typical feature: very small neutralino–chargino mass difference

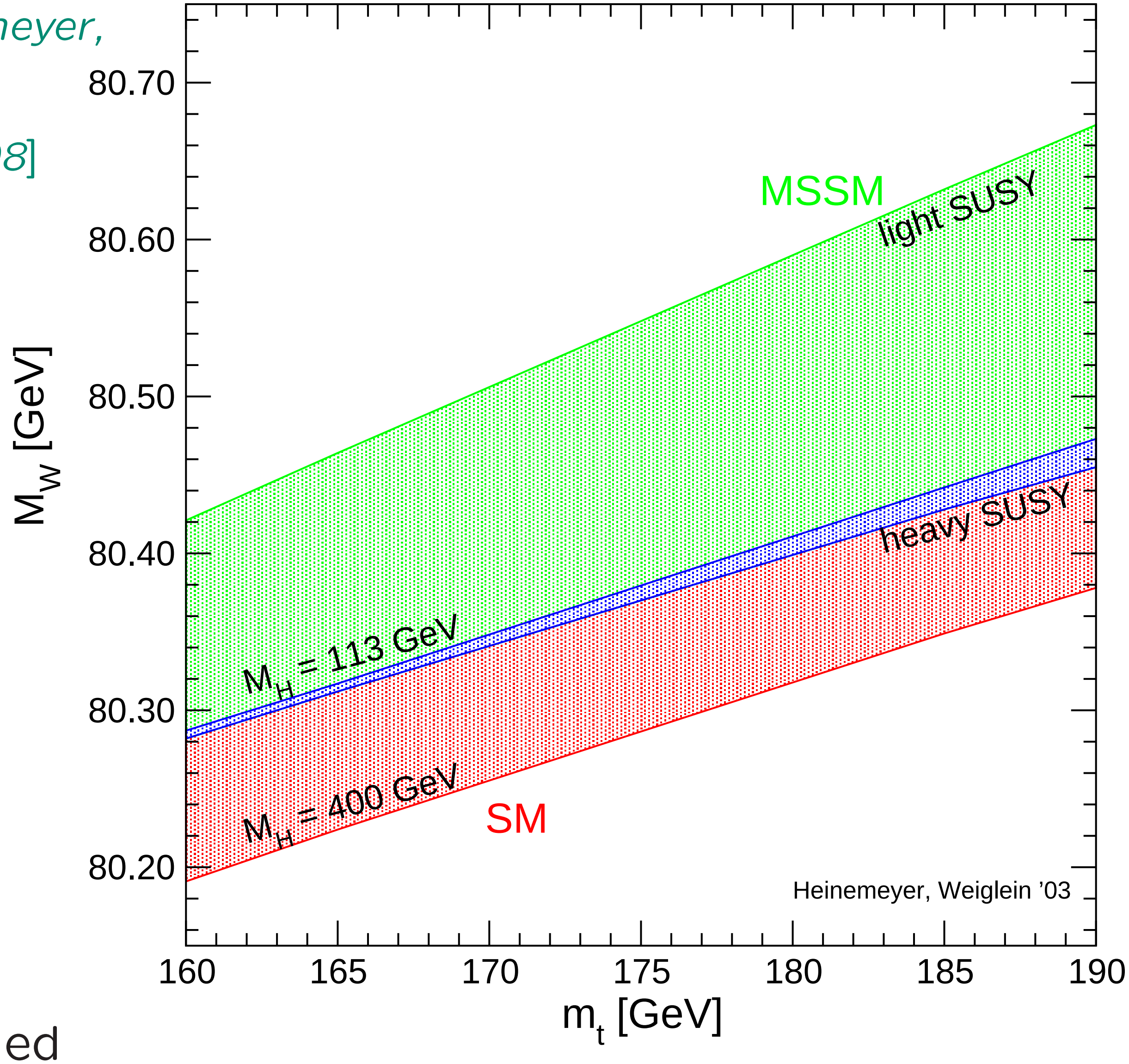


Electroweak precision tests: SM vs. MSSM

Prediction for M_W in the SM and the MSSM:

[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]

[S. Heinemeyer, W. Hollik, G. W. '98]



SM: M_H varied

MSSM: SUSY parameters varied

Electroweak precision tests: SM vs. MSSM

Prediction for M_{W} in the SM and the MSSM:

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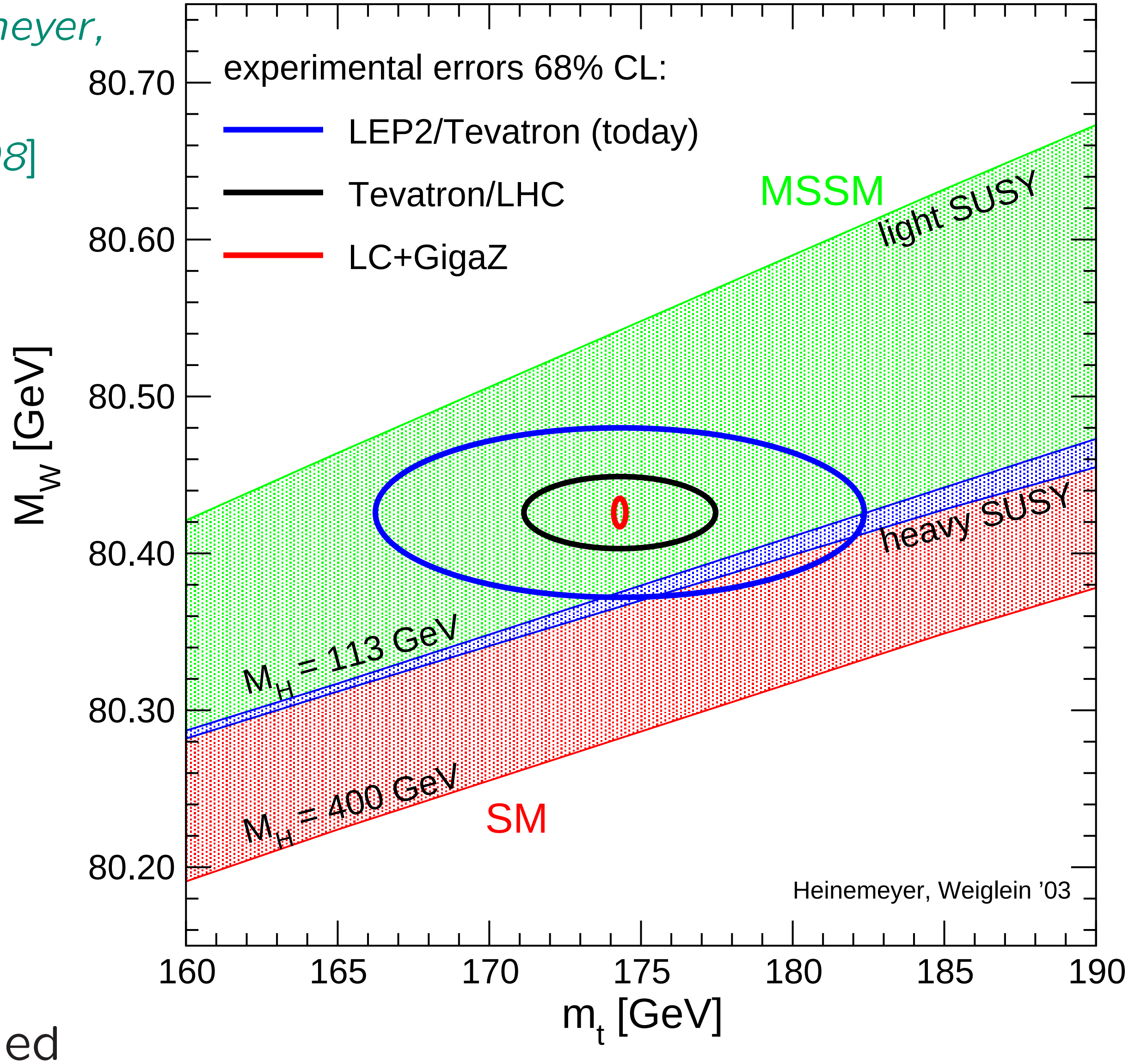
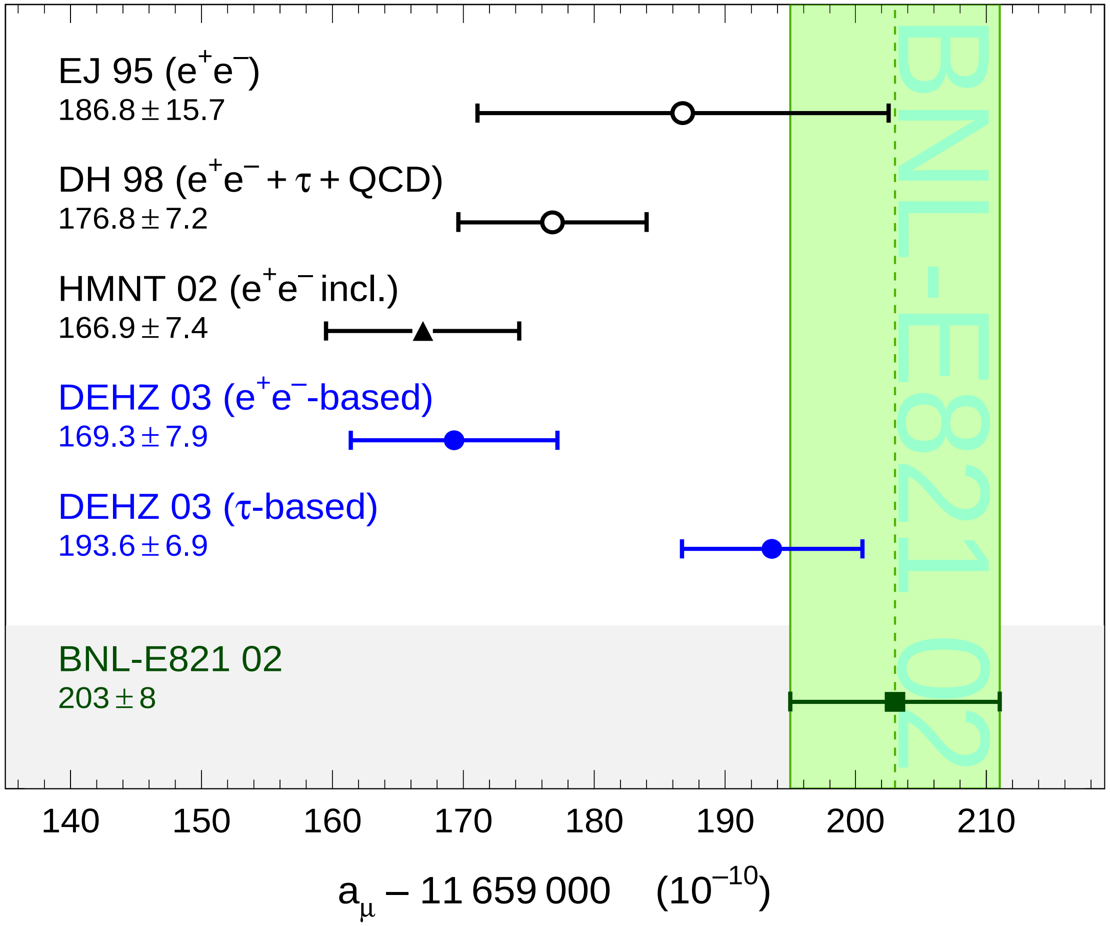


SM: M_{H} varied

MSSM: SUSY parameters varied

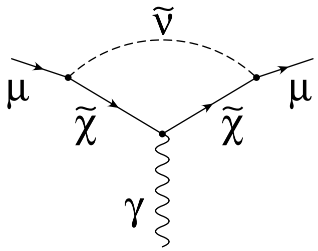
Experimental result for anomalous magnetic moment of the muon vs. SM prediction:

[*Muon* $(g-2)$ *Collaboration '02*]



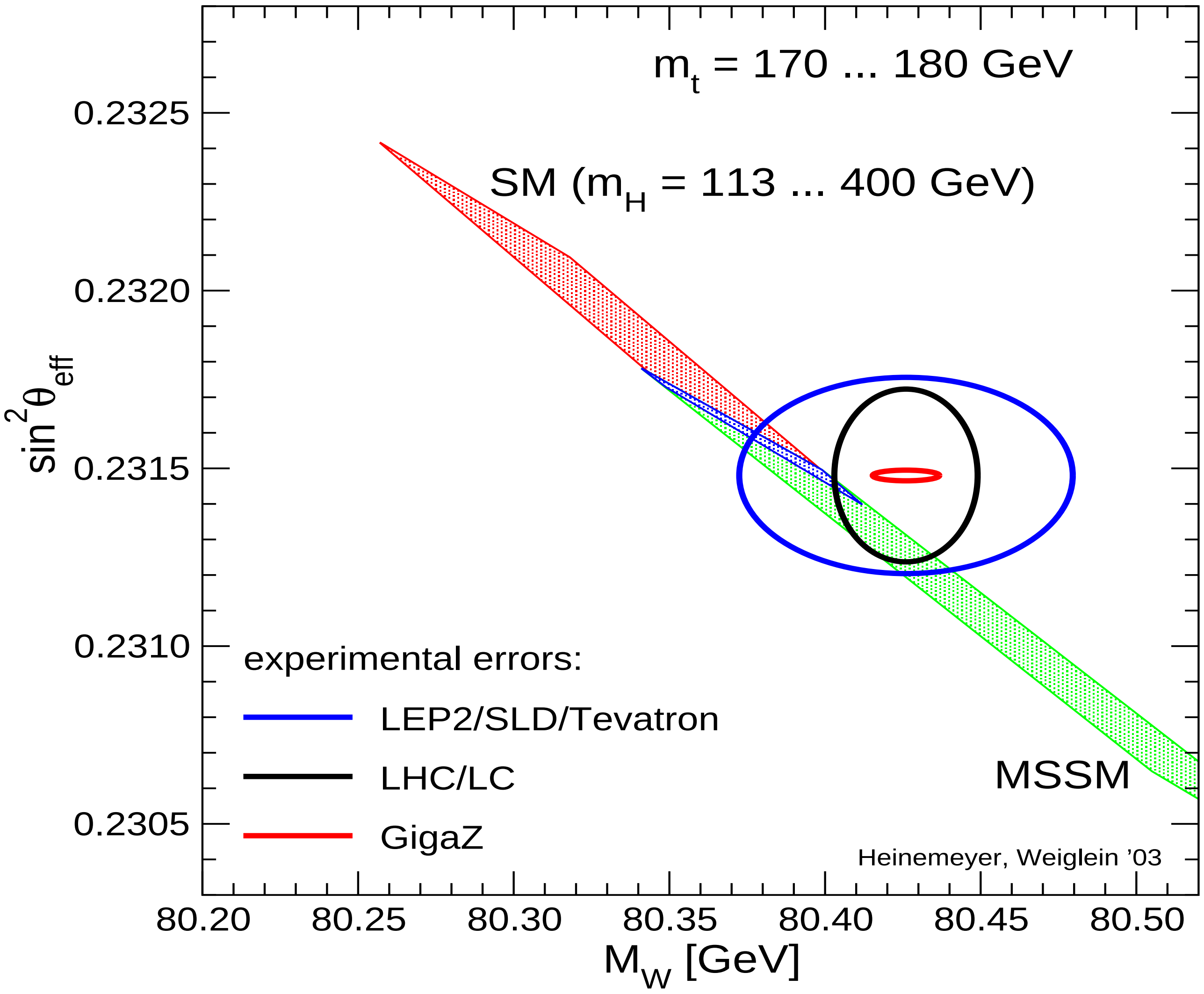
$\Rightarrow a_\mu(\text{exp}) - a_\mu(\text{SM})$: 1–3 σ deviation (sizable hadronic uncertainties)

Compatible with effect of SUSY contributions to a_μ:



Prediction for M_W, $\sin^2\theta_\mathrm{eff}$ in SM and MSSM:

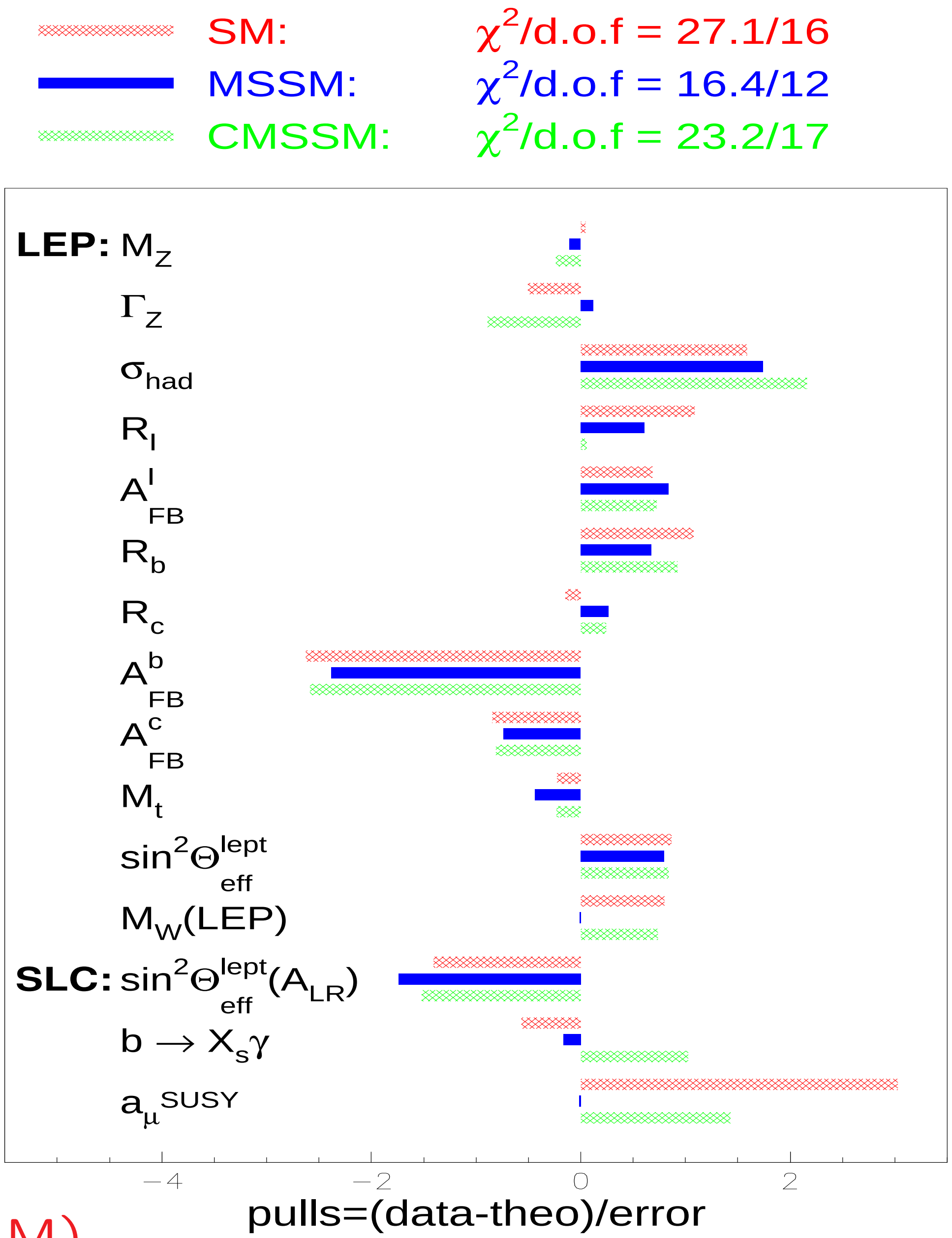
[*S. Heinemeyer, G. W. '01*]



$\Rightarrow$ High sensitivity to deviations both from the SM and the MSSM

Global fit: SM / unconstrained MSSM / mSUGRA

[*W. de Boer, C. Sander '02*]



⇒ MSSM contributions can improve agreement with data for $g_\mu - 2$ (for large $\tan\beta$) and M_W

no significant improvement for A^b_{FB} and neutrino–nucleon cross section

Similar quality of global fit (slightly better in MSSM than in SM)

The LSP as a dark matter candidate

Astrophysical data (cosmic microwave background, . . .) $\Rightarrow$ existence of non-baryonic cold dark matter in the Universe

SUSY with R parity conservation $\Rightarrow$ LSP relic density falls naturally in favoured range if $m_{\mathrm{LSP}} \lesssim 1$ TeV

Most recent result (WMAP)
[*C. Bennet et al. '03*], [*D. Spergel et al. '03*]

$\Rightarrow$ cold dark matter density: $\Omega_{\mathrm{CDM}} h^2 = 0.1126^{+0.0161}_{-0.0181}$ at 95% C.L.
(h: expansion rate)

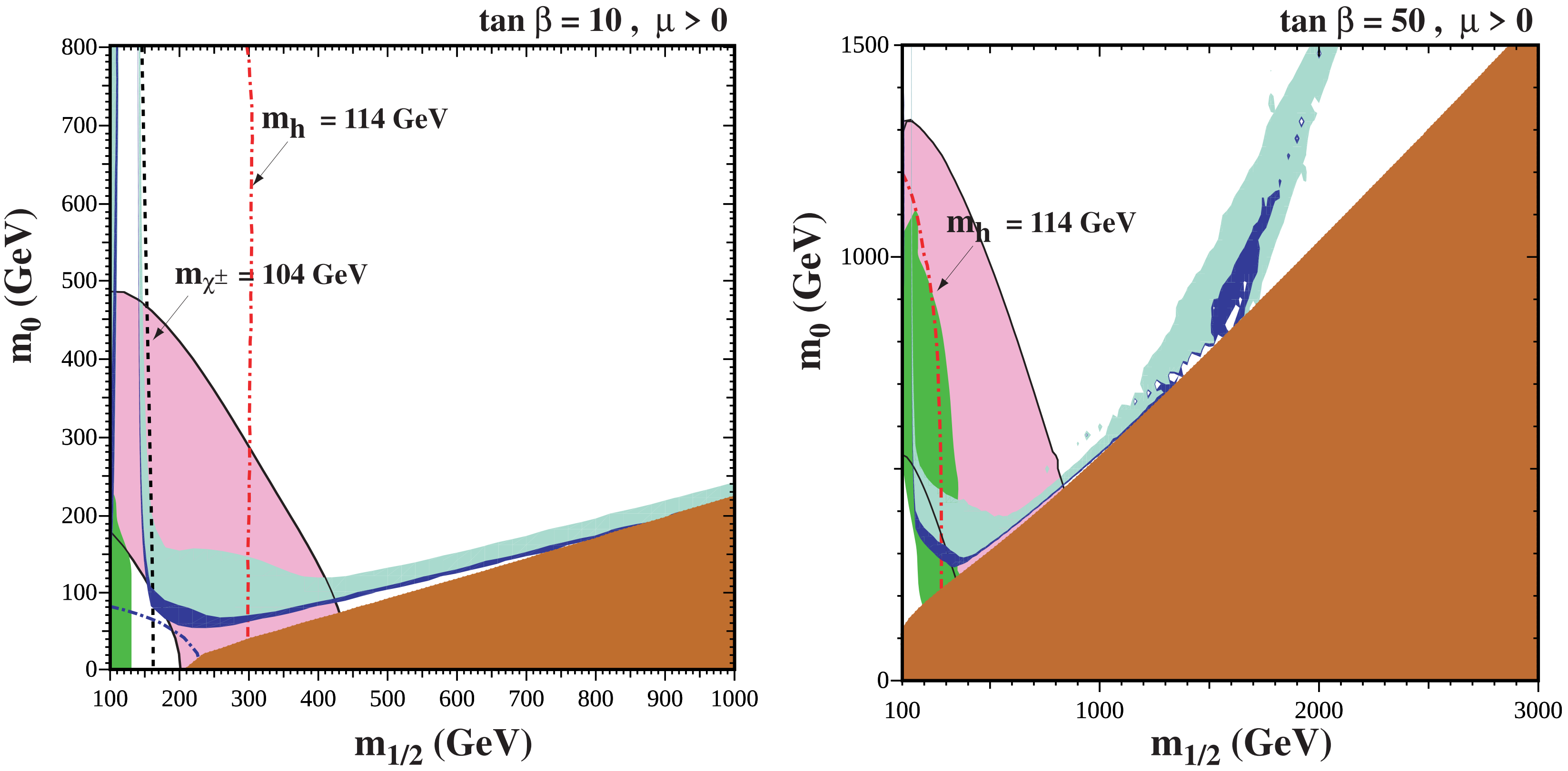
$\Rightarrow$ Constraints on the SUSY parameter space

Example: mSUGRA scenario with $\mu > 0$, $A_0 = 0$, $\tan\beta = 10, 50$

Parameter region allowed by the CDM constraint in the $m_{1/2}$–m_0 plane with and without the new WMAP data

(also shown: favoured region from $(g_\mu - 2)$, region excluded by $b \to s\gamma$)

[J. Ellis, K. Olive, Y. Santoso, V. Spanos '03]



$\Rightarrow$ mSUGRA scenario consistent with all experimental constraints

Predictions of SUSY theories

Relations between SUSY parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.:

gauge boson–fermion coupling

=

gaugino–fermion–sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters
(except M_W–M_Z interdependence)

MSSM:

- Upper bound on mass of lightest $\mathcal{CP}$-even Higgs boson

- Relations between neutralino and chargino masses

- Sfermion mass relations, e.g.

$$m^2_{\tilde{e}_L} = m^2_{\tilde{\nu}_L} - M_W^2 \cos(2\beta)$$

All relations receive corrections from loop effects
$\Leftrightarrow$ effects of soft SUSY breaking, electroweak symmetry breaking

$\Rightarrow$ Experimental verification of parameter relations is a crucial test of SUSY!

The Higgs sector of the MSSM

Two Higgs doublets:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Mixing of $\mathcal{CP}$-even, $\mathcal{CP}$-odd, charged fields:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

$$\tan(2\alpha) = \tan(2\beta)\frac{M_\mathrm{A}^2 + M_\mathrm{Z}^2}{M_\mathrm{A}^2 - M_\mathrm{Z}^2}$$

Three Goldstone bosons (as in SM): G^0, $G^\pm$

$\longrightarrow$ longitudinal components of $W^\pm$, Z

$\Rightarrow$ Five physical states: $h^0, H^0, A^0, H^\pm$

h, H: neutral, $\mathcal{CP}$-even, A^0: neutral, $\mathcal{CP}$-odd, $H^\pm$: charged

Gauge-boson masses:

$$M_\mathrm{W}^2 = \frac{1}{2}g'^2(v_1^2 + v_2^2), \quad M_\mathrm{Z}^2 = \frac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2), \quad M_\gamma = 0$$

Parameters in MSSM Higgs potential V (besides g, g'):

$$v_1,\ v_2,\ m_1,\ m_2,\ m_{12}$$

relation for M_W^2, M_Z^2 $\Rightarrow$ 1 condition

minimization of V w.r.t. neutral Higgs fields H_1^1, H_2^2 $\Rightarrow$ 2 conditions

$\Rightarrow$ only two free parameters remain in V, conventionally chosen as

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{H^\pm}$: no free parameters, can be predicted

In lowest order:

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1\phi_2}^2 \\ m_{\phi_1\phi_2}^2 & m_{\phi_2}^2 \end{pmatrix} =$$

$$\begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2) \sin\beta \cos\beta \\ -(M_A^2 + M_Z^2) \sin\beta \cos\beta & M_A^2 \cos^2\beta + M_Z^2 \sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \ \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

Tree-level result for m_h, m_H:

$$m_{H,h}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_h, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

Higgs mass bounds in SUSY theories

MSSM predicts upper bound on m_{h}:

tree-level bound: $m_{\mathrm{h}} < M_{\mathrm{Z}}$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{\mathrm{e}\, m_{\mathrm{t}}}{2 M_{\mathrm{W}} s_{\mathrm{W}}}$, $\frac{\mathrm{e}\, m_{\mathrm{t}}^2}{M_{\mathrm{W}} s_{\mathrm{W}}}$, ...

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_{\mathrm{t}}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_{\mathrm{t}}^2}\right)$

Present status of m_{h} prediction in the MSSM:

Complete one-loop + "almost complete" two-loop result available

Used for LEP analyses: *FeynHiggs* (Feynman-diagrammatic result: complete one-loop, leading + subleading two-loop)
[*S. Heinemeyer, W. Hollik, G. W. '98, '00, '02*]

Upper bound on m_{h} in the MSSM:

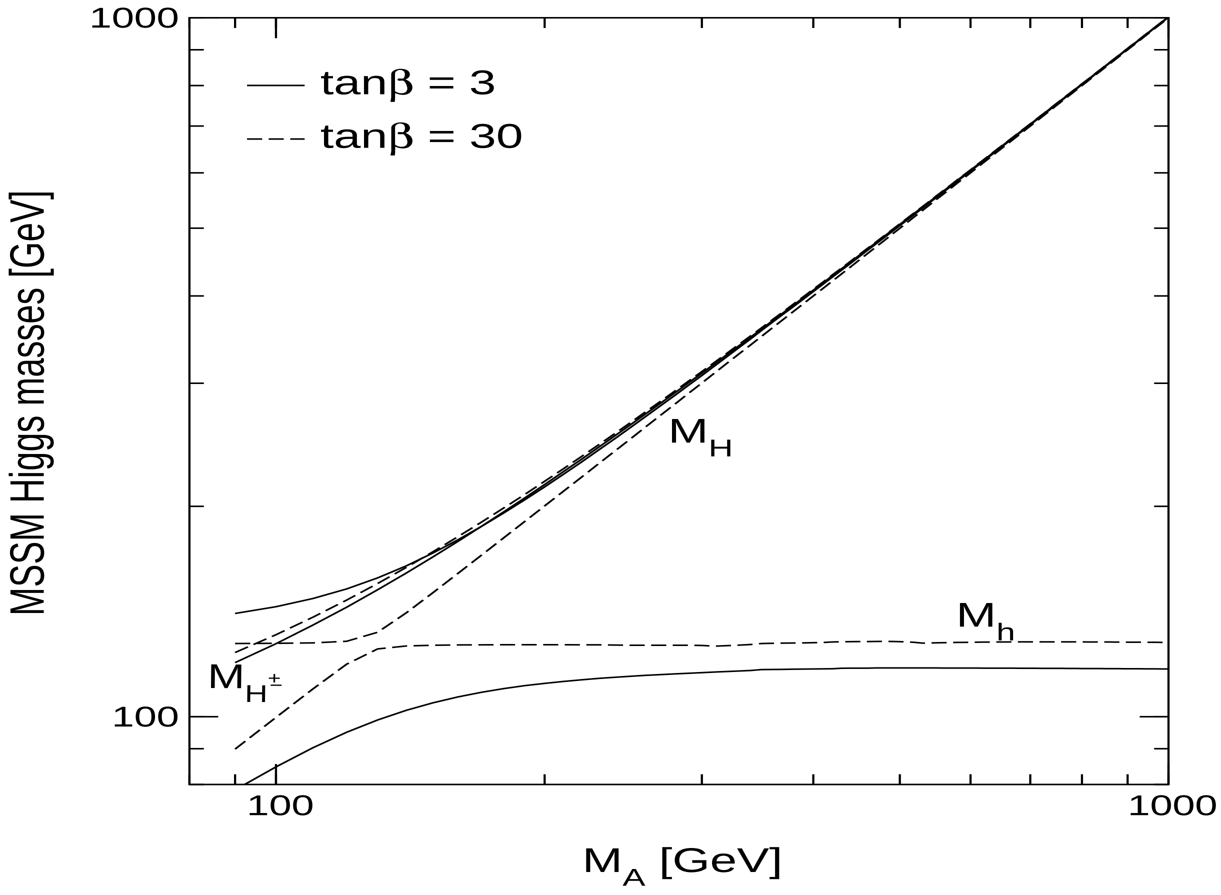
"Unconstrained MSSM": M_{A}, $\tan\beta$, 5 parameters in $\tilde{\mathrm{t}}$–$\tilde{\mathrm{b}}$ sector, μ, $m_{\tilde{\mathrm{g}}}$, M_2

$$m_{\mathrm{h}} \lesssim 135\,\mathrm{GeV}$$

[*S. Heinemeyer, W. Hollik, G. W. '99*], [*M. Frank, S. Heinemeyer, W. Hollik, G. W. '02*]

for $m_{\mathrm{t}} = 175\,\mathrm{GeV}$, no theoretical uncertainties included

Upper bound on m_{h} saturated for large M_{A}, large $\tan\beta$, significant mixing in $\tilde{\mathrm{t}}$ sector



Remaining theoretical uncertainties in prediction for m_{h}:
[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]

– From unknown higher-order corrections: $\Rightarrow \Delta m_{\mathrm{h}} \approx 3$ GeV

– From uncertainties in input parameters
$m_{\mathrm{t}}, \ldots, M_{\mathrm{A}}, \tan\beta, m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}, m_{\tilde{\mathrm{g}}}, \ldots$: $\Delta m_{\mathrm{t}} \approx 5$ GeV $\Rightarrow \Delta m_{\mathrm{h}} \approx 5$ GeV

Upper bound $m_{\mathrm{h}} \lesssim 135$ GeV reduced by

$\approx 6, 11, 8$ GeV in mSUGRA, GMSB, AMSB scenarios
[*S. Ambrosanio, A. Dedes, S. Heinemeyer, S. Su, G. W. '01*]

Upper bound on m_{h} in extensions of MSSM: $m_{\mathrm{h}} \lesssim 200$ GeV
[*G. Kane, C. Kolda, J. Wells '93*] [*J. Espinosa, M. Quirós '93, '98*]

$\Rightarrow$ **SUSY requires light Higgs boson:**

definite and robust prediction of SUSY models

testable at next generation of colliders

Higgs couplings, tree level:

$$
\begin{aligned}
g_{\mathrm{hVV}} &= \sin(\beta-\alpha)\, g^{\mathrm{SM}}_{\mathrm{HVV}}, \qquad V = W^{\pm}, Z \\
g_{\mathrm{HVV}} &= \cos(\beta-\alpha)\, g^{\mathrm{SM}}_{\mathrm{HVV}} \\
g_{\mathrm{hAZ}} &= \cos(\beta-\alpha)\, \frac{g'}{2\cos\theta_{\mathrm{W}}} \\
g_{\mathrm{hb\bar{b}}}, g_{\mathrm{h}\tau^+\tau^-} &= -\frac{\sin\alpha}{\cos\beta}\, g^{\mathrm{SM}}_{\mathrm{Hb\bar{b}},\mathrm{H}\tau^+\tau^-} \\
g_{\mathrm{ht\bar{t}}} &= \frac{\cos\alpha}{\sin\beta}\, g^{\mathrm{SM}}_{\mathrm{Ht\bar{t}}} \\
g_{\mathrm{Ab\bar{b}}}, g_{\mathrm{A}\tau^+\tau^-} &= \gamma_5 \tan\beta\, g^{\mathrm{SM}}_{\mathrm{Ab\bar{b}}}
\end{aligned}
$$

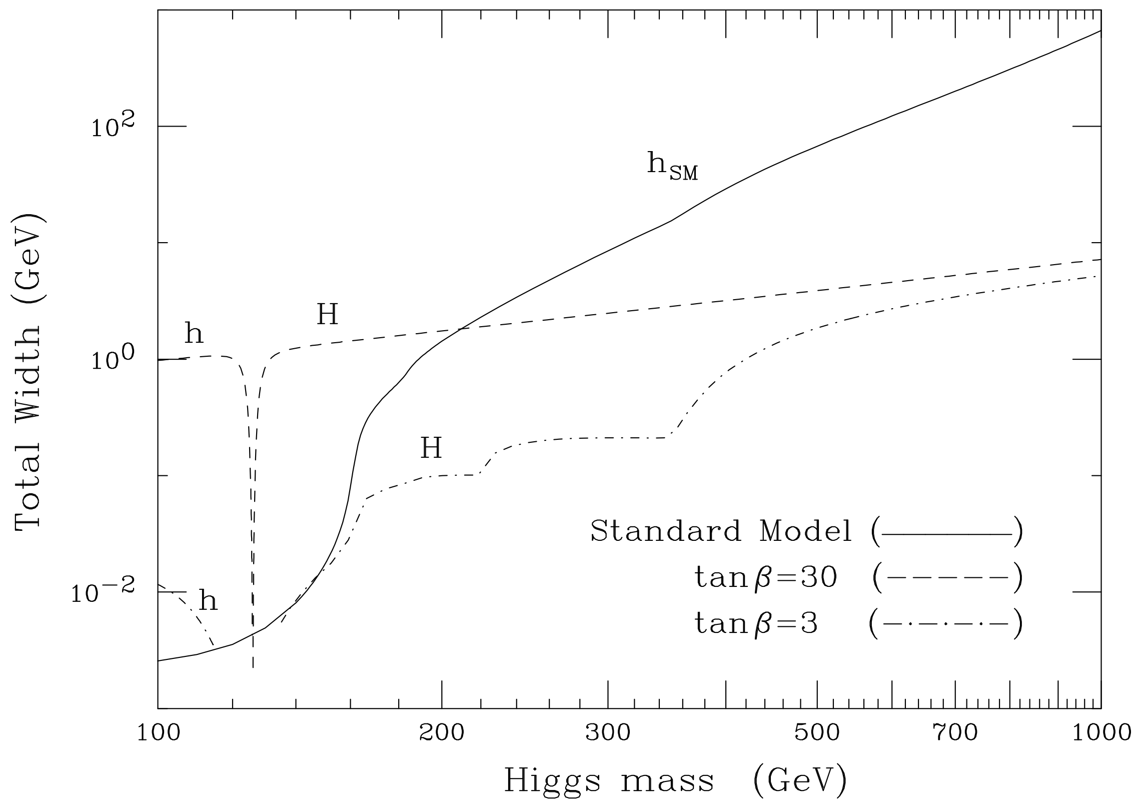
$\Rightarrow$ $g_{\mathrm{hVV}} \leq g^{\mathrm{SM}}_{\mathrm{HVV}}$, $\quad g_{\mathrm{hVV}}, g_{\mathrm{HVV}}, g_{\mathrm{hAZ}}$ cannot all be small

$g_{\mathrm{hb\bar{b}}}, g_{\mathrm{h}\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling possible

For $M_A \lesssim 200$ GeV, $\tan\beta \gtrsim 10$:

large enhancement of
$\Gamma(h \to b\bar{b})$, $\Gamma(h \to \tau^+\tau^-)$, . . .

$\Rightarrow$ Total Higgs width can be much larger than in SM



[*M. Carena, H. Haber '03*]

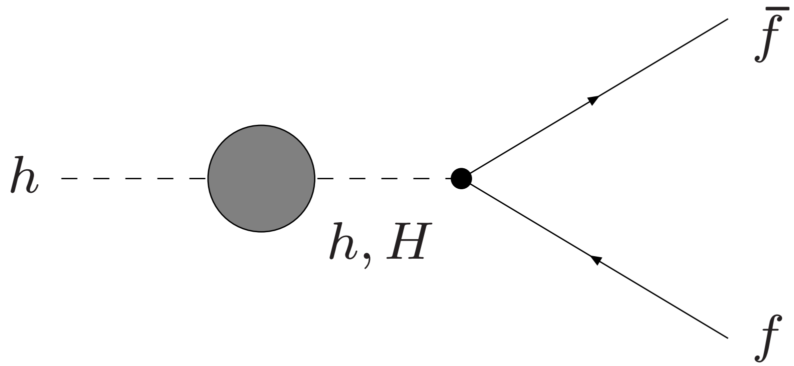
$\Rightarrow$ Large suppression of BR($h \to \gamma\gamma$), BR($h \to WW^*$), . . .

"Pathological" Higgs decays:

SUSY Higgs h can have sizable branching fraction into invisible final state: $h \to \tilde{\chi}_1^0\tilde{\chi}_1^0$

Higgs couplings, production cross sections also affected by large SUSY loop corrections

$hf\bar{f}$ coupling:



$$A(h \to f\bar{f}) = \sqrt{Z_h}\left(\Gamma_h - \frac{\hat{\Sigma}_{\mathrm{hH}}(m_{\mathrm{h}}^2)}{m_{\mathrm{h}}^2 - m_{\mathrm{H}}^2 + \hat{\Sigma}_{\mathrm{HH}}(m_{\mathrm{h}}^2)}\Gamma_H\right)$$

$\Rightarrow$ Effective $hf\bar{f}$ coupling can vanish for large $\hat{\Sigma}_{\mathrm{hH}}$: $\sin\alpha_{\mathrm{eff}} \to 0$

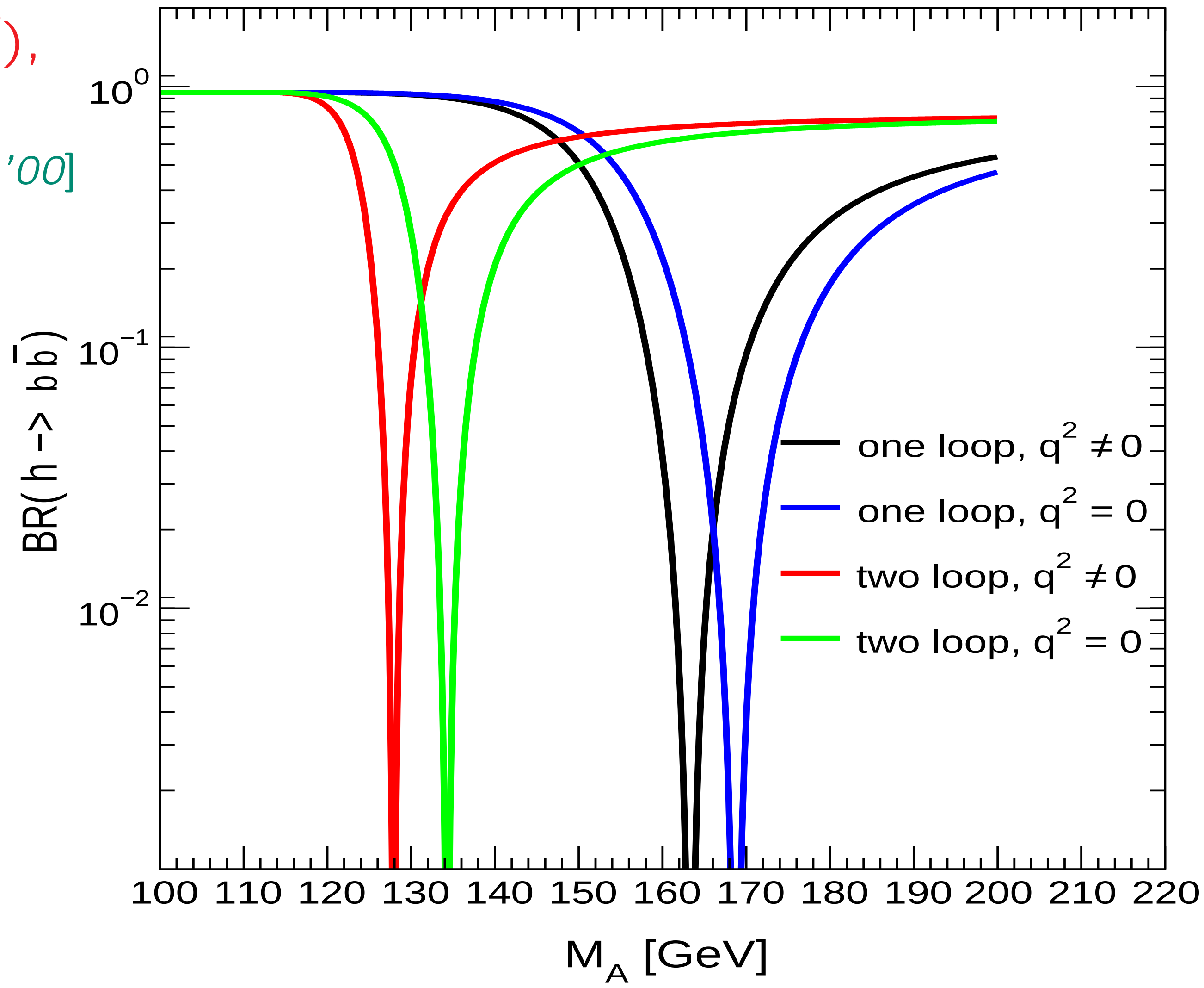
Effective $hf\bar{f}$ coupling can go to zero for large $\hat{\Sigma}_{\rm hH}$

$\Rightarrow$ "Pathological regions"

[*W. Loinaz, J. Wells '98*] [*M. Carena, S. Mrenna, C. Wagner '99*]

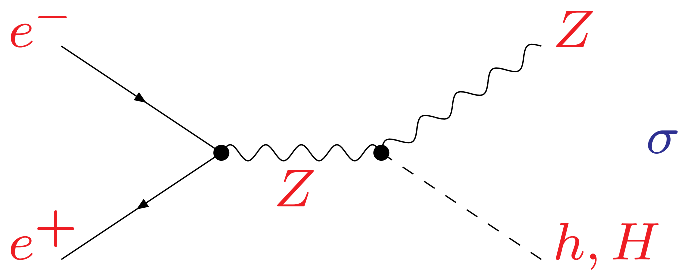
$\Rightarrow$ Suppression of BR($h \to b\bar{b}$),
BR($h \to \tau\tau$), . . .

[*S. Heinemeyer, W. Hollik, G. W. '00*]



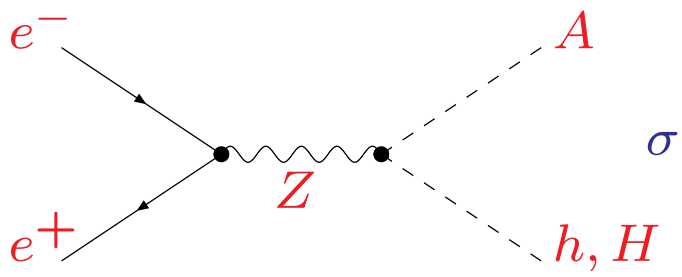
Search for neutral MSSM Higgs bosons at LEP:

$e^+e^- \rightarrow Zh, ZH$



$\sigma_{\mathrm{hZ}} \approx \sin^2(\beta - \alpha_{\mathrm{eff}})\sigma_{\mathrm{hZ}}^{\mathrm{SM}}$

$e^+e^- \rightarrow Ah, AH$

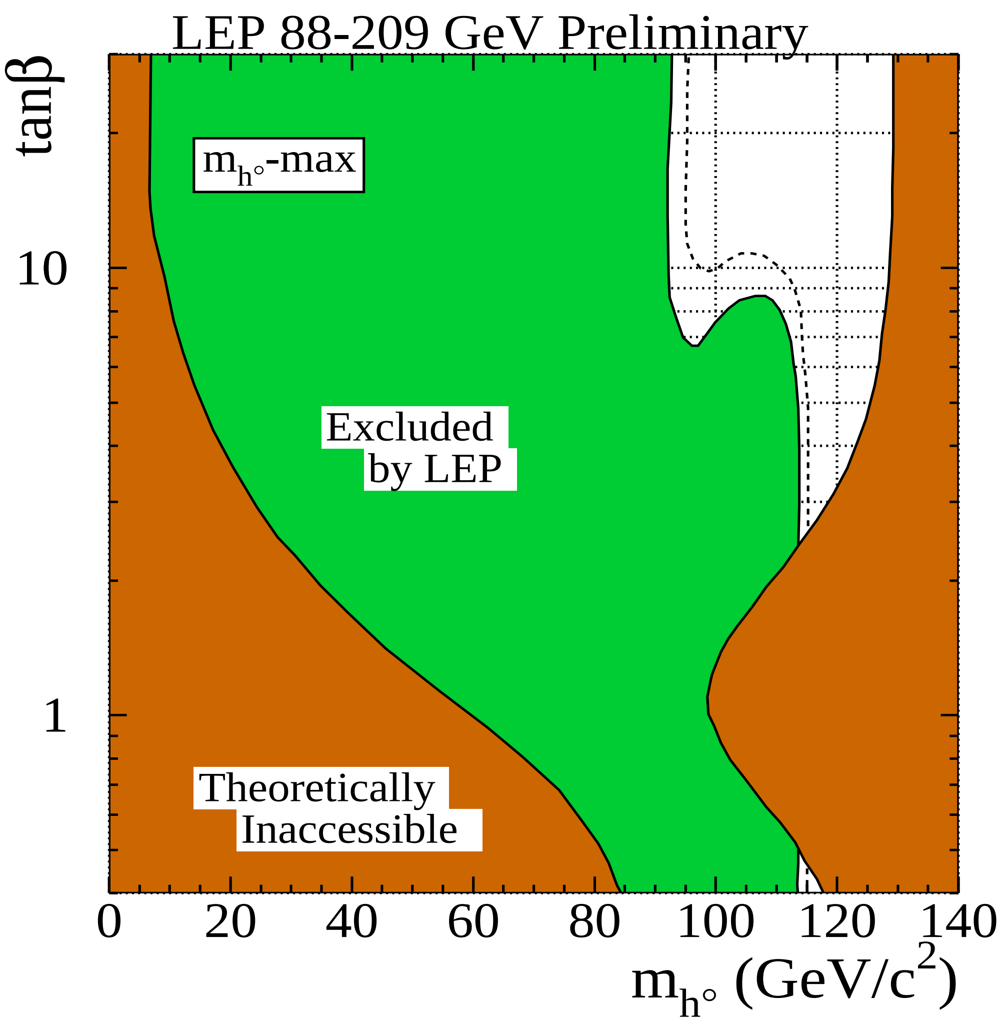


$\sigma_{\mathrm{hA}} \propto \cos^2(\beta - \alpha_{\mathrm{eff}})\sigma_{\mathrm{hZ}}^{\mathrm{SM}}$

Constraints from the Higgs search at LEP [*LEP Higgs Working Group '01*]

Experimental search vs. upper m_h-bound (*FeynHiggs* 1.0)

m_h^{max}-scenario ($m_t = 174.3$ GeV, $M_{SUSY} = 1$ TeV):



$m_h > 91.0$ GeV
(expected: 94.6 GeV), 95% C.L.

$M_A > 91.9$ GeV
(expected: 95.0 GeV)

Parameter region where experimental lower bound on m_h is significantly lower than SM bound, $M_\mathsf{H} > 114.4$ GeV, corresponds to $\sin^2(\beta - \alpha_\mathsf{eff}) \ll 1$

"Excluded" $\tan\beta$ region:

$$0.5 < \tan\beta < 2.4$$

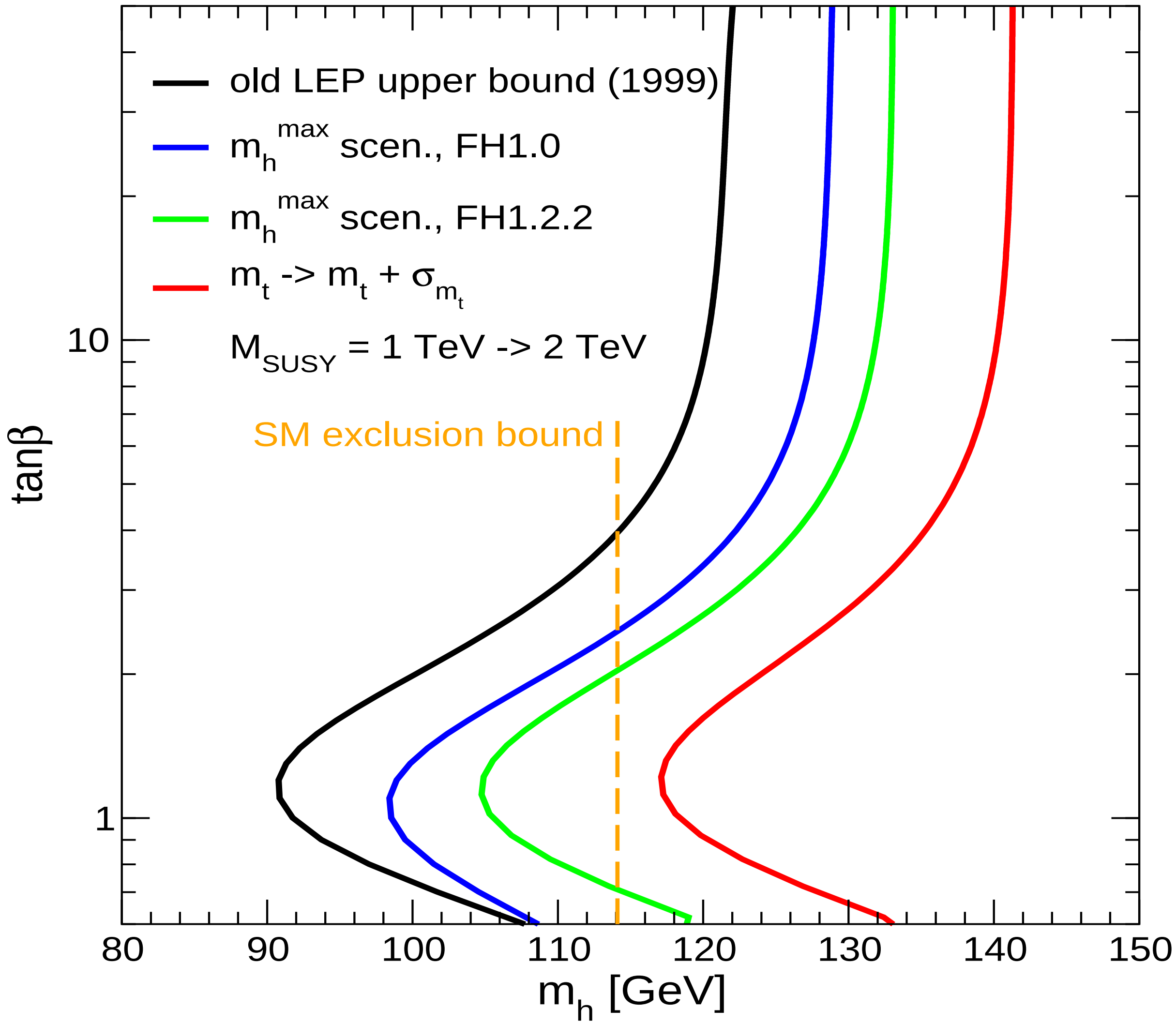
Note: this exclusion bound assumes

m_t, M_SUSY fixed, $m_\mathsf{t} = 174.3$ GeV, $M_\mathsf{SUSY} = 1$ TeV

no theoretical uncertainties included

Effect of new corrections and $m_\mathrm{t} \to m_\mathrm{t} + \sigma_{m_\mathrm{t}}$, $M_\mathrm{SUSY} = 1$ TeV $\to$ 2 TeV

[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]



$\Rightarrow$ no $\tan\beta$ exclusion for $m_\mathrm{t} \to m_\mathrm{t} + \sigma_{m_\mathrm{t}}$, $M_\mathrm{SUSY} = 1$ TeV $\to$ 2 TeV

$\Rightarrow$ Low $\tan\beta$ region not fully excluded by LEP!

Prospects for SUSY searches at the next generation of colliders

Limits on SUSY particles from LEP, Tevatron Run I: $\mathcal{O}(100\,\mathrm{GeV})$

But: some SUSY particles can be much lighter if they have small coupling to Z boson

E.g.:

– no strict lower bound on lightest neutralino if "GUT relation" between M_1 and M_2 is relaxed

– A light sbottom with $m_{\tilde{b}_1} \approx 5$ GeV is in agreement with electroweak precision tests, Higgs searches, . . .
[*M. Carena, S. Heinemeyer, C. Wagner, G. W. '01*]

– . . .

SUSY searches at the Tevatron, Run II:

compared to Run I: $\approx 100\times$ higher luminosity, slightly increased energy (1.8 $\rightarrow$ 2 TeV)

Limited mass window in which discovery of SUSY particles above Run I is possible

Best prospects for:

– 'Trilepton signal': $\tilde{\chi}_2^0 \tilde{\chi}_1^+ \rightarrow \ell^+ \ell^- \tilde{\chi}_1^0 \ell^+ \nu \chi_1^0$

– $\tilde{t}$, $\tilde{b}$ searches

– light SUSY Higgs h in region of large $\tan\beta$

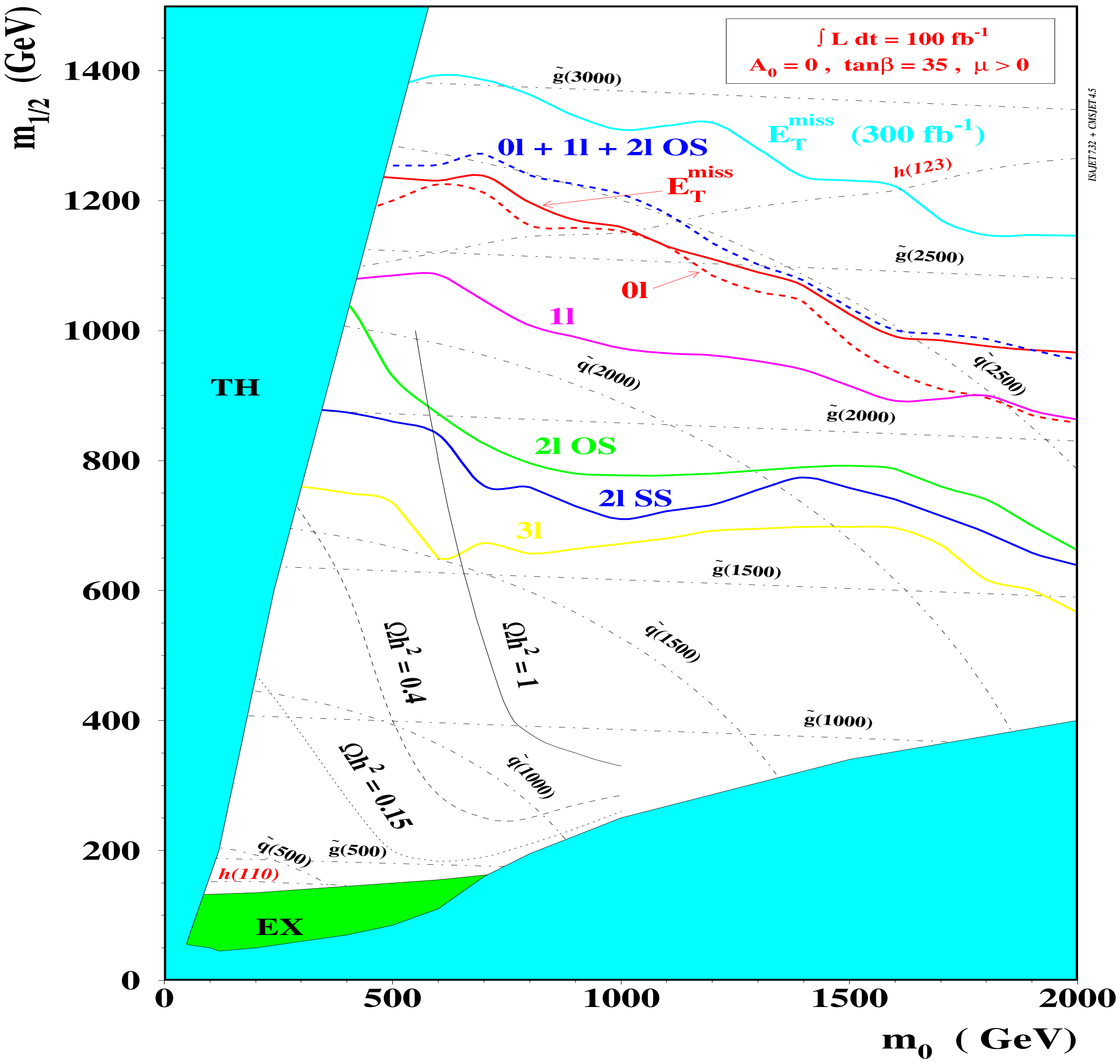
SUSY searches at the LHC:

Dominated by production of colored particles: gluino, squarks

Very large mass range in the searches for jets + missing energy

$\Rightarrow$ gluino, squarks accessible up to 2–3 TeV

Discovery reach contours in m_0–$m_{1/2}$ plane (mSUGRA scenario) for various final states with 100 fb^{-1}: [*CMS '99*]



⇒ discovery of SUSY particles expected if low-energy SUSY is realized

Production of SUSY particles at the LHC will in general result in complicated final states, e.g.

$$\tilde{g} \longrightarrow \bar{q}\tilde{q} \longrightarrow \bar{q}q\tilde{\chi}_2^0 \longrightarrow \bar{q}q\tilde{\tau}\tau \longrightarrow \bar{q}q\tau\tau\tilde{\chi}_1^0$$

Production of uncolored particles via cascade decays often dominates over direct production

Many states are produced at once

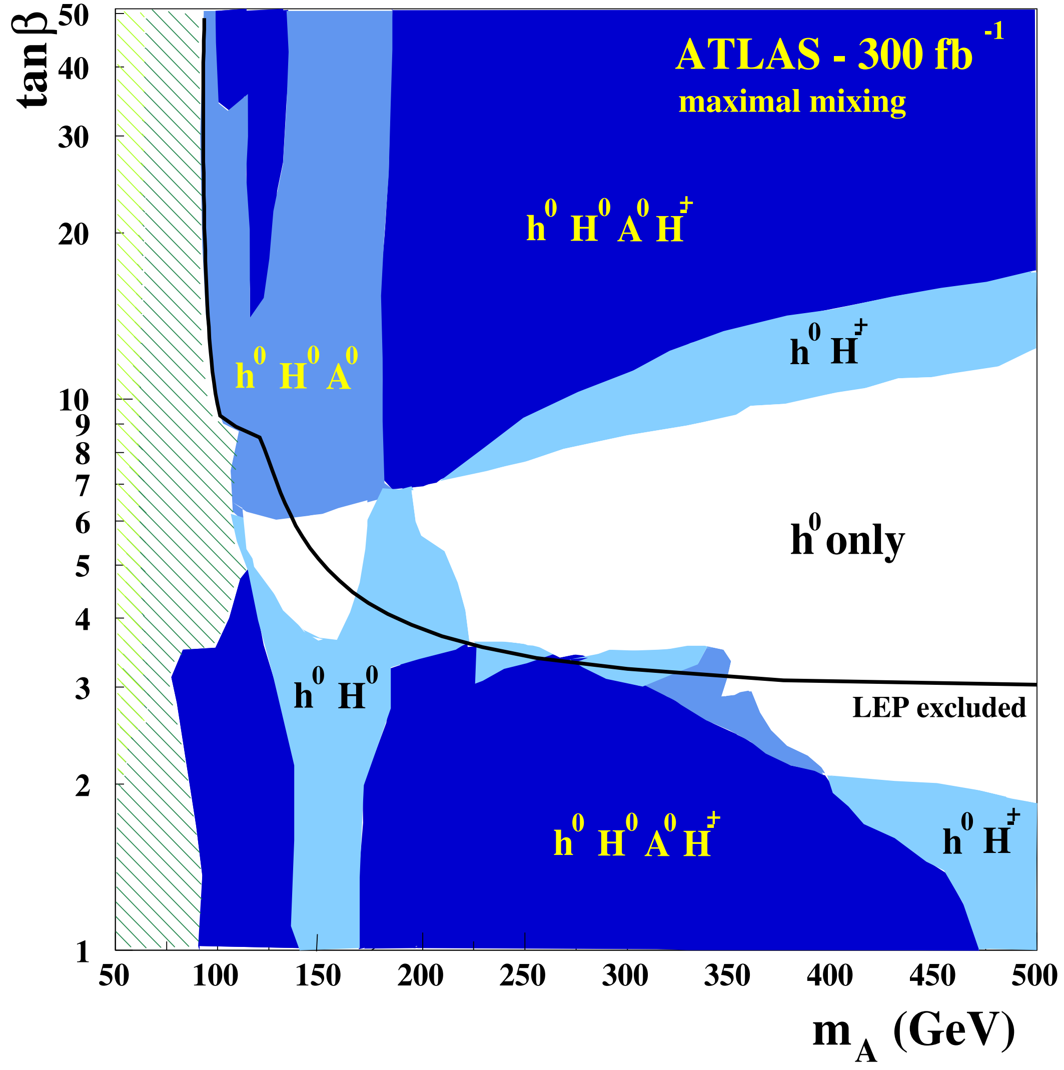
$\Rightarrow$ **Main background for SUSY is SUSY itself!**

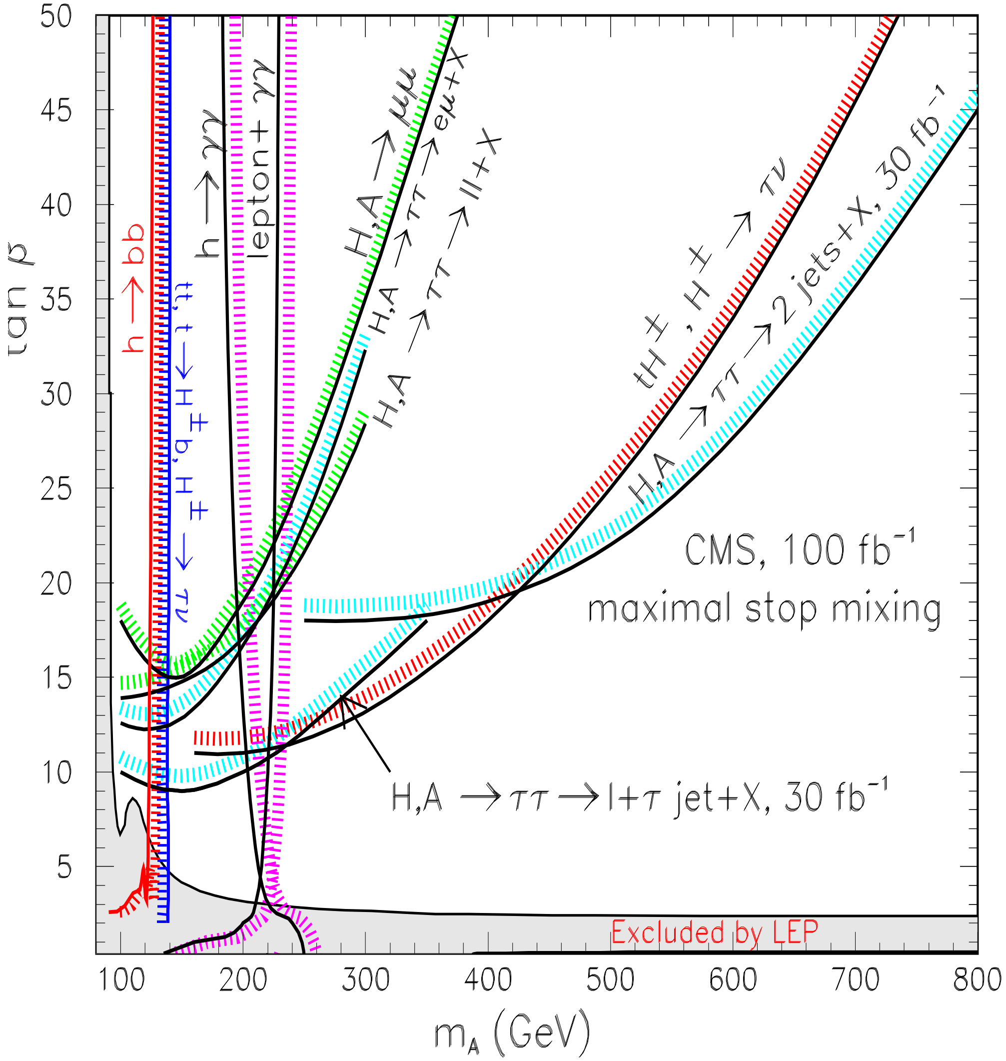
Searches for MSSM Higgs bosons:

good prospects for detecting light Higgs h

H/A discovery possible in significant part of parameter space

MSSM Higgs discovery contours in M_A–$\tan\beta$ plane (m_h^{max} benchmark scenario): [ATLAS '99] [CMS '03]





In order to establish SUSY experimentally:

Need to demonstrate that:

– every particle has superpartner

– their spins differ by 1/2

– their gauge quantum numbers are the same

– their couplings are identical

– mass relations hold

...

$\Rightarrow$ Precise measurements of masses, branching ratios, cross sections, angular distributions, ... mandatory for

– establishing SUSY experimentally

– disentangling patterns of SUSY breaking

SUSY searches at the LC:

Clean signatures, small backgrounds

Thresholds for pair production of SUSY particles

$\Rightarrow$ precise determination of mass and spin of SUSY particles, mixing angles, complex phases, . . .

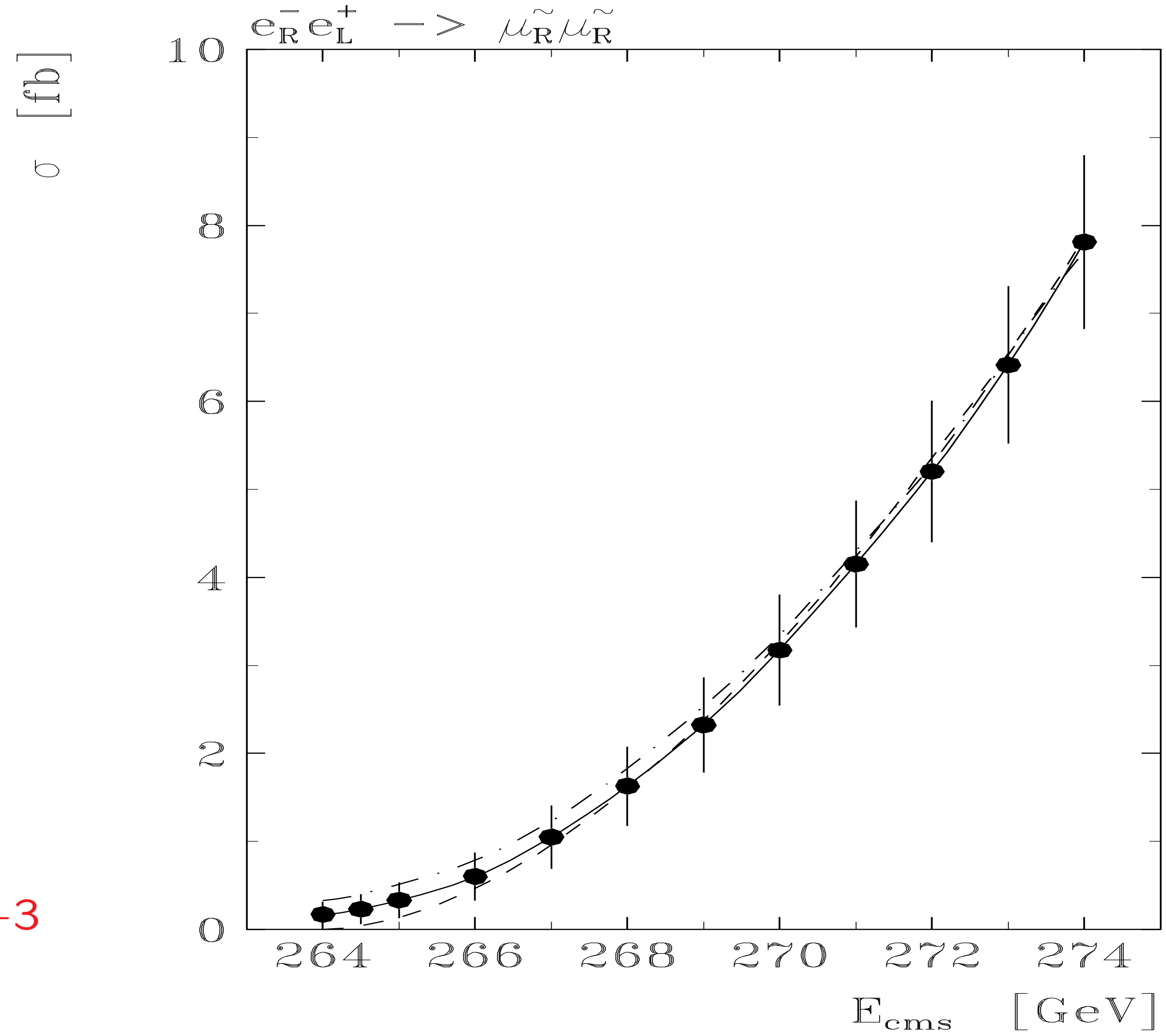
Limited by kinematic reach

Good prospects for production of uncolored particles

$\Rightarrow$ LHC / LC complementarity

Examples for SUSY physics at the LC:

Determination of mass and spin of $\tilde{\mu}_R$ from production at threshold:
[*TESLA TDR '01*]

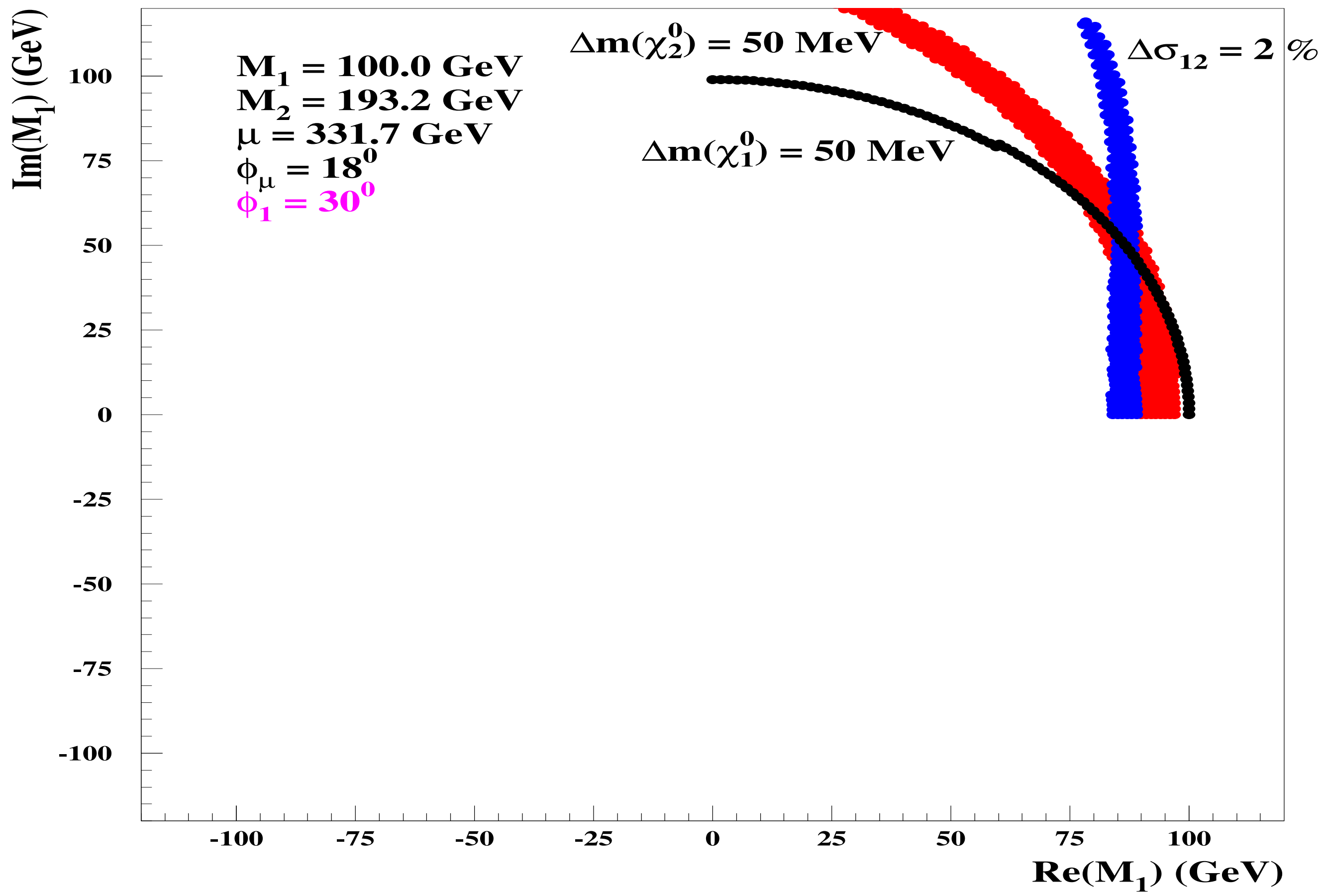


$$\Rightarrow \quad \frac{\Delta m_{\tilde{\mu}_R}}{m_{\tilde{\mu}_R}} < 1 \times 10^{-3}$$

$\Rightarrow$ test of $J = 0$ hypothesis

Determination of phase ϕ_1 in neutralino sector from measurement of $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\mathcal{L} = 500$ fb^{-1}:

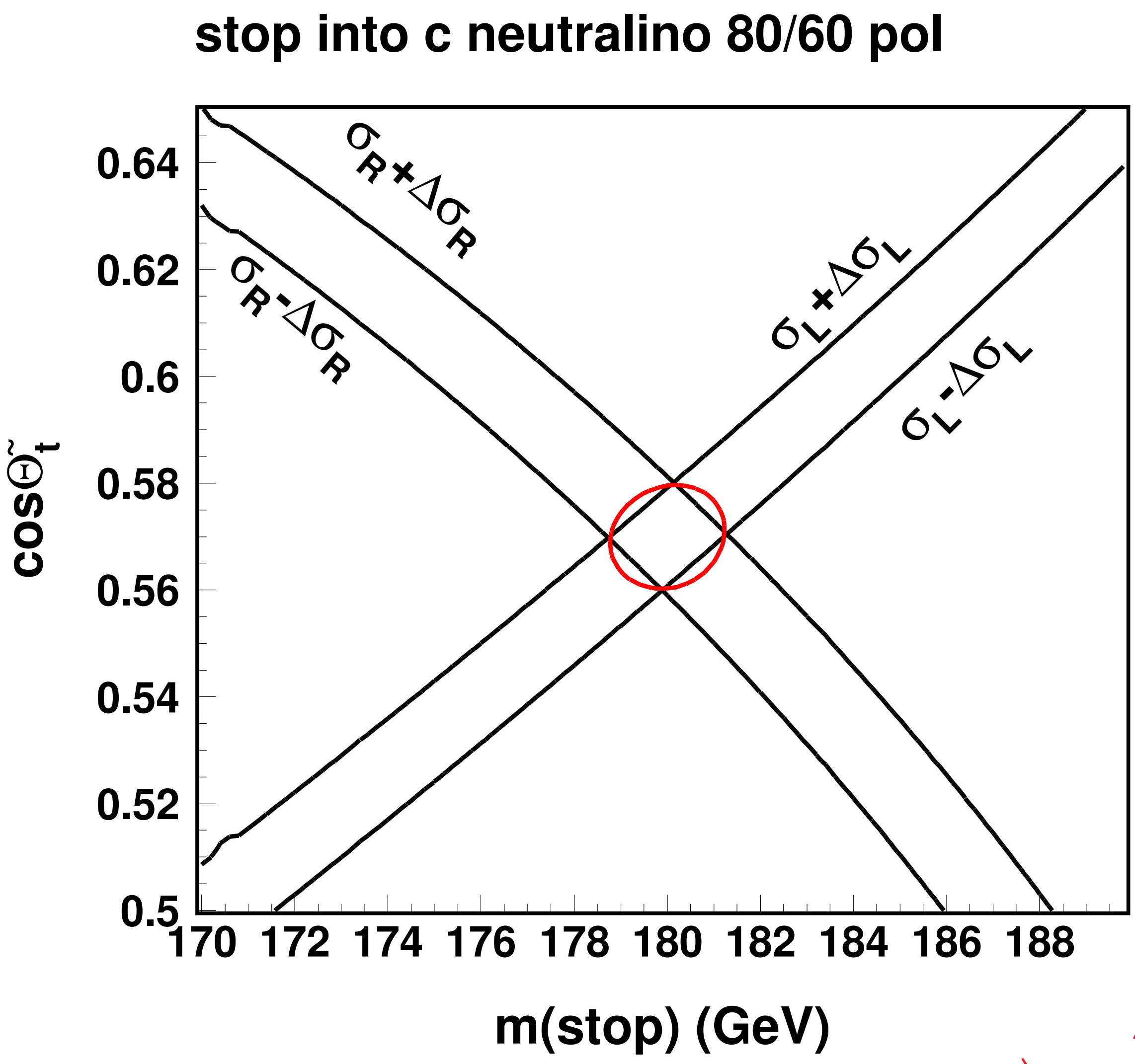
$m_{\tilde{\chi}_1^0} = 96.12 \pm 0.05$ GeV, $m_{\tilde{\chi}_2^0} = 177.13 \pm 0.05$ GeV, $m_{\tilde{\chi}_1^\pm} = 174.90 \pm 0.05$ GeV, $\Delta\sigma(\tilde{\chi}_1^0\tilde{\chi}_2^0) = \pm 2\%$ *[K. Desch, G. Moortgat-Pick '02]*



$\Rightarrow \Phi_1 = 30^0 \pm 3^0$

Determination of $m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$ from $\sigma(e^+e^- \to \tilde{t}_1\tilde{t}_1)$ with polarized beams:

[R. Keränen, H. Nowak, A. Sopczak '00]



$$\Rightarrow \quad \frac{\Delta m_{\tilde{t}_1}}{m_{\tilde{t}_1}} \approx 0.5\%, \qquad \frac{\Delta \cos\theta_{\tilde{t}}}{\cos\theta_{\tilde{t}}} \approx 1.5\%$$

Complementarity of LHC and LC:

⇒ Results obtained at one collider can be used for improving experimental analyses at the other

⇒ investigated in “LHC / LC Study Group”

`www.ippp.dur.ac.uk/~georg/lhclc`

Collaborative effort of Hadron Collider (HC) and Linear Collider (LC) community

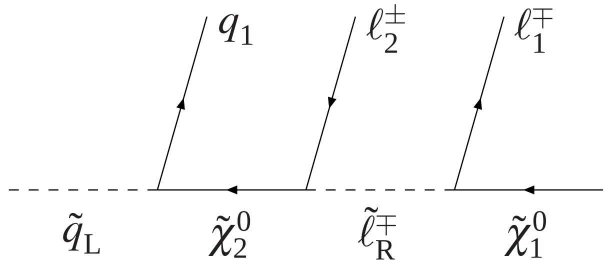
Started in spring 2002, currently about 190 working group members from ATLAS, CMS, LC working groups, theory + Tevatron contact person

Example: SUSY parameters at LHC and LC

Reconstruction of sparticle masses at the LHC

[*B. Gjelsten, E. Lytken, D. Miller, P. Osland, G. Polesello, M. Chiorboli, A. Tricomi*]

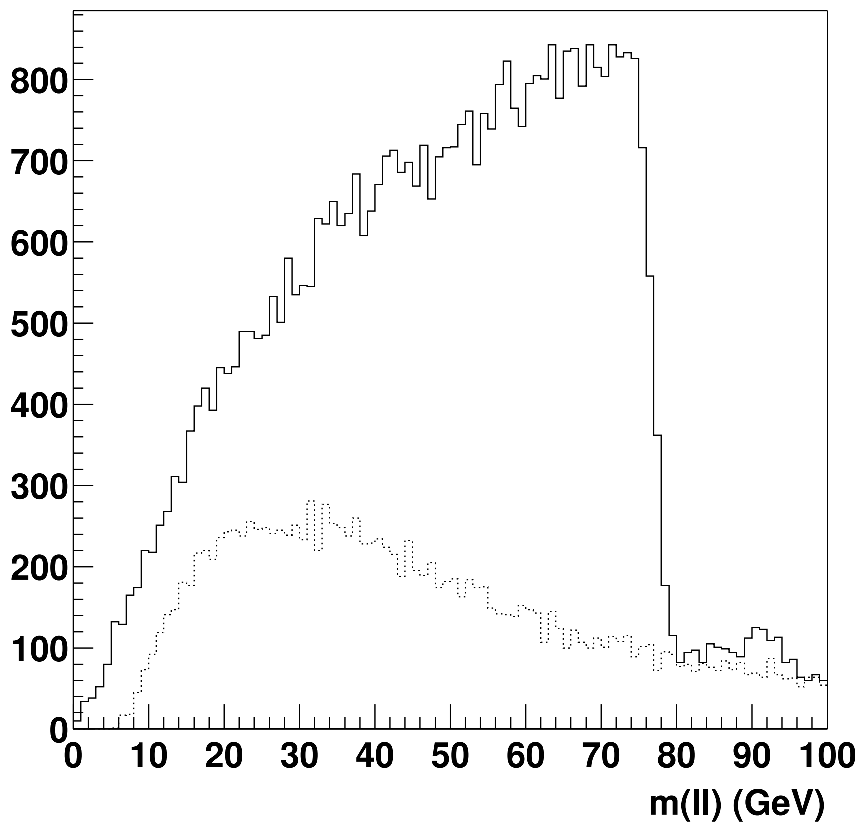
Complicated decay chains for squarks and gluinos

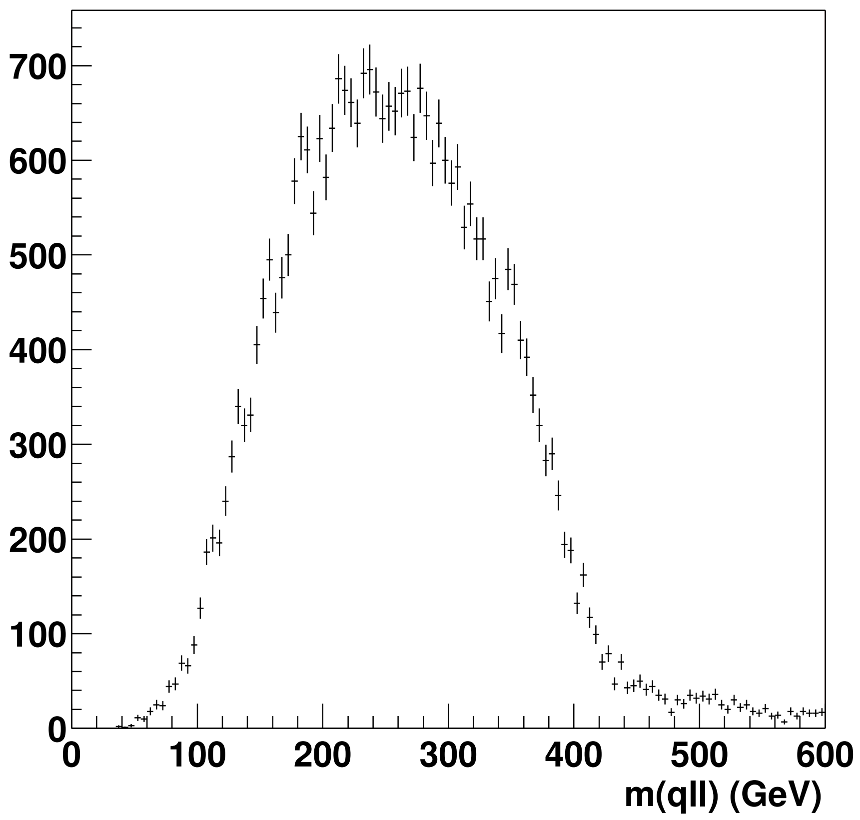


Examples worked out for SPS1a from ATLAS and CMS
main tool: dilepton "edge" from $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$

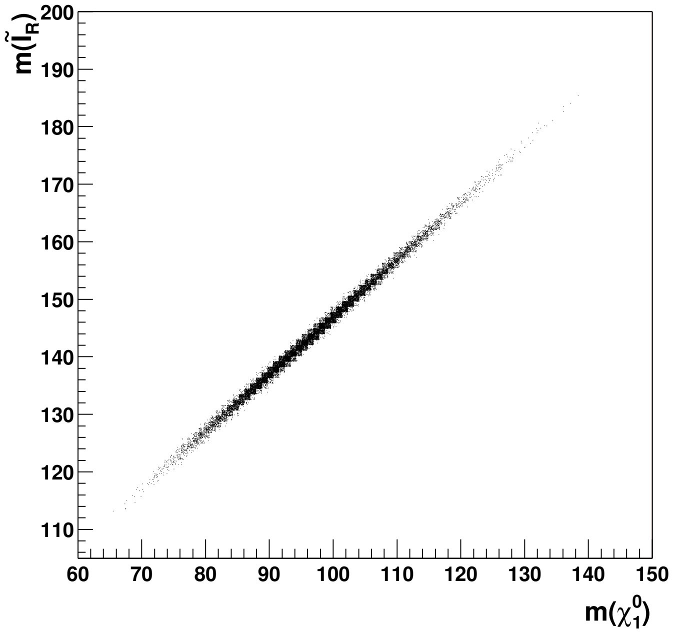
Sbottom/squark and gluino reconstruction:

Edge in same flavour-opposite sign lepton distribution (left), invariant mass distributions with kinematical endpoints (right)





Strong correlation between slepton mass and LSP mass, LSP mass can be constrained at LHC at the 10% level only:



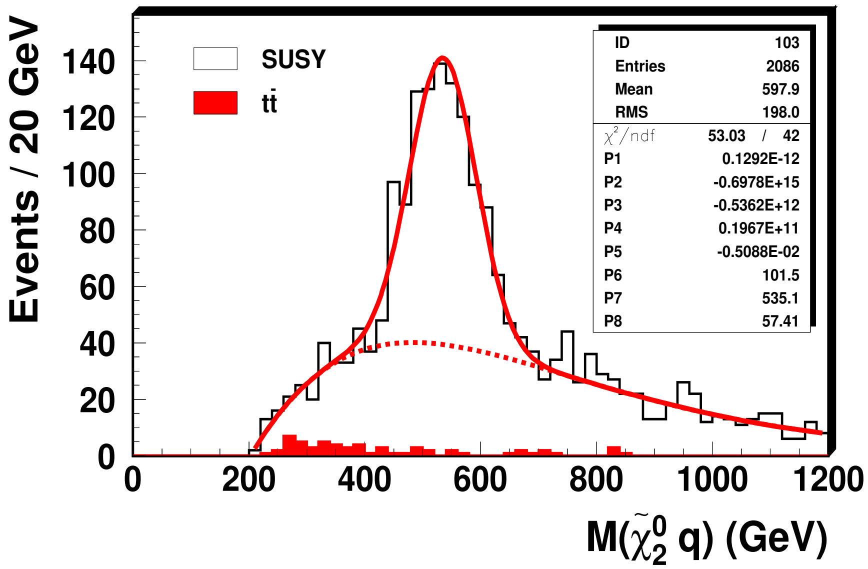
$\Rightarrow$ Take LSP mass as input from LC

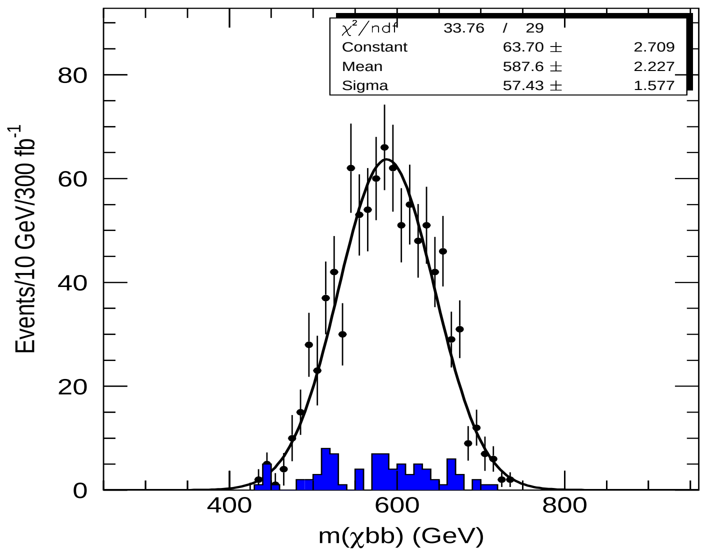
Select events close to the edge and combine with b-jet / q-jet

$$\vec{p}(\tilde{\chi}_2^0) = \left(1 - \frac{m(\tilde{\chi}_1^0)}{m(\ell\ell)}\right) \vec{p}_{\ell\ell}$$

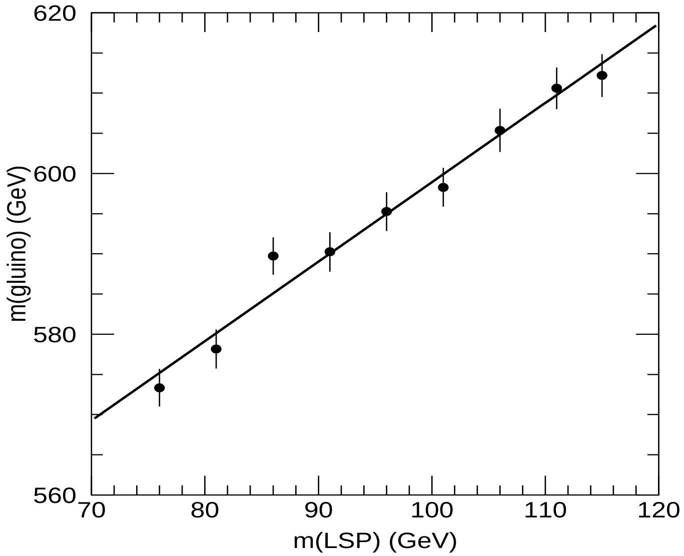
$\Rightarrow$ Get sbottom/squark mass if LSP mass is known

Squark peak (left) and gluino reconstruction from $(\chi_2^0 bb)$ invariant mass distribution (right):





$m_{\tilde{g}}$ as function of the LSP mass:



$\Rightarrow \Delta m_{\tilde{g}} \approx \Delta m_{\mathrm{LSP}}$

Accuracies for the case of the LHC alone (left) and with the LC measurement of the LSP mass with 0.2% accuracy (middle) and 1.0% accuracy (right column) in GeV:

	LHC	LHC+LC (0.2%)	LHC+LC (1.0%)
$\Delta m_{\tilde{\chi}_1^0}$	9.2	0.2	1.0
$\Delta m_{\tilde{l}_R}$	9.2	0.5	1.0
$\Delta m_{\tilde{\chi}_2^0}$	9.0	0.3	1.0
$\Delta m_{\tilde{b}_1}$	23.1	16.9	17.0
$\Delta m_{\tilde{q}_L}$	15.0	5.1	5.3

$\Rightarrow$ LC input improves accuracy significantly

Many more examples of LHC / LC interplay, see
`www.ippp.dur.ac.uk/~georg/lhclc`

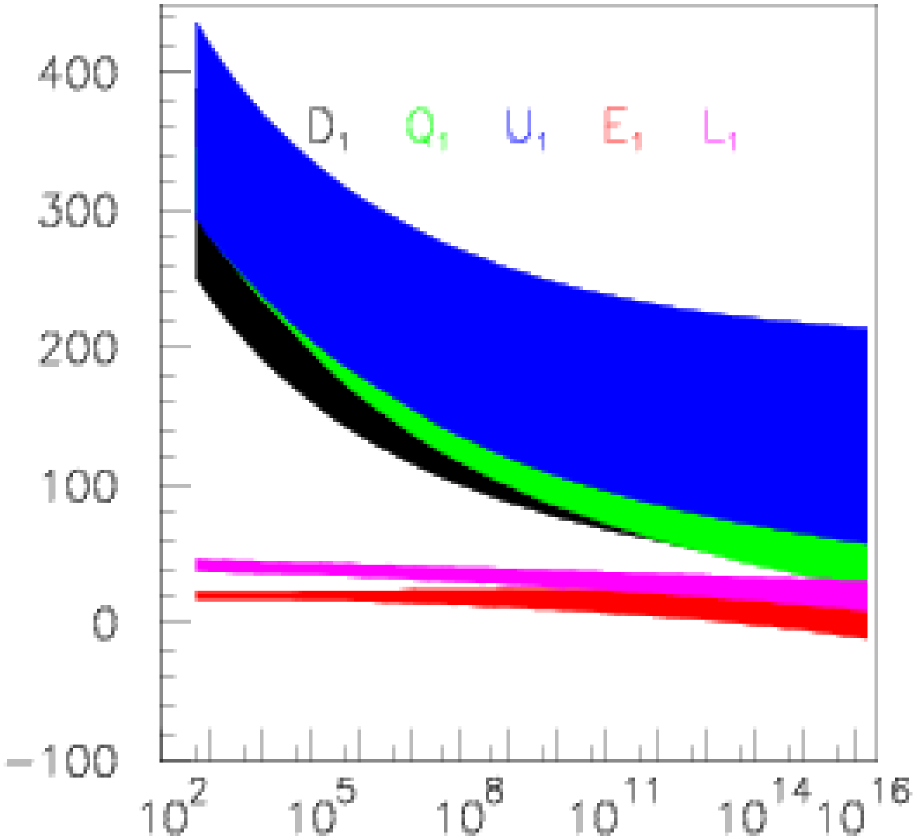
Extrapolation to physics at high scales

[*G. Blair, U. Martyn, G. Polesello, W. Porod, P. Zerwas*]

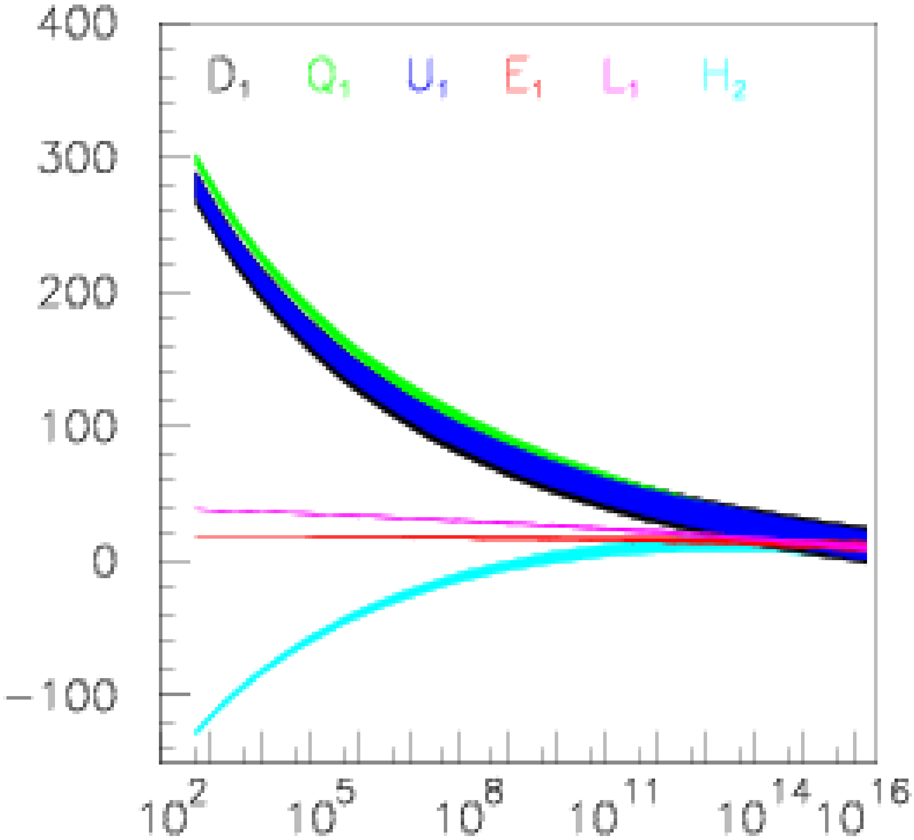
from combination of LHC and LC results, precise measurement of masses of SUSY particles, couplings

E.g.: evolution of first-gen. sfermion mass parameters, Higgs mass param.

LHC



LHC $\oplus$ LC



Outlook:

- "Golden era" of the Standard Model is likely to end soon

- Low-energy Supersymmetry continues to be our best bet for physics beyond the Standard Model

- Data rules:

  We need experimental information from Tevatron, LHC, LC,
  ν experiments, dark matter searches, low-energy experiments, . . .
  to verify / falsify models of new physics

- The experiments in the next years will bring a decisive test of our ideas about physics beyond the SM

$\Rightarrow$ Very exciting prospects for the coming years

Expect the unexpected!